Covariate adjustment II

1. Covariate adjustment – part 2
2. Direct causal effects
3. Frontdoor criterion (下次talk)

- 汇报人：刘思璇

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回顾

◆ 混杂（Confounding）

混杂是指存在一个变量同时影响原因和结果，导致错误估计因果效应

◆ 选择偏差（Selection bias）

- ✓ 选择偏差是指样本的选取方式导致分析结果与真实因果关系不符
- ✓ 这与"碰撞节点"（Collider）相关，当我们对一个同时被两个变量影响的节点进行条件化时，会人为制造虚假关联

◆ 结构方程模型（SEM）

SEM用数学方程明确描述变量间的因果机制

形式： 每个变量由其直接原因和随机误差决定

优点：不仅告诉我们谁影响谁，还告诉我们如何影响、影响多大

◆ 干预/调整（Interventions）

干预是因果推断的核心操作，用 do(·) 表示，P(Y | do(X=1))

◆ 总因果效应（Total Causal Effect）

X对Y的完整因果影响，包括所有因果路径。对应直接因果效应：只考虑x->y的影响

◆ 路径方法

在线性SEM中，总因果效应可以通过路径系数计算

◆ 协变量调整第一部分（Covariate adjustment）

调整 (Adjustment)：使用线性回归，移除协变量对因变量的线性影响，从而“校正”因变量的分数

1. Covariate adjustment

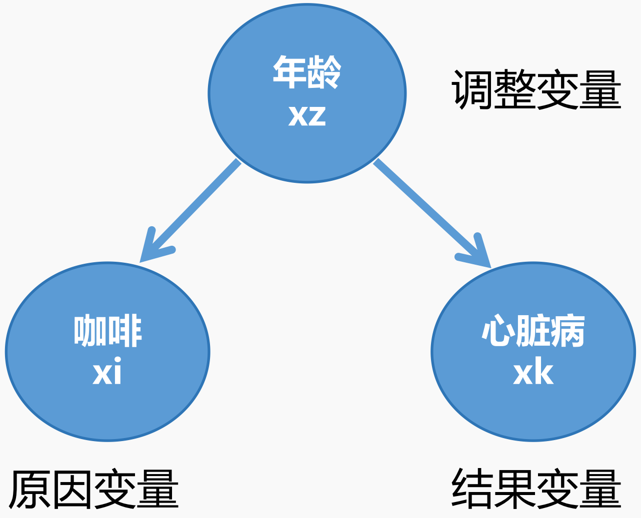
◆ 确定调整集

由一个有向无环图（DAG）和一个概率分布组成，图中的箭头表示因果关系方向

- Let (G, P) with $G = (\boldsymbol{V}, \boldsymbol{E})$ be a causal Bayesian network, $(i, k) \in \boldsymbol{V}, i \neq k$
- Adjustment formula

$$p(x_k | do(x_i)) = \int_{x_Z} p(x_k | x_i, x_Z) p(x_Z) dx_Z \qquad (1)$$

- Sets $\boldsymbol{Z}$ satisfying Eq. (1) are called valid adjustment sets
- If no proper subset of $\boldsymbol{Z}$ satisfies (1), $\boldsymbol{Z}$ is called a minimal adjustment set



年龄	P(年龄)	P(心脏病｜喝咖啡, 年龄)
年轻	0.6	0.05
老年	0.4	0.30

有效调整集（Valid Adjustment Set）：

满足上述公式的变量集Z

P(心脏病|do(喝咖啡))

=0.05×0.6+0.30×0.4=0.03+0.12=0.15

最小调整集（Minimal Adjustment Set）：

没有多余变量的有效调整集

调整集：用来消除混杂，让我们能从观察数据中提取因果信息

1. Covariate adjustment

◆ 能否通过看图就判断哪些变量集Z可以用于调整?

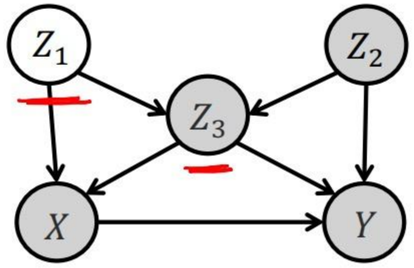
Example

- Interested in the causal effect of X on Y
- Parent adjustment implies controlling for

$$\boldsymbol{Z} = \{Z_1, Z_3\}$$

- Can we compute $p(y|do(x))$ if (only) Z_1 is not measured?
  - I.e., is $p(y|do(x))$ identifiable if (only) Z_1 is not measured?

Shaded nodes are observed



引出

如果z1无法测量，还能计算因果效应吗 → 后门准则

1. Covariate adjustment

◆ 后门准则（Backdoor Criterion）

- Let $G=(\boldsymbol{V},\boldsymbol{E})$ be a DAG and $i,k\in\boldsymbol{V}, i\neq k$. A set $\boldsymbol{Z}\subset\boldsymbol{V}$ (not containing i and k) satisfies the backdoor criterion relative to (i,k) in G if:
  i. $\boldsymbol{Z}\cap\mathrm{desc}(i)=\emptyset$, and
  ii. $\boldsymbol{Z}$ blocks all "backdoor paths" from i to k in G, i.e., all paths between i and k that start with an arrow into i $(i\leftarrow\cdots k)$

后门路径是从 X 到 Y 的路径，其中第一条边的箭头指向X

- If $\boldsymbol{Z}\subset\boldsymbol{V}$ satisfies the backdoor criterion relative to (i,k) in a DAG $G=(\boldsymbol{V},\boldsymbol{E})$ then for all $p(\cdot)$ such that (G,P) is a causal Bayesian network, we have:

$$p(x_k|do(x_i))=\int_{x_{\boldsymbol{Z}}}p(x_k|x_i,x_{\boldsymbol{Z}})p(x_{\boldsymbol{Z}})dx_{\boldsymbol{Z}}$$

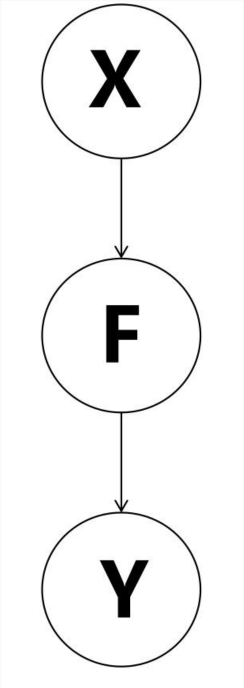
  - The backdoor criterion is sufficient for adjustment

