

# Numerical Ordinary Differential Equations

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1. Introductory Lecture
2. Review of Polynomial Interpolation
3. Numerical Differentiation
4. Complex Function Method of Approximating Derivatives
5. Richardson Extrapolation
6. Roundoff Error and Higher-Order Methods
7. Numerical Quadrature based on Interpolation
8. Newton-Cotes Formula  $n = 2$
9. Method of undetermined coefficients and Simpson's rule
10. Composite Quadrature Rules
11. Romberg Integration
12. Pseudocode for Romberg Algorithm
13. Gaussian Quadrature - Motivation and Example
14. Gaussian Quadrature - Analysis
15. Adaptive Quadrature
16. Adaptive Quadrature - Pseudocode
17. Existence and Uniqueness of Solutions of Differential Equations
18. Well-posedness of initial value problem
19. Euler's method
20. Error Analysis - Local vs. Global Error
21. Discrete Gronwall Lemma
22. Convergence of Euler's method
23. Taylor series method

24. [Runge-Kutta Method of Order 2](#)
25. [Implicit Runge-Kutta methods - Introduction](#)
26. [Butcher Tableaus and Examples of Runge-Kutta Methods](#)
27. [Stepsize Control](#)
28. [4th/5th order Runge-Kutta-Fehlberg Method](#)
29. [Multistep Methods](#)
30. [Examples of Adams-Bashforth Methods](#)
31. [Adams-Moulton Methods](#)
32. [Order of multistep methods](#)
33. [Order of Adams-Bashforth methods](#)
34. [Solution theory for linear difference equations](#)
35. [Root condition and Dahlquist equivalence theorem](#)
36. [Local truncation error and order of accuracy](#)
37. [Global truncation error](#)
38. [Systems of higher-order ODEs](#)
39. [Autonomous differential equations](#)
40. [Taylor method applied to a system of ODEs](#)
41. [Introduction to boundary-value problems](#)
42. [Existence and uniqueness of boundary-value problems](#)
43. [Shooting method for boundary-value problems](#)
44. [Linear two-point boundary-value problems](#)
45. [Newton's Method Applied to Shooting Method for BVPs](#)
46. [Multiple shooting for BVPs](#)
47. [Discretizing Boundary-Value Problems using Finite-Differences](#)
48. [Convergence of finite-difference discretization of boundary-value problem](#)

49. Systems of linear differential equations
50. Solutions of systems of linear differential equations in terms of matrix exponentials
51. How to compute the matrix exponential in special cases
52. Using the matrix exponential to solve an IVP, and the general case with the Jordan canonical form
53. Stiff equations
54. Modified Euler's method
55. General linear multistep methods and A-stability
56. Region of absolute stability and nonlinear systems
57. Classification of linear partial differential equations
58. Parabolic equations: Explicit methods
59. Parabolic equations: Explicit methods (Stability Analysis)
60. Parabolic equations: Explicit methods (Fourier stability analysis)
61. Parabolic equations: Implicit methods
62. Parabolic equations: Crank-Nicolson method
63. Parabolic equations: Error analysis of explicit method
64. Problems without Time Dependence: Finite-Differences
65. Problems without Time Dependence: Finite-Differences (Algorithm)
66. Problems without Time Dependence: Galerkin Methods
67. Galerkin method applied to the Dirichlet problem
68. Poisson equation
69. Rayleigh-Ritz method
70. Characteristic curves
71. Quasilinear second-order equations: Characteristics