Calculus and Analysis

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Calculus I

1. System of real number 1
2. System of real number 2
3. System of real number 3
4. Completeness of Real Number, Weierstrass Theorem 1
5. Weierstrass Theorem 2, define subsequence, Cauchy Sequence
6. Least Upper Bound & Greatest Lower Bound
7. Continuous function, uniformly continuous, Lipchitz continuous
8. Intermediate value theorem, Extreme Value Theory
9. Rieman sum
10. Properties of integral, MVT/general MVT for integral, logarithm
11. Logarithm and exponential 1
12. Logarithm and exponential 2
13. Fundamental Theorem of Calculus 1, Rolle's Theorem, MVT for derivative 1
14. MVT for derivative 2, approximation by linear function, FTC 2
15. Properties of derivative, composite functions, chain rule
16. First/second derivative test
17. Order of magnitude, introduce an smooth function
18. Some example of oscillating functions, chain rule, method of substitution
19. Integration by parts
20. Integration of rational functions 1
21. Integration of rational functions 2, Improper integral 1

22. Improper integral 2
23. Taylor's series: Gamma function, power series, expansion of the logarithm
24. Taylor's series: Taylor's theorem, Cauchy's & Lagrange's remainder
25. Taylor's series: estimate remainder term, expansion of elementary
26. Taylor's series: examples
27. Taylor's polynomial: interpolation 1
28. Taylor's polynomial: interpolation 2, approximation in first order
29. Taylor's polynomial: approximation in second order
30. Taylor's polynomial: fixed point approximation, Stirling's formula
31. Series: concepts of convergence and divergence 1
32. Series: rearrangement, tests for absolute convergence and divergence 1
33. Series: tests for abs. convergence and divergence 2, sequences of functions
34. Series: pointwise/uniform convergence 1
35. Series: pointwise/uniform convergence 2
36. Series: power series, interval of convergence
37. Series: product of two power series
38. Series: expansion of given power series, infinite product
39. Fourier series: periodic function, complex notation 1
40. Fourier series: complex form 2, trigonometric formula, Riemann Lebesgue Lemma
41. Fourier series: examples of Fourier expansion
42. Fourier series: main theorem on Fourier expansion
43. Fourier series: examples of Fourier series 1
44. Fourier series: examples of Fourier series 2, Bessel's Inequality
45. Approximation by trigonometric and rational polynomial 1

46. Approximation by trigonometric and rational polynomial 2
47. Approximation by trigonometric and rational polynomial 3
48. Inner product
49. Bernoulli polynomial and their applications 1
50. Bernoulli polynomial and their applications 2

Calculus II

1. functions of multiple variables & partial derivative
2. continuity
3. differentiability & directional derivative (1)
4. directional derivative (2) & tangent plane
5. change of variables & Taylor series
6. MVT & Taylor expansion & Integral
7. double integral & length of curve
8. curvature & linear differential one form
9. line integral
10. Heine-Borel theorem
11. compact subset & Implicit Function theorem (1)
12. Implicit Function theorem (2)
13. Inverse Function theorem (1)
14. Inverse Function theorem (2) & extreme value
15. Lagrange Multiplier method
16. examples & Hölder Inequality
17. sufficient condition for local max/min
18. Jordan measurable

19. Jordan area (1)
20. Jordan area (2)
21. double integral (1)
22. double integral (2)
23. transformation of multiple integrals
24. improper multiple integrals (1)
25. improper multiple integrals (2) & volumes
26. surface area
27. surface area formula
28. multiple integral in curved coordinate
29. extend to R^n
30. integral of unbounded set
31. Fourier integral theorem (1)
32. Fourier integral theorem (2)
33. Fourier transform - decay property & Parseval's identity
34. Fourier transform of several variables
35. Green theorem
36. Divergence theorem
37. prove Inverse Function theorem by Green theorem
38. orientation of surface in R^3 (1)
39. orientation of surface in R^3 (2)
40. Gauss theorem in R^3 (1)
41. Gauss theorem in R^3 (2)
42. application of fluid

43. Stoke's theorem
44. general surface (partition of unit) (1)
45. general surface (partition of unit) (2)
46. general surface (partition of unit) (3) & functions of one complex variable
47. complex function - power series
48. complex function - differentiability
49. conformal map & integration of analytic functions - Cauchy's theorem
50. complex integral theorem & Cauchy integral formula
51. zeros, poles and residues

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