回顾三种结构

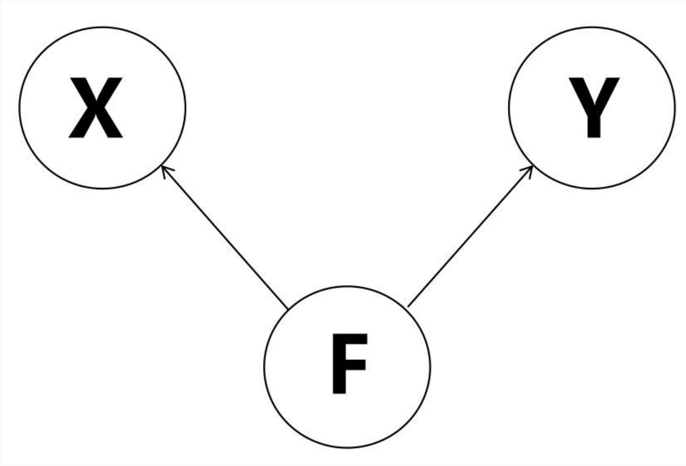
什么是"阻断"?

核心思想：信息（或关联）可以沿着路径"流动"。阻断就是切断这种流动

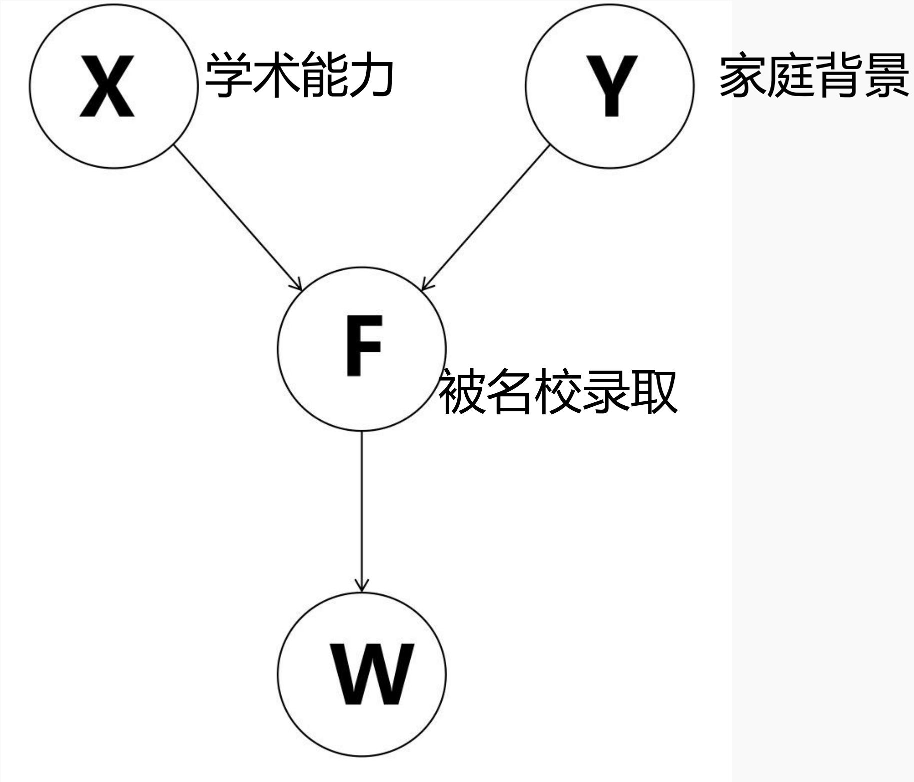
链 chain



叉 fork

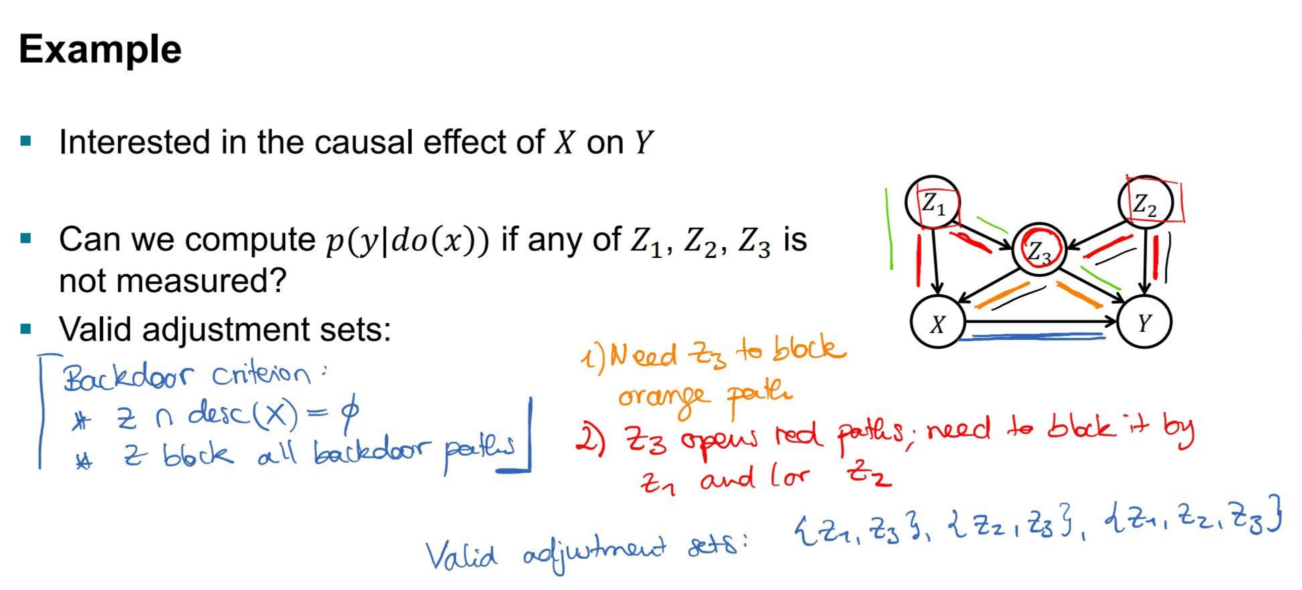


碰撞 collider



1. Covariate adjustment

◆ 后门准则（Backdoor Criterion)



