Appendix

Traditional binary decision rule

Here we will show why the traditional binary decision rule fails to capture the operational flexibility of hydroelectric units. Let $(w_{h,t}, \boldsymbol{W}_{h,t}))^\top$ be the coefficients of the linear function associated with the linear decision rule for $p_{h,t}$. Combining the water discharge bound constraint for $q_{h,t}$ and its linear decision rule, $(w_{h,t}, \boldsymbol{W}_{h,t})^\top$ must belong to the following set:

$$\mathcal{W}_{h,t} = \Big\{ (w_{h,t}, \boldsymbol{W}_{h,t})^\top : p_{h,t} = w_{h,t} + \boldsymbol{W}_{h,t}^\top \boldsymbol{u}_{[t]},$$
$$q_{i,t} \in \{0\} \cup \left\{Q_i^{\min}, Q_i^{\max}\right\} \Big\}, h \in H, t = 1, \ldots, T \tag{1}$$

We will show that $y_{h,t} = 0$ if and only if when $w_{h,t} = 0$ and $\boldsymbol{W}_{h,t} = \boldsymbol{0}$. When $w_{h,t} = 0$ and $\boldsymbol{W}_{h,t} = \boldsymbol{0}$, we have $q_{h,t} = 0$ and thus $y_{h,t} = 0$ according to constraint (**??**). Then we consider the case when $y_{h,t} = 0$. Suppose that there exist some $\tilde{w}_{h,t} \in \mathbb{R}$ and $\tilde{\boldsymbol{W}}_{h,t} \in \mathbb{R}^{(|H_p|+1)\times t}$ satisfying $\left(\tilde{w}_{h,t}, \tilde{\boldsymbol{W}}_{h,t}\right)^\top \neq (0, \boldsymbol{0})^\top$. Let $\tilde{\boldsymbol{u}}_{[t]}$ be an uncertain realization such that $q_{h,t} = \tilde{w}_{h,t} + \tilde{\boldsymbol{W}}_{h,t}\tilde{\boldsymbol{u}}_{[t]} = 0$. Since $\boldsymbol{u}$ is continuous over the polyhedral uncertainty set $\mathcal{U}$, there must exists some $\boldsymbol{u}'_{[t]} \in N_\epsilon\left(\tilde{\boldsymbol{u}}_{[t]}\right)$, such that $q_{h,t} = \tilde{w}_{h,t} + \tilde{\boldsymbol{W}}_{h,t}\boldsymbol{u}'_{[t]} \neq 0$, where $N_\epsilon()$ denote $\epsilon-$ neighborhood and ϵ is a small positive constant. Since affine function is continuous, we can let ϵ be sufficiently small such that $p_{h,t} = \tilde{w}_{h,t} + \tilde{\boldsymbol{W}}_{h,t}\boldsymbol{u}'_{[t]} \in \left(0, Q_h^{\min}\right)$. According to (1), this contradicts the fact that $\left(\tilde{w}_{h,t}, \tilde{\boldsymbol{W}}_{h,t}\right)^\top$ is a feasible decision rule for $q_{h,t}$. Hence, $q_{h,t} = 0$ if and only if when $w_{h,t} = 0$ and $\boldsymbol{W}_{i,t} = \boldsymbol{0}$. In this case, we will have either $q_{h,t} = 0$ or $Q_h^{\min} \leq q_{h,t} \leq Q_h^{\max}$ for $\forall \boldsymbol{u}_t \in \mathcal{U}$. This justifies that linear decision rule and binary decision rule fail to capture the flexible adjustment of start-up and shut-down decisions of hydroelectric units.

