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Analogue Computation with Microwaves

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We describe preliminary studies of a microwave-based analogue computer with an alternative set of basic components from Shannon's general purpose analogue computer (GPAC). Motivated by continuous variable quantum computing (CVQC), we are developing a universal set of components for classical computing that is as close as possible to the CVQC set. We demonstrate the basic behaviour of the experimental devices in two frequency regimes, 10 MHz propagating in co-axial cable, and 20 GHz propagating in rectangular waveguides.

Keywords: Analog computation, experimental devices, GPAC, microwaves.

1 INTRODUCTION

Quantum computation was first proposed in the 1980s [1–3] and has been thoroughly studied for the past fifteen years. Although the theory of digital quantum computers is highly developed, only very small proof-of-concept devices have so far been built [4]. In contrast, analogue quantum computation has been developed only as far as proving that universal quantum computation can in principle be performed with information encoded in continuous variable properties of quantum systems [5]. Using position and momentum, or equivalent conjugate variables, a set of universal operations is known for various choices of data encoding. It has also been proved that without a non-linearity cubic or higher in the variables, such systems can be simulated efficiently with classical computers [6]. A review of quantum information processing using continuous variables by Braunstein and van Loock [7] concentrates on quantum communications tasks, for which such systems are highly suitable. Devices such as cryptographic key distribution systems [8, 9] and quantum repeaters [10] and hybrid systems in which the continuous variables form a "bus" to communicate between qubits [11] demonstrate the potential of CV quantum computation.

Part of the difficulty in developing larger CV quantum computing schemes arises because the underlying theory of classical analogue computation is also much less developed compared to digital computation [12]. Some universal differential equations are known, for example, [13], but whether these are useful or common or optimal is almost completely unexplored.

Our motivation with these studies of classical analogue computation using microwaves is to study a more quantum-compatible version of analogue computation, in which the addition of a nonlinear quantum operation would lift the system into the fully quantum domain. The paper is organised as follows: we provide an overview of the relevant theory in section 2. Our choice of experimental system is explained in section 3 along with details of the actual setup we are working with. Our first results are given in section 4 and our plans for further work in section 5.

2 THEORY OF CLASSICAL AND QUANTUM ANALOGUE COMPUTATION

2.1 Classical analogue computation

Computation using continuous variables (CV) to encode the data is over a century old but fell out of favour to its discrete cousin. Analogue computation is less well defined than digital computation in the sense that there is no single concise model, instead there are several different models available. One famous early model is that of the General Purpose Analogue Computer (GPAC), as given by Claude Shannon [14], which is a mathematical description of the Differential Analyser. The GPAC embodies what we consider to be the basics of CV computation – operations on polynomials of continuous variables. As this model is quite simple in its essence, efforts have been made to expand the model by adding more operations to the available set e.g., [15–17], or take other approaches, such as a CV version of Recursion Theory [18].

The GPAC consists of a set of black boxes, each having a number of inputs and outputs. The boxes each perform one of the following operations:

1. Constant – Producing a constant Real output.
2. Adder – Takes two inputs and outputs their sum.
3. Multiplier – Takes one input and outputs it multiplied by a constant.
4. Integrator – Takes two inputs and outputs the integral of one with respect to the other over the time of the computation.

The integrator is the most complicated of these operations, mathematically it does

$$a + \int_{y(t_0)}^{y(t)} X(t)\mathrm{dy(t)} = \mathrm{a} + \int_{\mathrm{t_0}}^{\mathrm{t}} \mathrm{X(t)}\frac{\mathrm{dy}}{\mathrm{dt}}\mathrm{dt}, \tag{1}$$

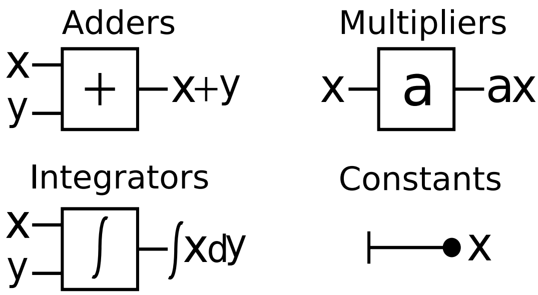

FIGURE 1
GPAC operation set.

where a is a constant. There are some simple rules for connecting the boxes: each output of a box goes to at most one input of another box; inputs cannot be interconnected, i.e., there is no "splitter" to allow the same input to go to more than one box; and similarly, outputs cannot be interconnected. These in effect say that each output of a box goes to either one box's input, or is an output of the computation.

