

Algebra

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Algebra I

1. Basic notion, Symmetry groups
2. Cosets, Normal Subgroups, Quotient Groups, Isomorphism Theorem
3. Isomorphism Theorem, Tower of Subgroups
4. Butterfly lemma, Jordan-Hölder theorem
5. Jordan-Hölder theorem, Cyclic groups
6. Group Action, Simplicity of A_n
7. Simplicity of A_n , Sylow theorems
8. Sylow theorems
9. Semidirect products
10. Structure theorem of finite abelian groups
11. Ring Isomorphism Theorem
12. Chinese Remainder Theorem, Localization
13. ED, PID, UFD
14. Gauss lemma, Criteria for irreducibility
15. Algebraic extensions
16. Algebraic closures
17. Algebraic closures, Normal extensions
18. Separable extensions
19. Primitive element theorem, Finite fields
20. Galois theory
21. Galois theory

22. Galois theory, roots of unity, cyclotomic extensions.
23. Norms and traces.
24. Purely inseparable extensions, cyclic extensions.
25. Solvable extensions, solvable by radicals.

Algebra II

1. Module theory.
2. Free modules, projective modules.
3. Modules over PID.
4. Modules over PID.
5. Tensor products.
6. Modules of fractions.
7. Noetherian modules.
8. Primary decompositions.
9. Primary decompositions, Nakayama's lemma.
10. Nakayama's lemma, filtered and graded modules, Artin-Rees.
11. Hilbert polynomial, Artinian modules.
12. Abelian categories, general homology theory.
13. Injective modules.
14. Homotopies of morphisms of complexes, derived functors.
15. Ext functor.
16. Ext functor, Ext^1 , Tor functor.
17. Group cohomology.
18. Group cohomology, Semisimple modules.

19. Semisimple modules, Semisimple rings.
20. Density theorem, Wedderburn theorem, Burnside theorem.
21. Representations, Characters.
22. Characters, Class functions.
23. Orthogonality relations.
24. Orthogonality relations.
25. Induced characters, Induced representations.
26. Induced representations, Mackey's criterion.
27. Artin theorem, Brauer theorem.

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