Int. Journ. of Unconventional Computing, Vol. 9, pp. 173–184
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Computation and Spacetime Structure*

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Received: July 26, 2012. Accepted: November 17, 2012.

We investigate the relationship between computation and spacetime structure, focussing on the role of closed timelike curves (CTCs) in promoting computational speedup. We note first that CTC traversal can be interpreted in two distinct ways, depending on ones understanding of spacetime. Focussing on one interpretation leads us to develop a toy universe in which no CTC can be traversed by a computer more than once, whence no direct computational speedup is possible. Focussing on the second (and more standard) interpretation leads to the surprising conclusion that CTCs may act as perfect information repositories: just as black holes have entropy, so do CTCs. If we also assume P $\neq$ NP, we find that all observers agree that, even if unbounded time travel existed in their youth, this capability eventually vanishes as they grow older. Thus the computational assumption P $\neq$ NP is also an assumption concerning cosmological structure.

1 INTRODUCTION

In the presence of spacetime curvature, the run-time of a program typically depends on who does the observing; the time registered by a clock co-moving with a computational system may differ from that registered by an observer watching the system from elsewhere. The existence of such discrepancies lies at the heart of *relativistic hypercomputation* schemes using e.g. Malament-Hogarth spacetimes [12], slow Kerr black holes [9, 15] and closed timelike

* This paper was partially completed whilst the author was a visiting fellow at the Isaac Newton Institute for the Mathematical Sciences, under the programme *Semantics and Syntax: A Legacy of Alan Turing*. Material in this paper has previously been discussed at *Physics and Computation* (P&C 2011, Turku, Finland) and *New Worlds of Computation* (NWC 11, Orléans, 2011).

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0 output v
1 start A and let it run to completion
2 let v be the result generated by A
3 send v back to time 0
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FIGURE 1
An algorithm (A') that exploits time travel to solve in constant time the same problem that A solves in superpolynomial time.

curves [4]. These schemes indicate that cosmological anomalies allow the resolution of formally undecidable problems, so it seems not unlikely that they would also allow problems in NP\P (if any) to be solved in polynomial time. Our results assume an inherently classical setting; for related quantum theoretical studies of CTC physics see e.g. [7, 8, 13] and Sect. 3 below.

Suppose, for the sake of argument, that we do indeed live in a universe containing closed timelike curves (CTCs). An observer who traverses a CTC considers himself to be doing nothing out of the ordinary; he travels forward in time as usual but eventually finds himself at a point in spacetime he has already visited previously. In a sense, then, the observer has "travelled into the past", but it is important to note that at no time does he violate any physical laws as viewed from his own co-moving frame of reference, nor does he consider himself to be moving "backwards in time". He is simply following a path through spacetime that happens to include a loop. Nonetheless, given this capacity for 'time travel', a simple argument then suggests that, for this observer, P = NP. For suppose A is a deterministic program for solving some problem in NP, and construct the algorithm A' in Figure 1. The total runtime of A' may be superpolynomial, since it includes step 1 (running A) but nonetheless the *result* is produced at step 0, and in this sense A' can be said to solve the problem in constant time, whence (loosely speaking) P = NP.

Unfortunately, this apparently simple argument is logically incomplete, since it relies on several unstated assumptions concerning the nature of CTCs, and as we discuss below these assumptions need not be generally valid.

2 CTC COMPUTATION

There are (at least) two essentially distinct ways in which CTCs might be exploited to implement computational speed-up. In the absence (so far) of experimental data confirming the existence of CTCs and the experiences of observers traversing them, the viability of these two computational schemes depends upon ones philosophical interpretation of relativity theory. In this paper we focus on one of these approaches; nonetheless we briefly discuss the consequences of choosing the other interpretation in Sec. 3.

Consider the following science-fiction cliché: a historian wants to make a clandestine visit to Ancient Rome, so he selects a suitable CTC and sets off on his journey. He makes detailed notes of Julius Cæsar's activities, and then returns to exactly the point in time and space from which he originally set out, so that his unauthorised absence cannot be detected. He repeats the same deception several days running. Using the information in his notes, he then writes an important academic paper and becomes famous.

Although this kind of story is familiar from science fiction, it requires a particular interpretation of what it means for a body to move in space and time. For consider what happens when the historian 'returns to the present'. At this point in the journey, he occupies exactly the same position in time and space as when he originally set off on his journey – *but he is not constrained to repeat the same behaviour,* for rather than endlessly repeating the journey to Ancient Rome, he chooses instead to write an academic paper. Moreover, since he occupies the same spacetime position at both points on his journey, and his notepad is in his pocket both times, its contents should be the same both times – but it contains notes when he returns which were not present when he set off.