有效调整集：{Z3}、{Z1,Z3}、{Z2,Z3}、{Z1,Z2,Z3}
最小有效调整集：{Z3}

如果z1无法测量，还能计算因果效应吗->可以

后门路径：从 X 到 Y 的路径，其中第一条边的箭头指向X

后门路径有三条：

X ← Z1 → Z3 → Y

X ← Z3 → Y

X ← Z3 ← Z2 → Y

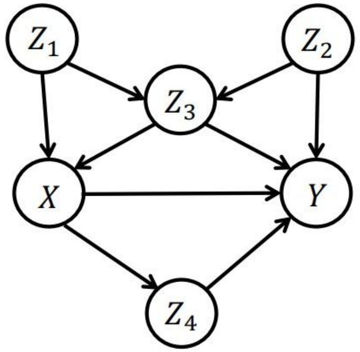
调整集	路径1	路径2	路径3	全部阻断?
{Z1}	阻断	开放	开放	
{Z2}	开放	开放	阻断	
{Z3}	阻断	阻断	阻断	是
{Z1,Z2}	阻断	开放	阻断	
{Z1,Z3}	阻断	阻断	阻断	是
{Z2,Z3}	阻断	阻断	阻断	是
{Z1,Z2,Z3}	阻断	阻断	阻断	是

1. Covariate adjustment

◆ 后门准则的直觉

Backdoor criterion

- Intuition behind backdoor criterion:
  - Backdoor paths carry spurious associations from X to Y
  - Paths directed along the arrows from X to Y carry causal associations
  - Blocking backdoor paths ensures that the measured association between X and Y is purely causal
  - Don't want to include descendants of X that are also ancestors of Y because this would block off a causal path
  - Don't want to include descendants of X that are also descendants of Y because this would introduce collider bias



◆ 后门路径携带虚假关联：不是X导致Y的真正原因

◆ 正向路径携带因果关联：X → Y 这条路径是我们想测量的

◆ 阻断后门路径确保了X和Y之间有单纯的因果关系

◆ 后门准则不包含X的后代：

✓ 如果后代也是Y的祖先：会阻断因果路径

✓ 如果后代也是Y的后代：会引入碰撞偏倚（collider bias）

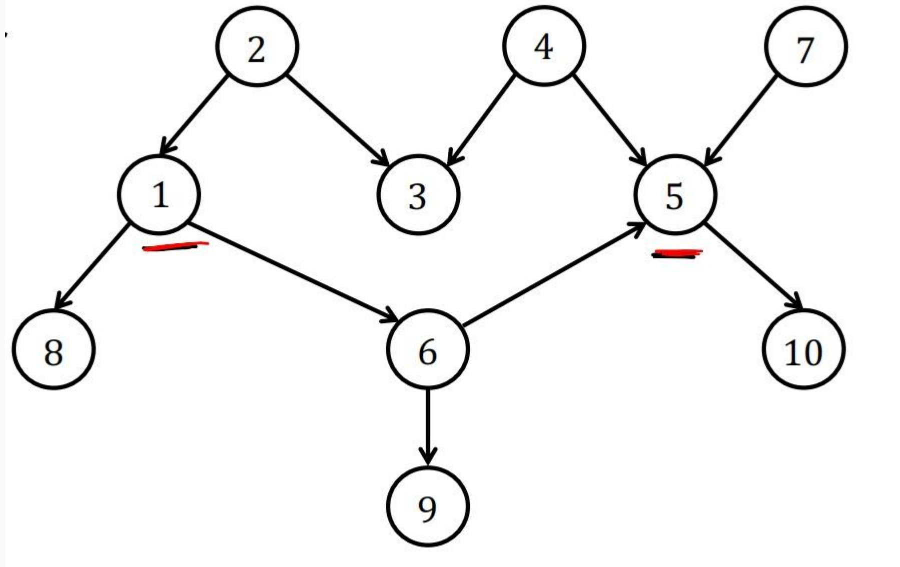
Backdoor criterion – proof

§ See Notes week 6 - I.pdf

1. Covariate adjustment

◆ 练习题（对1->5的因果关系感兴趣，找到符合后门准则的集合）

- Let $G = (\boldsymbol{V}, \boldsymbol{E})$ be a DAG and $i, k \in \boldsymbol{V}, i \neq k$. A set $\boldsymbol{Z} \subset \boldsymbol{V}$ (not containing i and k) satisfies the backdoor criterion relative to (i, k) in G if:
  i. $\boldsymbol{Z} \cap \mathrm{desc}(i) = \emptyset$, and
  ii. $\boldsymbol{Z}$ blocks all "backdoor paths" from i to k in G, i.e., all paths between i and k that start with an arrow into i $(i \leftarrow \cdots k)$



◆ 节点1的所有后代（descendants）： desc(1)={6,8,5,9,10}，满足后门准则的点集不包括这些

◆ 从 1 到 5 的所有路径

Directed path（因果路径）： $1 \rightarrow 6 \rightarrow 5$

Backdoor path（后门路径，即第一条边指向1）： $1 \leftarrow 2 \rightarrow 3 \leftarrow 4 \rightarrow 5$

集合	空集	{2}	{4}	{7}	{2,4}	{2,7}	{4,7}	{2,4,7}
分析	对撞3默认阻断	在节点2阻断	在节点4阻断	不在路径，因此阻断	两处都阻断	2阻断7无关	4阻断	2 4都能阻断7无关
有效	是	是	是	是	是	是	是	是

1. Covariate adjustment

◆ 正性假设

核心思想：我们想通过观测数据来估计干预效果
如果在观测数据中，某些 (X, Z)的组合从来没有出现过，即P(X, Z) = 0，
那么我们就无法估计P(Y | X,Z)

Positivity

- General requirement for identifiability:
  - Empirical basis for estimating the consequences of the contemplated interventions
  - Combinations of values under the interventional regime must also be possible under the observational regime
- Adjustment formula: $p(x_k|do(x_i)) = \int_{x_Z} p(x_k|x_i, x_Z)p(x_Z)dx_Z$
- In absence of further assumptions, positivity assumption requires:

$$p(x_i, x_Z) > 0 \; \forall x_i \in \mathcal{X}_i,\; x_Z \in \mathcal{X}_Z$$

  - E.g. violation if we want to compare "treatment" with "no treatment" in a patient group where some patients are so ill that they are never left untreated in practice

可识别性的一般要求：
- ◆ 需要有经验基础来估计所设想干预的后果
- ◆ 在干预条件下的变量取值组合，在观测条件下也必须是可能发生的

1. Covariate adjustment

◆ 多元高斯分布的假设 如果数据服从多元高斯分布，调整公式可以大大简化

- Adjustment formula

$$p(x_k|do(x_i)) = \int_{x_Z} p(x_k|x_i, x_Z)p(x_Z)dx_Z$$

- May be hard to compute, especially in the case of continuous variables and high-dimensional $\boldsymbol{Z}$
- Simplification if the joint distribution p is Gaussian
- Let

$$p(x_k|do(x_i)) = \int_{x_Z} p(x_k|x_i, x_Z)p(x_Z)dx_Z$$

and let $p(x_V)$ be multivariate Gaussian. Then

$$E(X_k|do(x_i = x_i' + 1)) - E(X_k|do(x_i = x_i')) = \gamma$$

where γ is the coefficient of X_i in the linear regression of X_k on X_i and X_z, i.e.

$$E(X_k|X_i, X_Z) = \alpha + \gamma X_i + \beta^T X_Z$$

for some α, β.

