

Exercise set #1

Solution should be submitted in teams of two. Please submit your solution online using the sciebo file-drop folder. The link is available in ILIAS. Please submit a single zip file with the following naming scheme: `username1-username2-ex#.zip` (e.g. `jadoe101-jodoe108-ex01.zip`). Allowed file extensions (of files within the zip file) are: `.pdf`, `.txt`, `.py` and `.ipynb`. Make sure the total file size does not exceed 10 MB. We will not accept photos of your solutions, scans produced by scanner apps are ok. In the case of multiple uploads we will consider the latest. Uploads after the deadline will be deleted without further notice.

1. Monster vs. mouse

Recall the example: “While you tried to sleep, you hear some noise under the bed...”
Following the lecture’s notation,

n = some noise under your bed
 M = a monster under your bed
 m = a mouse under your bed
 e = something else (e.g. only air) under your bed

express your beliefs about monsters, mice and noise by assigning numbers to $p(M)$, $p(m)$, $p(e)$, $p(n|M)$, $p(n|m)$, $p(n|e)$ and calculate the probability that given some noise under your bed, there is a monster, i.e. calculate $p(M|n)$ using Bayes’ rule. Calculate the results with pencil and paper, use the **02-Monster-vs-mouse** notebook that you can find on sciebo to check your results.

30 points

2. Plausible inference

Without assumptions the plausibility of B is $p(B)$. Assuming that A is true, the plausibility of B is $p(B|A)$. Thus we can translate the phrase “if A is true, B becomes more plausible” into the inequality $p(B|A) \geq p(B)$. Assuming that inequality and using Bayes’ rule and the basic laws of probability show the following:

$$\begin{array}{ll} \text{(a) } p(B|A) \geq p(B) & \text{(c) } p(A|B) \geq p(A) \\ \text{(b) } p(B|\neg A) \leq p(B) & \text{(d) } p(A|\neg B) \leq p(A) \end{array}$$

30 points

3. Medical inference

Imagine you are an oncologist. Your current patient is a 45 year-old woman without symptoms, and with no history of cancer in her family, who came in for a routine checkup. Among patients with this background, 0.8% have breast cancer. But the routine mammogram of this patient is positive. Among women with this specific background who have breast cancer, 90% have a positive mammogram, while only 7% of these patients without breast cancer have positive mammograms.¹ Given these observations, what is the probability that your patient has breast cancer?

20 points

¹numbers based on K.-A. Phillips, G. Glendon, J.A. Knight; *Putting the risk of breast cancer into perspective*. New England J of Medicine **340** (1999), pp. 141–144

4. **Three prisoners problem**²

Three prisoners, A , B and C , are in separate cells and sentenced to death. The governor has selected one of them at random to be pardoned. The warden knows which one is pardoned, but is not allowed to tell. Prisoner A begs the warden to let him know the identity of one of the others who is going to be executed. “If B is to be pardoned, give me C ’s name. If C is to be pardoned, give me B ’s name. And if I’m to be pardoned, flip a coin to decide whether to name B or C .”

The warden tells A that B is to be executed. Prisoner A is pleased because he believes that his probability of surviving has gone up from $1/3$ to $1/2$, as it is now between him and C . Prisoner A secretly tells C the news, who is also pleased, because he reasons that A still has a chance of $1/3$ to be the pardoned one, but his chance has gone up to $2/3$. What is the correct answer?

20 points

²copied from http://en.wikipedia.org/wiki/Three_Prisoners_problem; think yourselves at least for five minutes before you look up the solution (if you don’t want to find out yourself)!