At first sight this seems to suggest a fundamental logical inconsistency, leading to the conclusion that this kind of CTC exploitation is impossible. But there is in fact no contradiction present, provided we think of spacetime as a surface across which observers move. The fact that a body can occupy a given position more than once, and be in different states each time, is hardly surprising given this interpretation; it is no different to a racing driver completing several laps of a Grand Prix, and then deciding on the next lap that he needs to take a pit stop so that his tyres can be replaced. He may pass through the same positions on the track several times, but he is not thereby constrained to repeat the same behaviour each time.

From the historian's point of view too there is no contradiction, because we have to ask ourselves *in what sense has he travelled back in time*? Certainly, he cannot have done so relative to his own clock, because he considers himself to be moving always forwards in time at sublight speeds. His judgment *I am in Ancient Rome* must therefore have been made relative to evidence provided by some independent witness (for example, he could ask a local trader what year it is, and whether the person standing in front of them is indeed Julius Cæsar). It is entirely possible, of course, that the witness might observe multiple copies of the historian, but this is not contradictory either, for each copy is in a different state (when asked how old he is, each copy of the historian will give a different answer). From the viewpoint of the witness, the various copies of the historian are distinct objects, and there is no sense in which the historian is observed in different places at the same time.

Thus, neither the witness nor the historian observes anything contradictory as a result of his re-occupying a point in spacetime without being constrained to repeat the same behaviour. By the same argument, a computer might feasibly be sent around a CTC several times without its computation being forced to loop, so we could run a program for as long as we like by repeatedly traversing (say) a 5-second CTC as often as we like. If we arrange for the machine to produce some observable output if and only if it eventually halts (e.g. on a printer) we can always decide after 5 seconds whether the program has halted, and if so what its result is, simply by checking the printer (which need not be on the CTC). Thus, not only does hypercomputation seem to be possible in this scenario, but *all* recursively enumerable problems can apparently be solved in constant time; in particular, P = NP.

This argument is, however, logically flawed, for although a CTC returns an observer to an earlier point on his worldline, *it does not follow that he can traverse the CTC a second time*. Moreover, we have no justification for assuming that arbitrarily complicated programs can be made to execute on a computer traversing a CTC. As we argue below, logical constraints on CTC computations suggest that the information storage capacity of a CTC is extremely limited, to the extent that only finitely many programs could ever be implemented via this paradigm.

2.1 Single-traversable CTCs

In this section we introduce a crossed-ribbon toy spacetime that includes a single inhabited CTC. It is impossible for the (unique) observer in this model to traverse more than one CTC, and no CTC can ever be traversed more than once. Our crossed-ribbon universe is in some respects non-standard, but we will nonetheless argue that it is a *reasonable* model.

Our description of the crossed-ribbon universe is given in three stages. First we construct a standard $(1+1)$-dimensional spacetime (M); then we describe the crossed-ribbon (*Ribbon*); and then we populate *Ribbon* with an (inertial) observer and consider its worldline. We argue that M can be populated with observers in such a way that observations in M and *Ribbon* are indistinguishable, that M gives an inherently acceptable description of spacetime, and hence that *Ribbon* (being observationally indistinguishable from M) is also a reasonable model.

Construction of M.

Following Andréka and her colleagues [2, 17] we assume that spacetime is coordinatized by an ordered Euclidean field Q of *quantities*. Our starting point is a ribbon-shaped manifold, $M = (-\alpha, \alpha) \times Q$, where $\alpha \in Q$

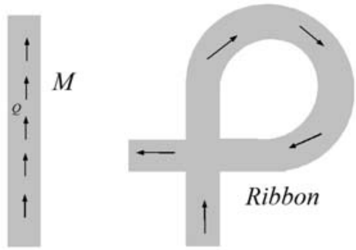

FIGURE 2
The $(1+1)$-dimensional manifold M, and the immersed manifold *Ribbon* generated by looping M in such a way that it intersects itself at right-angles.

(Figure 2). We regard this as a $(1+1)$-dimensional Minkowskian spacetime of infinite length and width 2α, with time flowing along the length of the ribbon, and space across its width. If we take $Q = \mathbb{R}$, then α should be finite, but for more general coordinatizations this need not be the case: for example, if Q contains infinitesimals, α could be an infinite value in Q.

Construction of Ribbon.
We now imagine wrapping M around in 3-dimensional space so that it self-intersects at right-angles. We identify the overlapping regions, and call the resulting manifold *Ribbon* (Figure 2).