- See `Notes week 6 - II.pdf`

1. Covariate adjustment

- 实际意义：不需要复杂计算，只需要跑一个普通的线性回归
- $Y = \alpha + \beta X + \gamma Z + \epsilon$，得到$\beta$就是X对Y的总因果效应估计

- 简化成立条件

（1）Z是有效调整集

（2）分布是多元高斯（或者更一般的，SEM是线性的）

Adjustment through regression

- Hence, we can then estimate the total causal effect of X_i on X_k in R by

```
coef(lm(xk ~ xi + xz))[2]
```

- See Jupyter notebook and R scripts

- Can show:
  The above also holds for linear SEMs with arbitrary error distributions

1. Covariate adjustment

◆ 调整准则（比后门准则 更强，是充要条件）

Adjustment criterion (Shpitser et al, Perkovic et al)

- Let $G = (\boldsymbol{V}, \boldsymbol{E})$ be a DAG and $i, k \in \boldsymbol{V}, i \neq k$. A set $\boldsymbol{Z} \subset \boldsymbol{V}$ (not containing i and k) satisfies the adjustment criterion relative to (i, k) in G if:
  - $\boldsymbol{Z}$ does not contain any descendants of nodes $r \neq i$ on a directed path from i to k in G
  - $\boldsymbol{Z}$ blocks all paths between i and k in G that are not directed from i to k

- A set $\boldsymbol{Z} \subset \boldsymbol{V}$ satisfies the adjustment criterion relative to (i, k) in a DAG $G = (\boldsymbol{V}, \boldsymbol{E})$ if and only if for all p such that (G, p) is a causal Bayesian network, we have:

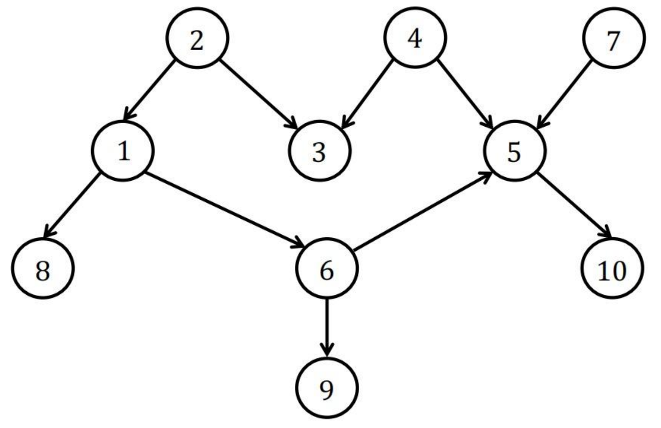
$$p(x_k|do(x_i)) = \int_{x_Z} p(x_k|x_i, x_Z)p(x_Z)dx_Z$$

◆ Z 不包含从 X 到 Y 的有向路径上任何节点 W≠X的后代

◆ Z 阻断所有从 X 到 Y 之间“非有向”的路径

	Backdoor Criterion	Adjustment Criterion
条件1	Z∩desc(X)=∅	不包含因果路径上中间节点的后代
条件2	阻断所有后门路径	阻断所有非有向路径
性质	充分条件	充分且必要条件

1. Covariate adjustment



◆ 调整准则 VS 后门准则

	Backdoor Criterion	Adjustment Criterion
条件1	X的后代一律不能调整	只有因果路径上的后代不能调整
条件2	阻断所有后门路径	阻断所有X到Y的非直接路径

因果路径 1 → 6 → 5
后门路径 1 ← 2 → 3 ← 4 → 5
X的后代 6，8，5，9，10
阻塞节点 3

符合后门准则的集合：（不包括1的全部后代）：

∅，{2}, {4}, {7}，{2,4}, {2,7}, {4,7}, {2,4,7}

符合调整准则的集合（不包括6的后代9）：
∅，{2}, {4}, {7}, {8}, {10}
{2,4}, {2,7}, {2,8}, {2,10}, {4,7}, {4,8}, {4,10}, {7,8}, {7,10}, {8,10}
{2,4,7}, {2,4,8}, {2,4,10}, {4,7,8}, {4,7,10}, {7,8,10}
{2,4,7,8}, {2,7,8,10}, {4,7,8,10}, {2,4,8,10}, {2,4,7,10}
{2,4,7,8,10}

1. Covariate adjustment

◆ 调整准则的重要性质

Adjustment criterion

- The adjustment criterion is necessary and sufficient for identifying total causal effects via adjustment
- It is only sufficient for the identification of total causal effects
  - Some effects are identified by other means, e.g., via the frontdoor criterion

必要且充分条件：调整准则是通过调整来识别总因果效应的充分必要条件

如果一个集合 Z 满足调整准则，就可以用调整公式计算因果效应
如果一个集合 Z 不满足调整准则，就不能用它来正确计算因果效应

局限性：调整准则只是识别总因果效应的充分条件，不是必要条件

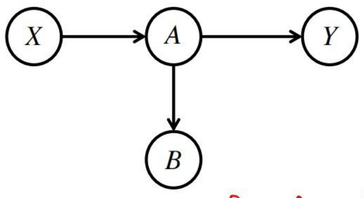
意味着：有些因果效应可以通过其他方法识别（如前门准则 frontdoor criterion）
即使没有有效的调整集存在，因果效应仍可能是可识别的

1. Covariate adjustment

Determining adjustment sets

- Should we adjust for as many variables as possible?

  - X: Smoking
  - Y: Future miscarriages
  - A: Physiological abnormality induced by smoking
  - B: Previous miscarriages



Do not adjust for A and/or B

是否应该调整尽可能多的变量?

X：吸烟
Y：未来流产
A：吸烟引起的生理异常
B：之前的流产

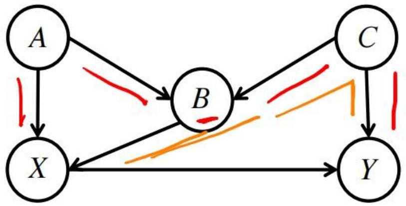
调整中介变量A会导致偏差!

Determining adjustment sets

- Is it always safe to adjust for “pre-treatment” variables?

