From Infinitely Small to Infinitely Big: The Universe as Computer

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INFINITY

Infinite space

Infinite time

Infinite complexity

UNCONVENTIONAL

COMPUTING

Contents

INTRODUCTION

- Conventional Computing

- Unconventional Computing

CONVENTIONAL COMPUTERS

CONVENTIONAL COMPUTATIONS

- Word processing
- Email
- Graphics
- Accounting (Payroll, taxes, ...)
- Business (Inventory, databases, ...)
- Engineering (Design, manufacturing, ...)
- Science (Marine biology, astrophysics, ...)
- Mathematics (Numerical computations, theorem proving, ...)

Is there anything wrong with conventional computers?

- Conventional computers are reaching their limit for

conventional problems

when it comes to

- processing huge amounts of data
- performing huge numbers of iterations

in a reasonable amount of time.

For example, virtual surgery

- Conventional computers are hopeless for solving

  unconventional problems

Unconventional Computations

1. When the data change with the passage of time

2. Computing in real time with deadlines

3. When the laws of nature control the outcome

4. Computing under mathematical constraints

Unconventional Computers

1. Parallel computers

2. Analog computers

3. Biological computers

4. Quantum computers

5. Accelerating computers

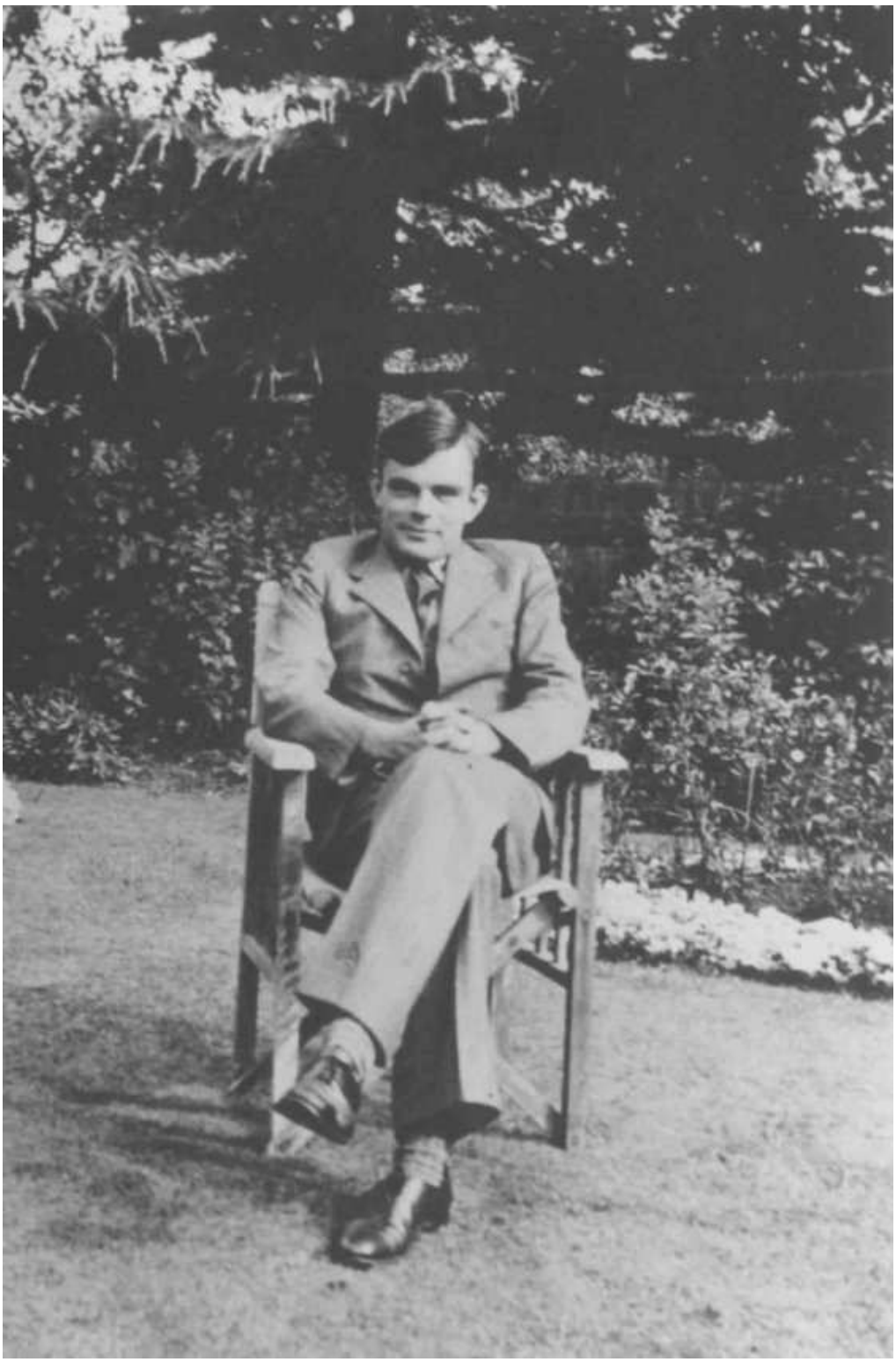
DAVID HILBERT (1862 - 1943)

Hilbert's Questions

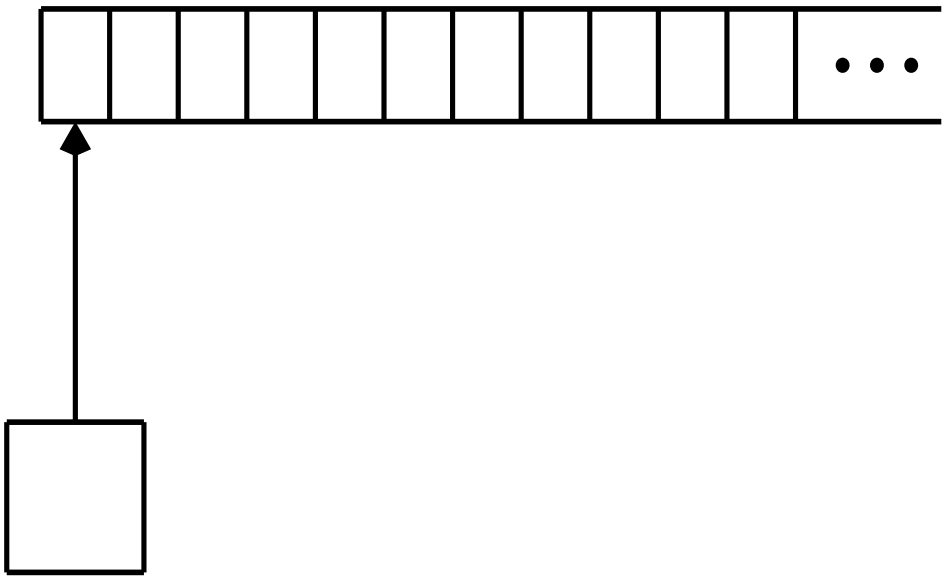
Question 1: Is there a fixed set of true mathematical statements that can be used to prove any new mathematical statement?

Question 2: Can such proofs be generated automatically?

ALAN TURING (1912 - 1954)