Data of PGE hybrid power system

Table 1: The parameters of the reservoirs

Reservoir	$V_{h_p}^{ini}$ (10^3 m^3)	$V_{h_p}^{\min}/V_{h_p}^{\max}$ (10^3 m^3)	Water time delay (h)
Round Butte	130,670	130,667/130,672	–
Pelton	1,160	1,158/1,163	1
Rereg	1,552	1,550/1,554	1

In water-to-power conversion constraints, we divide reservoir volume interval $\left[V_{h_p}^{\min}, V_{h_p}^{\max}\right]$ into three sub-intervals with equal range, corresponding to “low volume”, “normal volume” and “high volume”. The third column of the following table gives the water-to-power conversion functions

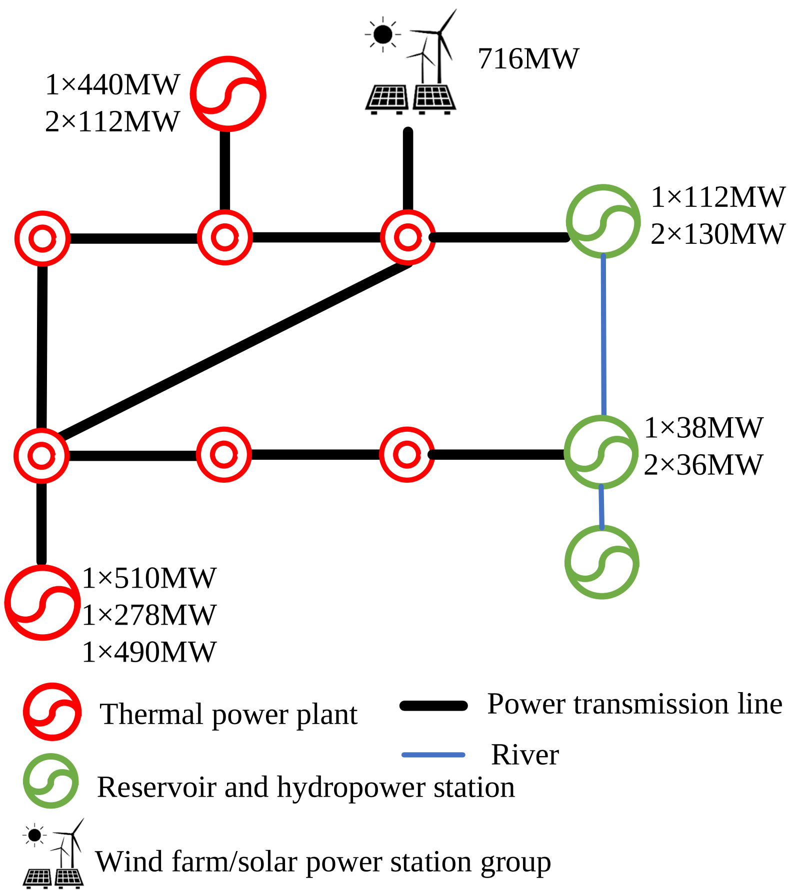


Figure 1: Schematic diagram of PGE power system

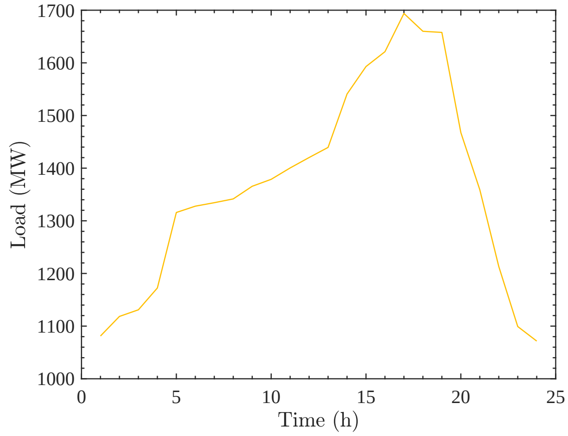


Figure 2: The electricity load of the hybrid power system

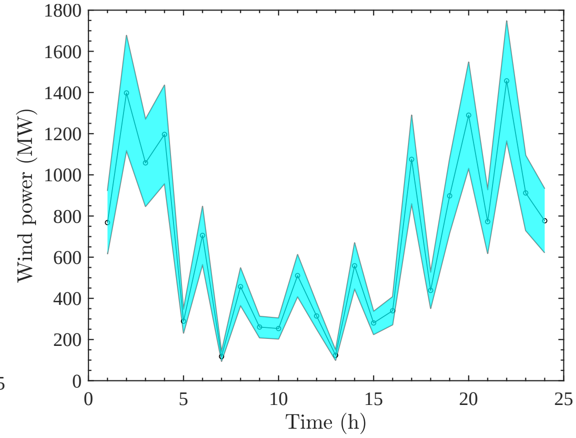
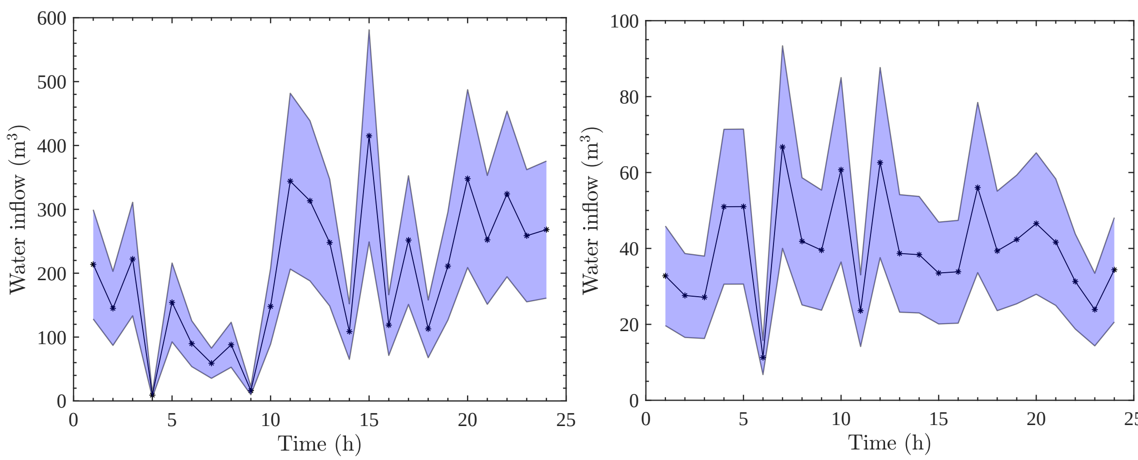


Figure 3: The uncertainty set of wind power



(a) Round Butte

(b) Pelton