  - X: Smoking
  - Y: Adult asthma
  - A: Parental smoking
  - B: Childhood asthma ← pre-treatment
  - C: Predisposition toward asthma

Only controlling for B would introduce "M-bias"



Adjust for {A, B} {C, B} {A, C} {A, B, C} {C}

后门路径
路径1：X ← A → B ← C → Y
路径2：X ← B ← C → Y

X：（你自己）吸烟
Y：（你的）成人哮喘
A：父母吸烟（Parental smoking）
B：（你的）儿童哮喘
C：（你的）哮喘易感性/遗传倾向

Pre-treatment：时间/因果上先于处理变量的变量
M-偏差：调整pre-treatment 的对撞节点反而引入偏差

调整一个本来阻断路径1的对撞节点B，反而打开了这条路径!

1. Covariate adjustment

Summary: Determining adjustment sets

- Should we adjust for as many variables as possible?
  - No. Adjusting for certain variables can create bias.
- Is it always safe to adjust for “pre-treatment” variables?
  - No. This can create so-called M-bias.
- If we want the total effect of X_i on X_k in G $(k \notin pa(i))$ then:
  - $\mathrm{pa}(i)$ is a valid adjustment set (this includes not adjusting for anything if $\mathrm{pa}(i) = \emptyset$).
  - Any set $\boldsymbol{Z}$ satisfying the backdoor criterion relative to (i, k) in G is a valid adjustment set.
  - A set $\boldsymbol{Z}$ is a valid adjustment set if and only if it satisfies the adjustment criterion relative to (i, k) in G.

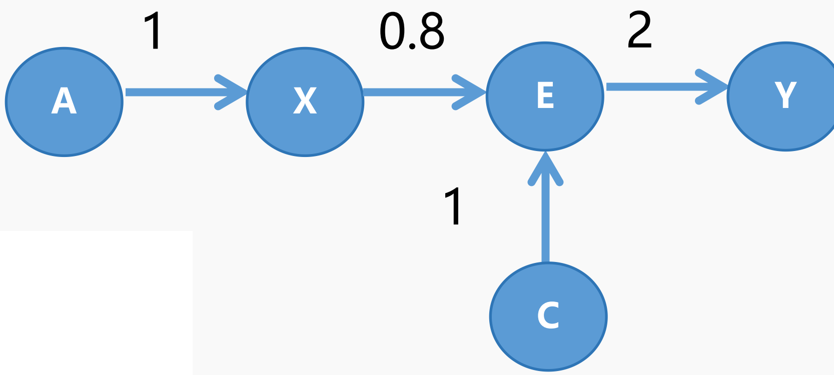
◆ 不应该调整尽可能多的变量 — 某些变量会引入偏差

◆ 调整"处理前"变量并非总是安全 — 可能产生 M-偏差

◆ 父节点调整
◆ 后门准则
◆ 调整准则

1. Covariate adjustment

◆ 线性SEM中的统计效率



真实因果效应：0.8 × 2 = 1.6

Statistical efficiency in linear SEMs

- We focused so far on sets that provide asymptotically correct causal effects
- We did not consider statistical efficiency
- Now: look at how to find the asymptotically most efficient valid adjustment set
  - I.e. which set yields estimator with the smallest asymptotic variance?
- Intuition for statistically efficient estimates in linear regression setting:
  - Try to avoid variables that are strongly correlated with X_i
    - This blows up the standard error
  - Try to use variables that help predict X_k
    - This decreases the residual variance and hence decreases the standard error
    - This may mean using optional variables that are not strictly needed

线性回归中的直觉

应该避免的变量：

◆ 与处理变量 X 强相关的变量

◆ 原因：多重共线性会增大标准误差

应该包含的变量：

◆ 有助于预测结果变量 Y 的变量

◆ 原因：减少残差方差，降低标准误差

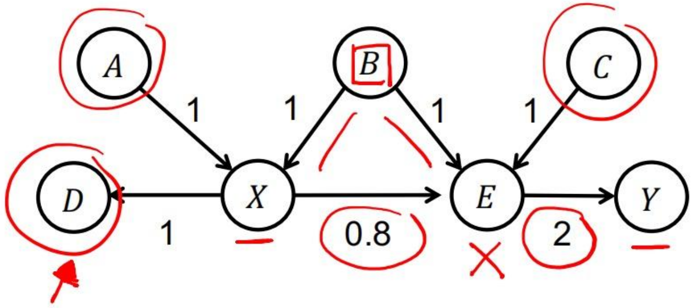
调整集	与X相关?	预测Y?
{A}	✓ 高度相关	✗
∅	✗	✗
{C}	不相关	✓

1. Covariate adjustment

◆ 线性SEM中的统计效率——示例

Example

- Total causal effect of X on Y is $0.8 \cdot 2 = 1.6$
- There are 8 valid adjustment sets $\boldsymbol{Z}$:
  - B has to be included ◆ 必须包含B
  - E may not be included ◆ 不能包含E
  - A, C, D may be in $\boldsymbol{Z}$ ◆ 可选变量ACD
- Which one should we use?
  - Variance varies significantly
    - See R script `06_adjustment_efficiency.R`
  - $\mathrm{pa}(X) = \{A, B\}$ large variance
  - Minimal set $\{B\}$ average
  - $\{B, C\}$ best



调整集	方差表现
{B}	最小调整集
{A，B}	父节点调整集，方差最大
{B，C}	最优调整集，方差小
{B，D}	
{A，B，C}	
{A，B，D}	
{B，C，D}	
{A，B，C，D}	

{B，C}调整集最优的原因：

◆ C 帮助预测 E，E 预测 Y

◆ 减少了 Y 的残差方差

◆ 而 C 与 X 不相关，不会增加共线性

1. Covariate adjustment

◆ 最优有效调整集策略

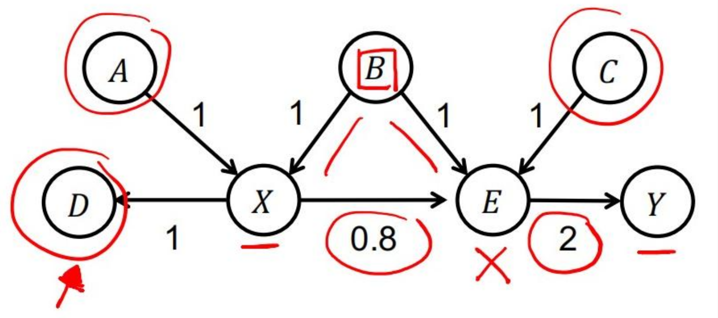
Optimal valid adjustment set in linear SEMs