This still allows more complex functions to be built. For example, if you take the both inputs of the integrator box to be the (separately generated) constant 1, then the output is $(t - t_0)$, since it integrates *over* real time t. This output $(t - t_0)$ can then be taken as the second input of another integrator to integrate the first input *with respect to* real time. If the first intput f is some output of another part of the circuit, and the second input is the constant one, the output is just $(t - t_0) \times f$, because it has just integrated *over* time, not with respect to time. It can be proved [19, 20] that the set of functions we can generate is the set of differentially algebraic functions.

This abstract notion of computation can be realised in many different physical systems, such as water in pipes or electronic circuits. Such systems were quite common in university science departments before digital computers became widespread.

2.2 Continuous variable quantum computation

Continuous variable quantum computation (CVQC) was first described by Lloyd and Braunstein [5]. They declare a computation to be enacting Hermitian Polynomials on the space of continuous variables, i.e., taking a continuous variable input x and making a polynomial $p(x)$ of it. The information itself is encoded in the eigenstates of some continuous-spectrum operator and computations on the information is embodied as physical manipulations which correspond to operations on the states. To encode our variables and to describe the physical modes involved we use the quadrature operators x and p which are orthogonal in the sense that $[\hat{x}, \hat{p}] = 1$ They act on the Hilbert Space $L2(\mathbb{R})$, the space of square-integrable functions over $\mathbb{R}$ (square-integrable is important as it corresponds to being normalisable). To perform a quantum computation, we create an initial state, evolve the state in a prescribed way and then perform a measurement from which we can extract the results.

Having described how to perform CVQC in general, we should say what it is for a machine to perform any CVQC – Universality. The notion of universality is important in all branches of computation theory as it simplifies discussions of theory and allows us to consider a physical computer to be powerful, or at least powerful enough. A computer can be called universal with respect to some class of systems if it can compute every function computable by systems in that class.

Universality in CV quantum systems has been solved and defined in simple mathematical terms [5]. A required computation can be achieved using an appropriate Hamiltonian on the states encoding the continuous variables, and any Hamiltonian can be written as a Hermitian polynomial in the position and momentum operators $\hat{x}$ and $\hat{p}$. Any polynomial in $\hat{x}$ and $\hat{p}$ can be generated given a certain set of available operators [5]:

> Simple linear operations on continuous variables, together with a non-linear operation and any interaction suffices to enact to an arbitrary degree of accuracy arbitrary Hermitian polynomials of the set of continuous variables.

In our terms, the simple linear operations are $\{\pm\hat{x}, \pm\hat{p}\}$, a non-linear operation might be the Kerr Hamiltonian $H_{Kerr} = (\hat{x}^2 + \hat{p}^2)^2$ and an interaction is to couple modes together – e.g. a C-NOT gate. If we have all of these available to us in a quantum computer then we can declare our computer to be Universal.

3 CHOICE OF EXPERIMENTAL SYSTEM

We are looking for a system that will ultimately be capable of Universal CV quantum computation. One simple choice is to use the cavity field in cavity QED systems, and one of the cleanest of these is the micromaser, or one-atom maser. A maser is like a laser but in the microwave frequency regime rather than visible (or near visible) light. However, the micromaser is a very demanding experimental setup [21, 22], and we wish to work with something simpler to start with. Thus we will work with microwaves travelling in waveguides, cables or free space, rather than trapped in a cavity.

Microwaves cover a large frequency range from radio waves to far infrared. The microwave synthesizer we have available can generate output from 10 MHz to around 20 GHz. At the low end, the microwaves behave much like ordinary electronics with alternating current (AC). They propagate through co-axial cable and interact with electronic components designed for AC as expected. At the high end, the microwaves propagate through small rectangular waveguides and in free space. The waveguides ensure the microwaves maintain a specific polarisation, giving them an extra property compared to the low frequency signals in co-axial cable. We are interested in both low and high frequency regimes.

