H7.

Problem 1

(a). $Pr(A \cap B) = Pr(A)Pr(B)$ if independent

Ω for $A = \{(1,2), (1,4), (1,6),$
$(2,1), (2,3), (2,5),$
$(3,2), (3,4), (3,6),$
$(4,1), (4,3), (4,5),$
$(5,2), (5,4), (5,6),$
$(6,1), (6,3), (6,5)\}$

18.

Ω for $B = \{(4,4), (4,5), (4,6),$
$(5,4), (5,5), (5,6)$
$(6,4), (6,5), (6,6)\}$.

9.

Total outcomes $= 6^2 = 36$ outcomes

$\therefore Pr(A) = \frac{18}{36} = \frac{1}{2}$ $\quad Pr(B) = \frac{9}{36} = \frac{1}{4}$

$Pr(A \cap B) = \frac{4}{36} = \frac{1}{9}$

$\therefore Pr(A)Pr(B) = \frac{1}{2}(\frac{1}{4}) = \frac{1}{8} \neq \frac{1}{9}$

$\therefore Pr(A \cap B) \neq Pr(A)Pr(B)$

so, not independent

(b). Ω for $C = \{(1,1), (2,2), (3,3), (4,4), (5,5), (6,6)\}$. 6

Ω for $D = \{(1,5), (1,6),$ $(4,5), (4,6)$
$(2,5), (2,6),$ $(5,5), (5,6)$
$(3,5), (3,6),$ $(6,5), (6,6)\}$.

12.

Total outcomes $= 6^2 = 36$

$\therefore Pr(C) = \frac{6}{36} = \frac{1}{6}$ $\quad Pr(D) = \frac{12}{36} = \frac{1}{3}$

$Pr(C \cap D) = \frac{2}{36} = \frac{1}{18}$

$\therefore Pr(C)Pr(D) = \frac{1}{6}(\frac{1}{3}) = \frac{1}{18}$

$\therefore Pr(C \cap D) = Pr(C)Pr(D)$

so, independent

Problem 2 $\quad Pr(BU) = 0.25 \quad Pr(NE) = 0.5 \quad Pr(BU \cap NE) = 0.2$

(a). $Pr(NE \mid BU) = \frac{Pr(BU \cap NE)}{Pr(BU)} = \frac{0.2}{0.25} = 0.8$

(b). $Pr(BU)Pr(NE) = 0.25(0.5) = 0.125 \neq 0.2$

$\because Pr(BU \cap NE) \neq Pr(BU)Pr(NE)$ $\quad \therefore$ not independent

(c). It reflects that if a student get accepted or rejected by any one of the two colleges, the probability of he/she accepted by the others will be affected.

Problem 3. $Pr(A\cap B)=Pr(A)Pr(B)$ $\quad Pr(A)=p \quad Pr(B)=q$

(a). $\because$ A and B independent

$\therefore Pr(A\cap B)=Pr(A)Pr(B)=$

$$Pr(A\setminus B)=Pr(A\cap\bar{B})=Pr(A)-Pr(A\cap B)$$
$$=Pr(A)-Pr(A)Pr(B)$$
$$=Pr(A)(1-Pr(B))$$
$$=Pr(A)Pr(\bar{B})$$

so, independent.

(b). $Pr(\bar{A}\cap\bar{B})=Pr(\bar{B}\cap\bar{A})=Pr(\bar{B}\setminus A)$

$\because$ A and $\bar{B}$ independent (proof in (a))

$\therefore Pr(A\cap\bar{B})=Pr(A)Pr(\bar{B})$

$$Pr(\bar{B}\setminus A)=Pr(\bar{A}\cap\bar{B})=Pr(\bar{B})-Pr(\bar{B}\cap A)$$
$$=Pr(\bar{B})-Pr(A\cap\bar{B})$$
$$=Pr(\bar{B})-Pr(A)Pr(\bar{B})$$
$$=Pr(\bar{B})(1-Pr(A))$$
$$=Pr(\bar{B})Pr(\bar{A})$$

so, independent.