- Let $G = (\boldsymbol{V}, \boldsymbol{E})$ be a DAG and $i, k \in \boldsymbol{V}, i \neq k$, and $k \in \mathrm{desc}(i)$
- Causal nodes $\mathrm{cn}(i, k)$: nodes $r \neq i$ on a directed path from i to k in G
- Forbidden nodes $\mathrm{forb}(i, k)$: descendants of causal nodes and node i
- Let $\hat{\tau}_{ik}^{\boldsymbol{Z}}$ denote the total causal effect estimator based on $\boldsymbol{Z}$

- Optimal valid adjustment set: $\boldsymbol{O}(i, k) = \mathrm{pa}(\mathrm{cn}(i, k)) \setminus \mathrm{forb}(i, k)$
  - If a valid adjustment set exists, $\boldsymbol{O}$ is one
  - For any valid adjustment set $\boldsymbol{Z}$

$$\mathrm{a.var}(\hat{\tau}_{ik}^{\boldsymbol{O}}) \leq \mathrm{a.var}(\hat{\tau}_{ik}^{\boldsymbol{Z}})$$

  where $\mathrm{a.var}$ denotes the asymptotic variance



定义几个关键概念

◆ 因果节点 cn(i, k)：从 i 到 k 的有向路径上，除了 i 以外的所有节点
◆ 禁止节点 forb(i, k)：因果节点的后代和节点 i 本身
◆ 最优有效调整集 O(i, k)

最优有效调整集：
因果节点的父节点，排除禁止节点

例子：
cn(X, Y)={E, Y}
pa(cn(X, Y))={B, X, C, E}
forb(X, Y)={X, E, Y}
O(X, Y)={B, C}

最优调整集（Optimal Adjustment Set）：在线性SEM中，能产生最小渐近方差的有效调整集

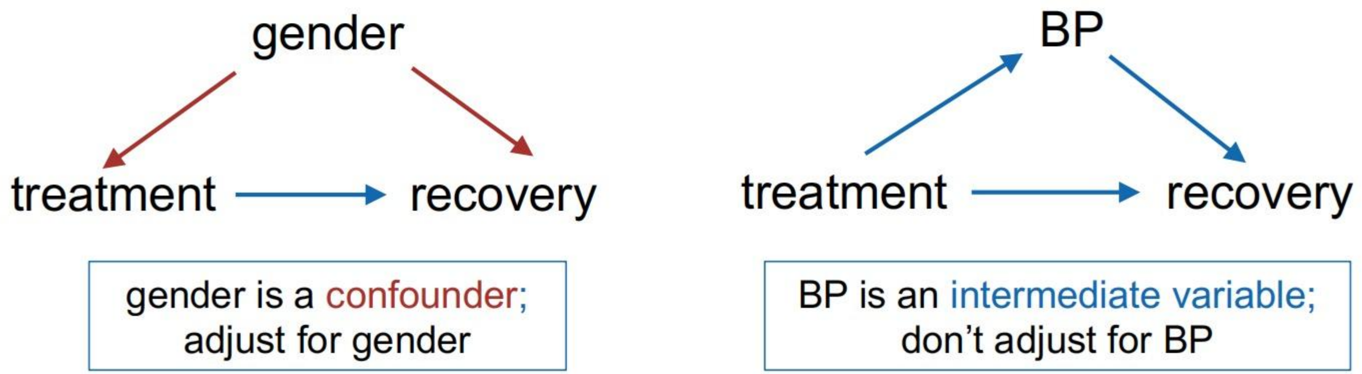
2. Direct causal effects

◆ 总因果效应VS直接因果效应

Total vs. direct causal effects

- Covariate adjustment concerns total causal effects
- Note that the causal question of interest is context-dependent
- Consider total causal effect of treatment on recovery
  - Possible scenarios:

两个对比场景（都是关于治疗对康复的因果效应）



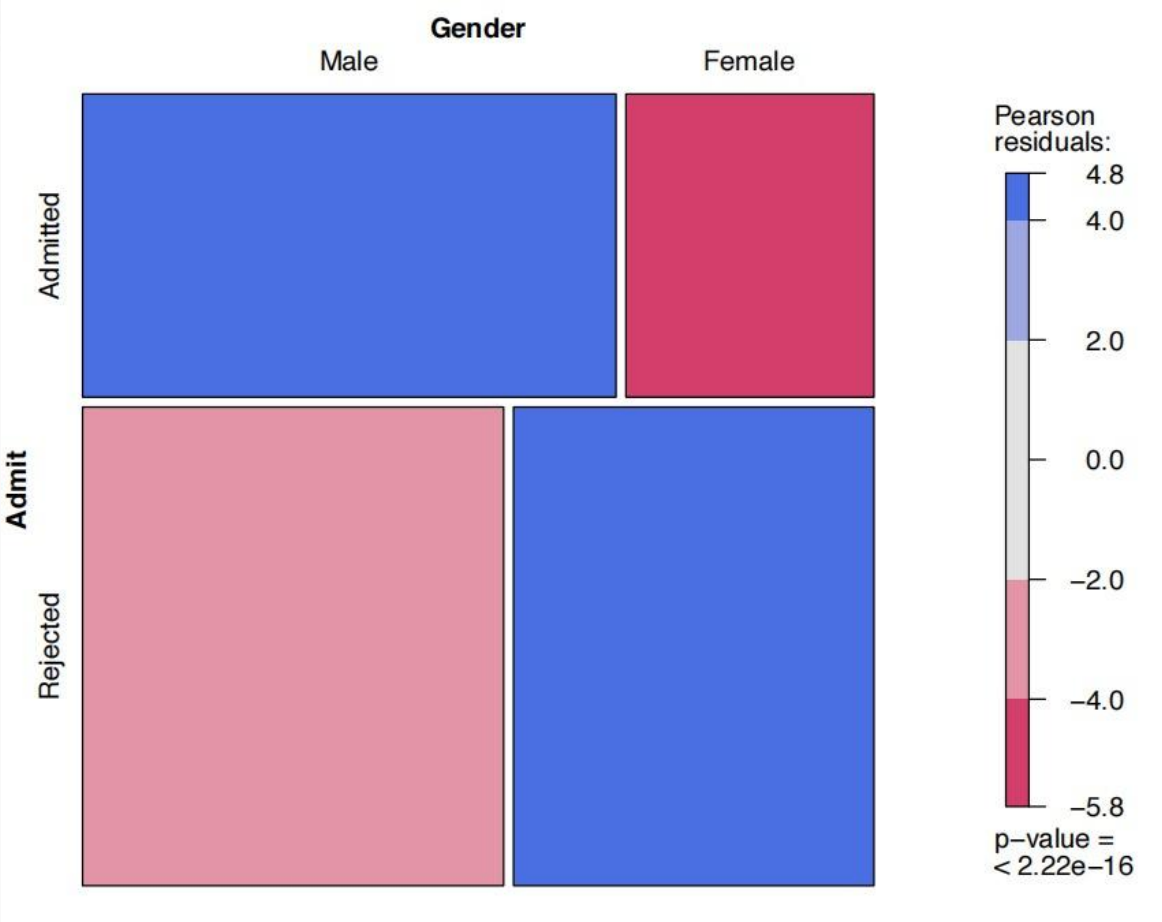
性别作为混淆因子（confounder）　血压（BP）作为中介变量（intermediate variable）