The Turing Machine

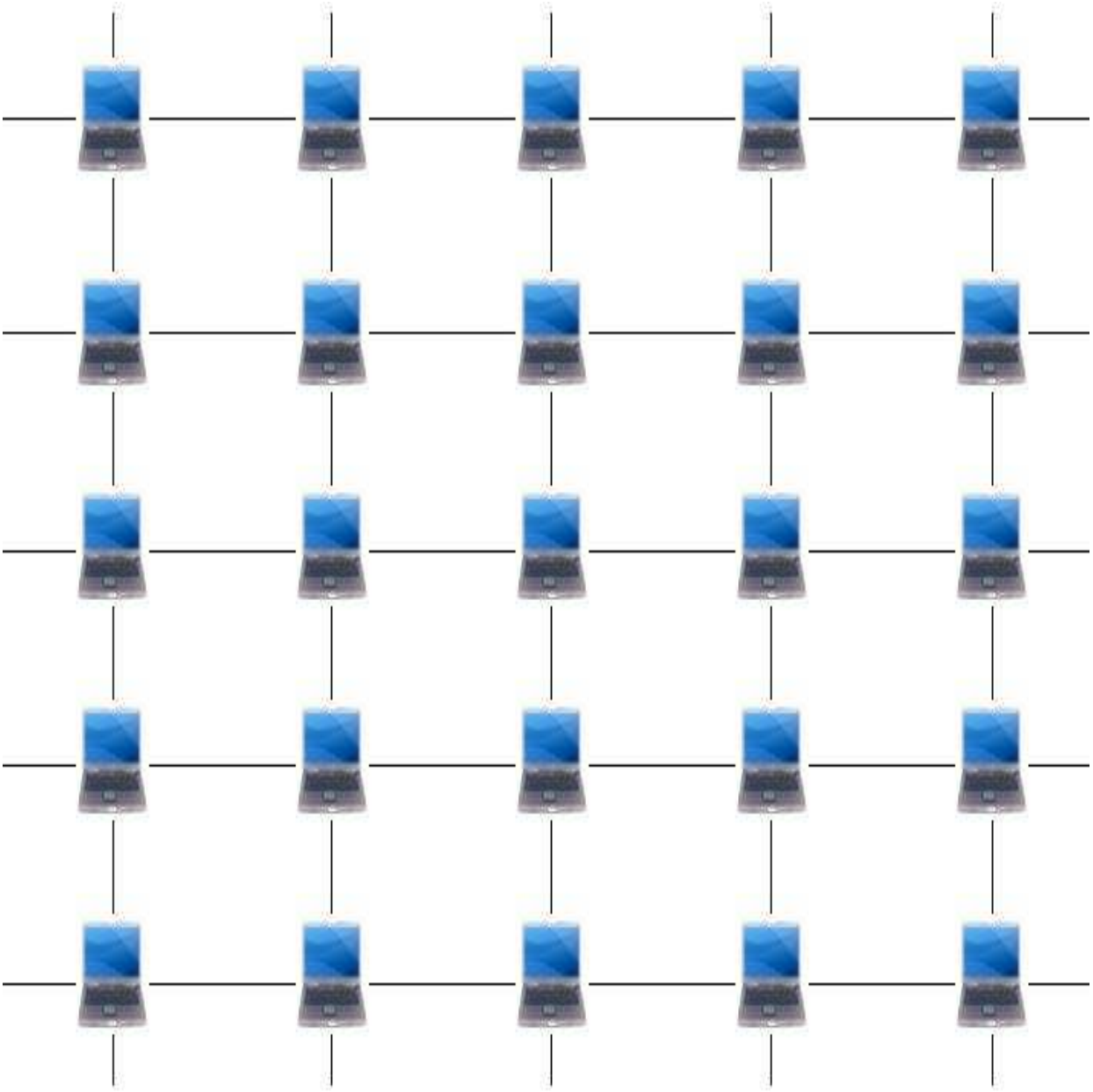


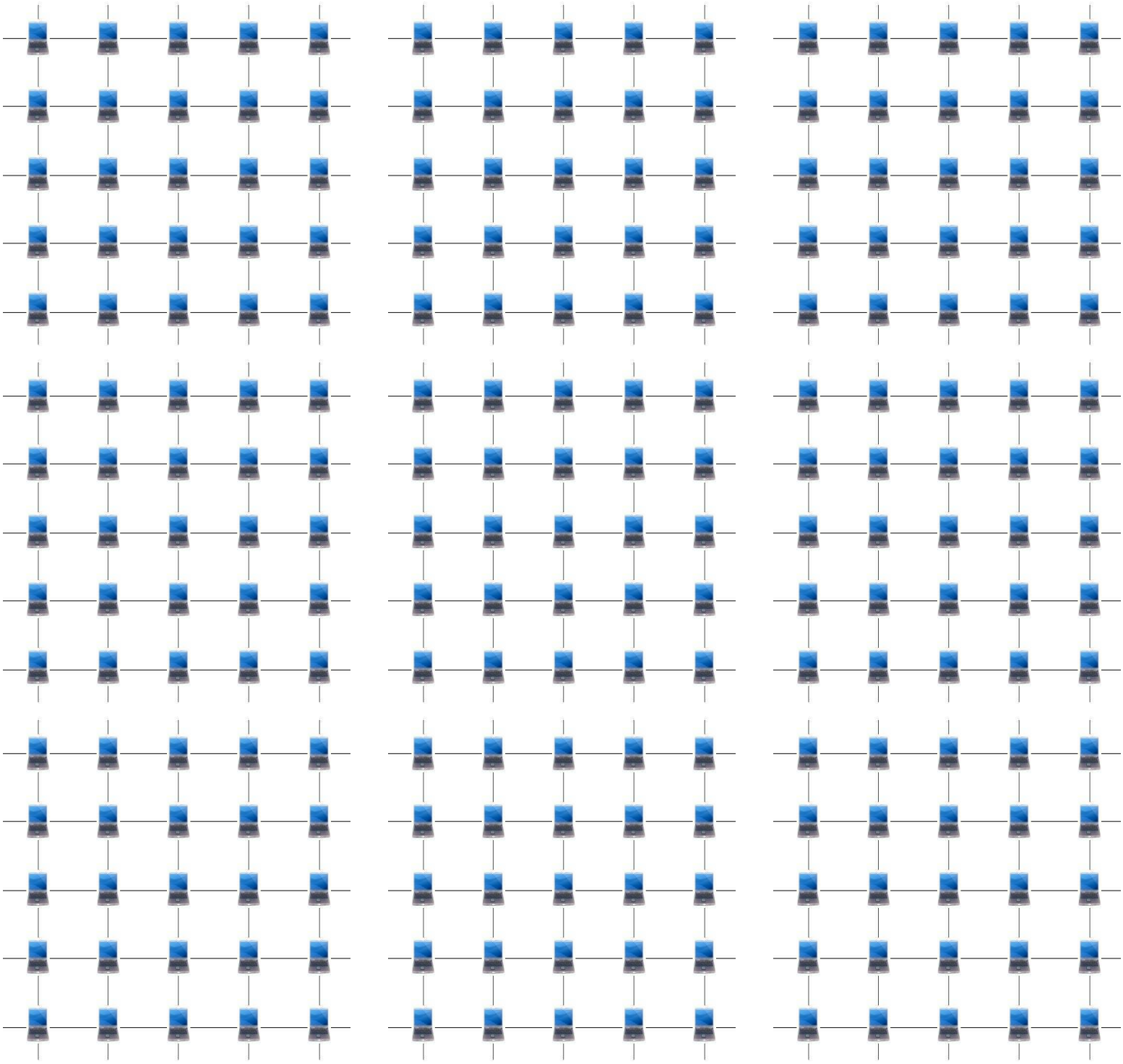
The Turing Machine is UNIVERSAL thanks to the

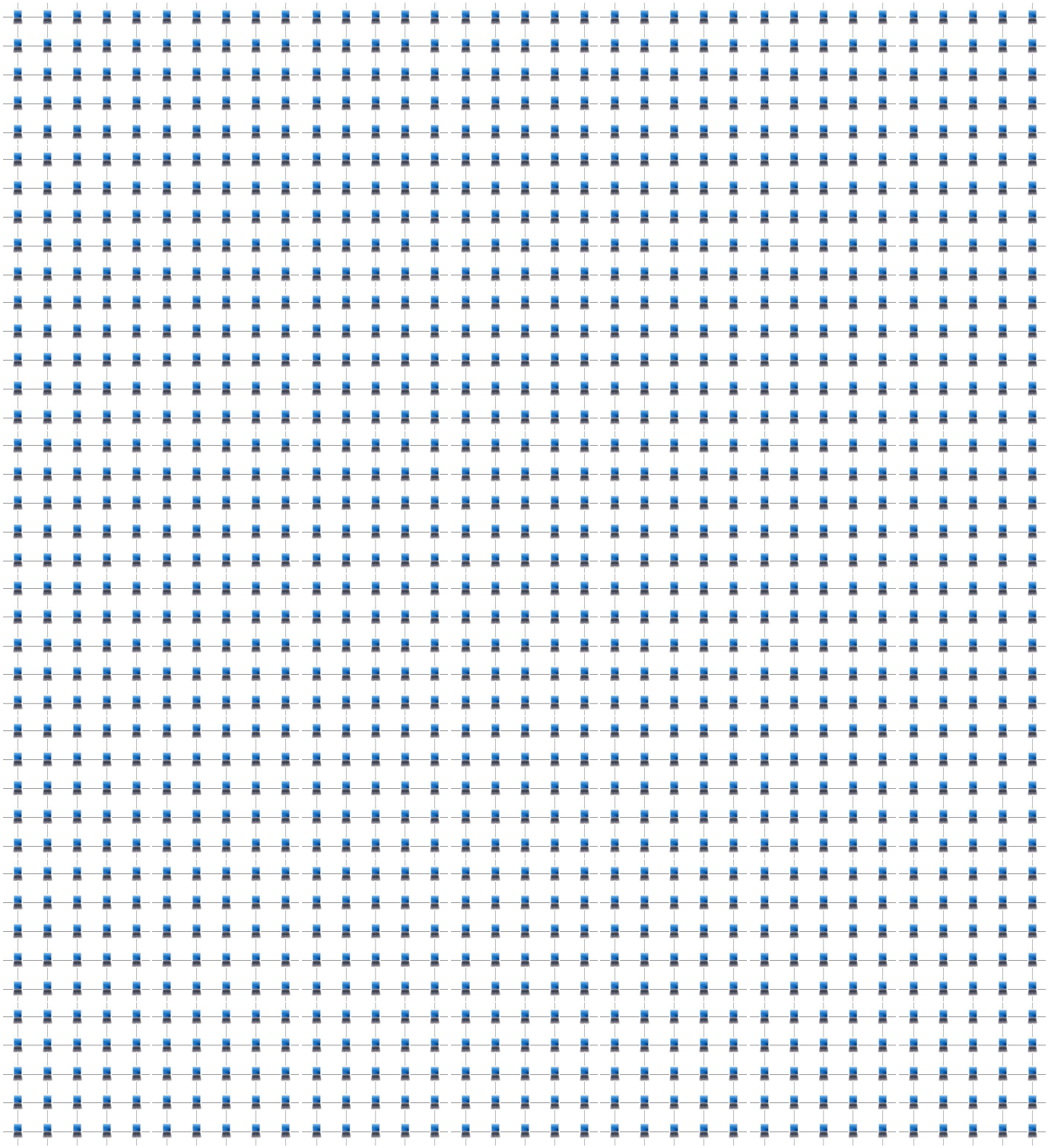
THE PRINCIPLE OF SIMULATION

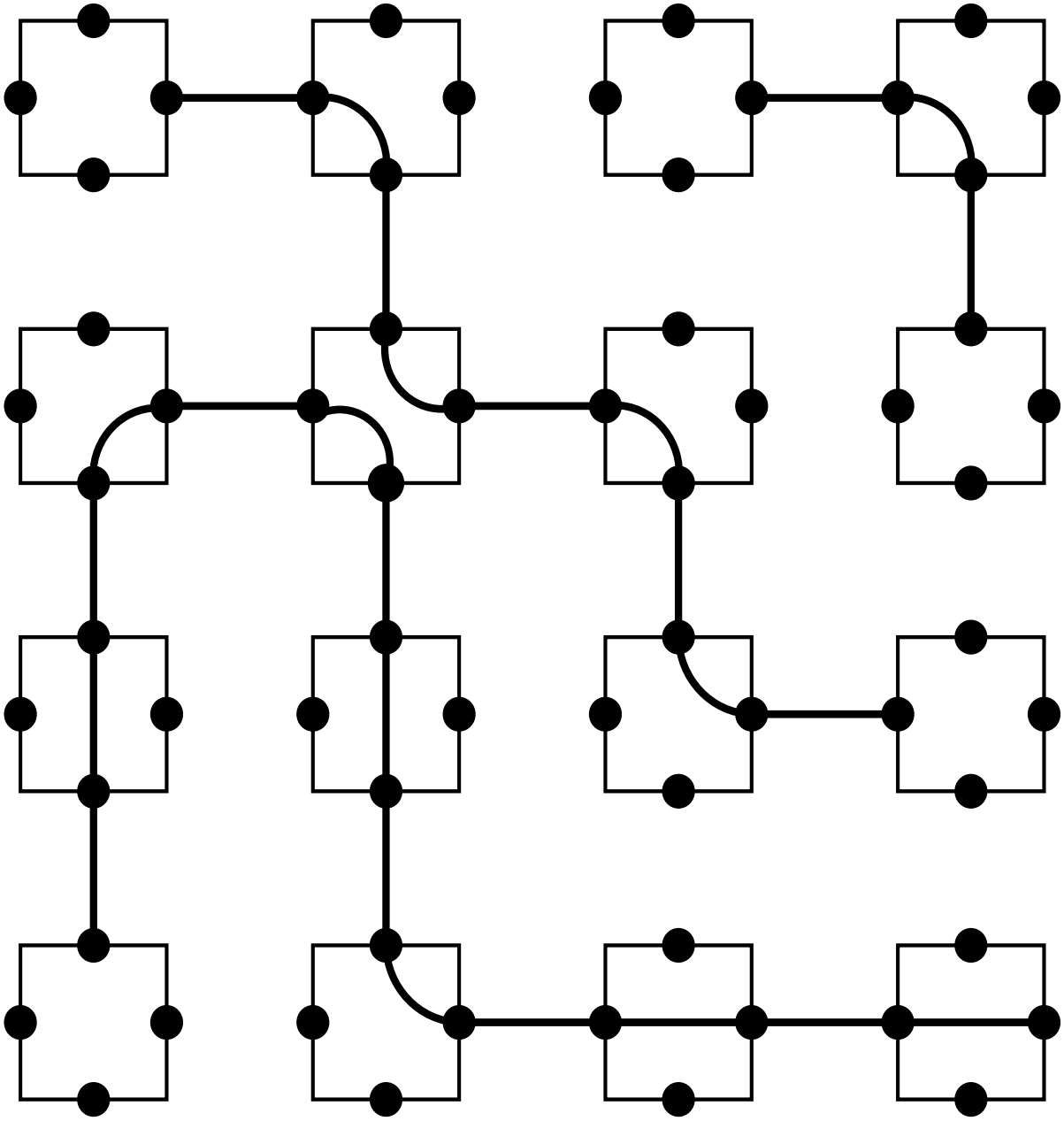
- Anything that can be computed can be computed on the Turing Machine
- If something cannot be computed on the Turing Machine then it cannot be computed at all