3.1 Microwaves in the low frequency regime

Since low frequency microwaves behave like AC signals in suitably designed circuits, we can replicate the known designs for electronic analogue computers in this regime. Since there is no polarisation, we must use amplitude or power (intensity) rather than phase or frequency to encode in. For example, we can produce a constant output of a particular size by simply adjusting the signal strength output by the synthesizer. We can multiply by a constant less than one by attenuating the signal using a rheostat. For constants greater than one, we can amplify the signal first then adjust using attenuation. An integrator in electronic circuits is usually constructed with an op-amp. Some precalculation is required to convert the differential equation into the right form. [23, 24].

The standard op-amp method uses Laplace transforms to convert the differential equation into an algebraic equation that can be solved to find the solution as an output of the circuit. We are interested in using Fourier transforms instead of Laplace transforms, because this is a natural operation once we get to higher frequencies where polarisation is possible. The Laplace and Fourier transforms are very closely related, for our purposes there are simply factors of $2\pi i$ in different places. So for the low frequency regime, we will build the standard op-amp based integrator and analyse its performance in our choice of experimental apparatus.

3.2 Microwaves in the high frequency regime

In the high frequency regime we need to use waveguides for connecting our circuit components. Producing a particular constant signal strength is still just a matter of setting the output power of the synthesizer, and multiplication by a constant less than one can be done with an attenuator designed to work with waveguides. Waveguides are designed to work with a single polarisation only (corresponding to the rectangular shape) so this can be used to keep the signal coherent, but we can't actually change the polarisation during the computation. For variable polarisation we will propagate the microwaves in free space between the components.

3.3 Experimental setup

For our initial studies, we are using a borrowed microwave synthesizer (with more capabilities than we actually need). It is the bottom box in Fig. 2.

It can produce microwaves at frequencies between 10 MHz (radio wave frequency) up to 20 GHz.

There are various ways to measure the microwaves that we are testing and comparing.

1. A Gigatronics Universal Power Meter (GUPM) can measure the power (root mean square of the sinusoidal signal intensity) ranging

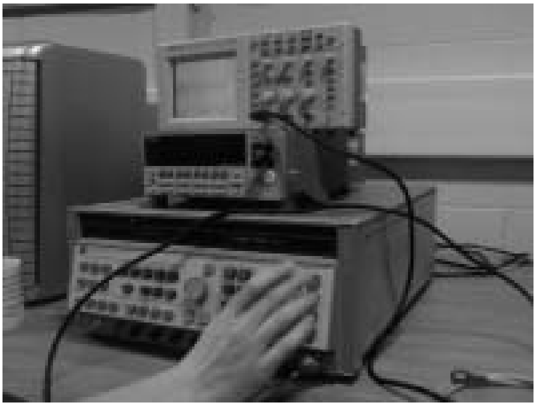

FIGURE 2
Microwave synthesizer on the bench, along with a signal generator providing a trigger, and a Tekronics oscilloscope acting as a detector.

from −60 dBm to 20 dBm. The relationship between dBm and Watts is

$$A = 10log_{10}(1000P), \quad (2)$$

where A is in dBm and P is in Watts. The GUPM conveniently can also display the power in milliWatts at the push of a button. While dBm are convenient for practical operation of the devices, the actual encoding will be in the amplitude (power/intensity is the square of the amplitude of the wave). The precision of the reading is around 2 decimal places (on the dBm scale), so it scales with the signal size. This will have important consequences for computing small differences, for example.

2. A Hameg Oscilloscope (HO), the top box in Fig. 3, can display the waveform for power from −70 dBm to 20dBm. At 10 MHz (the lowest operating frequency of the microwave synthesizer) the maximum power without signal distortion is around 15 dBm. A Tektronics Oscilloscope (TO) was also tested, but it has a lower update rate than the HO, because it has an LCD screen rather than a CRT.
3. At low frequencies, a multimeter can measure the voltage between the central pin and outer sheath of the co-axial cable. This also corresponds to the power of the microwave signal, since it is the root mean square (RMS) average voltage that is measured.