Figure 4: The uncertainty set of natural water inflow

under normal volume. The generation efficiency will increase by 5% under high volume and decrease by 8% under low volume.

Table 2: The parameters of hydroelectric units

Units	$p_h^{\min}/p_h^{\max}$ (MW)	Water-to-power function (m^3, MWh)	Location
1	1/112	-4.55E-05x^3+7.62E-03$x^2+0.76x-7.25$	Round Butte
2	1/130	-4.10E-05x^3+7.62E-03$x^2+0.71x-95.98$	Round Butte
3	1/130	-4.55E-05x^3+1.02E-02$x^2+0.54x+15.98$	Round Butte
4	12/38	-9.11E-05x^3+1.27E-02$x^2-0.34x+374.56$	Pelton
5	12/36	-9.11E-05x^3+1.27E-02$x^2-0.34x+374.56$	Pelton
6	12/36	-9.11E-05x^3+1.27E-02$x^2-0.34x+374.56$	Pelton

The following table gives the parameters of thermal power units:

Table 3: The parameters of thermal power units

Units	Min/Max power output (MWh)	Min on/off time (h)	Max ramping rate (MW/h)	Cost ($/MWh)
1	6/112	1/1	70	30
2	6/112	1/1	70	30
3	140/278	48/3	195	20
4	1/440	1/1	290	20
5	1/490	1/1	360	20
6	5/510	2/1	365	20