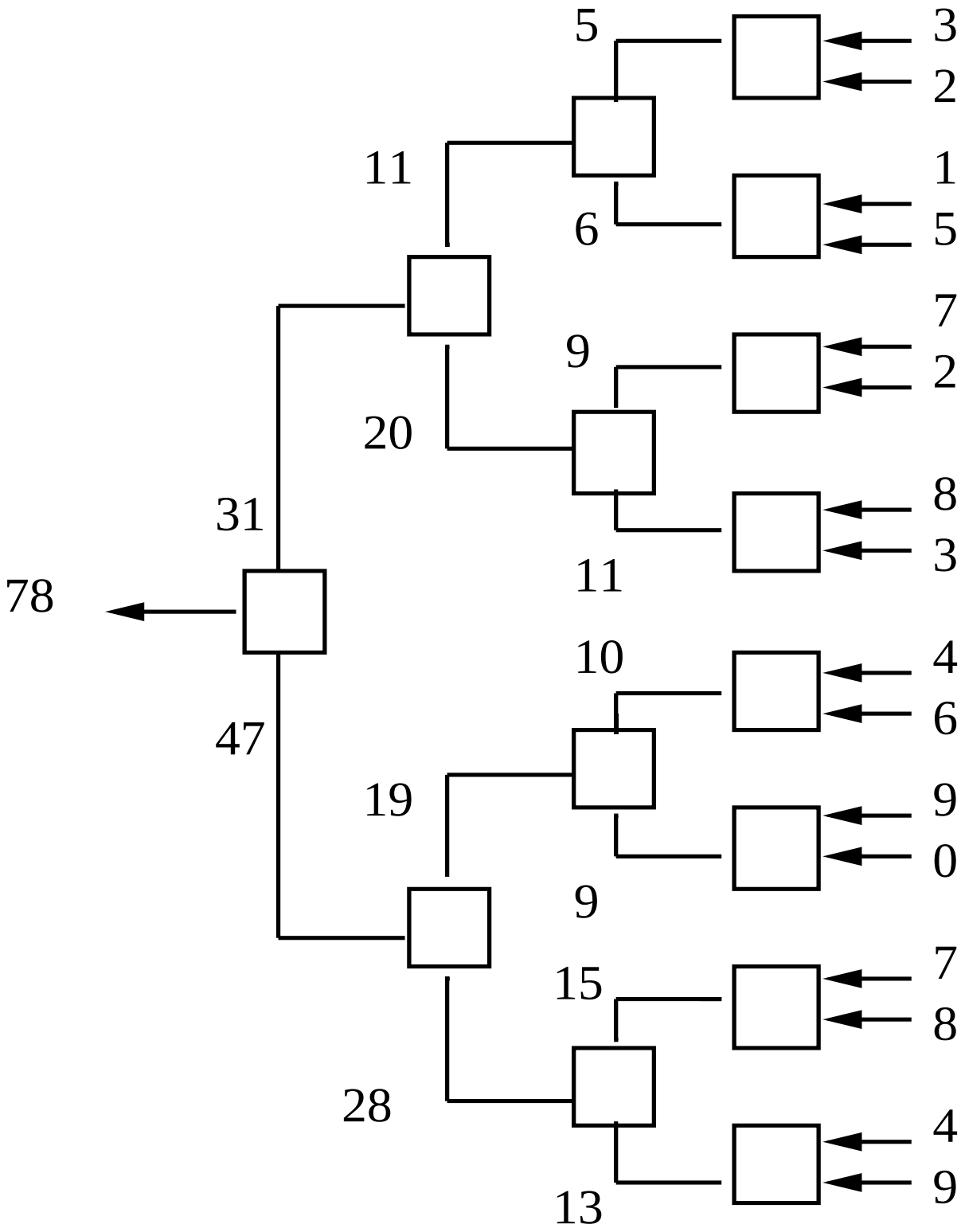
PARALLEL COMPUTING










78
31
47
11
20
9
5
6
11
9
10
19
9
15
28
13
3
2
1
5
7
2
8
3
4
6
9
0
7
8
4
9

"SPEEDUP RULE"

Time to solve a problem on N computers

is never smaller than

$$\frac{\text{Time to solve the problem on 1 computer}}{N}$$

IS THE SPEEDUP RULE

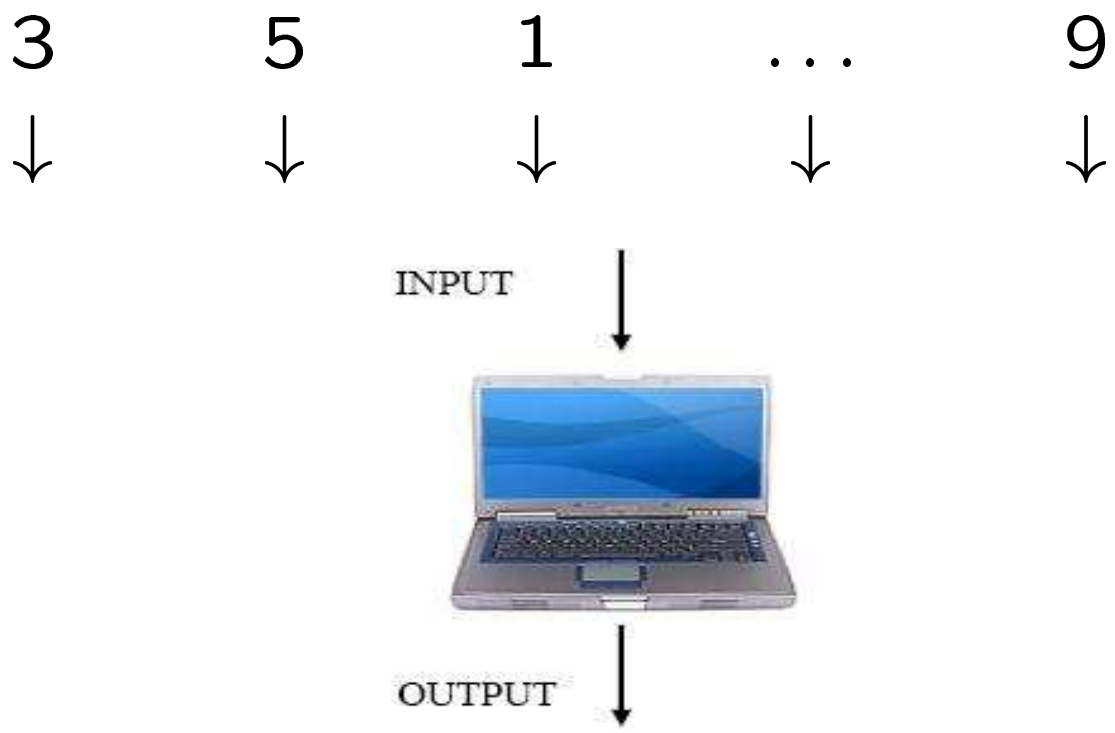
ALWAYS TRUE?

Snow Shoveling and Data Accumulation

4	1	6	...	8
⋮	⋮	⋮	...	⋮
5	2	7	...	0
1	4	8	...	6
3	5	1	...	9
↓	↓	↓	↓	↓

- 50 new numbers are received every 2 seconds
- If all numbers currently in store have been processed, computation terminates
  regardless of whether more numbers arrive later
- If a new set arrives before the previous one is completely processed, the new set must also be processed
- 2^{50} sets are received in all

Conventional Solution



Conventional Solution

1	4	8	. . .	6
3	5	1	. . .	9
↓	↓	↓	↓	↓

Conventional Solution

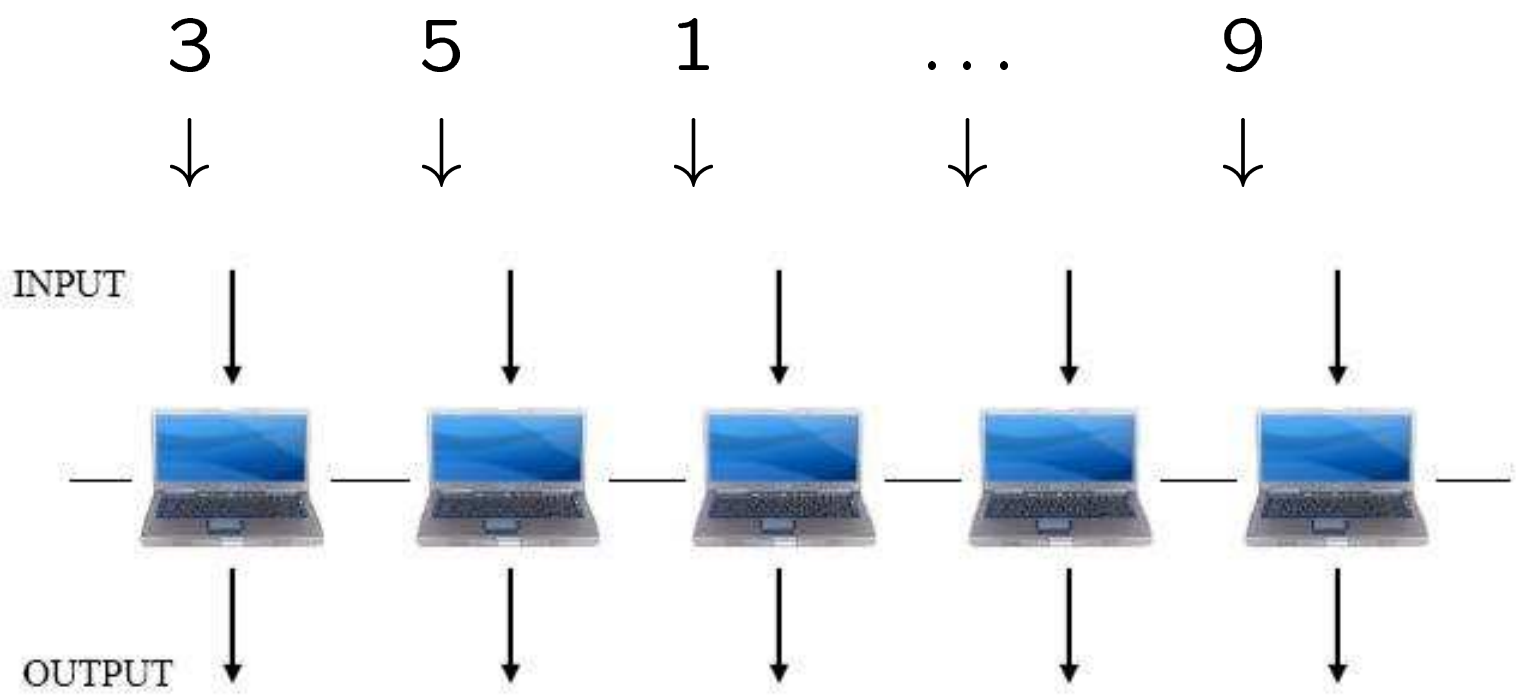
5	2	7	...	0
1	4	8	...	6
3	5	1	...	9
↓	↓	↓	↓	↓

