Blatt 4.

Aufgabe 1. $\boxed{p_y(y) = p_x(x(y)) \cdot \frac{dx(y)}{dy}}$ mit $y = ax+b$

(a) $p_x(x) = \mathcal{U}(x \mid c, d) \iff f(x) = \begin{cases} \frac{1}{c-b} & (b \le x \le c) \\ 0 & \text{otherweise} \end{cases}$

$\Rightarrow y_c = ac+b,\ y_d = ad+b,\ y \in [ac+b, ad+b]$

$\iff f(y) = \begin{cases} \frac{1}{a(d-c)} & ; (ac+b \le y \le ad+b) \\ 0 & \text{otherweise} \end{cases}$

(b) $p_x(x) = \mathcal{N}(x \mid \mu, \sigma^2)$

Let characteristic function $\phi_X(t) = E[\exp(it^T X)]$

Then $\Rightarrow \phi_x(t) = \exp(it^T \mu - \frac{1}{2} t^T \sigma^2 t)$.

mit $y = ax+b$,

$$\begin{aligned} \Longrightarrow \quad \phi_y(t) &= E[\exp(it^T(ax+b))] \\ &= E[\exp(it^T b)\exp(it^T ax)] \\ &= \exp(it^T b)\, \phi_x(A^T t) \\ &= \exp(it^T b) \exp(i(a^T t)^T \mu - \frac{1}{2}(a^T t)^T \sigma^2 (a^T t)) \\ &= \exp(it^T(a\mu + b) - \frac{1}{2} t^T a \sigma^2 a^T t) \end{aligned}$$

$\iff$

$p_y(y) = \mathcal{N}[a\mu + b,\ a\sigma^2 a^T]$

Aufgabe 2. $p(w) = \mathcal{N}(w | w_0, V_0)$

$p(y|X,w) = \mathcal{N}(y | Xw, \Sigma)$

$p(w|X,y) = \mathcal{N}(w | w_n, V_n)$ mit $V_n = (X^T\Sigma^{-1}X + V_0^{-1})^{-1}$

$w_n = V_n(V_0^{-1}w_0 + X^T\Sigma^{-1}y)$

① $$p(w|y,x) = \frac{p(y|X,w)\cdot p(w)}{p(y|X)}$$

weil $p(y|X)$ unabhängig mit w $\Rightarrow$ sei $p(y|X) = \frac{1}{a}$.

Dann:

② $$p(w|y,x) = a\cdot \exp((-\tfrac{1}{2}((y-Xw)^T\Sigma^{-1}(y-Xw))\cdot \exp(-\tfrac{1}{2}((y-w_0)^T V_0^{-1}(y-w_0))$$
$$= a\cdot \exp(-\tfrac{1}{2}((y-Xw)^T\Sigma^{-1}(y-Xw) + (y-w_0)^T V_0^{-1}(y-w_0))$$
$$= a\cdot \exp(-\tfrac{1}{2}(y\Sigma^{-1}y - y^T\Sigma^{-1}Xw - (Xw)^T\Sigma^{-1}y + (Xw)^T\Sigma^{-1}Xw + y^TV_0^{-1}y - y^TV_0^{-1}w_0 - w_0^TV_0^{-1}y + w_0^TV_0^{-1}w_0)$$
$$= a\cdot \exp(-\tfrac{1}{2}(w^T(V_0^{-1} + X^T\Sigma^{-1}X)w - w^T\cdot V_n(V_0^{-1}w_0 + X^T\Sigma^{-1}y)\cdot X + C)$$

Dann: $V_n^{-1} = (X^T\Sigma^{-1}X + V_0^{-1})$

$V_n^{-1}\cdot w_n = V_n(V_0^{-1}w_0 + X^T\Sigma^{-1}y)$

$\Leftrightarrow$

$p(w|X,y) = \mathcal{N}(w | w_n, V_n)$

mit $V_n = (X^T\Sigma^{-1}X + V_0^{-1})^{-1}$

$w_n = V_n(V_0^{-1}w_0 + X^T\Sigma^{-1}y)$

Aufgabe 3.

(a) The last column (MEDV) is the target, the rest are features.

(b) / (c) in .py