Worldlines in Ribbon.
The manifold M is a standard $(1+1)$-dimensional spacetime model, and we can populate it with observers in the usual way. As usual, we shall assume for convenience that time flows up the page, and define a valid *initial* worldline to be any path followed by a body that always travels at subluminal speed (its tangential motion always points into the interior of its colocated future lightcone). Given any such worldline w in M, we define w' to be its image in *Ribbon*. We then *reflect* the worldline back into M. In other words, we determine what other observers would need to be present in M, and following what worldlines, if a body following w is to observe exactly the same series of events as a body moving along the corresponding path w' in *Ribbon* (Figure 3).

2.2 Is *Ribbon* a reasonable model of spacetime?

Any observer following w' will encounter itself twice (once each time it crosses the overlap region), but it is important to realise that on each occasion the body considers its other incarnation to be travelling faster than light (FTL), because of the way *Ribbon* intersects itself at right angles (which

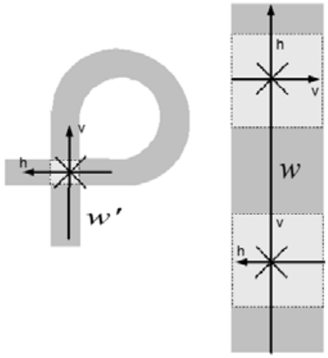

FIGURE 3
Schematic showing how a worldline w' in *Ribbon* corresponds to three intersecting worldlines in M, and how the 'horizontal' (h) and 'vertical' (v) axes in *Ribbon*'s overlap region appear in M: they are identical except for a 90° rotation, corresponding to an interchange of the observer's space and time directions after traversing the CTC.

ensures that the time and space axes have been interchanged when the body encounters its past self). Reflecting this back into the original manifold M, the body traversing w meets two FTL versions of itself as it moves along its worldline. The paths followed by these FTL observers are fully determined once w is specified.

Apart from the existence of observers apparently moving at FTL speeds relative to one another, there is nothing unusual about this three-observer version of M, and indeed FTL motion has long been a research topic in cosmological theory [11, 14, 16]. We therefore claim that populating M with these additional FTL observers yields an entirely reasonable (toy) universe. By construction, however, the flow of events observed by the observer following w' in *Ribbon* is identical to the flow observed by the observer following w in the three-observer variant of M, whence *Ribbon* must also be a reasonable (toy) universe.

We reiterate that M necessarily contains observers travelling FTL with respect to one another, and that the trajectories through the overlap regions, both of which are 'squares' of side 2α, fully determine one another. As shown in Figure 3, the upper overlap region is generated by 'rotating' the lower overlap region through 90° so as to interchange the locally 'horizontal' (h) and vertical (v) vectors. Consequently, we cannot simply populate M with arbitrary FTL bodies; if their trajectories fail to match up correctly, the scenario will fail to correspond to the desired CTC motion in *Ribbon*.

2.3 Existence of single-traversable CTCs

The significance of *Ribbon* lies in the nature of the path w'. Since this path includes a self-intersection, it implements a CTC. But as we have noted above, when the observer meets its former self, its time and space axes have been interchanged, and it considers its other selves to be travelling at FTL speeds. In order to re-traverse the CTC it would need to complete the ‘90 degree turn’ at the point of intersection, thereby crossing from one quadrant of the local lightcone to another. Assuming that motion is smooth, this is impossible because the observer would at some point need to move “at light speed” (tangentially to its local light cone).

Thus *Ribbon* provides an example of a spacetime containing a CTC which *cannot* be traversed a second time by any observer. Notice, incidentally, that the existence of single-traversable CTCs is a *global* property of the spacetime in question. We could, for example, join the two ‘open ends’ of *Ribbon* to form a figure-of-eight configuration, thereby converting the observer’s *entire* trajectory into a (multiply traversable) CTC. The original overlap region would remain unchanged by this manoeuvre; in effect, we can construct a multiply-traversable CTC by joining together two ‘locally single-traversable’ CTCs.

3 DISCUSSION

We have described an interpretation of CTC traversal in which spacetime exists as an independent entity across which observers move subject to various laws; an observer could potentially pass through a given location several times without being constrained to display identical behaviour subsequently. An alternative viewpoint is that a point in spacetime is fully defined by the set of observers that exist there [3]. According to this viewpoint, when the historian returns to the present, he has no choice but to re-traverse the CTC back to Ancient Rome. Since he occupies the same location in spacetime as his past self, he *is* his past self and must behave accordingly. Similarly, since his notes are colocated with his notepad when he returns from Rome, they must also have been present before he set off.

