Numerical Computation

A. Numerical Computation by Wen Shen

1. Computer Arithmetic

1. Introduction to Numerical Computation
2. Repre. of numbers in different bases
3. Floating point representation
4. Introduction to Matlab
5. Loss of significance
6. More examples on lost of significant digits
7. Review of Taylor Series
8. Finite Difference Approximation

2. Polynomial Interpolation

1. Poly. inter., Van der Monde matrix
2. Polynomial interpolation, Lagrange form
3. Barycentric forms of Lagrange polynomials
4. Poly. inter., Newton's divided diff. 1
5. Poly. inter., Newton's divided diff. 2
6. Poly. inter., Newton's divided diff. 3
7. Poly. inter., existence and uniqueness Thm
8. Error Thm for Polynomial Interpolation
9. Examples for Error Theorem
10. Uniform Grid
11. Chebyshev nodes
12. Matlab simulation for VdM matrix
13. Matlab Simulation for Chebyshev nodes
14. Matlab simulation with uniform grid
15. Aitken-Neville Interpolation
16. Hermite interpolation

3. Piecewise Polynomial Interpolation: Splines

1. Splines
2. Examples of Spline Functions
3. Linear Splines
4. Quadratic Splines
5. Natural cubic spline
6. Natural Cubic Splines, Derivation of Algo.
7. Smoothness Thm for Natural Cubic Spline
8. Matlab Simulation
9. Other Types of BCs for Cubic Splines
10. Cubic Hermite Spline
11. Bézier curves

4. Numerical Integration

1. Numerical integration: Trapezoid Rule
2. Example and sample codes for TR
3. Error estimate for trapezoid rule
4. Simpson's rule, derivation
5. Ex. and sample code for Simpson's rule
6. Error estimate for Simpson's Rule
7. Recursive trapezoid, composite schemes
8. Richardson extrapolation
9. Romberg algorithm
10. Adaptive Simpson's Quadrature
11. Gaussian quadrature 1
12. Gaussian quadrature 2
13. Matlab
14. Numerical integration rules in a more abstract setting
15. Integrals over Infinite Intervals, Gauss Laguerre, Gauss Hermite
16. Monte Carlo Integration

5. Numerical Solutions of Non-linear equations

1. Numerical Solutions of nonlinear equations
2. Bisection method
3. Fixed point iteration, algorithm
4. Fixed Point iteration, convergence
5. Fixed point iteration, error analysis
6. Newton's iteration
7. Newton's iteration, convergence
8. Newton's iteration, example, code
9. Secant method
10. Aitken Method and Acceleration
11. Halley's Method: an improved version of Newton's method
12. Roots of Polynomials, Horner's Algorithm
13. Continuation Method

6. Direct Methods for Systems of Linear Equations

1. System of linear eq.: Gaussian Elimination
2. LU-Factorization
3. Cholesky Factorization
4. Vector Norms
5. Matrix norms
6. Condition number of a matrix
7. Overdetermined Systems and QR fact.
8. SVD and Image compression

7. Fixed Point Iterative Solvers for Linear and Non-linear Systems

1. Iterative Solvers: Jacobi Iterations
2. Example
3. Gauss-Seidal iterations
4. SOR iterations
5. Linear Fixed Point Iteration for systems
6. Convergence Analysis
7. Matlab
8. Systems of Non-linear Equations, Fixed Point iterations
9. Systems of Non-linear Equations, Newton iterations

8. The Method of Least Squares

1. Least Squares Method: Linear Regression
2. Linear Least Squares with three functions
3. General Linear Squares Method
4. Nonlinear Least Squares Method
5. Least Squares Method for continuous functions
6. Examples of orthogonal basis functions
7. Matlab Examples on Least Squares Method

9. Numerical Solutions for ODEs

1. Numerical solutions for ODEs
2. Taylor series methods for ODEs
3. Examples of Taylor Series Method
4. Error analysis for Taylor Series Methods
5. RK Methods, Euler step and Heun step
6. The classical 4th order RK method
7. Numerical Simulations of RK methods
8. Adaptive RKF method
9. Explicit AB method for ODEs
10. Examples of explicit AB methods
11. Implicit ABM methods
12. Multistep ABM methods for ODEs
13. First order systems of ODEs
14. Higher order ODEs and systems
15. Stiffness of ODEs, Scalar ODEs
16. Systems of ODEs
17. Stiff system, Implicit method
18. Geometric Int.: Symplectic, Hamiltonian preserving method

10. Numerical Methods for Two-point Boundary Value Problems

1. Two-point Boundary value problems
2. Shooting method
3. Linear Shooting method, extensions
4. Nonlinear shooting method
5. FDM for two-point BVP
6. Finite Difference Methods in 1D
7. Neumann BC, Poisson's equation
8. Robin BC for Poisson Equation

