

Functional Analysis

Claudio Landim

1. Linear Spaces: Definition, Examples and Linear Span
2. Linear Spaces: Quotient Spaces and Convex Sets
3. Normed Linear Spaces: Definition and Basic Properties
4. Completing a Normed Linear Space
5. Finite dimensional linear spaces
6. Examples of normed linear spaces
7. In infinite dimensions the unit ball is not compact
8. Zorn's lemma
9. The Hahn-Banach theorem
10. Convex sets and gauge functions
11. Geometric Hahn-Banach theorems
12. Dual of a normed linear space
13. Extension of bounded linear functionals, closed linear spans
14. Reflexive spaces
 - (a) The dual space of $C([a,b])$
 - (b) An application of the Hahn-Banach theorem: the moment problem and Chebyshev approximation.
 - (c) A dual variational problem in optimal control
 - (d) An application of the Hahn-Banach theorem: the existence of a Green function.
15. Hilbert spaces
16. Closed convex subsets of a Hilbert space
17. Riesz and Lax-Milgram representation theorems
18. Orthonormal sets and closed linear spans
19. Orthonormal bases

- (a) A quadratic variational problem
 - (b) The Dirichlet principle
 - (c) Generalized derivatives and Sobolev spaces
20. Uniform boundedness principle
 21. Weak convergence
 22. Uniform boundedness of weak converging sequences
 23. Weak sequentially compactness
 24. Weak* topology
 25. Applications of weak convergence
 26. Bounded linear operators
 27. Transpose of Bounded linear operators
 28. Strong and weak convergence of operators
 29. Principle of uniform boundedness for maps and compositions
 30. Open map principle.
 31. The closed graph theorem.
 32. Examples of bounded linear maps: Integral operators
 33. Symmetric operators
 34. Eigenvalues of compact symmetric operators
 35. The Fredholm alternative
 36. An application to integral operators
 37. Materials