H10.

- The assignment is due at Gradescope on Monday, November 22, 2021 at 10pm. Submit early and often.
- Read and sign the collaboration and honesty policy. Submit the signed policy to Gradescope before submitting any work.
- Unless otherwise specified, you can leave your answer in closed form (e.g. $1 - \binom{120}{7}(0.1)^{200}$).
- *Show your work.* Answers without justification will be given little credit. Justify each step in your solutions e.g. by stating that the step follows from an axiom of probability, a definition, algebra, etc.; for example, your answer could include a line like this:

$$\Pr(X \cap Y \cap Z) \cdot \Pr(A \cup B) = \Pr(X \cap Y \cap Z) \cdot (\Pr(A) + \Pr(B)) \qquad (A \text{ and } B \text{ are disjoint})$$

- The syllabus has some pointers on using LaTeX and Python.

In answering some of the following problems, you may find it helpful to connect the random variables to the distributions we have covered in class, and use the properties of the distributions in your answers.

PROBLEM 1. A new promotion at Insomnia Cookies lets you play the following game: by paying a fixed amount of money, you can get a free cookie every day, selected uniformly at random from chocolate chunk, double chocolate mint, and peanut butter chip, until you get a chocolate chunk (after which the promotion will stop). Suppose you decide to play this game.

(a) Let X be the number of free cookies you obtain with this promotion. Find $\text{Ex}(X)$.

(b) Let Y be the number of free cookies you get **that are not chocolate chunk**. Find $\text{Ex}(Y)$.

Solution:

PROBLEM 2. Suppose that the price of a stock starts at \$10. Each day, the price of the stock increases by \$1 with probability 1/3 and decreases by \$1 with probability 2/3, independently of other days. Find the probability that after six days the stock has its original price of \$10.

Solution:

PROBLEM 3. Charles claims that he can distinguish between beer and ale 75 percent of the time. Ruth bets that he cannot and, in fact, just guesses. To settle this, a bet is made: Charles is to be given ten small glasses, each having been filled with beer or ale, chosen by tossing a fair coin. He wins the bet if he gets seven or more correct. Find the probability that Charles wins if he has the ability that he claims. Find the probability that Ruth wins if Charles is guessing.

Solution:

PROBLEM 4 (Problem 19.19 in the textbook). BU students sometimes delay doing laundry until they finish their problem sets. Assume all random values described below are mutually independent.

(a) A busy student must complete 3 problem sets before doing laundry. Each problem set requires 1 day with probability $2/3$ and 2 days with probability $1/3$. Let B be the number of days a busy student delays laundry. What is $\text{Ex}(B)$?

Example: If the first problem set requires 1 day and the second and third problem sets each require 2 days, then the student delays for $B = 5$ days.

(b) A relaxed student rolls a fair, 6-sided die in the morning. If he rolls a 1, then he does his laundry immediately (with zero days of delay). Otherwise, he delays for one day and repeats the experiment the following morning. Let R be the number of days a relaxed student delays laundry. What is $\text{Ex}(R)$?

Example: If the student rolls a 2 the first morning, a 5 the second morning, and a 1 the third morning, then he delays for $R = 2$ days.

(c) Before doing laundry, an unlucky student must recover from illness. The number of days the student is ill is determined as follows. We toss a fair coin and we roll a fair 6-sided die. If the coin is heads, the student is ill for n days, where n is the outcome of the die roll. If the coin is tails, the student is ill for $2n$ days, where n is the outcome of the die roll. What is $\text{Ex}(U)$?

Example: If the die roll is 5, the student delays for $U = 5$ days if the coin toss is heads and $U = 10$ days if the coin toss is tails.

(d) A student is busy with probability $1/2$, relaxed with probability $1/3$, and unlucky with probability $1/6$. Let D be the number of days the student delays laundry. What is $\text{Ex}(D)$?

Solution:

PROBLEM 5. The company RandomCereal places figurines in cereal boxes via the following process: in each box, with probability 1/2 it places no figurine in the box, and with probability 1/2 it places some figurine in the box. When it places a figurine, it chooses one of 4 possible types uniformly and random.

(a) Suppose you have collected k of the 4 types of figurines, where $k \in \{0, 1, 2, 3\}$ (collecting a type of figurine means that you have at least one figurine of that type). What is the expected number of cereal boxes you need to open until you obtain a new type of figurine (that you have not already collected)? Your answer should be in terms of k.

(b) Suppose you do not have any of the figurines. What is the expected number of boxes you need to open to collect all 4 types of figurines?

Solution:

PROBLEM 6 (Simulations). Download the HW10 Jupyter notebook. Complete all the exercises in the notebook. Submit the Jupyter notebook with your solutions to the Homework 10 Programming assignment on Gradescope. Your submission should be a single .ipynb file.