4. Three prisons problem

C is right.

Weil Sei Event A = ' A is to be pardoned'
Event B = ' B is to be pardoned'
Event C = ' C is to be pardoned'
Event W = ' warden tells B is to be executed'

then we have

$P(A|W) \Leftrightarrow$ 'The probability that A pardoned under the condition that B will be executed'

so $P(A|W) = \frac{P(A \cap W)}{P(W)}$

$= \frac{P(A)\,P(W|A)}{P(A)P(W|A)+P(B)P(W|B)+P(C)P(W|C)}$

$\Leftrightarrow$

first we know $P(A) = P(B) = P(C) = \frac{1}{3}$

then $P(W|A) \Leftrightarrow$ since the warden say either B or C is executed is same.

so. $P(W|A) = \frac{1}{2}$

$P(W|B) = 0$, weil warden tell is true

$P(W|C) = 1$, weil if C pardoned, B will executed.

then $P(W) = \frac{1}{2} \Rightarrow P(A|W) = \frac{\frac{1}{6}}{\frac{1}{2}} = \frac{1}{3}$.

so. A is to be pardoned $P(A|W) = P(A) = \frac{1}{3}$. not change.

But $P(C|W) = 1 - P(A|W) - P(B|W) = 1 - \frac{1}{3} - 0 = \frac{2}{3}$

$\Rightarrow$ C is right.