3.1 The Entropy of a CTC

This second interpretation severely challenges certain key assumptions of everyday computer science. For simplicity let us assume that one program statement can be executed every second, and that it takes precisely n seconds to traverse some given CTC exactly once. If we run a program on a computer following this CTC, then once the n^{th} statement has been executed, the entire system will have returned to its original spacetime location, and so must have returned to its original state. It follows that no irreversible process can be implemented on a computer following a CTC. And yet there is no obvious reason we shouldn’t be able to load our computer with any program we like.

To avoid the apparent contradictions inherent in this situation, we need to re-appraise the nature of CTC computation. Since reversibility requires that no information is lost from the system, we have to conclude that when an irreversible procedure is executed on the computer, the information lost during program execution must be preserved somehow in the computer's environment, i.e. the CTC itself. It is well-known that black holes have an associated entropy, proportional to the area of their event horizon [5]. What we are suggesting here is that, given this second interpretation of CTC traversal, a CTC can also have an associated entropy, indeed *CTCs are perfect information repositories*. More than this, the CTC actually overrides the intended behaviour of the program, since the computer is forced to re-accept the information stored in the CTC when it returns to its initial state, regardless of the underlying program specification. Indeed, this can be seen as a mechanism enforcing the Novikov self-consistency principle [10] in the context of CTC computation.

3.2 Single-traversable CTCs and computational speed-up

Given our original interpretation of CTC traversal, *Ribbon* shows that CTCs need not be traversable more than once. This second interpretation of CTC traversal likewise concludes that repeated traversal of a CTC cannot (of itself) lead to computational speed-up, because any lengthy computation would be forced to return to its initialisation state rather than running to completion. The question remains whether it is possible to use CTCs to speed up computation, where we *voluntarily* restrict ourselves to traversing CTCs no more than once. Indeed such schemes are described in the literature, but these schemes make the additional assumption of *causal consistency*, using it to deduce that CTC-computation can solve e.g. PSPACE-problems in polynomial time [1, 6].

What, then, can we deduce if we impose no additional constraints, and simply regard CTCs to be used as an implementation of time travel. As we illustrated in Figure 1, the availability of time travel can be used to show that P = NP. But the situation is not entirely clearcut, because we have assumed in Figure 1 that information can be sent back to time 0 no matter how long we have to wait for A to complete its execution, and there is no guarantee that this is the case. For example, suppose the maximum time any CTC can take a traveller back is 5 seconds; then as soon as an input is provided which causes A to run for more than 5 seconds, algorithm A' will be invalid.

This suggests that, even in the presence of time travel, we cannot necessarily reduce problems in NP\P (if any) to problems in P. For the remainder of this paper we therefore assume, to the contrary, that P $\neq$ NP and ask what consequences this assumption entails.

CTCs and P $\neq$ NP.

We assume the existence of some arbitrary observer (typically a computer) O. Given any spacetime location X on O's worldline, write X^+ for the set of timelike paths starting at X that are traversable (in theory) by observers co-moving with O at X. From O's viewpoint, these paths are all future-pointing, and it is possible for O to send information (e.g. by rocket) along any of these paths without requiring lightspeed or faster-than-light travel.

Some of these paths may intersect O's worldline at points other than X. If these points lie to the past of X from O's point of view, then CTCs are present, and we can ask to what extent they can be exploited computationally. For simplicity, we will assume that O can identify whether any given path in X^+ leads to a point Y in his past, and can also identify the point Y itself (i.e., how far back into his past the path takes him). It is extremely unlikely, of course, that such properties of CTCs would ever be so conveniently decidable.

Write $\mathit{Past}(X)$ for the set of all such points Y, i.e. those points on O's past worldline that he can revisit by following paths in X^+. For each such intersection Y, write $|X - Y|$ for the amount of time that originally passed, from O's point of view, in travelling from Y to X. In other words, if O chooses to follow the path in question, how far into his own past will it take him? For simplicity we shall assume that all durations are measured in seconds.

Given any time t, write $X(t)$ for the point X on O's worldline that has time coordinate t as coordinatized by O, and define the set of *time differences* available at time t to be the set

$$D(t) = \{|X(t) - Y| : Y \in \mathit{Past}(X(t))\}$$

and let

$$D^*(t) = \begin{cases} \sup D(t) & \text{if } D(t) \text{ is bounded above} \\ \text{undefined} & \text{otherwise} \end{cases} .$$

In essence, the function $D^*(t)$ tells us how far back O can travel into his own past if he sets off on his journey at time t. If $D^*(t)$ is undefined, there is no limit to how far back O can travel. Likewise we write $R(t)$ for the set of past times O on his own worldline that are *reachable* by setting off from $X(t)$, and $R^*(t)$ for the infimum of these reachable times, i.e.

$$R(t) = \{t - t' : t' \in D(t)\}$$

and

$$R^*(t) = \begin{cases} \inf R(t) & \text{if } R(t) \text{ is bounded below} \\ \text{undefined} & \text{otherwise} \end{cases} .$$

We show that when P $\neq$ NP, $R^*(t)$ (equivalently, $D^*(t)$) must be defined for all sufficiently large t.

