

# Analysis

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## Analysis I

1. Real Numbers and the completeness
2. Limits and continuity in metric space
3. Open and closed sets in metric space
4. Topological space & The closure of the set
5. Equivalence of continuous functions & Inheritance Principle & Homeomorphism
6. Product metric spaces & Completeness & Compactness (sequentially compact)
7. Compactness (2) & Bolzano-Weierstrass Thm & Nested seq of nonempty compact sets
8. continuous functions and Homeomorphisms on compact sets & Connectedness
9. Some properties in topological spaces
10. Accumulation points and isolated points & Dense & Hausdorff space
11. Properties of Connectedness
12. Path connected & Compactness in metric space, Hausdorff space, Topological space(1)
13. Compactness (2) & Sequentially compact implies compact(open cover) (1)
14. Sequentially compact implies compact(open cover) (2) & Totally bounded (1)
15. Totally bounded (2) & Perfect & Cantor set
16. Cantor surjection Theorem
17. Peano Curves & Riemann integrable & Darboux integrable(1)
18. Darboux integrable(2) & Riemann integrable iff Darboux integrable(1)
19. Riemann integrable iff Darboux integrable(2) & Some integrable functions (1)
20. Some integrable functions (2) & Lebesgue number & Zero set
21. Oscillation & Riemann-Lebesgue Theorem

22. [Some applications of R-L theorem](#)
23. [Fundamental Theorem of Calculus & Convergence in functional space](#)
24. [Complete functional space & Convergence function series & Integrals wst unif conv.](#)
25. [Derivatives w.s.t uniform convergence & Equicontinuity and Arzela-Ascoli Theorem](#)
26. [Heine-Borel Theorem in a function space & Stone-Weierstrass Theorem](#)
27. [Fixed point, contraction and ODE](#)
28. [Nowhere differentiable continuous function & Baire' s Theorem](#)
29. [Corollary of Baire' s Theorem](#)
30. [Multiple integral and Fubini' s Theorem](#)
31. [Change of variables formula \(1\)](#)
32. [Change of variables formula \(2\)](#)
33. [Banach spaces](#)
34. [Banach algebra](#)
35. [Differentiation in Banach Spaces](#)
36. [Some examples & Integration in Banach spaces \(1\)](#)
37. [Integration in Banach spaces \(2\)](#)
38. [Inverse function theorem](#)
39. [Lebesgue Outer measure](#)
40. [Lebesgue measurable set & Abstract outer measure](#)
41. [Measurable Sets form Sigma-Algebra & Measure continuity](#)
42. [Regularity \(1\)](#)
43. [Regularity \(2\)](#)
44. [Every open set is a countable disjoint union of balls plus a zero set](#)
45. [Measure Product & Inner measure \(1\)](#)
46. [Inner measure \(2\) & Slice Measure \(1\)](#)

47. Slice Measure (2) & Lebesgue integrable functions & Monotone Convergence Theorem
48. Completed undergraph & Dominated Convergence Thm and Fatou's Lem & Basic properties
49. Some basic properties of measurable functions (2)
50. Cavalieri's principle & Equivalence measurable function definition & Tonelli Thm
51. Vitali covering Lemma
52. Density Theorem

## Analysis II

1. Lebesgue Differentiation Theorem (1)
2. Lebesgue Differentiation Theorem (2) & Absolutely continuous functions
3. Indefinite integral is abs conti. & A monotone function has a derivative a.e. (1)
4. A monotone function has a derivative almost everywhere (2)
5. Riemann integrals vs Lebesgue integrals
6. Bounded variation
7. Equi relation of B-V functions and abs conti functions & R-S integrals (1)
8. Riemann-Stieltjes integral (2)
9. Riemann-Stieltjes integral (3)
10. Riemann-Stieltjes integral (4)
11.  $L^2 [a, b]$  space
12. Properties of  $L^2 [a, b]$  space
13. Orthogonal basis in  $L^2$
14. Fourier coefficients and Parseval's formula
15. A necessary condition of Fourier series converges in  $L^1$  & Riemann-Lebesgue lemma
16. Dirichlet kernel

17. Dini Theorem & There is a continuous function whose Fourier series diverges (1)
18. There is a continuous function whose Fourier series diverges (2) & U-B Principle
19. There is a continuous function whose Fourier series diverges (3)
20. Some properties of Fourier coefficients & Cesàro summability
21. Fejér's Theorem & Approximate integrable functions by tri-polynomial in  $L^1$  norm
22. The average of Fourier series converges in  $L^p$  norm
23. Equi. of the average of Fourier series converges unif & Weakly convergence(1)
24. Weakly convergent(2) & Equi condition for tri-series to be the Fourier series
25. Convergence of Cesàro mean of Fourier series (1)
26. Part2 Conv of Cesàro mean of Fourier series(2), Conjugate Fourier series & Dini theorem
27. Singular integrals and conjugate function
28. Bound of the  $L^2$  norm of conjugate function & Hardy-Littlewood Maximal function
29. Bound of the norm of the supremum of conjugate functions
30. A counterexample & C-Z Decomposition & The conjugate function exists a.e. in  $L^1$
31. The conjugate function exists a.e. in  $L^1$  (2)
32. The conjugate function exists a.e. in  $L^1$  (3) & Weakly estimate
33. Applications of Fourier series -isoperimetric inequality
34. Applications of Fourier series -continuous but nowhere differentiable function
35. Fourier transform and some properties
36. Fourier inversion formula and Minkowski's inequality
37. Poisson summation formula & Roth's theorem on 3-term AP
38. Discrete Fourier series, Fourier transform and some properties (1)
39. Discrete Fourier series, Fourier transform and some properties (2)
40. Bourgain's theorem
41. Proof of Roth's theorem on 3-terms AP (step 1)

- 42. Proof of Roth's theorem on 3-terms AP (step 2 to step 7)
- 43. Proof of Roth's theorem on 3-terms AP (step 9 , step 8)
- 44. Proof of Roth's theorem on 3-terms AP (step 8)
- 45. There is  $f$  in  $L^1$  such that for a.e.  $x$ , the Fourier series diverges (1)
- 46. There is  $f$  in  $L^1$  such that for a.e.  $x$ , the Fourier series diverges (2)
- 47. There is  $f$  in  $L^1$  such that for a.e.  $x$ , the Fourier series diverges (3)
- 48. There is  $f$  in  $L^1$  such that for a.e.  $x$ , the Fourier series diverges (4)

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