4	1	6	...	8
⋮	⋮	⋮	...	⋮
5	2	7	...	0
1	4	8	...	6
3	5	1	...	9
↓	↓	↓	↓	↓

Time required =

$2^{50} \times 50$ seconds.

Unconventional Solution



Time required in parallel =

one second!

By the "speedup rule":

Time to solve the problem using 50 computers

cannot be smaller than

$$\frac{2^{50} \times 50}{50} = 2^{50} \qquad \text{seconds}$$

In other words, with 50 computers

we should not hope to solve the problem in less than

five million years.

Yet, our parallel solution runs in

one second!

This represents a speedup of 2^{50}.

That's more than

10,000,000,000,000,000 times faster

with only 50 computers!

What is the meaning of "to compute"?

- Electronic computers
- Neurons
- Chemical reactions
- DNA
- Subatomic particles

Any form of information processing is a computation:

- Measuring data
- Combining data
- Transforming data

Computation

is a fundamental category

in Nature

HOW DOES NATURE COMPUTE?

Does Nature use infinite-precision real numbers?

0.472854017384561749356193526201373 81...

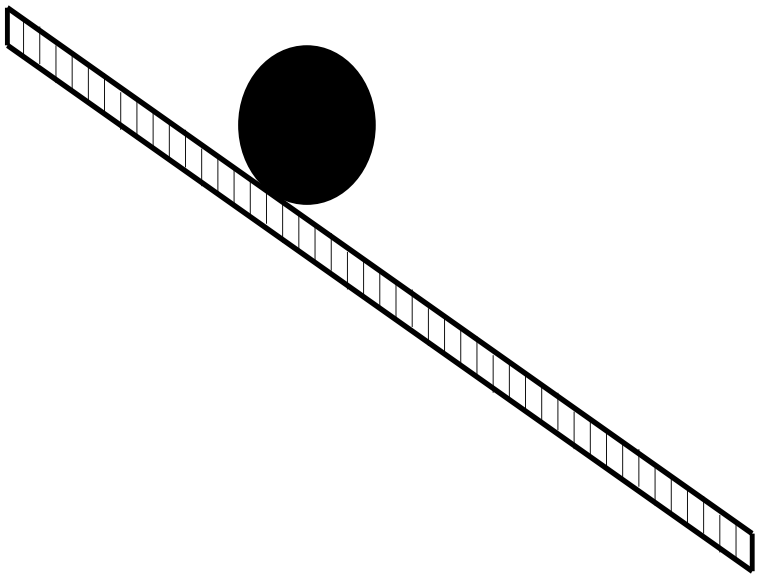
Conventional computers cannot handle such numbers.

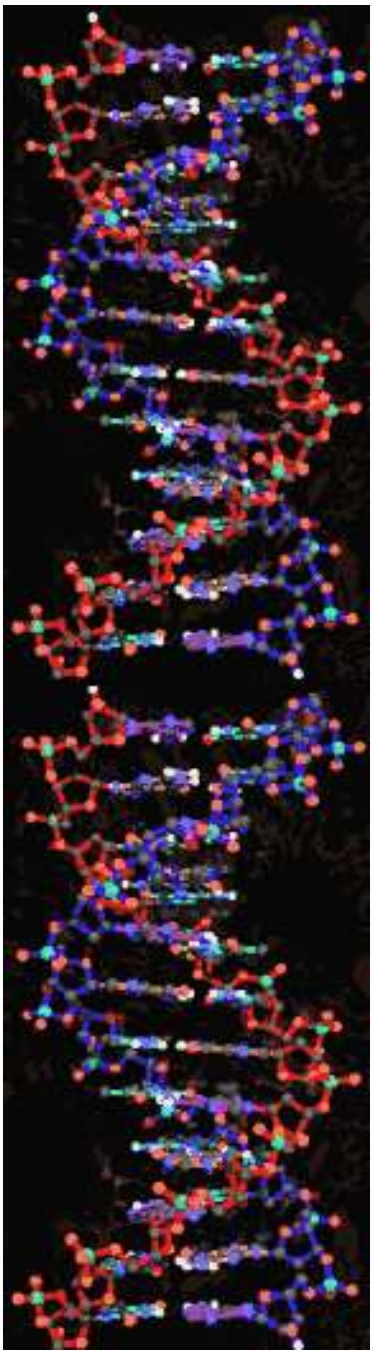
ANALOG COMPUTING

Consider the infinite interval between 0 and 1:

0 ... 0.25 ... 0.5 ... 0.75 ... 1

Can we search this interval in finite time?





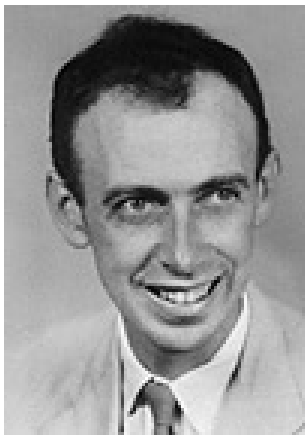
Deoxyribonucleic Acid

DNA

The Nobel Prize in Physiology or Medicine 1962

"for their discoveries concerning the molecular structure of nucleic acids and its significance for information transfer in living material"

Francis Crick



James Watson

Maurice Wilkins

One gram of DNA
can hold as much
information as

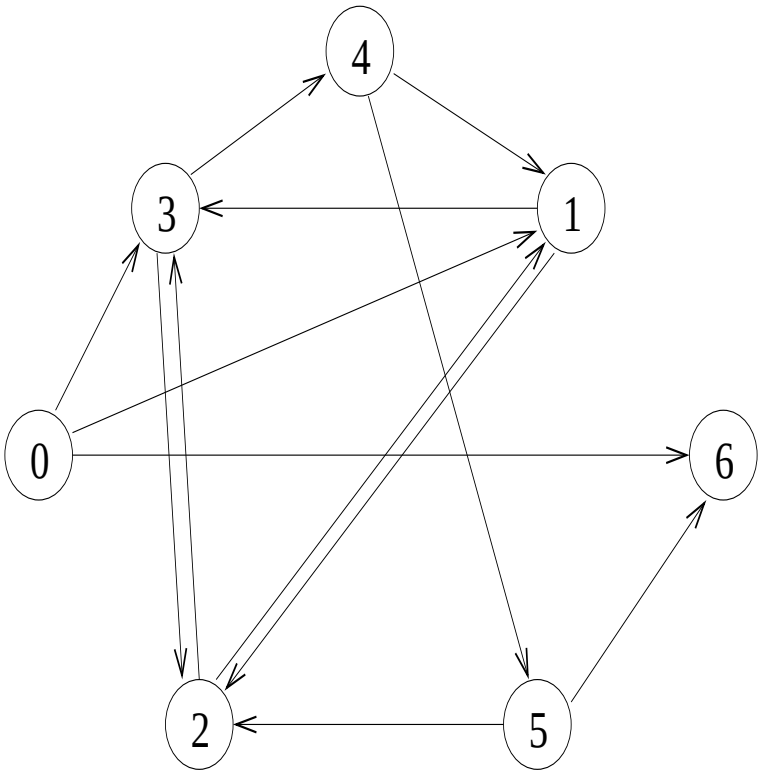
a trillion compact
discs.

	Conventional Computer	*DNA Computer*
Storage (space for one bit 0 or 1)	10^{12} nm^3	1 nm^3
Speed (millions of instructions per second)	10^3	10^{14} (test tube full)
Energy (number of operations per joule)	10^9	2×10^{19}

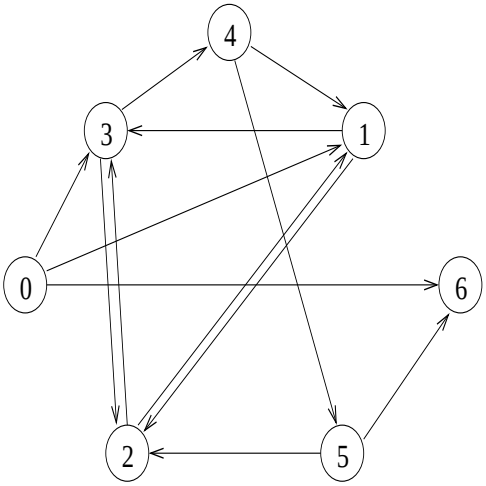
DNA computation has been used to solve

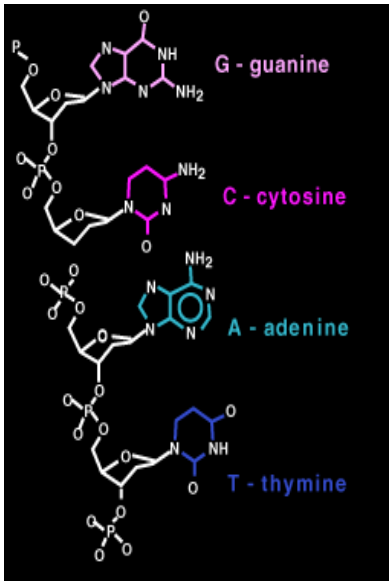
hard combinatorial problems.

