Calculus and Analysis

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Calculus I

22. Improper integral (2)
23. Taylor's series: Gamma function, power series, expansion of the logarithm
24. Taylor's series: Taylor's theorem, Cauchy's & Lagrange's remainder
25. Taylor's series: estimate remainder term, expansion of elementary
26. Taylor's series: examples
27. Taylor's polynomial: interpolation (1)
28. Taylor's polynomial: interpolation (2), approximation in first order
29. Taylor's polynomial: approximation in second order
30. Taylor's polynomial: fixed point approximation, Stirling's formula
31. Series: concepts of convergence and divergence (1)
32. Series: rearrangement, tests for absolute convergence and divergence (1)
33. Series: tests for abs. convergence and divergence (2), sequences of functions
34. Series: pointwise/uniform convergence (1)
35. Series: pointwise/uniform convergence (2)
36. Series: power series, interval of convergence
37. Series: product of two power series
38. Series: expansion of given power series, infinite product
39. Fourier series: periodic function, complex notation (1)
40. Fourier series: complex form(2), trigonometric formula, Riemann Lebesgue Lemma
41. Fourier series: examples of Fourier expansion
42. Fourier series: main theorem on Fourier expansion
43. Fourier series: examples of Fourier series (1)
44. Fourier series: examples of Fourier series (2), Bessel's Inequality
45. approximation by trigonometric and rational polynomial (1)

46. approximation by trigonometric and rational polynomial (2)
47. approximation by trigonometric and rational polynomial (3)
48. inner product
49. Bernoulli polynomial and their applications (1)
50. Bernoulli polynomial and their applications (2)

Calculus II

1. functions of multiple variables & partial derivative
2. continuity
3. differentiability & directional derivative (1)
4. directional derivative (2) & tangent plane
5. change of variables & Taylor series
6. MVT & Taylor expansion & Integral
7. double integral & length of curve
8. curvature & linear differential one form
9. line integral
10. Heine-Borel theorem
11. compact subset & Implicit Function theorem (1)
12. Implicit Function theorem (2)
13. Inverse Function theorem (1)
14. Inverse Function theorem (2) & extreme value
15. Lagrange Multiplier method
16. examples & Hölder Inequality
17. sufficient condition for local max/min
18. Jordan measurable
19. Jordan area (1)
20. Jordan area (2)
21. double integral (1)
22. double integral (2)
23. transformation of multiple integrals

24. improper multiple integrals (1)
25. improper multiple integrals (2) & volumes
26. surface area
27. surface area formula
28. multiple integral in curved coordinate
29. extend to R^n
30. integral of unbounded set
31. Fourier integral theorem (1)
32. Fourier integral theorem (2)
33. Fourier transform - decay property & Parseval's identity
34. Fourier transform of several variables
35. Green theorem
36. Divergence theorem
37. prove Inverse Function theorem by Green theorem
38. orientation of surface in R^3 (1)
39. orientation of surface in R^3 (2)
40. Gauss theorem in R^3 (1)
41. Gauss theorem in R^3 (2)
42. application of fluid
43. Stoke's theorem
44. general surface (partition of unit) (1)
45. general surface (partition of unit) (2)
46. general surface (partition of unit) (3) & functions of one complex variable
47. complex function - power series

48. complex function - differentiability
49. conformal map & integration of analytic functions - Cauchy' s theorem
50. complex integral theorem & Cauchy integral formula
51. zeros, poles and residues

Analysis I

1. Real Numbers and the completeness
2. Limits and continuity in metric space
3. Open and closed sets in metric space
4. Topological space & The closure of the set
5. Equivalence of continuous functions & Inheritance Principle & Homeomorphism
6. Product metric spaces & Completeness & Compactness (sequentially compact)
7. Compactness (2) & Bolzano-Weierstrass Thm & Nested seq of nonempty compact sets
8. ontinuous functions and Homeomorphisms on compact sets & Connectedness
9. Some properties in topological spaces
10. Accumulation points and isolated points & Dense & Hausdorff space
11. Properties of Connectedness
12. Path connected & Compactness in metric space, Hausdorff space, Topological space(1)
13. Compactness (2) & Sequentially compact implies compact(open cover) (1)
14. Sequentially compact implies compact(open cover) (2) & Totally bounded (1)
15. Totally bounded (2) & Perfect & Cantor set
16. Cantor surjection Theorem
17. Peano Curves & Riemann integrable & Darboux integrable(1)
18. Darboux integrable(2) & Riemann integrable iff Darboux integrable(1)
19. Riemann integrable iff Darboux integrable(2) & Some integrable functions (1)
20. Some integrable functions (2) & Lebesgue number & Zero set
21. Oscillation & Riemann-Lebesgue Theorem
22. Some applications of R-L theorem
23. Fundamental Theorem of Calculus & Convergence in functional space