- ◆ 协变量调整（covariate adjustment）关注的是总因果效应
- ◆ 但我们感兴趣的因果问题取决于具体情境

2. Direct causal effects

◆ 辛普森悖论案例

◆ 针对UC Berkeley 1973年的调查 声称存在性别歧视

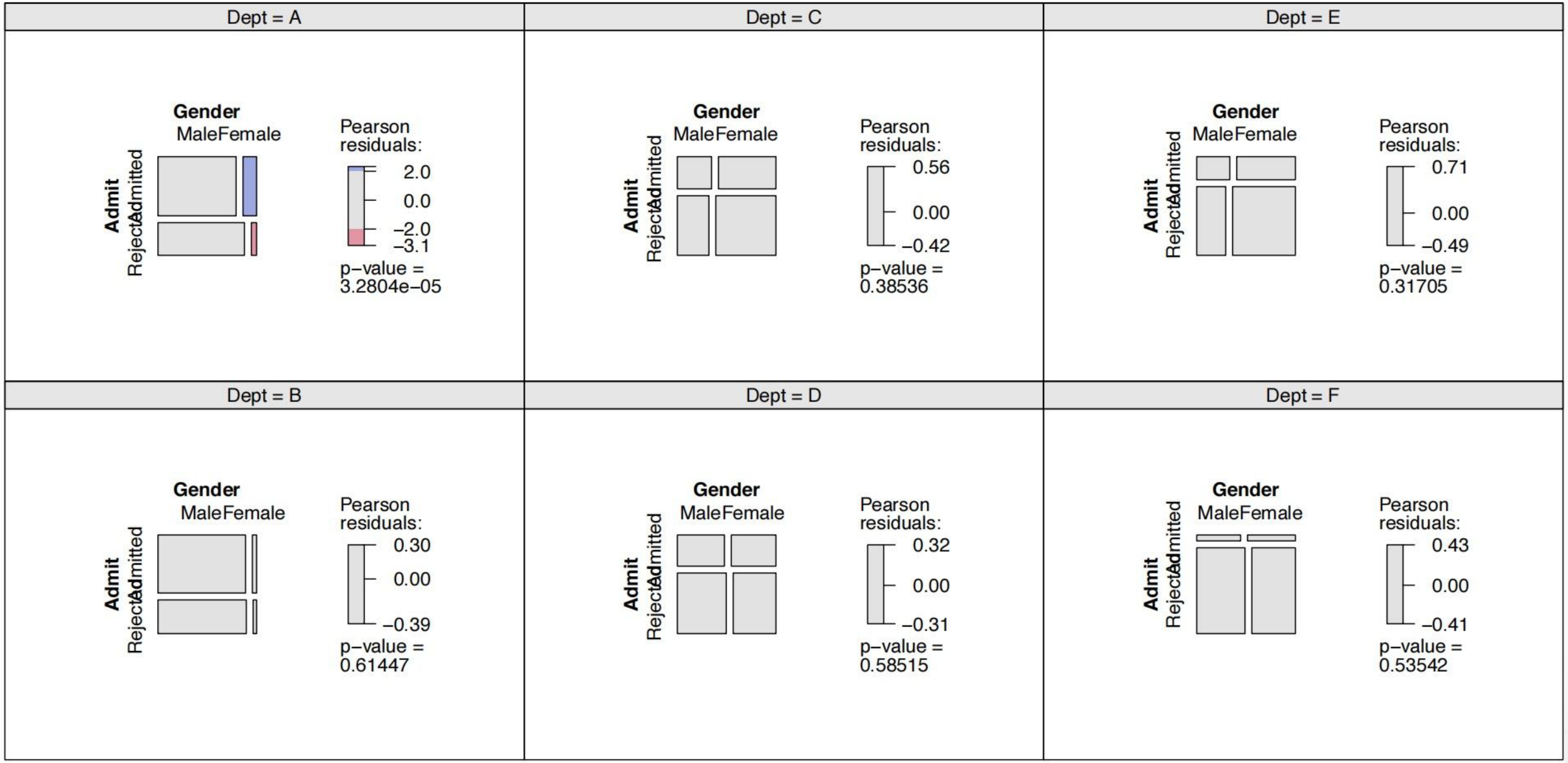


2. Direct causal effects

◆ 辛普森悖论案例

◆ 按院系A-F分层后，大多数院系没有性别差异

大多数院系的p值都不显著（>0.05）
院系A甚至显示女性更有优势



2. Direct causal effects

◆ 总因果效应VS直接因果效应

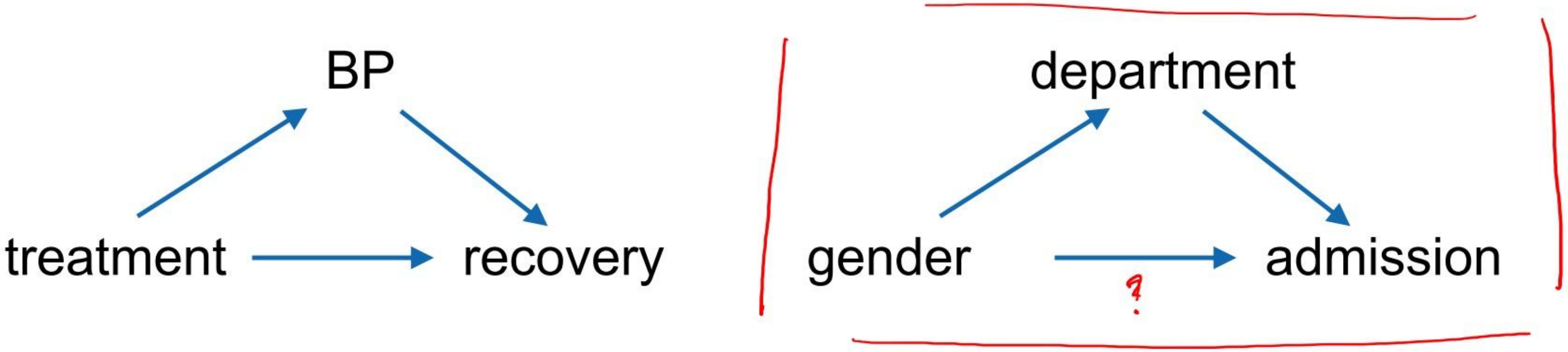
两个图结构相同，但问题不同

◆ 血压例子：想知道治疗的总因果效应（包括通过血压的路径）

◆ Berkeley例子：想知道性别的直接因果效应（排除院系选择的影响）

Total vs. direct causal effects

- Covariate adjustment concerns total causal effects
- Note that the causal question of interest is context-dependent
- Recall Berkeley admissions example from first lecture:



