

# Numerical Partial Differential Equations

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## 1 Finite Difference Methods by Jon Shiach

1. Introduction to PDEs unit I
2. Introduction to PDEs unit II
3. Finite-Difference Approximations
4. Finite Difference Schemes
5. Elliptic PDEs I
6. Elliptic PDEs II
7. Hyperbolic PDEs I
8. Hyperbolic PDEs II
9. Extension to Multidimensions
10. Systems of PDEs

## 2 Finite Volume Methods by Randy LeVeque

1. Derivation of Conservation Laws
2. Variable Coefficient Advection default
3. Linearization of Nonlinear Systems
4. Linear Hyperbolic Systems
5. Linear Systems - Riemann Problems
6. Linear Systems Nonhyperbolic Cases
7. Introduction to Finite Volume Methods
8. Accuracy, Consistency, Stability, CFL Condition
9. Dissipation, Dispersion, Modified Equations
10. High resolution TVD methods
11. TVD Methods and Limiters
12. Nonlinear Scalar PDEs, Traffic flow
13. Nonlinear scalar and rarefaction waves
14. Finite Volume Methods for Scalar Conservation Laws
15. Admissible Solutions and Entropy Functions
16. Convergence to Weak Solutions and Nonlinear Stability
17. Nonlinear systems - Shock Waves and Hugoniot Loci
18. Rarefaction waves and integral curves
19. Gas dynamics and Euler equations
20. Finite volume methods for nonlinear systems
21. Approximate Riemann solvers
22. Multidimensional hyperbolic problems
23. Fractional step methods
24. Multidimensional finite volume methods
25. Acoustics in Heterogeneous Media

### 3 Boundary Element Methods by CBMS/NSF

1. Introduction to Fast Direct Solvers for Elliptic PDEs
2. The Classical Fast Multipole Method
3. The Interpolative Decomposition
4. Introduction to Structured Matrix Algebra
5. Randomized Methods for Low-Rank Approximation
6. Fast Direct Solvers for Sparse Matrices
7. The Hierarchical Poincare-Steklov Scheme
8. Boundary Integral Equations and the Nystrom Method
9. Fast Direct Solvers for Integral Equations
10. Scattering Matrices
11. Boundary Integral Equations
12. The Hierarchical Poincare-Steklov Scheme

## 4 Finite Element Methods by Patrick E. Farrell

- |       |        |         |         |         |
|-------|--------|---------|---------|---------|
| 1. 1A | 9. 4A  | 17. 8A  | 25. 11B | 33. 14A |
| 2. 1B | 10. 4B | 18. 8B  | 26. 12A | 34. 14B |
| 3. 1C | 11. 5A | 19. 9A  | 27. 12B | 35. 14C |
| 4. 2A | 12. 5B | 20. 9B  | 28. 12C | 36. 15A |
| 5. 2B | 13. 6A | 21. 10A | 29. 13A | 37. 15B |
| 6. 2C | 14. 6B | 22. 10B | 30. 13B | 38. 15C |
| 7. 3A | 15. 7A | 23. 10C | 31. 13C |         |
| 8. 3B | 16. 7B | 24. 11A | 32. 13D |         |

## 5 Spectral Method by Bartosz Protas

1. 01	6. 06	11. 11	16. 16	21. 21
2. 02	7. 07	12. 12	17. 17	22. 22
3. 03	8. 08	13. 13	18. 18	23. 23
4. 04	9. 09	14. 14	19. 19	24. 24
5. 05	10. 10	15. 15	20. 20	25. 25

## 6 Machine Learning based Methods by F. Xiong

1. 物理信息神经网络, PINN 原文, 综述, PINN 代码详细讲解
2. 区域分解: CPINN, XPINN. 并行, 自适应激活函数
3. 残差点采样方法: DeepXDE(自适应加密), gPINN, 系统研究
4. 改善网络训练方式: bcPINN, Seq2seq\_Curriculum learning, Causality
5. Loss 中的权重设计: 动态权重, point-weighted, gwPINN
6. 变分法框架: 基础知识回顾, DGM, Deep Ritz, hp-VPINNs
7. PINN 与离散数值格式结合
8. PINN 失效的探索和改进 1: Loss 的 landscape, 代码, 频率原则简介
9. PINN 失效的探索和改进 2: NTK 理论, 傅里叶特征嵌入, Multi-scale DNN
10. PINN 解双曲守恒律方程 1: cvPINN, discrete divergence operator
11. PINN 解双曲守恒律方程 2: PINN 解欧拉方程文献综述, 反问题上的应用
12. 算子学习 1: 泛函基础知识回顾, Chen & Chen(1995), DeepONet 原文
13. 算子学习 2: Physics-Informed DeepONet, V-DeepONet
14. 算子学习 3: MIONet, DeepM & Mnet, Multifidelity DeepONet
15. 算子学习 4: FNO, DeepONet 与 FNO 的对比
16. 算子学习, 课程回顾, 互动交流

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