SECI1013:DISCRETE STRUCTURES
SESSION 2025/2026 SEMESTER 1
ASSIGNMENT 3 (CHAPTER 3 – COUNTING & DISCRETE PROBABILITY THEORY)

INSTRUCTIONS:

i. This assignment must be conducted in group. Please clearly write the group member's names & matric numbers on the front page of the submission.

ii. Solutions for each question must be readable and neatly written on plain A4 paper or digitally written. Every step or calculation should be properly shown. Failure to do so will result in the rejection of the submission of the assignment.

iii. This assignment consists of 5 questions (53 marks), contributing 5 % of overall course marks.

QUESTION 1 **[10 Marks]**

a) Ali has 5 certificates to be stored in 3 different folders. Show that at least 2 certificates share the same folder. (4 marks)

b) In a group of 20 people, each person has one pet that is a cat, a dog or a goat. What is the minimum number of people you need to be sure two of them have the same type of pet.

(6 marks)

QUESTION 2 **[8 Marks]**

a) A 4-digit code is formed using digits 0–9, with repetition allowed. If a code is chosen at random, find the probability that it contains **exactly two distinct digits.** (2 marks)

b) A random variable (X) has the probability mass function:

X	**0**	**1**	**2**	**3**
P(X)	0.1	0.3	0.4	0.2

i. Find $P(X \geq 2)$. (2 marks)

ii. Find $P(3 > X \geq 0)$ (4 marks)

QUESTION 3 **[8 Marks]**

A shipment contains defective items with a probability of 0.05.

- A defective item is returned with probability 0.9
- A non-defective item is returned with probability 0.1

a) Find the probability that a randomly selected item is returned. (4 marks)
b) Given that an item is returned, find the probability that it is defective. (4 marks)

QUESTION 4 **[10 Marks]**

Consider a family that has two children. Assume that the four possible outcomes (BB, BG, GB, GG) are equally likely.

Let the following events be defined:

E: The family has at least one boy
F: The family has children of both sexes (one boy and one girl)
G: The family's firstborn is a boy

Find the following probabilities:

a) Find the probability of event E, P(E). (2 marks)
b) Find the probability of event F, P(F). (2 marks)
c) Find the probability of the intersection of events E and F, $P(E \cap F)$ (2 marks)
d) **Determine if E and F are independent.** Use probabilities to formally justify your conclusion. (4 marks)

QUESTION 5 **[7 Marks]**

A factory sources components from three suppliers:

Supplier	Share	Defect rate
A	40%	2%
B	35%	4%
C	25%	1%

a) Find the probability that a randomly chosen component is defective. (4 marks)
b) Given that a component is defective, find the probability that it came from Supplier B. (3 marks)

QUESTION 6 **[10 Marks]**

An email filter identifies spam as follows:

- 15% of emails are spam
- Spam is correctly flagged with probability 0.95
- Legitimate emails are incorrectly flagged with probability 0.08

a) Find the probability that an email is flagged as spam. (5 marks)
b) Given that an email is flagged, find the probability that it is spam. (5 marks)

Q1

a. 5 pigeons, 3 pigeonholes

$$\left\lceil \frac{5}{3} \right\rceil = \lceil 1.67 \rceil = 2$$

$\therefore$ According to the Pigeonhole principle, there will be at least 2 certificates share the same folder.

b. Assume the worst case scenario:

The first 3 people have different pets.

Hence, the 4th person must have the same type of pet with either one of the first 3 people because there are only 3 types of pets.

$3 + 1 = 4$

$\therefore$ minimum 4 people so that 2 of them have the same type of pet.

Q 2

a. total = 10^4 = 10 000

choose 2 digits = ${}^{10}C_2 = 45$

arange the 2 digits = $2^4 - 2 = 14$

$45 \times 14 = 630$

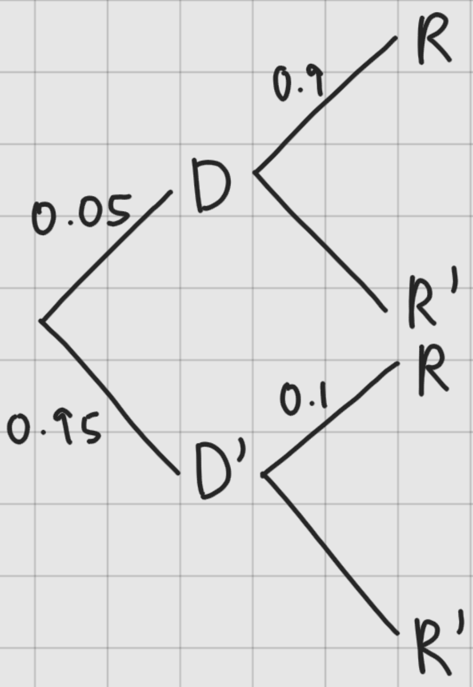
$P(\text{exactly 2 distinct digits}) = \frac{630}{10000} = 0.063$

