Discrete Structure Assignment 1 (Section 02)

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1.(a) $x^2 + x - 6 = 0$
$(x-2)(x+3) = 0$
$x = -3,\ x = 2$

$A = \{-3, 2\}$
$\therefore A' = \{x \in \mathbb{Z} \mid x \neq -3, x \neq 2\}$

(b) $(B - A) \cap C = \{1,4\} \cap \{2,3,4\}$
$= \{4\}$.

(c) $B \cap C = \{2,4\}$
$n = 2$

$|P(B \cap C)| = 2^n = 2^2 = 4$

2. $(((P \cup Q) \cap R)' \cup Q')' = \cancel{((P \cup Q) \cap R) \cap Q}$ ~~DeMorgan~~
$= (((P \cup Q) \cap R)')' \cap (Q')'$ (DeMorgan's Law)
$= ((P \cup Q) \cap R) \cap Q$ (Double Complement Law)
$= Q \cap ((P \cup Q) \cap R)$ (Commutative Law)
$= (Q \cap (P \cup Q)) \cap R$ (Associative Law)
$= Q \cap R$ (Absorption Law)

3(a) $|A|$ = student in art class
$|S|$ = student in science class
$|A \cup S| = |A| + |S| - |A \cap S|$
$= 35 + 57 - 12$
$= 80$ $\therefore$ 80 students

(b) $|A \cup S| = |A| + |S| + |A \cap S|$
$= 35 + 57$
$= 92$

$\therefore$ 92 students

4.(a) p: you try hard
q: you have a talent
r: you will get rich

$(p \wedge q) \rightarrow r$

(b) The statement false implies that r must be false and $(p \wedge q)$ must be true, because a statement $X \rightarrow Y$ is false when X is true and Y is false.

Since p is true (you try hard) and r is false, then q must be true as

$$(p \wedge q) \rightarrow r = (T \wedge T) \rightarrow F = T \rightarrow F = F.$$

∴ You try hard and have a talent but you do not get rich.

(c) r is true (you are rich)

since r is true, the statement $(p \wedge q) \rightarrow r$ must be true, because a statement $X \rightarrow Y$ is ~~false~~ true whenever Y is true.

∴ statement is true.

5.

p	q	r	$r \rightarrow p$	$q \wedge (r \rightarrow p)$	$p \vee (q \wedge (r \rightarrow p))$	A	$\sim p$	$q \rightarrow r$	B
F	F	F	T	F	F	T	T	T	T
F	F	T	F	F	F	T	T	T	T
F	T	F	T	T	T	F	T	F	F
F	T	T	F	F	F	T	T	T	T
T	F	F	T	F	T	F	F	T	F
T	F	T	T	F	T	F	F	T	F
T	T	F	T	T	T	F	F	F	F
T	T	T	T	T	T	F	F	T	F

∴ $A \equiv B$

Subject:

No:

Date:

6. Assume x is odd and y is even.

Since x is odd, $x = 2h+1$ for some integer h and since y is even, $y = 2k$ for some integer k.

$$x^2 - 2y = (2h+1)^2 - 2(2k)$$
$$= 4h^2 + 4h + 1 - 4k$$
$$= 2(2h^2 + 2h - 2k) + 1$$
$$= 2r + 1, \text{ where } r = 2h^2 + 2h - 2k \text{ and } r \in \mathbb{Z}$$

$\therefore$ since $x^2 - 2y$ is in the form of $2r+1$ where r is an integer, so $x^2 - 2y$ is odd.

The statement is true and is proven.