

Functional Analysis

Claudio Landim

1. Linear Spaces: Definition, Examples and Linear Span.
2. Linear Spaces: Quotient Spaces and Convex Sets.
3. Normed Linear Spaces: Definition and Basic Properties.
4. Completing a Normed Linear Space.
5. Finite Dimensional Linear Spaces.
6. Examples of Normed Linear Spaces.
7. In Infinite Dimensions the Unit Ball is not Compact.
8. Zorn's Lemma.
9. The Hahn-Banach Theorem.
10. Convex Sets and Gauge Functions.
11. Geometric Hahn-Banach Theorems.
12. Dual of a Normed Linear Space.
13. Extension of Bounded Linear Functionals, Closed Linear Spans.
14. Reflexive Spaces.
 - (a) The Dual Space of $C([a,b])$.
 - (b) An Application of the Hahn-Banach Theorem: the Moment Problem and Chebyshev Approximation.
 - (c) A Dual Variational Problem in Optimal Control.
 - (d) An Application of the Hahn-Banach Theorem: the Existence of a Green Function.
15. Hilbert Spaces.
16. Closed Convex Subsets of a Hilbert Space.
17. Riesz and Lax-Milgram Representation Theorems.
18. Orthonormal Sets and Closed Linear Spans.
19. Orthonormal Bases.

- (a) A Quadratic Variational Problem.
 - (b) The Dirichlet Principle.
 - (c) Generalized Derivatives and Sobolev Spaces.
20. Uniform Boundedness Principle.
 21. Weak Convergence.
 22. Uniform Boundedness of Weak Converging Sequences.
 23. Weak Sequentially Compactness.
 24. Weak* Topology.
 25. Applications of Weak Convergence.
 26. Bounded Linear Operators.
 27. Transpose of Bounded Linear Operators.
 28. Strong and Weak Convergence of Operators.
 29. Principle of Uniform Boundedness for Maps and Compositions.
 30. Open Map Principle.
 31. The Closed Graph Theorem.
 32. Examples of Bounded Linear Maps: Integral Operators.
 33. Symmetric Operators.
 34. Eigenvalues of Compact Symmetric Operators.
 35. The Fredholm Alternative.
 36. An Application to Integral Operators.
 37. Materials