Machine Learning übung 1

一: Monster vs. mouse

weil $p(M) + p(m) + p(e) = 1$ (before hearing the noise)

$p(M|n) + p(m|n) + p(e|n) = 1$ (after hearing the noise)

und $p(n,M) = p(n|M) \cdot p(M)$

Dann mit $p(M) = 1 \times 10^{-3} = 0.001$ $\quad p(m) = 0.5 \quad p(e) = 1 - p(M) - p(m) = 0.499$

$p(n|M) = 0.99 \quad p(n|m) = 0.2 \quad p(n|e) = 0.1$

haben wir: $p(M|n) = \dfrac{p(n|M)p(M)}{p(n)}$

$$= \frac{p(n|M)\,p(M)}{p(n|M)p(M) + p(n|m)p(m) + p(n|e)p(e)}$$

$$= \frac{0.99 \times 0.001}{0.99 \times 0.001 + 0.2 \times 0.5 + 0.1 \times 0.499}$$

$$= \frac{0.00099}{0.00099 + 0.1 + 0.0499}$$

$$= \frac{0.00099}{0.15089} \approx 0.0065610$$

$$= 6.56 \times 10^{-3}$$

2. Plausible inference

(a) Assuming that A is true $\Rightarrow P(B|A) \geq P(B)$ ✓

(c) mit (a) und $P(B|A) = \frac{P(A|B)P(B)}{P(A)}$ (Bayes')

haben wir $\frac{P(A|B)P(B)}{P(A)} \geq P(B)$

$\Rightarrow P(A|B) \geq P(A)$ ✓

(b) weil $P(B|\neg A) \leq P(B) \Leftrightarrow \frac{P(\neg A|B)P(B)}{P(\neg A)} \leq P(B)$ (Bayes's rules)

$\Leftrightarrow P(\neg A|B) \leq P(\neg A)$

$\Leftrightarrow 1 - P(A|B) \leq 1 - P(A)$

$\Leftrightarrow P(A) \leq P(A|B)$ ✓

same as (c).

(d) weil $P(A|\neg B) \leq P(A) \Leftrightarrow \frac{P(\neg B|A)P(A)}{P(\neg B)} \leq P(A)$

$\Leftrightarrow P(\neg B|A) \leq P(\neg B)$

$\Leftrightarrow 1 - P(B|A) \leq 1 - P(B)$

$\Leftrightarrow P(B) \leq P(B|A)$

same as (a) ✓.

3. Medical inference (P(A)= no history,but result is positiv)

sei $P(A)$ is woman have no history (background)

$P(B)$ is woman have cancer

so $P(B|A) = 90\%$ $P(A) = 0.8\%$

$P(B|\neg A) = 7\%$

~~P(B) =~~

we must caculate $P(A|B)$

well: $$P(B|A) = \frac{P(A|B)P(B)}{P(A)} = 90\% \quad (1)$$

$$P(B|\neg A) = \frac{P(\neg A|B)\cdot P(B)}{1-P(A)} = 7\% \quad (2)$$

(1)+(2) gilt:

$$P(A|B)P(B) + P(\neg A|B)\cdot P(B) = 90\%\cdot P(A) + 7\%\cdot(1-P(A))$$

$$\Leftrightarrow \underbrace{(P(A|B) + P(\neg A|B))}_{=1}\cdot P(B) = 0.9\cdot 0.8\% + 0.07\cdot 0.992$$

$$\Leftrightarrow P(B) = 0.07664$$

Dann

$$P(A|B) = \frac{P(B|A)P(A)}{P(B)} = \frac{90\%\cdot 0.8\%}{0.07664} = 0.09394$$

$$\approx 9.39\%$$

4. Three prisons problem

C is right.

weil sei Event A = 'A is to be pardoned'

Event B = 'B is to be pardoned'

Event C = 'C is to be pardoned'

Event W = 'warden tells B is to be executed'

then we have

$P(A|W) \Leftrightarrow$ 'The probability that A pardoned under the condition that B will be executed'

so $$P(A|W) = \frac{P(A \cap W)}{P(W)} = \frac{P(A)\,P(W|A)}{P(A)\,P(W|A) + P(B)P(W|B) + P(C)P(W|C)}$$

$\Leftarrow$

first we know $P(A) = P(B) = P(C) = \frac{1}{3}$

then $P(W|A) \Leftrightarrow$ Since the warden say either B or C is executed is same.

so. $P(W|A) = \frac{1}{2}$

$P(W|B) = 0$, weil warden tell is true

$P(W|C) = 1$, weil if C pardoned, B will executed.

then $P(W) = \frac{1}{2} \Rightarrow P(A|W) = \frac{\frac{1}{6}}{\frac{1}{2}} = \frac{1}{3}$.

so. A is to be pardoned $P(A|W) = P(A) = \frac{1}{3}$. not change.

But $P(C|W) = 1 - P(A|W) - P(B|W) = 1 - \frac{1}{3} - 0 = \frac{2}{3}$

$\Rightarrow$ C is right.