Calculus and Analysis

Calculus I by Siao-Hao Guo

1.	01	11. 11	21. 21	31. 31	41. 41	51. 5 1
2.	02	12. 12	22. 22	32. 32	42. 42	52. 52
3.	03	13. 13	23. 23	33. 33	43. 43	53. 53
4.	04	14. 14	24. 24	34. 34	44. 44	54. 54
5.	05	15. 15	25. 25	35. 35	45. 45	55. 55
6.	06	16. 16	26. 26	36. 36	46. 46	56. 56
7.	07	17. 17	27. 27	37. 37	47. 47	
8.	08	18. 18	28. 28	38. 38	48. 48	
9.	09	19. 19	29. 29	39. 39	49. 49	
10.	10	20. 20	30. 30	40. 40	50. 50	

Calculus II by Siao-Hao Guo

- 1. 01
- 2. 02
- 3. 03
- 4. 04
- 5. 05
- 6. 06
- 7. 07
- 8. 08
- 9. 09
- 10. 10
- 11. Topology in \mathbb{R}^n

12. Topology in \mathbb{R}^n 13. Topology in \mathbb{R}^n 14. Topology in \mathbb{R}^n 15. Topology in \mathbb{R}^n 16. Partial, Directional Derivatives 17. Continuity, Differentiability 18. Continuity, Differentiability 19. Differentiability and Its Application 20. Twice Differentiation, Clairaut's Theorem, Chain Rule 21. Chain Rule 22. Applications of the Chain Rule, Maximum, Minimum 23. **23** 24. **24** 25. **25** 26. **26** 27. **27** 28. **28** 29. **29** 30. <mark>30</mark> 31. 31 32. 32

33. **33**

34. **34**

35. **35**

36. <mark>36</mark>

- 37. **37**
- 38. **38**
- 39. **39**
- 40. 40
- 41. 41
- 42. **42**
- 43. 43
- 44. 44
- 45. Line Integral
- 46. Line Integral
- 47. Green's Theorem
- 48. Green's Theorem
- 49. Surface Integral
- 50. Surface Integral
- 51. Stokes's Theorem
- 52. Stokes's Theorem
- 53. Divergence Theorem
- 54. Divergence Theorem

Analysis I by Chun-Yen Shen

- 1. Real Numbers and the completeness
- 2. Limits and continuity in metric space
- 3. Open and closed sets in metric space
- 4. Topological space & The closure of the set
- 5. Equivalence of continuous functions & Inheritance Principle & Homeomorphism

- 6. Product metric spaces & Completeness & Compactness (sequentially compact)
- 7. Compactness (2) & Bolzano-Weierstrass Thm & Nested seq of nonempty compact sets
- 8. ontinuous functions and Homeomorphisms on compact sets & Connectedness
- 9. Some properties in topological spaces
- 10. Accumulation points and isolated points & Dense & Hausdorff space
- 11. Properties of Connectedness
- 12. Path connected & Compactness in metric space, Hausdorff space, Topological space(1)
- 13. Compactness (2) & Sequentially compact implies compact(open cover) (1)
- 14. Sequentially compact implies compact(open cover) (2) & Totally bounded (1)
- 15. Totally bounded (2) & Perfect & Cantor set
- 16. Cantor surjection Theorem
- 17. Peano Curves & Riemann integrable & Darboux integrable(1)
- 18. Darboux integrable(2) & Riemann integrable iff Darboux integrable(1)
- 19. Riemann integrable iff Darboux integrable(2) & Some integrable functions (1)
- 20. Some integrable functions (2) & Lebesgue number & Zero set
- 21. Oscillation & Riemann-Lebesgue Theorem
- 22. Some applications of R-L theorem
- 23. Fundamental Theorem of Calculus & Convergence in functional space
- 24. Complete functional space & Convergence function series & Integrals wst unif conv.
- 25. Derivatives w.s.t uniform convergence & Equicontinuity and Arzela-Ascoli Theorem
- 26. Heine-Borel Theorem in a function space & Stone-Weierstrass Theorem
- 27. Fixed point, contraction and ODE
- 28. Nowhere differentiable continuous function & Baire's Theorem
- 29. Corollary of Baire's Theorem
- 30. Multiple integral and Fubini's Theorem

