3. Medical inference

sei $P(A)$ is woman have no history (background)

$P(B)$ is woman have cancer

so $P(B|A) = 90\%$ $\quad P(A) = 0.8\%$

$P(B|\neg A) = 7\%$

~~$P(B) =$~~

we must caculate $P(A|B)$

well: $P(B|A) = \frac{P(A|B)P(B)}{P(A)} = 90\%$ (1)

$P(B|\neg A) = \frac{P(\neg A|B)\cdot P(B)}{1 - P(A)} = 7\%$ (2)

(1)+(2) gilt:

$$P(A|B)P(B) + P(\neg A|B)\cdot P(B) = 90\%\cdot P(A) + 7\%\cdot(1-P(A))$$

$$\Leftrightarrow \quad \underbrace{(P(A|B) + P(\neg A|B))}_{=1}\cdot P(B) = 0.9\cdot 0.8\% + 0.07\cdot 0.992$$

$$\Leftrightarrow \quad P(B) = 0.07664$$

Dann

$$P(A|B) = \frac{P(B|A)P(A)}{P(B)} = \frac{90\%\cdot 0.8\%}{0.07664} = 0.09394$$

$$\approx 9.39\%$$