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Machine Learning
Exercise Sheet 1
(4 Exercises, 100 Points)
Due: 18.10.2022, 10:00

Exercise 1: Monster vs. mouse (30 Points)

Recall the example: "While you tried to sleep, you hear some noise under the bed..."
Following the lecture's notation, consider the following events:

$$
\begin{aligned}
n &= \text{some noise under your bed} \\
M &= \text{a monster under your bed} \\
m &= \text{a mouse under your bed} \\
e &= \text{something else (e.g. only air) under your bed}
\end{aligned}
$$

Express your beliefs about monsters, mice and noise by assigning numbers to $p(M), p(m), p(e), p(n|M), p(n|m), p(n|e)$ and calculate the probability that given some noise under your bed, there is a monster, i.e. calculate $p(M|n)$ using Bayes' rule. Calculate the results with pencil and paper, then use the 02-Monster-vs-mouse notebook that you can find on Sciebo to check your results.

Exercise 2: Plausible Inference (30 Points)

Without assumptions the plausibility of B is $p(B)$. Assuming that A is true, the plausibility of B is $p(B|A)$. Thus we can translate the phrase "if A is true, B becomes more plausible" into the inequality $p(B|A) \geq p(B)$. Assuming that inequality and using Bayes' rule and the basic laws of probability show the following:

(a) $p(B|A) \geq p(B)$

(b) $p(B|\neg A) \leq p(B)$

(c) $p(A|B) \geq p(A)$

(d) $p(A|B) \leq p(A)$

Exercise 3: Medical Inference (20 Points)

Imagine you are an oncologist. Your current patient is a 45 year-old woman without symptoms, and with no history of cancer in her family, who came in for a routine checkup. Among patients with this background, 0.8% have breast cancer. But the routine mammogram of this patient is positive. Among women with this specific background who have breast cancer, 90% have a positive mammogram, while only 7% of these patients without breast cancer have positive mammograms [1]. Given these observations, what is the probability that your patient has breast cancer?

[1] numbers based on K.-A. Phillips, G. Glendon, J.A. Knight; *Putting the risk of breast cancer into perspective.* New England J of Medicine **340** (1999), pp. 141–144

Exercise 4: Three Prisoners[2] (20 Points)

Three prisoners, A, B and C, are in separate cells and sentenced to death. The governor has selected one of them at random to be pardoned. The warden knows which one is pardoned, but is not allowed to tell. Prisoner A begs the warden to let him know the identity of one of the others who is going to be executed. "If B is to be pardoned, give me C's name. If C is to be pardoned, give me B's name. And if I'm to be pardoned, flip a coin to decide whether to name B or C."

The warden tells A that B is to be executed. Prisoner A is pleased because he believes that his probability of surviving has gone up from 1/3 to 1/2, as it is now between him and C. Prisoner A secretly tells C the news, who is also pleased, because he reasons that A still has a chance of 1/3 to be the pardoned one, but his chance has gone up to 2/3. What is the correct answer?

[2] A famous problem copied from `http://en.wikipedia.org/wiki/Three_Prisoners_problem`; think yourselves at least for 30 minutes before you look up the solution (if you don't want to find out yourself)!