11. Finite Difference Methods for Some Partial Differential Equations

1. FDM for Laplace Equation in 2D
2. System of Linear Equations for Discrete Laplace Equation with FDM
3. Laplace equation with non-homogeneous Dirichlet BCs
4. Poisson equation on a unit square
5. Laplace equation with Neumann BC
6. Heat equation in 1D, forward Euler method
7. Heat equation, CFL stability condition for explicit forward Euler method
8. Heat equation, implicit backward Euler step, unconditionally stable
9. Heat equation, Crank-Nicholson scheme
10. Heat equation with Neumann BC

B. Numerical Analysis by Ching-hsiao Cheng

1. Mathematical Preliminaries

1. 数值分析课程介绍与函数的极限、连续、可微性质
2. 均值定理、中间值定理、函数的可积性、Lebesgue 定理与广义积分均值定理
3. 单变数函数的泰勒定理
4. 多变数函数的泰勒定理
5. 计算多变数函数泰勒多项式之实例, 与 Big O 符号

2. Solutions of Nonlinear Equations

1. 第二章的介绍, 与二分法求根
2. 二分法误差估计与固定点迭代
3. Banach 固定点定理的证明
4. 使用固定点迭代找函数零根的实例，以及牛顿法的进一步介绍
5. 牛顿法二次收敛的证明以及割线法的介绍
6. 割线法求根的复习、两个未知数与方程求根的牛顿法与 matlab 实作
7. matlab 实作、多个未知数与方程求零根的牛顿法

3. Interpolation and Polynomial Approximation

1. 第三章主要内容的介绍: 关于插值
2. Lagrange 插值多项式、一个函数与其 Lagrange 插值多项式的误差
3. Newton's divided difference 插值多项式
4. 使用高低 divided difference 的关系求 Newton's 插值多项式的系数, 以及 Hermite 插值多项式的介绍
5. Hermite 多项式的形式与余项的证明
6. Spline 插值的介绍, 以及决定 cubic spline 的条件
7. Natural cubic spline 与 Clamped cubic spline 的唯一性

4. Numerical Differentiation and Integration

1. 数值微分中的 central difference formula 与误差
2. 使用 Richarson extrapolation 由低阶演算法得到高阶演算法
3. Richardson extrapolation 的一般式、使用 Lagrange 插值多项式求一阶导数的演算法
4. 数值积分: 以 Lagrange 插值多项式推导梯形法与 Simpson 法、以 Taylor 定理推导中点法
5. 使用泰勒定理与柯西均值定理推导 Simpson 法, 以及 composite 数值积分法
6. Legendre 多项式两个性质的证明, 与使用 Gauss quadrature 做积分的证明

5. Direct and Iterative Methods for Solving Linear Systems

1. 线性代数关于可逆矩阵的复习
2. 线性代数关于基本列 (行) 运算与 elementary matrix 等价关系的复习
3. 当 A 是对角矩阵、上下三角矩阵时, 解 Ax=b 的演算法与其计算量
4. 以高斯消去做矩阵的 LU 分解之演算法的 order, 与赋范向量空间
5. 有限维向量空间上的任意两个 norms 都等价的证明
6. Induced matrix norm 与其性质, 以及一个 $m \times n$ 矩阵的 infinity norm
7. 矩阵的 1-norm, 2-norm 还有 spectral radius
8. 矩阵的 Frobenius norm 与 spectural radius 是所有 norm 中的 inf 的证明
9. Spectural radius 是所有 norm 中的 inf 的证明、收敛矩阵的定义, 与收敛矩阵的等价性质
10. 解 Ax=b 系统之迭代法初探: Jacobi method 与 Gauss-Seidel method 的介绍
11. x = Tx + c 迭代格式收敛之充分必要条件的证明、严格对角优势矩阵之 Jacobi method 与 Gauss-Seidel method 的收敛性, 以及 SOR 的概念
12. SOR 补完、绝对误差、相对误差与条件数

6. Numerical Ordinary Differential Equations

1. 数值常微分方程的开头
2. 一些与数值解初始值问题相关术语的介绍

3. 边界值问题与泰勒方法
4. 泰勒方法的应用例、泰勒方法的优缺点、Euler 方法与 Runge-Kutta 方法的介绍
5. 二次 Runge-Kutta 法的进一步说明
6. 四次 Runge-Kutta 法的说明, 以及 Collocation 方法解初始值问题
7. 使用 Collocation 法与有限差分法解边界值问题, 以及有限元素法解边界值问题的初步介绍
8. 边界值问题之 variation (weak) form 与某最佳化问题之间的等价性
9. 变分形式 (Variational form) 有唯一解的证明, 以及有限元素法求解变分形式解的实作方法
10. 有限元素法的具体实作方法, 以及有限元素法与有限差分法的比较

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