24. Complete functional space & Convergence function series & Integrals wst unif conv.
25. Derivatives w.s.t uniform convergence & Equicontinuity and Arzela-Ascoli Theorem
26. Heine-Borel Theorem in a function space & Stone-Weierstrass Theorem
27. Fixed point, contraction and ODE
28. Nowhere differentiable continuous function & Baire' s Theorem
29. Corollary of Baire' s Theorem
30. Multiple integral and Fubini' s Theorem
31. Change of variables formula (1)
32. Change of variables formula (2)
33. Banach spaces
34. Banach algebra
35. Differentiation in Banach Spaces
36. Some examples & Integration in Banach spaces (1)
37. Integration in Banach spaces (2)
38. Inverse function theorem
39. Lebesgue Outer measure
40. Lebesgue measurable set & Abstract outer measure
41. Measurable Sets form Sigma-Algebra & Measure continuity
42. Regularity (1)
43. Regularity (2)
44. Every open set is a countable disjoint union of balls plus a zero set
45. Measure Product & Inner measure (1)
46. Inner measure (2) & Slice Measure (1)
47. Slice Measure (2) & Lebesgue integrable functions & Monotone Convergence Theorem

48. Completed undergraph & Dominated Convergence Thm and Fatou' s Lem & Basic properties

49. Some basic properties of measurable functions (2)

50. Cavalieri's principle & Equivalence measurable function definition & Tonelli Thm

51. Vitali covering Lemma

52. Density Theorem

Analysis II

1. Lebesgue Differentiation Theorem (1)
2. Lebesgue Differentiation Theorem (2) & Absolutely continuous functions
3. Indefinite integral is abs conti. & A monotone function has a derivative a.e. (1)
4. A monotone function has a derivative almost everywhere (2)
5. Riemann integrals vs Lebesgue integrals
6. Bounded variation
7. Equi relation of B-V functions and abs conti functions & R-S integrals (1)
8. Riemann-Stieltjes integral (2)
9. Riemann-Stieltjes integral (3)
10. Riemann-Stieltjes integral (4)
11. L^2 $[a, b]$ space
12. Properties of L^2 $[a, b]$ space
13. Orthogonal basis in L^2
14. Fourier coefficients and Parseval's formula
15. A necessary condition of Fourier series converges in L^1 & Riemann-Lebesgue lemma
16. Dirichlet kernel
17. Dini Theorem & There is a continuous function whose Fourier series diverges (1)
18. There is a continuous function whose Fourier series diverges (2) & U-B Principle
19. There is a continuous function whose Fourier series diverges (3)
20. Some properties of Fourier coefficients & Cesàro summability
21. Fejér's Theorem & Approximate integrable functions by tri-polynomial in L^1 norm
22. The average of Fourier series converges in L^p norm
23. Equi. of the average of Fourier series converges unif & Weakly convergence(1)

24. Weakly convergent(2) & Equi condition for tri-series to be the Fourier series
25. Convergence of Cesàro mean of Fourier series (1)
26. Part2 Conv of Cesàro mean of Fourier series(2), Conjugate Fourier series & Dini theorem
27. Singular integrals and conjugate function
28. Bound of the L^2 norm of conjugate function & Hardy-Littlewood Maximal function
29. Bound of the norm of the supremum of conjugate functions
30. A counterexample & C-Z Decomposition & The conjugate function exists a.e. in L^1
31. The conjugate function exists a.e. in L^1 (2)
32. The conjugate function exists a.e. in L^1 (3) & Weakly estimate
33. Applications of Fourier series -isoperimetric inequality
34. Applications of Fourier series -continuous but nowhere differentiable function
35. Fourier transform and some properties
36. Fourier inversion formula and Minkowski's inequality
37. Poisson summation formula & Roth's theorem on 3-term AP
38. Discrete Fourier series, Fourier transform and some properties (1)
39. Discrete Fourier series, Fourier transform and some properties (2)
40. Bourgain's theorem
41. Proof of Roth's theorem on 3-terms AP (step 1)
42. Proof of Roth's theorem on 3-terms AP (step 2 to step 7)
43. Proof of Roth's theorem on 3-terms AP (step 9 , step 8)
44. Proof of Roth's theorem on 3-terms AP (step 8)
45. There is f in L^1 such that for a.e. x, the Fourier series diverges (1)
46. There is f in L^1 such that for a.e. x, the Fourier series diverges (2)
47. There is f in L^1 such that for a.e. x, the Fourier series diverges (3)

48. There is f in L^1 such that for a.e. x, the Fourier series diverges (4)

September 16, 2025