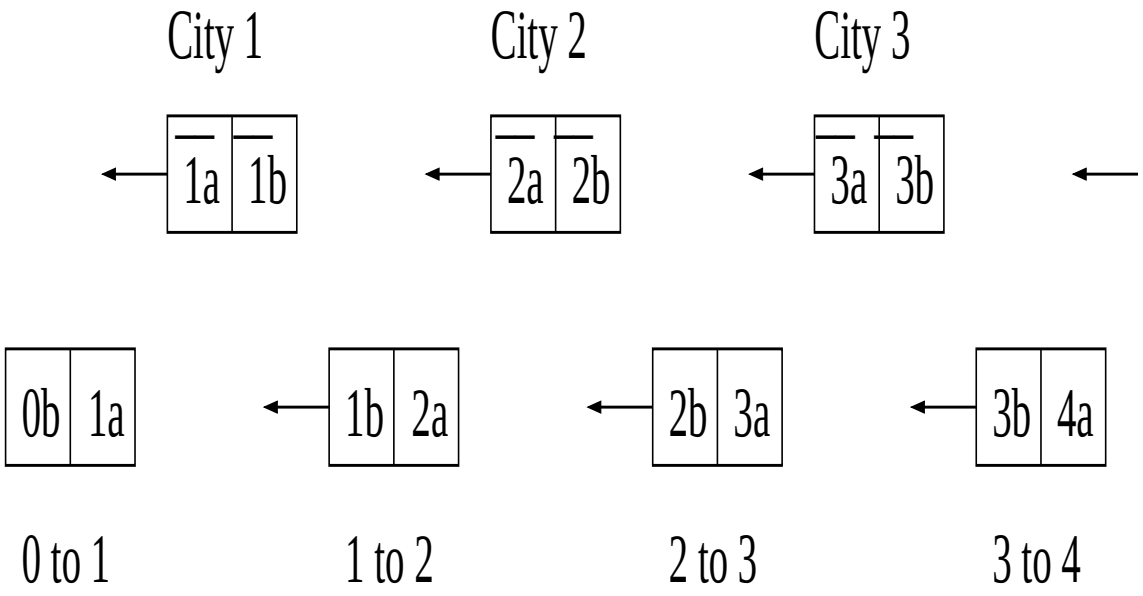
Adleman (1994)



Hamilton Cycle Problem


4
3
1
0
6
2
5

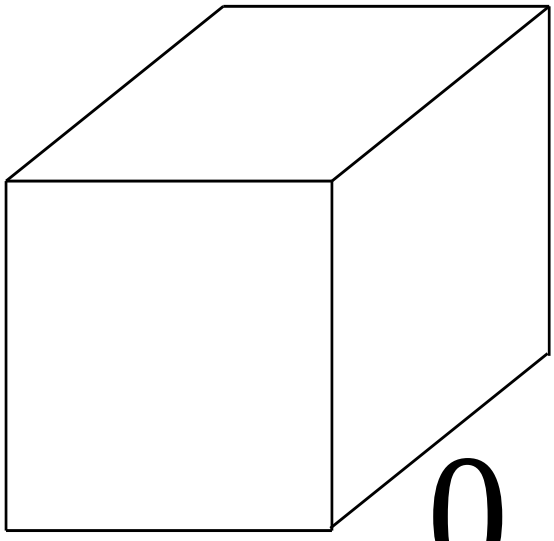

G - guanine
C - cytosine
A - adenine
T - thymine


City 1
City 2
City 3
1a
1b
2a
2b
3a
3b
0b
1a
1b
2a
2b
3a
3b
4a
0 to 1
1 to 2
2 to 3
3 to 4

"There's plenty of room at the bottom."

Richard Feynman, 1959

Assume one bit (0 or 1) is stored in one atom.

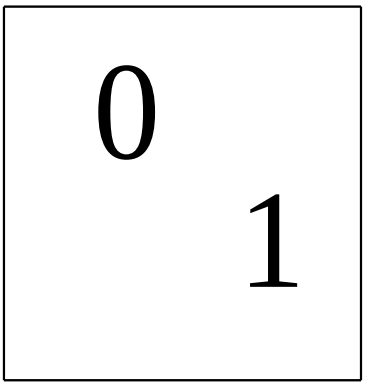


0.1 mm

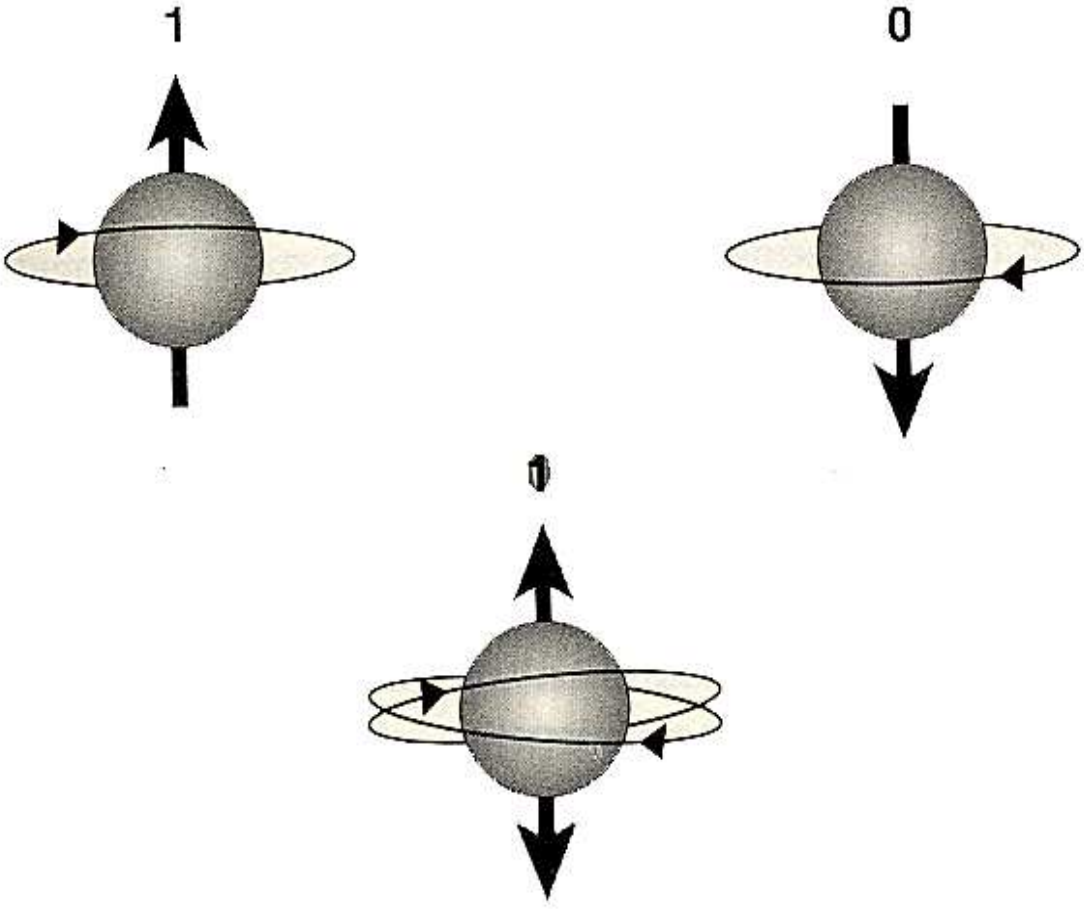
EVERYTHING EVER PRINTED!

INFINITELY SMALL SPACE!

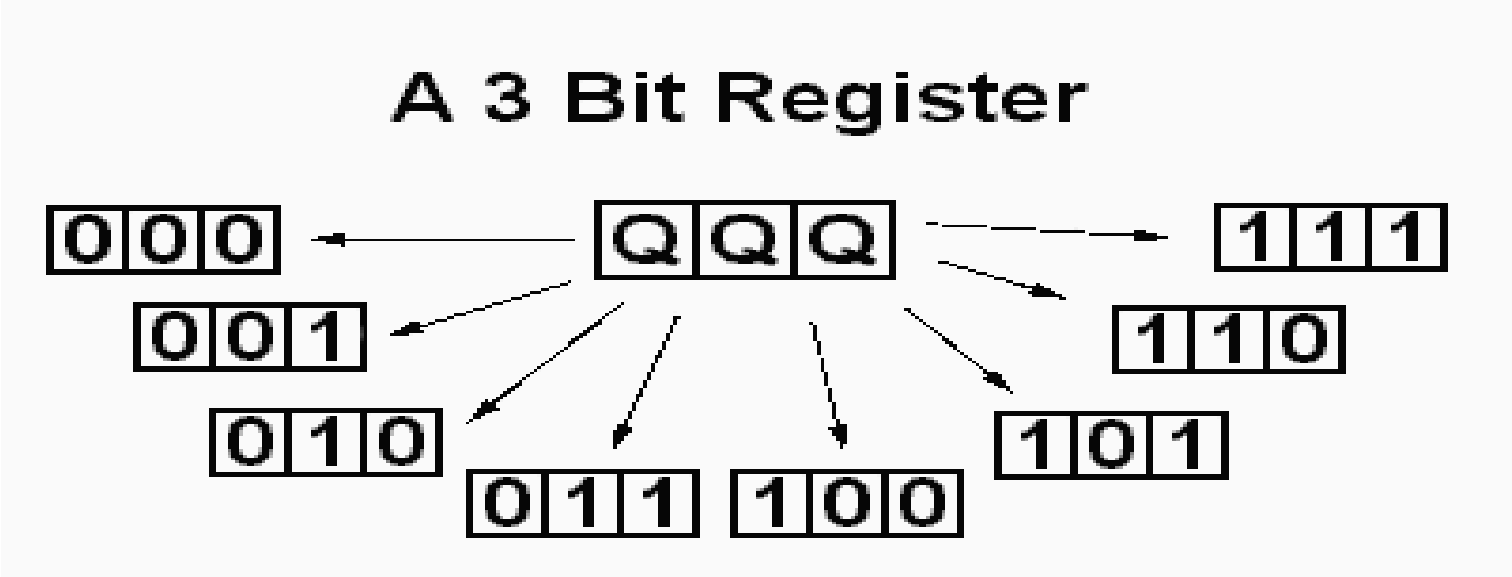
Conventional bit



Quantum bit


1
0
1

WHAT CAN QUANTUM COMPUTERS DO?



1024 qubits define simultaneously

2^{1024} quantum states.