Data of a provincial hybrid power system in southwest China

Table 4: The parameters of reservoirs

Reservoir	$V_{h_p}^{ini}$ (10^6 m^3)	$V_{h_p}^{\min}/V_{h_p}^{\max}$ (10^6 m^3)	$QS_{h_p}^{\min}/QS_{h_p}^{\max}$ (m^3/s)	Water time delay (h)
#1	2.86	2.01/4.23	724.9	–
#2	3.43	2.56/4.97	790.3	3/5/7
#3	8.01	5.58/8.93	1853.1	1/2/3
#4	5.78	3.41/6.36	1807.9	1/2/3
#5	13.24	10.60/17.24	3181.2	2/3/4
#6	20.99	11.09/34.12	4608.4	1/2/3
#7	17.21	8.77/20.53	5276.1	2/3/4
#8	12.32	9.05/15.46	5994.5	1/2/3

We assume that each uncertain parameter fluctuates over the range between 80% and 120% of its nominal value.

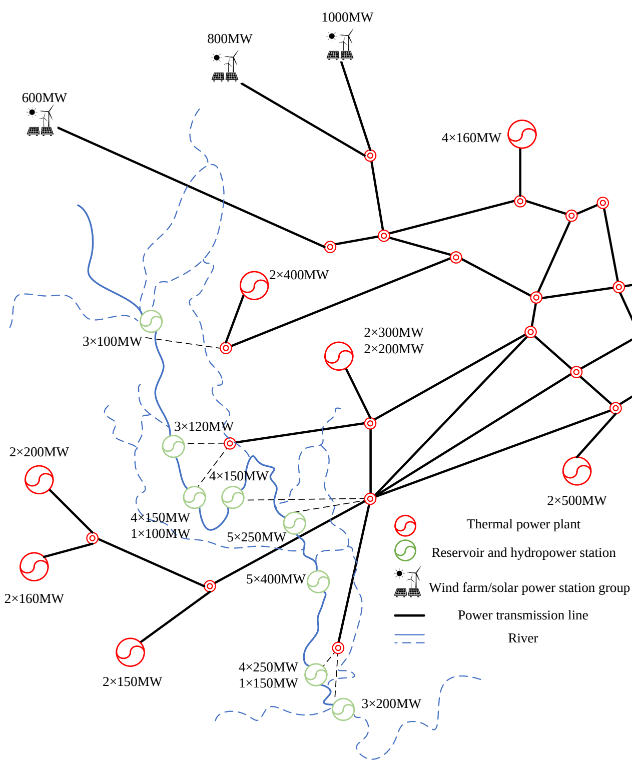


Figure 5: Schematic diagram of the provincial power system in southwest China

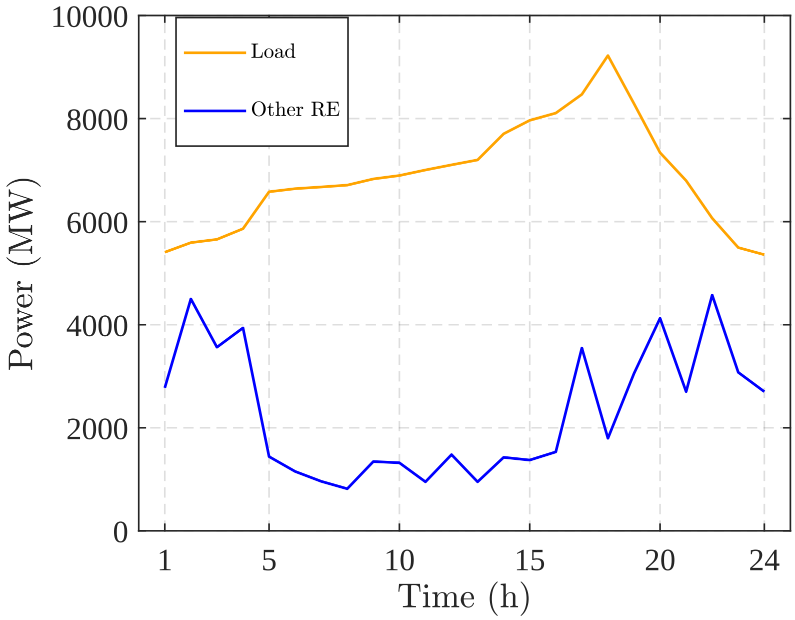


Figure 6: Electricity load and other renewable energy output except hydroelectric generation