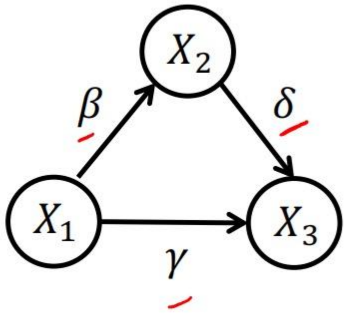
  - Same graph structure
  - Berkeley example: want to evaluate the existence of a direct causal effect

Berkeley问题本质是：在相同院系内，性别是否影响录取？ 这是直接效应问题

2. Direct causal effects

◆ 如何计算直接因果效应

Direct causal effects



- How do we compute the direct causal effect?
  - In a linear SEM: given by the edge weights

    线性SEM中： 直接因果效应就是边的权重
- There are different definitions for a direct causal effect
  - E.g. "controlled direct effect", "natural direct effect"
    - Will not go into much detail in this course
  - In a linear SEM, these definitions coincide – hence, speak of "the" direct causal effect

- Controlled direct effect: 控制直接效应：固定所有其他变量后，将xi 增加1单位对 xk 的影响

$$\text{CDE} = E(X_k | do(x_i), do(pa(k)\backslash i)) - E(X_k | do(x_i + 1), do(pa(k)\backslash i))$$

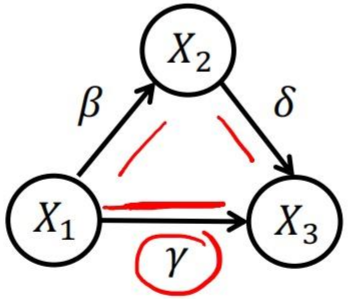
  - Question: What level to choose for $pa(k)\backslash i$?

对于k的其他父节点，要怎么赋值呢？

2. Direct causal effects

◆ 线性SEM中的直接因果效应

Direct causal effects



- How do we compute the direct causal effect?
  - In a linear SEM: given by the edge weights
- Controlled direct effect:

$$\text{CDE} = E(X_k | do(x_i), do(pa(k)\backslash i)) - E(X_k | do(x_i + 1), do(pa(k)\backslash i))$$

  - Example:

$$\boxed{\begin{aligned} X_1 &\leftarrow \epsilon_1 \\ X_2 &\leftarrow \beta X_1 + \epsilon_2 \\ X_3 &\leftarrow \gamma X_1 + \delta X_2 + \epsilon_3 \end{aligned}}$$

$$E(X_3 | do(x_1 + 1), do(x_2)) - E(X_3 | do(x_1), do(x_2)) = \gamma(x_1 + 1) + \delta x_2 - (\gamma x_1 + \delta x_2) = \gamma$$

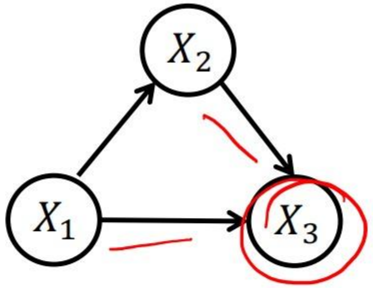
  - In a linear SEM, CDE does not depend on the level of $pa(k)\backslash i$

结论：在线性SEM中，直接因果效应CDE不依赖于固定变量的水平，直接等于边权重γ

2. Direct causal effects

◆ 当变量间存在交互作用时

Direct causal effects



- How do we compute the direct causal effect?
  - What if there is an interaction between X_1 and X_2?
- Controlled direct effect:

$$\mathrm{CDE} = E(X_k | do(x_i), do(pa(k)\backslash i)) - E(X_k | do(x_i + 1), do(pa(k)\backslash i))$$

  - Question: What level to choose for $pa(k)\backslash i$?

- Can show: In the given graph, CDE from X_1 on X_3

$$\mathrm{CDE} = E(X_3 | x_1, x_2) - E(X_3 | x_1 + 1, x_2)$$

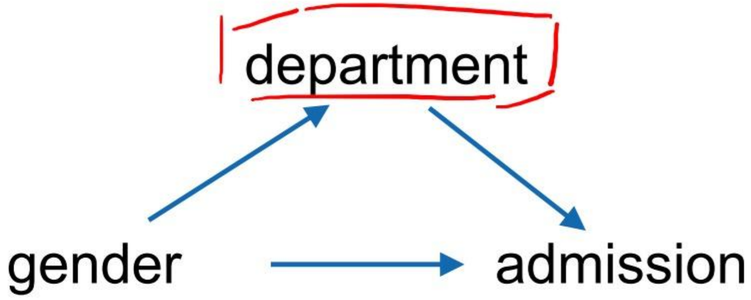
  - Question: What level to choose for X_2?

2. Direct causal effects

◆ Berkeley案例的直接因果效应

Direct causal effects

- How do we compute the direct causal effect?



- Controlled direct effect:
  In the given graph, CDE:

$$E(Admission|Gender = Female, Dept = d) - E(Admission|Gender = Male, Dept = d)$$

  - Question: What level to choose for d?
    - In this example: Can report the controlled direct effect for every department
    - Not always practical or possible

◆ 可以报告每个院系的控制直接效应
◆ 但这并不总是实际可行的

- Further optional reading:
  - Petersen, Sinisi, and van der Laan. Estimation of Direct Causal Effects.

https://journals.lww.com/epidem/fulltext/2006/05000/estimation_of_direct_causal
effects.12.aspx

References and acknowledgments

- Slides adapted from M. Maathuis, Emilija Perković, and Leonhard Henckel
- Shalizi (2019). Chapter 20.3 - 20.3.1
- Some examples from
  - Pearl and Mackenzie (2018). The Book of Why.
  - Pearl (2009). Causality: Models, Reasoning and Inference.

Thanks for your　listening！