INFINITELY BIG COMPLEXITY!

$$15 = 3 \times 5$$

$$144 = 2 \times 2 \times 2 \times 2 \times 3 \times 3$$

$$819,351,275,419,745,$$

$$314,593,354,912,643,$$

$$546,738,219,652,817$$

$$=? \times ?$$

Factoring a 1024-bit number

1) On a conventional computer

$$(1024)^2 \times 2^{1024/4} \quad \text{bit operations on average}$$

Assuming

2^{30} bit operations per second

2^{25} seconds per year we need

$$2^{221} \quad \text{years}$$

The age of the known universe is estimated to be less than

$$2^{35} \quad \text{years!}$$

(at most 15 billion years)

Factoring a 1024-bit number

2) On a quantum computer

$$(1024)^3 = 2^{30} \quad \text{bit operations}$$

Assuming

2^{30} bit operations per second

the time required is

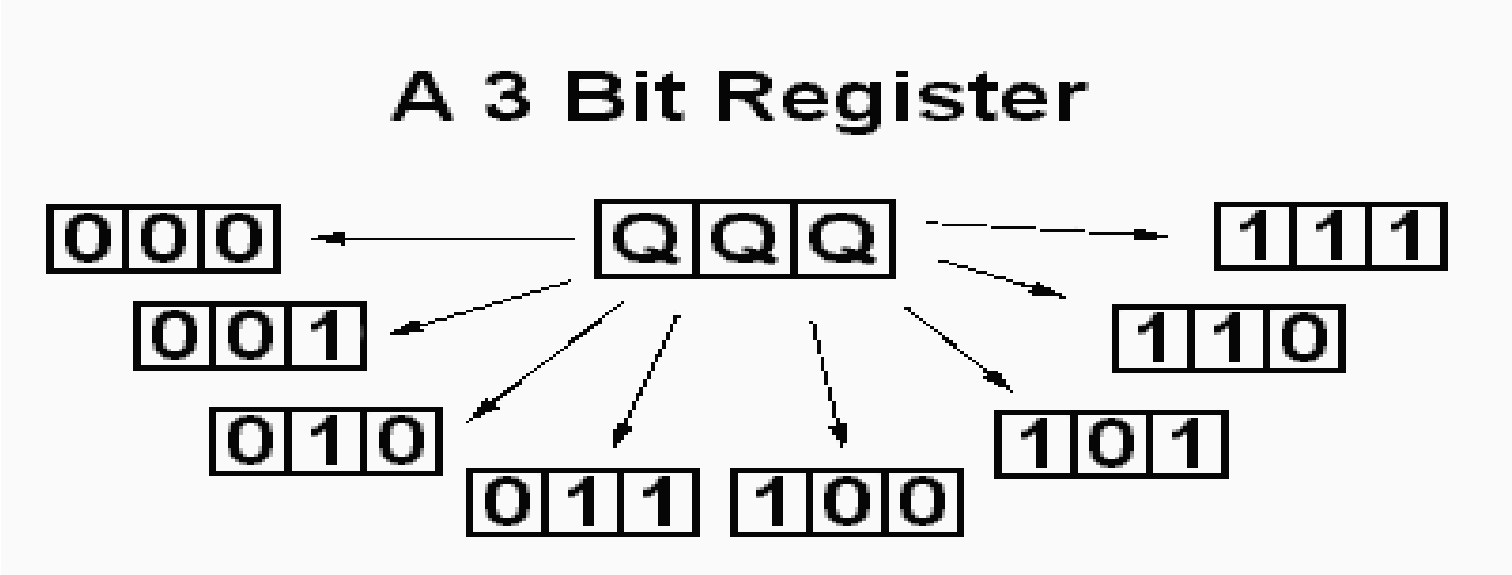
ONE SECOND!!

Distinguishing among 2^N quantum states

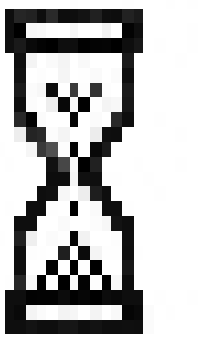
A computation that

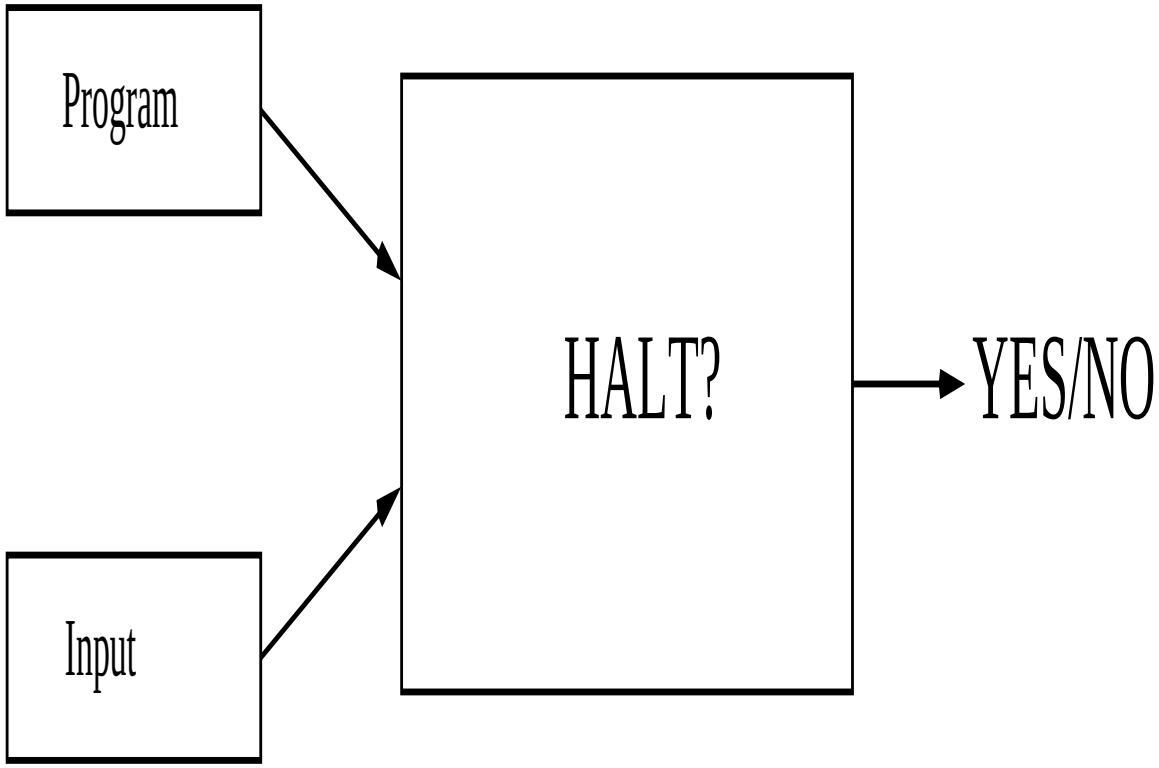
can be done on a quantum computer,

but cannot be done on any conventional computer.

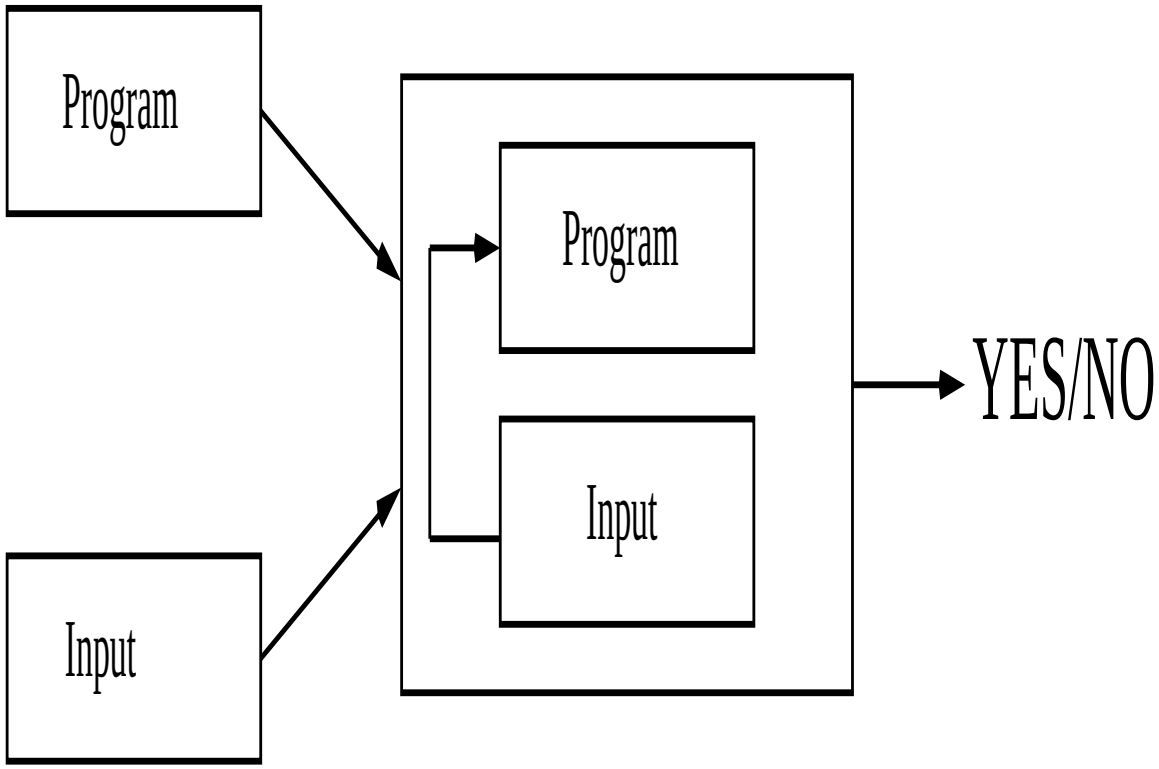


THE HALTING PROBLEM




Program
Input
HALT?
YES/NO

ACCELERATING MACHINE



$$1 + 1/2 + 1/4 + 1/8 + \ldots = 2.$$

INFINITELY SMALL TIME

INFINITELY BIG COMPLEXITY

UNIVERSALITY

"Any computation that can be performed by any physical computing device can be performed by any universal computer, as long as the latter has sufficient time and memory."

D. Hillis, *The Pattern on the Stone*, Basic Books, New York, 1998, pp. 63-64.

NO

FINITE COMPUTER

CAN BE

UNIVERSAL

COMPUTING SUBJECT TO THE LAWS OF NATURE

- Heisenberg's uncertainty principle of quantum physics

- Le Châtelier's principle of chemical systems under stress

- The homeostatic principle in biology which maintains the equilibrium necessary for the survival of organisms

An Example:
Mutually dependent variables

In a laboratory, N living organisms are under observation in a closed environment that they share.

The organisms depend on each other for survival.

It is required to perform a measurement on each organism.

CONVENTIONAL SOLUTION

- as each organism

  is measured separately

- equilibrium is disturbed:
  the remaining organisms may be altered irreparably, or may even die.

PARALLEL SOLUTION

Using a computer capable of

N simultaneous operations

- Measurements on all N organisms

  are performed at the same time,

  thus avoiding harming any of them.

While mother Nature's laws are immutable, they seem to favor parallel computation!

However,

- Any computer capable of
  fewer than N operations
  in one step fails to perform
  the computation successfully.

- Simulating the parallel solution
  on any computer capable of
  fewer than N operations per step
  is impossible,
  regardless of how much time is
  available to perform the simulation.

