Ex1. (a) ① $A \to B$

P(A)	T	F
	0.3	0.7

P(A) \ P(B)	T	F
T	0.2	0.8
F	0.4	0.6

mit $P(A,B) = P(A) \times P(B|A)$

Dann $P(B) = P(B=T) = \sum_{A \in \{T,F\}} P(B=T, A)$

$= P(B_T, A_T) + P(B_T, A_F)$

$= 0.3 \times 0.2 + 0.7 \times 0.4$

$= 0.34$

② $B \to C$

P(B)	T	F
	0.34	0.66

P(B) \ P(C)	T	F
T	0.7	0.3
F	0.5	0.5

mit $P(B,C) = P(B) \times P(C|B)$

Dann $P(C) = P(C=T) = \sum_{B \in \{T,F\}} P(C_T, B_F)$

$= 0.34 \times 0.7 + 0.66 \times 0.5$

$= 0.568$

(b) ① $A \to B$ same as (a) $\Rightarrow P(B) = 0.34$

② $A \to C$

P(A)	T	F
	0.3	0.7

P(A) \ P(C)	T	F
T	0.7	0.3
F	0.6	0.4

$P(C) = \sum_{A \in \{T,F\}} P(C=T, A) = 0.3 \times 0.7 + 0.7 \times 0.6 = 0.63$

P(C)	P(B)	P(D) T	P(D) F
T	T	0.9	0.1
T	F	0.3	0.7
F	T	0.5	0.5
F	F	0.3	0.7

③ $C \to D \leftarrow B$ mit head to head haben wir

$P(B,C,D) = P(B) \times P(C) \times P(D|B,C)$

$P(B,C) = P(B) \times P(C)$

Dann $P(D) = P(D=T) = \sum_{B \cdot C \in \{T,F\}} P(D=T, B, C)$

$= (0.9 \times 0.63 \times 0.34)_{TTT} + (0.3 \times 0.63 \times 0.66)_{TTF}$

$+ (0.5 \times 0.37 \times 0.34)_{TFT} + (0.3 \times 0.37 \, 0.66)_{TFF}$

$= 0.19278 + 0.12474 + 0.0624 + 0.07326$

$= 0.45368$

(discrete)

EX2. (a) weil $E(X) = \sum_{x \in X} x \cdot p_X(x) = x_1 \cdot p_X(x_1) + \dots + x_k \cdot p_k(x_k)$

Dann (a) $E(aX+bY) = aE(X) + bE(Y)$ können wir $\Rightarrow \begin{cases} E(bY) = bE(Y) \\ E(aX) = aE(X) & (1) \\ E(X+Y) = E(X) + E(Y) & (2) \end{cases}$

In (1): $E(aX) = \sum_{x \in X} ax \cdot p_X(x) = a_x \cdot p_X(x) = ax_1 \cdot p_X(x_1) + \dots + ax_k \cdot p_X(x_k)$

$= a(x_1 p_X(x_1) + \dots + x_k \cdot p_X(x_k))$

$= a \cdot E(X)$

analog zu $E(bY) = bE(Y)$

In (2): $E(X+Y) = E(X) + E(Y)$

$\Rightarrow \sum_{1 \le i,j \le n} (x_i + y_j) = P(X = x_i, Y = y_i) = \sum_{1 \le i,j \le n} (x_i + y_i) P(X = x_i) P(Y = y_i)$

$= \sum_{1 \le i \le n} x_i P(X = x_i) + \sum_{1 \le j \le n} y_j P(Y = y_i) = \sum_{1 \le i \le n} x_i P(X = x_i) + E(Y) = E(X) + E(Y)$ $\square$

(b) $Var(aX) = a^2 Var(X)$

weil $Var(X) = E((X-\mu)^2) = E((X - E(X))^2)$ mit $E(aX) = aE(X)$

haben wir $Var(aX) = E((aX - E(aX))^2)$

$= E((aX - aE(X))^2)$

$= E(a^2(X - E(X))^2)$

$= a^2 E((X - E(X))^2)$

$= a^2 Var(X)$

EX. 3 $\mathcal{N}(x|\mu, \sigma^2) = \frac{1}{\sqrt{2\pi\sigma^2}} \exp\left(\frac{-(x-\mu)^2}{2\sigma^2}\right)$

(1) sei $f(x) = \frac{1}{\sigma\sqrt{2\pi}} \exp\left(\frac{-(x-\mu)^2}{2\sigma^2}\right)$

Dann $\log f(x) = \log \frac{1}{\sigma\sqrt{2\pi}} - \frac{1}{2\sigma^2}(x-\mu)^2$ (mit $\log e^k = k$)

Dann $\frac{f'(x)}{f(x)} = 0 - \frac{1}{\sigma^2}(x-\mu) = 0$ $f'(x) = 0,\ x = \mu$

$f'(x) = -\frac{1}{\sigma^2}(x-\mu) \cdot f(x)$ (1)

$f''(x) = -\frac{1}{\sigma^2}[1 \cdot f(x) + (x-\mu) f'(x)]$

$= -\frac{f'(x)}{\sigma^2}(1 - (x-\mu) \cdot \frac{(x-\mu)^2}{\sigma^2} \cdot f(x)]$

$= -\frac{f(x)}{\sigma^2}[1 - \frac{(x-\mu)^2}{\sigma^2}]$

$f''(\mu) = -\frac{f(\mu)}{\sigma^2} = \frac{1}{\sigma^2} \cdot \frac{1}{\sigma\sqrt{2x}} < 0$

and $\Rightarrow x = \mu$ $\boxed{\text{(Avg = mode}}$

μ is also the mode of Gaussian normal distribution