31. Change of variables formula (1) 32. Change of variables formula (2) 33. Banach spaces 34. Banach algebra 35. Differentiation in Banach Spaces 36. Some examples & Integration in Banach spaces (1) 37. Integration in Banach spaces (2) 38. Inverse function theorem 39. Lebesgue Outer measure 40. Lebesgue measurable set & Abstract outer measure 41. Measurable Sets form Sigma-Algebra & Measure continuity 42. Regularity (1) 43. Regularity (2) 44. Every open set is a countable disjoint union of balls plus a zero set 45. Measure Product & Inner measure (1) 46. Inner measure (2) & Slice Measure (1) 47. Slice Measure (2) & Lebesgue integrable functions & Monotone Convergence Theorem 48. Completed undergraph & Dominated Convergence Thm and Fatou's Lem & Basic properties 49. Some basic properties of measurable functions (2) 50. Cavalieri's principle & Equivalence measurable function definition & Tonelli Thm 51. Vitali covering Lemma 52. Density Theorem

Analysis II by Chun-Yen Shen

1. Lebesgue Differentiation Theorem (1)

- 2. Lebesgue Differentiation Theorem (2) & Absolutely continuous functions
- 3. Indefinite integral is abs conti. & A monotone function has a derivative a.e. (1)
- 4. A monotone function has a derivative almost everywhere (2)
- 5. Riemann integrals vs Lebesgue integrals
- 6. Bounded variation
- 7. Equi relation of B-V functions and abs conti functions & R-S integrals (1)
- 8. Riemann-Stieltjes integral (2)
- 9. Riemann-Stieltjes integral (3)
- 10. Riemann-Stieltjes integral (4)
- 11. $L^{2}[a,b]$ space
- 12. Properties of L^2 [a, b] space
- 13. Orthogonal basis in L^2
- 14. Fourier coefficients and Parseval's formula
- 15. A necessary condition of Fourier series converges in L^1 & Riemann-Lebesgue lemma
- 16. Dirichlet kernel
- 17. Dini Theorem & There is a continuous function whose Fourier series diverges (1)
- 18. There is a continuous function whose Fourier series diverges (2) & U-B Principle
- 19. There is a continuous function whose Fourier series diverges (3)
- 20. Some properties of Fourier coefficients & Cesàro summability
- 21. Fejér's Theorem & Approximate integrable functions by tri-polynomial in L^1 norm
- 22. The average of Fourier series converges in L^p norm
- 23. Equi. of the average of Fourier series converges unif & Weakly convergence(1)
- 24. Weakly convergent(2) & Equi condition for tri-series to be the Fourier series
- 25. Convergence of Cesàro mean of Fourier series (1)
- 26. Part2 Conv of Cesàro mean of Fourier series(2), Conjugate Fourier series & Dini theorem

- 27. Singular integrals and conjugate function
- 28. Bound of the L^2 norm of conjugate function & Hardy-Littlewood Maximal function
- 29. Bound of the norm of the supremum of conjugate functions
- 30. A counterexample & C-Z Decomposition & The conjugate function exists a.e. in L^1
- 31. The conjugate function exists a.e. in L^1 (2)
- 32. The conjugate function exists a.e. in L^1 (3) & Weakly estimate
- 33. Applications of Fourier series -isoperimetric inequality
- 34. Applications of Fourier series -continuous but nowhere differentiable function
- 35. Fourier transform and some properties
- 36. Fourier inversion formula and Minkowski's inequality
- 37. Poisson summation formula & Roth's theorem on 3-term AP
- 38. Discrete Fourier series, Fourier transform and some properties (1)
- 39. Discrete Fourier series, Fourier transform and some properties (2)
- 40. Bourgain's theorem
- 41. Proof of Roth's theorem on 3-terms AP (step 1)
- 42. Proof of Roth's theorem on 3-terms AP (step 2 to step 7)
- 43. Proof of Roth's theorem on 3-terms AP (step 9, step 8)
- 44. Proof of Roth's theorem on 3-terms AP (step 8)
- 45. There is f in L^1 such that for a.e. x, the Fourier series diverges (1)
- 46. There is f in L^1 such that for a.e. x, the Fourier series diverges (2)
- 47. There is f in L^1 such that for a.e. x, the Fourier series diverges (3)
- 48. There is f in L^1 such that for a.e. x, the Fourier series diverges (4)

June 5, 2025