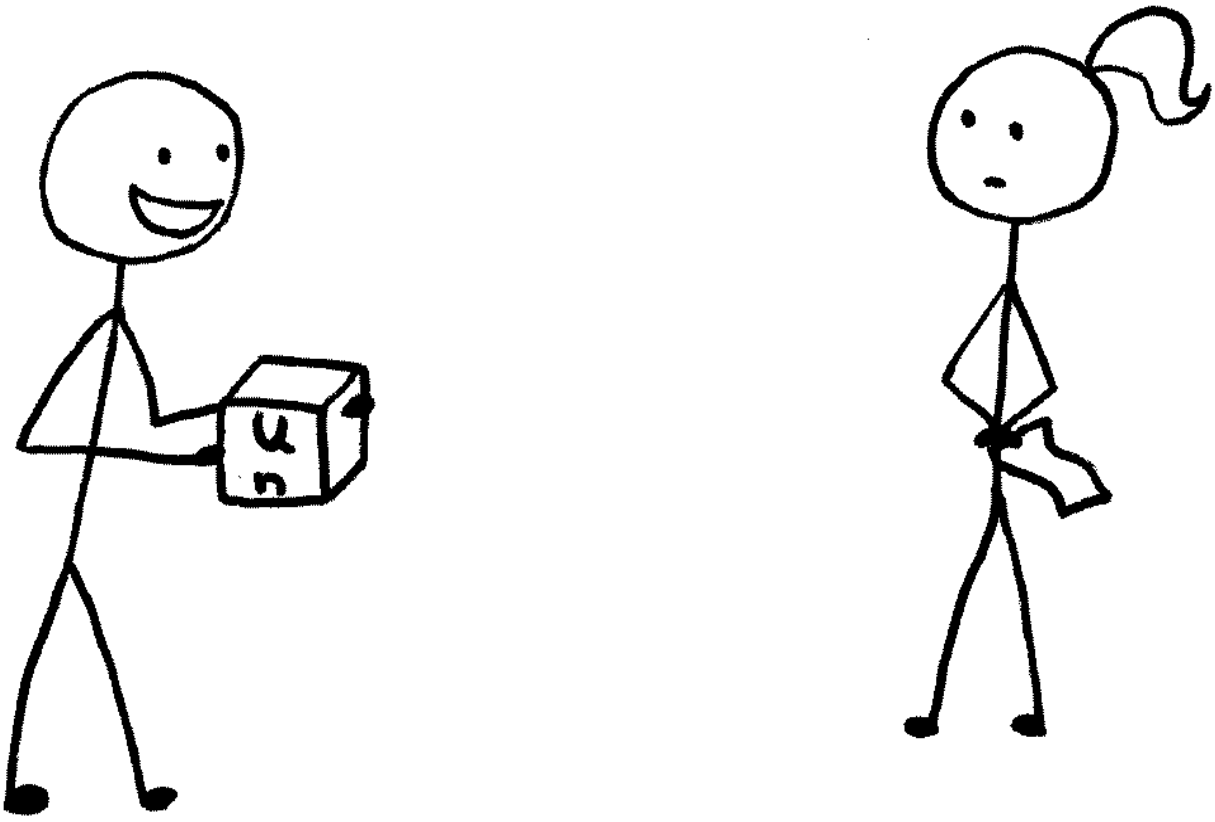
- These observations hold regardless of the

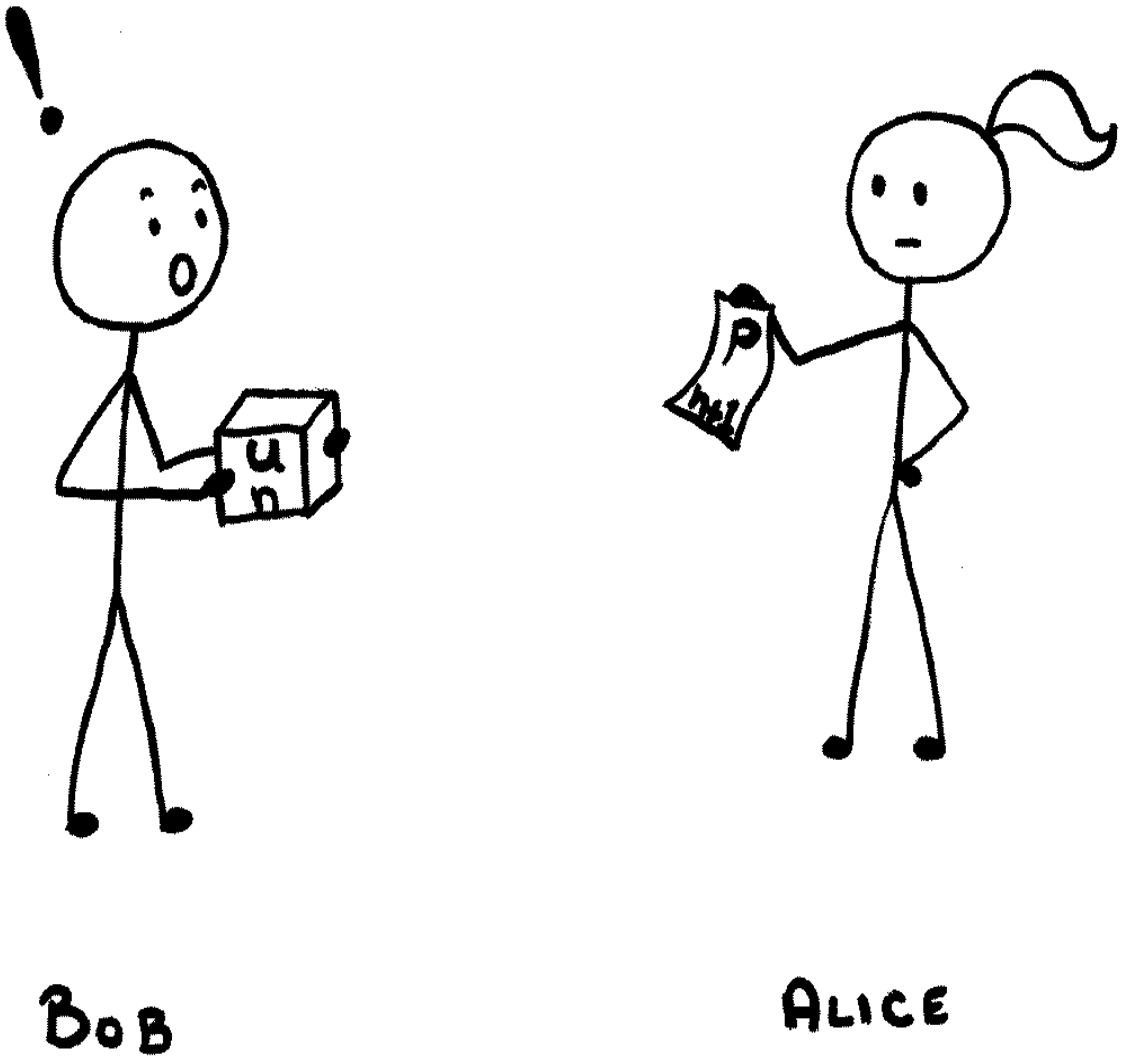
  technology

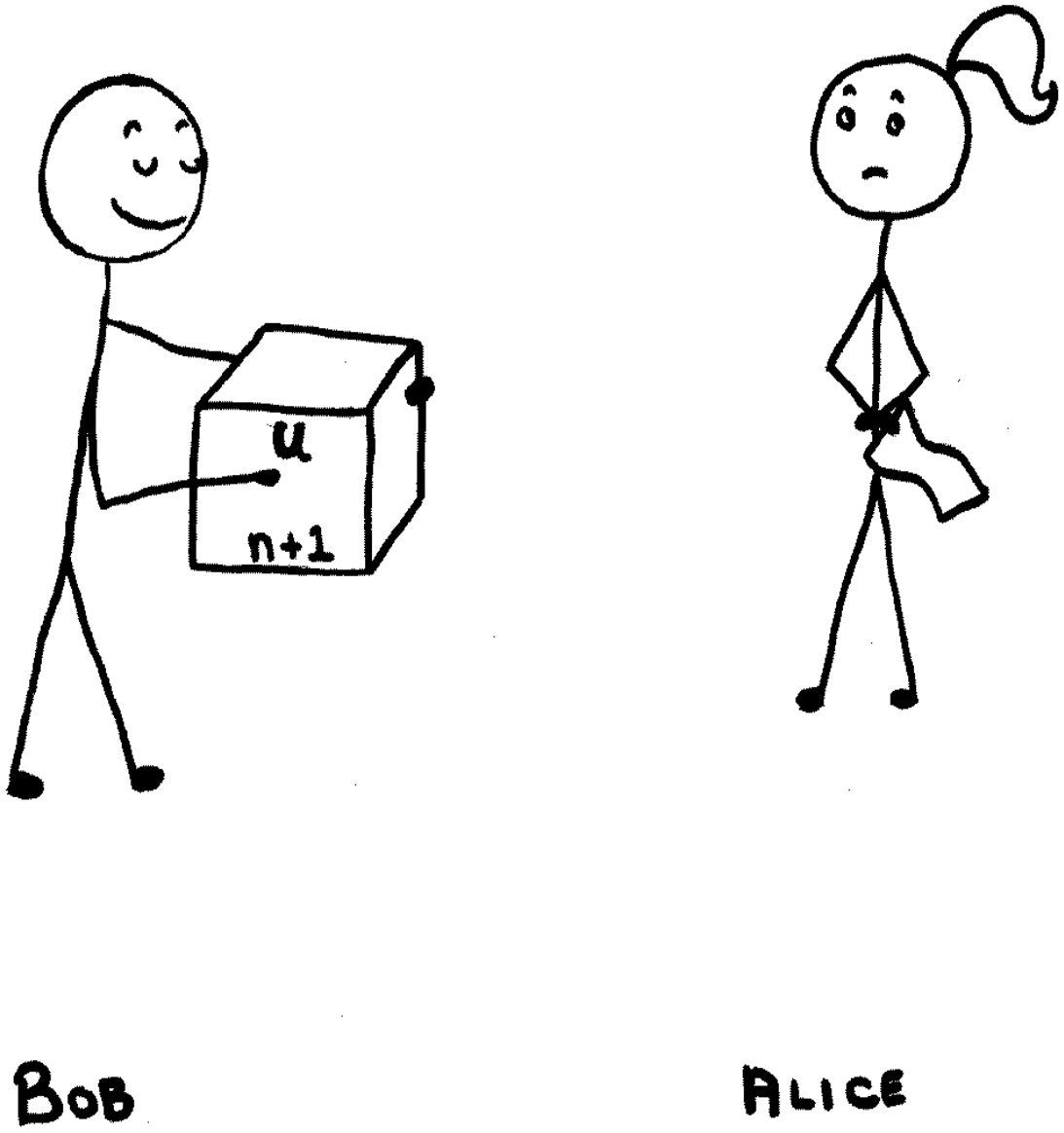
  used to build the computer.