The hydroelectric units with the same installed capacity share the same water-to-power conversion function.

Table 5: The nominal values of natural water inflow (m^3/s)

Time \ Reservoir	#1	#2	#3	#4	#5	#6	#7	#8
1	53.49	10.79	30.73	32.96	59.12	116	112.84	117.76
2	32.77	6.92	22.56	19.1	45.86	85.97	86.54	83
3	66.99	14.89	43.89	50.13	106.61	220.12	237.52	211.69
4	168.93	34.3	82.54	124.05	171.72	372.16	311.12	366.58
5	476.73	69.44	177.67	273	334.12	664.16	415.61	590.82
6	642.35	86.65	187.25	270.72	396.24	718.64	388.85	639.88
7	478.48	73.1	127.37	192.17	265.82	486.45	271.14	440.16
8	355.19	56.09	96.31	119.68	202.58	314.01	187.14	325.81
9	241.02	39.19	75.12	90.29	145.13	285.92	185	234.92
10	125.65	23.17	49.14	58.23	103.77	209.05	159.61	184.18
11	52.57	10.63	28.07	29.2	54.11	100.26	84.28	99.29
12	36.43	7.17	20.01	20.42	37.43	73.44	64.19	74.03
13	33.64	6.8	19.13	20.55	37.08	72.65	70.64	73.33
14	32.43	6.9	22.27	18.92	45.39	85	85.93	82.39
15	43.29	9.63	28.32	32.45	69.11	142.34	153.59	136.76
16	200.83	40.72	98.29	147.5	204.28	444.49	369.52	436.46
17	559.62	81.48	208.7	319.17	393.65	780.91	489.4	697.93
18	842.94	113.49	245.38	355.53	518.95	940.43	510.35	837.21
19	552.52	84.79	147.48	222.7	309.09	563.47	311.82	509.22
20	413.26	64.9	111.48	138.53	234.67	362.93	216.67	377.27
21	206.02	33.73	64.56	77.28	124.21	245.15	158.42	201.21
22	116.03	21.43	45.4	53.95	96.08	193.31	147.49	170.19
23	49.54	9.73	27.2	27.9	51.03	100.08	87.47	100.48
24	45.72	9.16	24.27	25.47	46.84	86.94	73.12	86.46

Table 6: The parameters of hydroelectric units

# of units	$p_h^{\min}/p_h^{\max}$ (MW)	Water-to-power function (m^3, MWh)
4	1/100	-4.61E-05x^3 7.31E-03x^2 0.75E-01x -7.25
3	1/120	-5.61E-05x^3 9.14E-03x^2 0.68E-01x -6.25
9	1/150	-7.79E-05x^3 1.49E-02x^2 0.23x 4.05
3	1/200	-1.51E-05x^3 -2.23E-04x^2 1.32x -18.1
9	1/250	-4.48E-05x^3 6.61E-03x^2 0.96x -8.85
5	1/400	-2.12E-05x^3 1.30E-03x^2 1.20x -1.63

Table 7: The parameters of thermal power units

# of units	Min/Max power output (MWh)	Min on/off time (h)	Max ramping rate (MW/h)	Cost ($/MWh)
2	10/150	1/1	80	25
6	10/160	1/1	90	25
4	30/200	2/1	120	24
4	40/300	2/2	140	22
2	45/400	2/2	200	20
2	60/500	2/2	240	20