Numerical Computation

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2. Repre. of numbers in different bases
3. Floating point representation
4. Introduction to Matlab
5. Loss of significance
6. More examples on lost of significant digits
7. Review of Taylor Series
8. Finite Difference Approximation

2. Polynomial Interpolation

1. Poly. inter., Van der Monde matrix
2. Polynomial interpolation, Lagrange form
3. Barycentric forms of Lagrange polynomials
4. Poly. inter., Newton's divided diff. 1
5. Poly. inter., Newton's divided diff. 2
6. Poly. inter., Newton's divided diff. 3
7. Poly. inter., existence and uniqueness Thm
8. Error Thm for Polynomial Interpolation
9. Examples for Error Theorem
10. Uniform Grid
11. Chebyshev nodes
12. Matlab simulation for VdM matrix
13. Matlab Simulation for Chebyshev nodes
14. Matlab simulation with uniform grid
15. Aitken-Neville Interpolation
16. Hermite interpolation

3. Piecewise Polynomial Interpolation: Splines

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2. Examples of Spline Functions
3. Linear Splines
4. Quadratic Splines
5. Natural cubic spline
6. Natural Cubic Splines, Derivation of Algo.
7. Smoothness Thm for Natural Cubic Spline
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9. Other Types of BCs for Cubic Splines
10. Cubic Hermite Spline
11. Bézier curves

4. Numerical Integration

1. Numerical integration: Trapezoid Rule
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3. Error estimate for trapezoid rule
4. Simpson's rule, derivation
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8. Richardson extrapolation
9. Romberg algorithm
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15. Integrals over Infinite Intervals, Gauss Laguerre, Gauss Hermite
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1. Numerical Solutions of nonlinear equations
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5. Fixed point iteration, error analysis
6. Newton's iteration
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9. Secant method
10. Aitken Method and Acceleration
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13. Continuation Method

6. Direct Methods for Systems of Linear Equations

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2. LU-Factorization
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7. Overdetermined Systems and QR fact.
8. SVD and Image compression

7. Fixed Point Iterative Solvers for Linear and Non-linear Systems

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8. Systems of Non-linear Equations, Fixed Point iterations
9. Systems of Non-linear Equations, Newton iterations

8. The Method of Least Squares

1. Least Squares Method: Linear Regression
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3. General Linear Squares Method
4. Nonlinear Least Squares Method
5. Least Squares Method for continuous functions
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9. Numerical Solutions for ODEs

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6. The classical 4th order RK method
7. Numerical Simulations of RK methods
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4. Poisson equation on a unit square
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