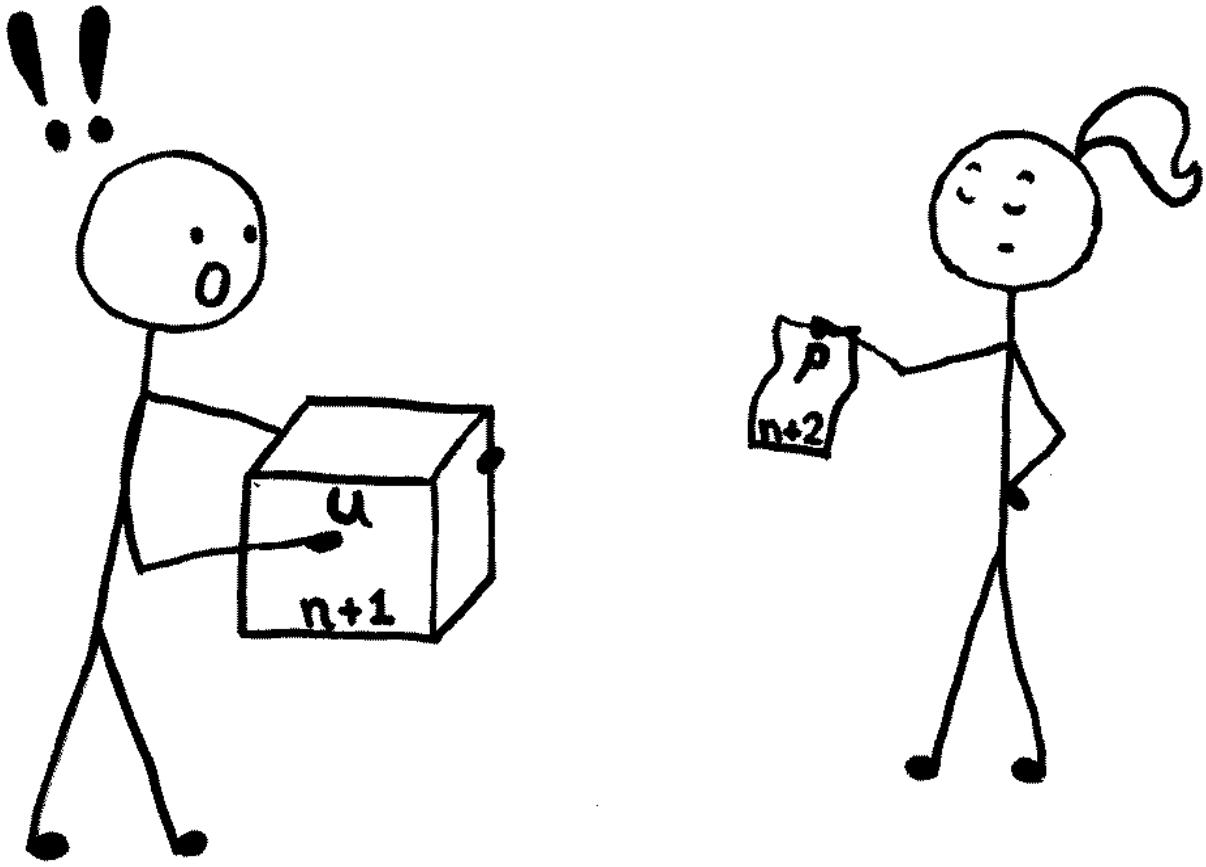
- In other words, the computer can be
  - mechanical,
  - electronic,
  - optical,
  - quantum,
  - chemical, or
  - biological,

  and the same limitations will be true.

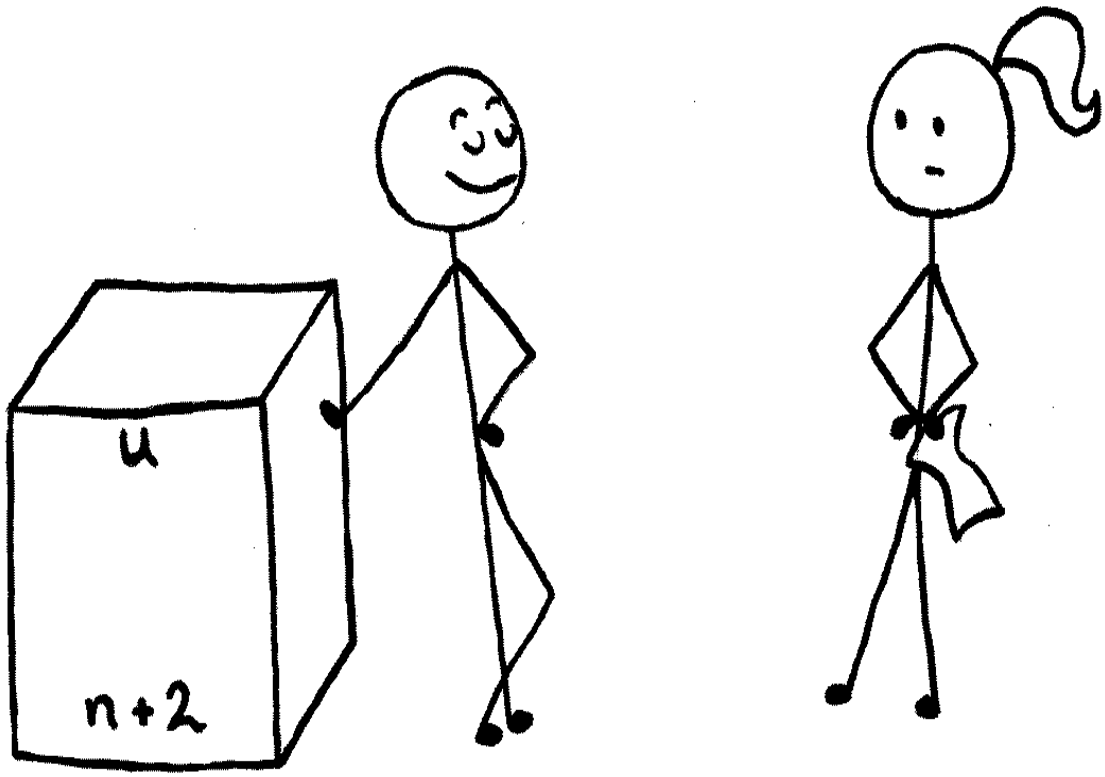

BOB
ALICE

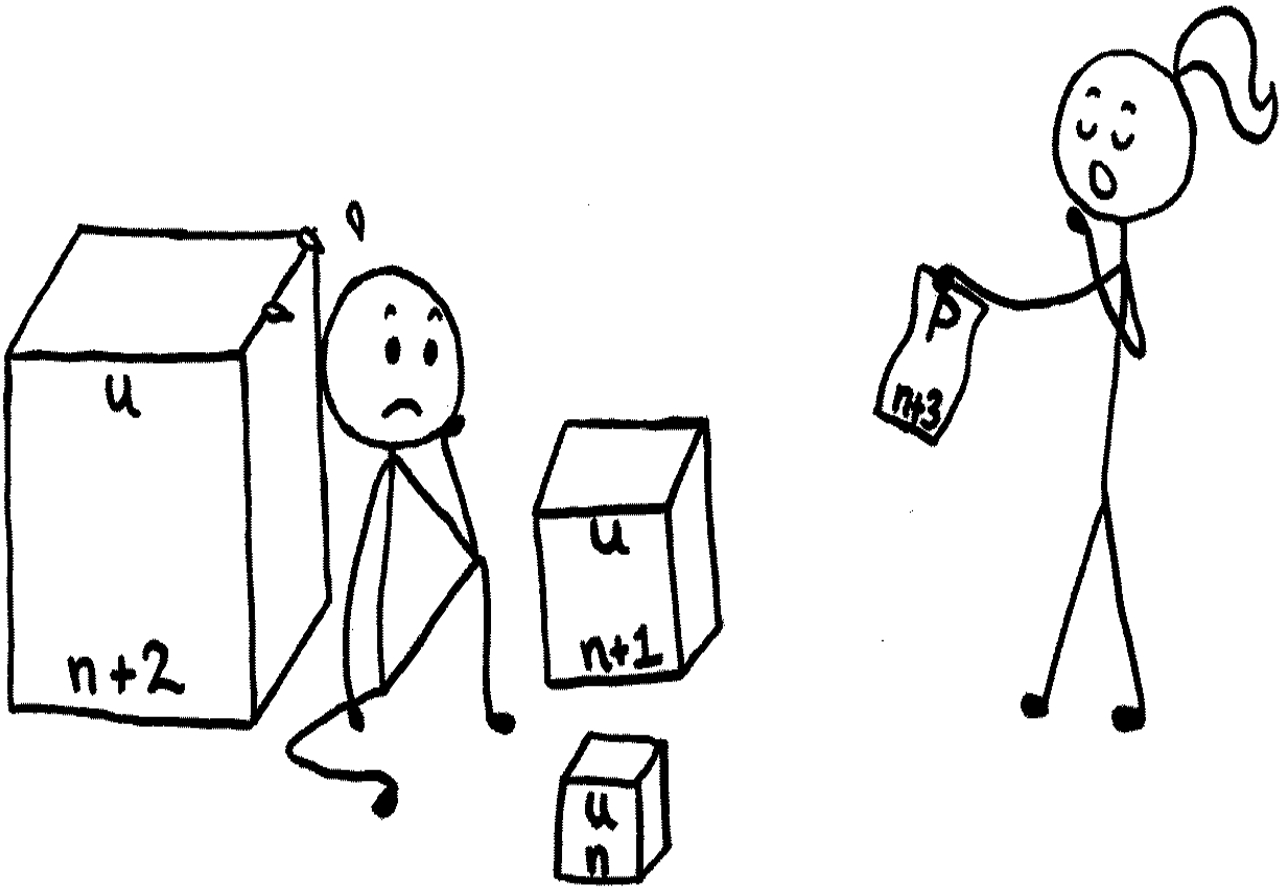

BOB
ALICE


u
n+1
BOB
ALICE



BOB ALICE


u
n+2
BOB
ALICE


u
n+2
u
n+1
u
n
p
n+3
BOB
ALICE

Will there ever be

a universal computer?

It appears that the only way

to conceive such a computer

is to allow it to have an

infinite number of processors

and thus be able of an

infinite number of operations per step.

INFINITELY BIG SPACE

INFINITELY BIG COMPLEXITY

Perhaps the only
universal computer possible is

THE UNIVERSE

ITSELF!