H6.

- The assignment is due at Gradescope on Monday October 18, 2021 at 10pm. Submit early and often.
- Read and sign the collaboration and honesty policy. Submit the signed policy to Gradescope before submitting any work.
- Unless otherwise specified, you can leave your answer in closed form (e.g. $1 - \binom{120}{7}(0.1)^{200}$).
- *Show your work.* Answers without justification will be given little credit. Justify each step in your solutions e.g. by stating that the step follows from an axiom of probability, a definition, algebra, etc.; for example, your answer could include a line like this:

$$\Pr(X \cap Y \cap Z) \cdot \Pr(A \cup B) = \Pr(X \cap Y \cap Z) \cdot (\Pr(A) + \Pr(B)) \qquad (A \text{ and } B \text{ are disjoint})$$

- The syllabus has some pointers on using LaTeX and Python.

PROBLEM 1. The Red Sox are playing the Yankees in a 4 game series during the regular season. 4 games will be played no matter the outcome of each game. Suppose that the Red Sox have $1/2$ probability of winning each game, independently of other games. Let R be a random variable that is equal to the number of games won by the Red Sox in the series.

(a) Recall from lecture that any assertion about a random variable defines an event. For example, the assertion $R = 0$ is the event that the Red Sox do not win any games in the series. For each possible value x of R, write down the outcomes in the event $R = x$.

(b) Find the PDF f_R of the random variable R.

(c) Find the CDF F_R of the random variable R.

(d) What is $F_R(10)$? Please explain your answer in 1-2 sentences.

Solution:

PROBLEM 2. (a) We toss a fair coin three times. Find the probability that exactly two heads occur, given that the first toss was a heads.

(b) We roll a standard 6-sided die twice. Find the probability that the sum of the faces is greater than 7, given that the first roll was less than 5.

(c) We roll two standard 6-sided dice once. Find the probability that the sum of the two rolls is 6 given that the dice land on different numbers.

(d) Let x be a point selected uniformly at random from the interval $[0, 1]$. Find the probability that $x > 1/2$, given that $x^2 - x + 2/9 < 0$.

(e) We toss a dart at a circular target of radius 4 inches. Given that the dart lands in the upper half of the target, find the probability that its distance from the center is greater than 2 inches.

Solution:

PROBLEM 3. Suppose you are competing in a trivia show. There are two jury members that ask you 4 questions each. In order to advance to the next stage, you have to correctly answer at least 3 questions from each jury member. Suppose that you have probability 0.75 of answering any given question correctly, independently of other questions. Given that you answered 6 questions correctly (and thus you answered 2 questions incorrectly), what is the probability that you advance to the next stage?

Solution:

PROBLEM 4 (Problem 18.5 in the textbook). There are two decks of cards. One is complete, but the other is missing the ace of spades ($A\spadesuit$). Alice picks one of the two decks with equal probability and then selects a card from that deck uniformly at random.

(a) What is the probability that Alice picked the complete deck, given that she selected the queen of diamonds ($Q\diamondsuit$)?

(b) What is the probability that Alice picked the complete deck, given that she selected a queen?

(c) What is the probability that Alice picked the complete deck, given that she selected the ace of diamonds ($A\diamondsuit$)?

(d) What is the probability that Alice picked the complete deck, given that she selected an ace?

Solution:

PROBLEM 5 (Problem 18.2 in the textbook). Dirty Harry places two bullets in random chambers of the six-bullet cylinder of his revolver. He gives the cylinder a random spin and says "Feeling lucky?" as he holds the gun against your heart.

(a) What is the probability that you will get shot if he pulls the trigger?

(b) Suppose he pulls the trigger and you don't get shot. What is the probability that you will get shot if he pulls the trigger a second time?

(c) Suppose you noticed that he placed the two shells next to each other in the cylinder. How does this change the answers to the previous two questions?

Solution:

PROBLEM 6 (Programming exercises). Download the HW 6 Jupyter notebook (coming soon!). Complete all the exercises in the notebook. Submit the Jupyter notebook with your solutions to the Homework 6 Programming assignment on Gradescope. Your submission should be a single .ipynb file.