# Partial Differential Equations

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1. A. Intro, definitions, elementary examples.

   B. Advection equations. Characteristic lines. General solutions of quasi-linear first order equations.

2. First order quasi-linear PDEs in two variables. Solution surface. Characteristic equations. Cauchy problem. Examples: advection equations and Burgers equation.

3. Burgers equation. Gradient catastrophe, weak solutions, shock formation, properties and propagation.

4. A. Shock waves, weak solutions, speed of propagation in Burgers equation. Conservation laws.

   B. Parametric solution to general first order PDEs. Monge cone. Characteristic equations.

5. Cauchy problem for general first order PDEs. Wave fronts on an ellipse, caustics. ODEs general solution for a linear system.

6. Second order PDEs intro. Wave equation D'Alembert solution. Changes of variables. Characteristic coordinates.

7. Linear and quasi-linear second-order equations. Characteristic curves.

8. Linear second-order equations. Example: Tricomi equations. Elliptic equations.

9. Parabolic linear second order equations. Examples. Rescaling the dependent variable to remove first order terms.

10. A. Elliptic equations. Laplace equation solutions and Cauchy's theorem. On a disk.

    B. Elliptic equations. Laplace equation solutions and Cauchy's theorem. On the upper half plane.

11. Uniqueness theorems, Laplace equation, divergence theorem, separation of variables, mean value theorem.

12. Maximum minimum principle for elliptic equations. Green's functions. Symmetrical solutions, boundary conditions.

13. Symmetry properties of the Green's function. Boundary conditions of the Green's function: Dirichlet, Robin, Neumann.

14. A. Green's function for Laplace equation on a disk.

    B. Green's function for Laplace equation on a rectangle.

15. Diffusion equation, self-similar solutions, uniqueness, min-max principle.

16. Green's function for the heat equation. Infinite and semi-infinite domains, Neumann and Dirichlet boundary conditions.

17. Green's function for the 1D diffusion equation on a finite domain. Wave equation in 2D and 3D, preliminaries. Uniqueness of solutions.

18. Fourier transform properties. Green's function for the wave equation in nD.

19. Green's function for the wave equation in 1D and 2D.

20. Wave equation. Green's function in 3D. Example: Radiation from an electron on an antenna.

21. Separation of variables in 2D. Laplace, Helmholtz, Schrödinger potentials. Separable coordinate systems in 2D.

22. Sturm-Liouville oscillation theory, Prüfer substitution, comparison theorems.

23. A. Sturm-Liouville theorem. Eigenvalue problem. Properties of zeros, dependence on the eigenvalue. Existence of unique orthogonal basis.

    B. Expanding functions on a Sturm-Liouville eigenbasis. Properties and method.

24. Wave equation on a disk, separation of variables.

25. Wave equation on a circular membrane, Bessel functions, zeros, generating function, integral representations.

26. Wave equation on a circular membrane, expansions using Bessel functions.

27. Solitons, Korteweg-de Vries equations. Advection nonlinear velocity, dispersion.

28. KdV travelling wave solutions, self-similar solutions, inverse scattering method, solitons.

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