b. i. $P(X \geq 2) = P(X=2) + P(X=3)$
$= 0.4 + 0.2$
$= 0.6$

ii $P(3 > X \geq 0) = 1 - P(X=3)$
$= 1 - 0.2$
$= 0.8$

Q 3

Let D = defective
R = return



a. $P(D') = 1 - 0.05 = 0.95$

$P(R) = (0.9)(0.05) + (0.1)(0.95)$

$= 0.14$

b. $P(D|R) = \dfrac{P(D \cap R)}{P(R)}$

$= \dfrac{(0.05)(0.9)}{0.14}$

$= \dfrac{9}{28}$

Q 4

a. $P(E) = \frac{1}{4} + \frac{1}{4} + \frac{1}{4} = \frac{3}{4}$

b. $P(F) = \frac{1}{4} + \frac{1}{4} = \frac{1}{2}$

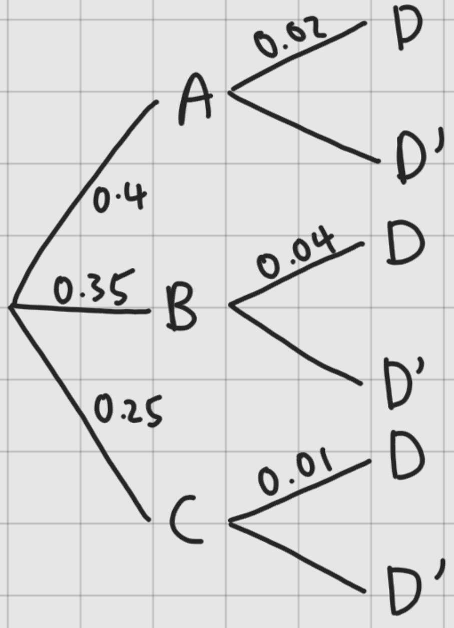
c. $P(E \cap F) = \frac{1}{4} + \frac{1}{4} = \frac{1}{2}$

d. $P(E) \times P(F) = \frac{3}{4} \times \frac{1}{2} = \frac{3}{8}$

since $P(E) \times P(F) \neq P(E \cap F)$,

E and F are not independant.

Q 5



a. $P(D) = 0.4(0.02) + 0.35(0.04) + 0.25(0.01)$
$= 0.0245$

b. $P(B|D) = \dfrac{P(B \cap D)}{P(D)}$

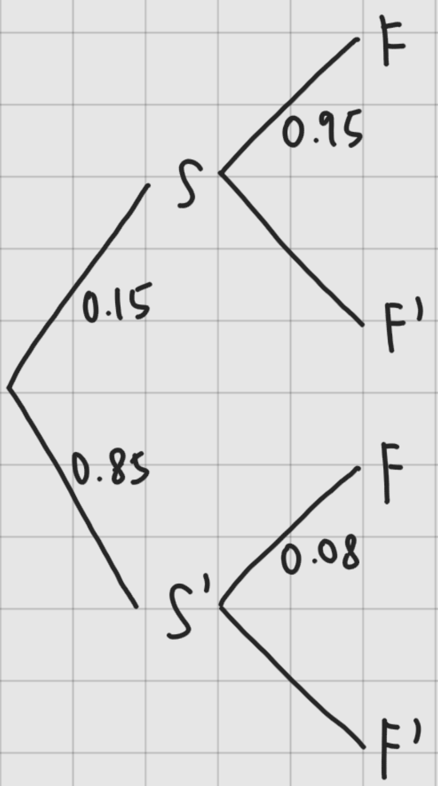
$= \dfrac{0.35(0.04)}{0.0245}$

$= \dfrac{4}{7}$

Q 6

Let $S =$ spam

$F =$ flagged



a. $P(S') = 1 - 0.15 = 0.85$

$P(F) = 0.15(0.95) + 0.85(0.08)$
$= 0.2105$

b. $P(S|F) = \dfrac{P(S \cap F)}{P(F)}$

$= \dfrac{0.15(0.95)}{0.2105}$

$= 0.677$