Lemma 1. *If $t' \leq t$, then $R(t) \subseteq R(t')$.*

Proof. Any path in $X(t)^+$ can be prepended by the section of O's worldline running between times t' and t to generate a path in $X(t')^+$. □

Theorem 1. *Suppose P $\neq$ NP. Then, for all sufficiently large t, $R(t)$ is bounded below, and hence $R^*(t)$ is defined.*

Proof. Suppose to the contrary that there exists an unbounded increasing sequence of times t at which $R(t)$ has no lower bound. By Lemma 1, $R(t)$ must be unbounded below for all t.

Let A be a deterministic algorithm for solving some problem in NP. We use A to define a new algorithm B to be implemented on a computer co-moving with O, with the following behaviour (essentially a generalisation of the algorithm presented in Sect. 1).

Given n, O resets his clock to 0, waits one second and then checks whether any output has yet been generated. One second later he starts running $A(n)$. After $A(n)$ has eventually halted at time t (say), B travels back to some $T < 0$ in $R(t)$, waits until he re-encounters time 0, and then publishes the result in time for his earlier incarnation to observe it at time 1. As before this implies that problems in NP can be solved deterministically in constant time, whence P = NP (contrary to assumption).

Therefore no unbounded increasing sequence of times t exists at which $R(t)$ is unbounded below, and the result follows. □

Corollary 1. *Suppose $R^*(t)$ is defined, and suppose $t' \in (R^*(t), t]$. Then $R^*(t')$ is also defined, and $R^*(t) = R^*(t')$.*

Proof. By assumption, $t' > R^*(t)$, so since $R^*(t) = \inf R(t)$ there must exist $T \in R(t)$ satisfying $t' > T \geq R^*(t)$. Consequently, O can travel back from $X(t)$ to arrive back on his past worldline at time T, then wait (if necessary) until time t' before setting off on any path in $R(t')$. Thus any past time reachable by O from $X(t')$ is also reachable from $X(t)$, whence $R(t') \subseteq R(t)$. Since $R(t)$ is bounded below, the same must be true of $R(t')$, whence $R^*(t')$ is defined, as claimed. Lemma 1 tells us conversely that $R(t) \subseteq R(t')$ (since $t' \leq t$), and combining the two inclusions gives $R(t) = R(t')$, whence the claim follows. □

Ours is not, of course, the first work to suggest that computability can be used to investigate the nature of physical theories. In his seminal study of quantum mechanics near CTCs, Deutsch used quantum computational considerations to analyse the physics of CTCs themselves, noting in particular that in the presence of CTCs a new experimental test of Everett's 'many worlds' interpretation of quantum mechanics would become possible [8]. Lloyd *et al.* subsequently re-examined the quantum theory of CTCs by considering a reformulation based on teleportation and postselection; their theory (which is inequivalent to Deutsch's) has been subjected to experimental simulation and shows how e.g., the grandfather paradox can be resolved [13].

More recently, Brun and Wilde [7] have considered the relationship between the "Deutschian" (D-CTC) and "postselected" (P-CTC) CTC formulations, showing that the computational power of P-CTCs is weaker than that of D-CTCs, and giving explicit circuit constructions of systems, using just one qubit travelling back in time, that can "efficiently solve any decision problem in the intersection of NP and coNP, and probabilistically solve any decision problem in NP".

Our own work, presented above, is inherently classical, and thus to some extent orthogonal to these quantum theoretical findings. Nonetheless, we may ask what our own simple results tell us about the physical world under the assumption P $\neq$ NP. Theorem 1 tells us that, even if an observer is able to travel arbitrarily far back in time when he is young, he will eventually lose that capability as he grows older, and his reach into the past will become finite. Corollary 1 then tells us as he grows older, he loses access to more and more of his past. This confirms that basic computational assumptions seem to be telling us something also about cosmological structure. The exact nature of the relationship depends on an analysis of how fast the function R^* grows, and remains an open question.

ACKNOWLEDGEMENTS

We would like to thank Gergely Székely, whose comments have greatly streamlined our thinking during the development of the ideas presented here; thanks also to the anonymous referees, for their careful work and helpful suggestions.

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