These devices all have SMA (SubMiniature version A) connectors or adapters, and lengths of thin co-axial cable (in the low frequency regime) or waveguide (high frequency regime) using SMA-to-waveguide adapters are used to construct the circuits. At low frequencies, co-axial t-junctions can be

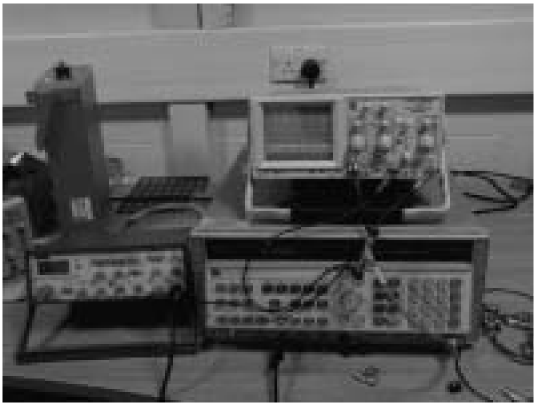

FIGURE 3
Microwave synthesizer with Hameg Oscilloscope (used with signal generator to provide a trigger).

used to split or join the signals. At high frequencies, more specialised waveguide devices are required for splitting and recombining the microwaves, all of which maintain the polarisation.

4 PRELIMINARY RESULTS

4.1 Identity operation, or production of a constant output

Our first experiment is to produce a constant output and measure it, to find out the range and precision available to us with this synthesizer. This also corresponds to the simplest of the basic GPAC operations, generation of a constant output. Taking the output signal straight from the synthesizer to the detector is the equivalent of performing the identity operation. This is usually regarded as trivial by theorists, but experimentally it can be one of the hardest operations to do accurately, because all parts of the circuit will have some losses and distortions. For example, we observe the output power is slightly less than the microwave synthesizer says it is producing, by about 0.1% in the middle of the power range. When used in within the specified frequency ranges, both co-axial cable and waveguides have very low losses, so the identity operation is, in fact, straightforward in our chosen systems.

At low frequencies: we have produced a good quality signal at 10 MHz that can be varied from −60 dBm to 15 dBm without distortion, and measured accurately with either a GUPM or HO. The HO can measure down to −70 dBm, which is equivalent to nanoWatts. The attenuation in short (20 cm to 50 cm) lengths of co-axial cable is minimal, and the signal strength can be measured to about one part in a thousand with the GUPM. The HO is less accurate because the signal must be read off the oscilloscope trace by eye, using the gradations marked on the screen. Accuracy is thus around two significant digits, or 1%.

At higher frequencies: we have produced good quality signal that can also be varied from −60 dBm to 15 dBm and measured with either the GUPM or HO. The losses are greater, around 5% in the middle of the range, mainly due at the connectors when converting between co-axial and waveguide.

4.2 Multiplication by a constant

To perform multiplication by a constant, another of the four basic GPAC operations, we can deliberately attenuate our signal by a chosen amount. This is equivalent to multiplying by a constant that is less than one. We can achieve fine control over the amount of attenuation. If we need to multiply by a constant greater than one, we can first amplify by a set amount (not so easy to control over a wide range) then attenuate to get the exact value required. An attenuation device is shown in Fig. 4, on the top of the stack.

This device is very accurate, but only works in the high frequency regime. For attenuation in the low frequency (MHz) regime, we can use a rheostat (variable resistor) and calibrate it.

FIGURE 4
Microwave synthesizer with Hameg Oscilloscope to the left, Gigatronics Universal power meter on top, and attenuator on top of that (not connected).

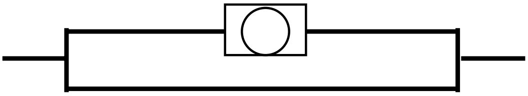

FIGURE 5
Circuit diagram for a simple interferometer. It shows co-axial cable or waveguide (thick black lines) consisting of an input, a t-junction, two arms, another t-junction and and output. The upper arm has a variable phase shifter in it.

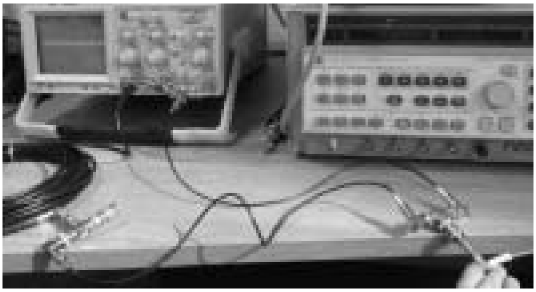

FIGURE 6
Interferometer in low frequency range using co-axial cable. The coil on the left is a π phase shifter, being exactly half a wavelength long (15m). The extra t-junction between the coil and the rest of the arm does not affect the operation. The active t-junction is attached to the HO, which has been turned to its highest sensitivity to see how close to extinction the signal is.

4.3 Interferometry

To investigate the coherence properties of our signals, the next device we tested was a simple interferometer, see Fig. 5.

If the two arms are exactly the same length, the signals arrive along both arms perfectly in phase and recombine to form the same signal as the input. If the phase of the signal in one arm is altered then the phases at the second t-junction will in general not match, and the output will be less than the input signal.

At low frequencies: Fig. 6 shows the interferometer for the low frequency regime. The first t-junction is on the right, connected to the output from the synthesizer (compare Fig. 4). Two short lengths of co-axial cable (20 cm) take the outputs from the t-junction to the second t-junction. After verifying that with the two arms the same length we obtained the same output signal as we input, we added an extra length of co-axial cable, equivalent to half a wavelength, into the left hand arm. This provides a π phase shift, causing the signals to be almost exactly out of phase when they recombine at the second t-junction. As expected, the output signal dropped close to zero, as the out of phase signals from each arm now cancel each other. The trace on the oscilloscope still shows a small signal because we have turned up the sensitivity to find out how close to full extinction the output is.

At higher frequencies: the splitters we currently have available for the waveguide are not ideal, because they have quite high losses. But this does not matter for the purpose of investigating the phase behaviour of the signal. Figure 7 shows the arrangement we were able to construct. The splitters have a straight arm and a second output curving off near one end. The two straight arms have been connected via a variable phase shifter, and the curved arms have a short length of flexible waveguide attached in a U-shape. The input

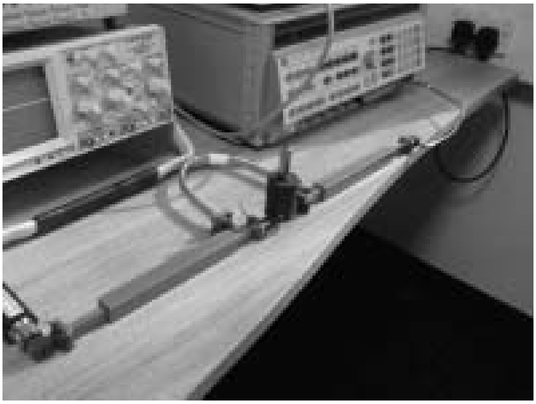

FIGURE 7
Interferometer in high frequency range. The device between the two splitters is a variable phase shifter.

FIGURE 8
Interferometer in high frequency range. The device between the two splitters is a variable phase shifter. An extra length of waveguide has been inserted into the other arm compared to Fig. 7.

from the synthesizer is connected on the right, and the output is connected on the left to the GUPM.

In this arrangement we can't exactly balance the arms of the interferometer since they are not symmetric. But by adjusting the phase shifter we can measure the maximum and minimum power transmitted through the interferometer. For 7 dBm input, we obtain a maximum of 2.6 dBm output (due to the lossy components) and a minimum of -13 dBm, which means the power has dropped to just below a tenth of the maximum, after converting from dBm.

We then added an extra short length of flexible waveguide into the curved arm, see Fig. 7. This changed the readings to a maximum output of 1.4 dBm and a minimum of -8.6 dBm. The drop in power is now less, most likely due to the extra losses and lack of symmetry in the arrangement of the interferometer.

5 FUTURE PLANS

Having shown that desktop systems are easily constructed and simple gates and devices can be built, we will expand our system to include all necessary components for classical analogue computation. We have shown that we have available a wide range of input signal strengths for encoding our data, and that we can measure the output signal in several useful ways to a useful precision. We have tested the phase coherence of our microwave signals using interferometry, and demonstrated good coherence over desktop distances at both low and high frequencies.

We will compare the functionality of both the GPAC and FT component sets, and develop benchmark computations that will facilitate comparisons of future systems. We will compare the operation of high frequency microwaves in free space with their behaviour in waveguides, which will allow us to make use of the polarisation as well as the phase and amplitude of the waves, bringing us a step closer to the requirements of a fully quantum CV computer.

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