(c) $Pr(A|\bar{B})=\frac{Pr(A\cap\bar{B})}{Pr(\bar{B})}$

$Pr(\bar{B})=1-Pr(B)=1-q$

$Pr(A\cap\bar{B})=p(1-q)$

$\therefore Pr(A|\bar{B})=\frac{p(1-q)}{q}=p$

(d). $Pr(A\cup\bar{B})=$ ✓

$$Pr(A)+Pr(\bar{B})-Pr(A\cap\bar{B})$$
$$=p+(1-q)-p(1-q)$$
$$=\cancel{p}+1-q-\cancel{p}+pq$$
$$=1-q+pq$$

Problem 4 $Pr(0\to1)=\frac{1}{4}$ $\quad Pr(0\to0)=\frac{3}{4}$ $\quad Pr(1\to0)=\frac{1}{3}$ $\quad Pr(1\to1)=\frac{2}{3}$

(a). $Pr(0)=Pr(1)=\frac{1}{2}$

$\frac{1}{2}$, 0	0 $\frac{3}{4}$	$\frac{1}{2}(\frac{3}{4})=\frac{3}{8}$	✓
	1 $\frac{1}{4}$	$\frac{1}{2}(\frac{1}{4})=\frac{1}{8}$	×
$\frac{1}{2}$, 1	0 $\frac{1}{3}$	$\frac{1}{2}(\frac{1}{3})=\frac{1}{6}$	×
	1 $\frac{2}{3}$	$\frac{1}{2}(\frac{2}{3})=\frac{1}{3}$	✓

$\therefore Pr(\text{Bob receives correctly})=\frac{3}{8}+\frac{1}{3}=\frac{17}{24}$

(b). $(\frac{2}{3})(\frac{3}{4})(\frac{2}{3})(\frac{2}{3})=\frac{2}{9}$

(c). $\Omega=\{(0,0,0),(0,0,1),(0,1,0),(1,0,0),$
$(0,1,1),(1,0,1),(1,1,0),(1,1,1)\}$ $\to 0$ $\to 1$

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$Pr(2\text{ 0s and 1 1 correctly})=(\frac{3}{4})(\frac{3}{4})(\frac{2}{3})=\frac{3}{8}$

$Pr(3\text{ 0s correctly})=(\frac{3}{4})(\frac{3}{4})(\frac{3}{4})=\frac{27}{64}$

$\therefore Pr(\text{at least 2 0s correctly})=\frac{3}{8}+\frac{27}{64}=\frac{51}{64}$

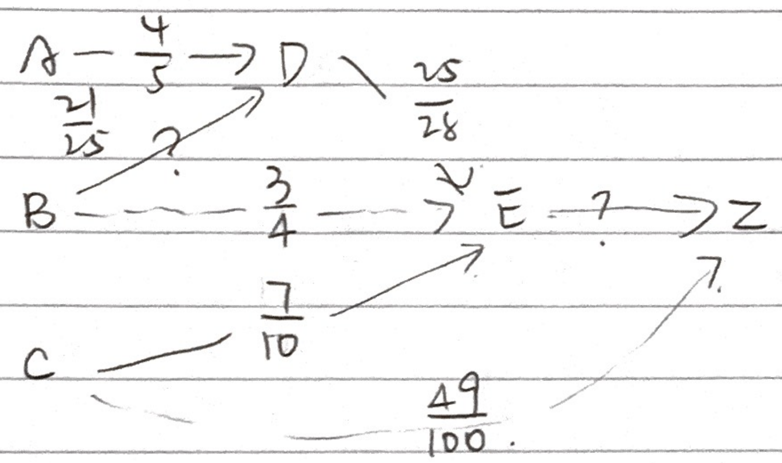
$\because Pr(101 \mid 2\ 1s \text{ and } 1\ 0) = \frac{1}{3}$

$\{(0,1,1), (1,0,1), (1,1,0)\}$.

$$\text{(d). } Pr(\overset{000}{\overset{\shortparallel}{0}} \mid 101) = \frac{Pr(0 \cap 101)}{Pr(101)} = \frac{Pr(0\cap 101)}{Pr(1\cap 101) + Pr(0 \cap 101)} \qquad \boxed{111 \to 101} \quad \boxed{000 \to 101}$$

$$= \frac{\overset{Pr(0)}{\frac{1}{2}}(\frac{1}{3})(\frac{1}{4})(\frac{3}{4})(\frac{1}{4})}{\underset{Pr(0)}{\frac{1}{2}}(\frac{1}{3})(\frac{1}{4})(\frac{3}{4})(\frac{1}{4}) + \underset{Pr(1)}{\frac{1}{2}}(\frac{1}{3})(\frac{2}{3})(\frac{1}{3})(\frac{2}{3})} = \frac{\frac{3}{64}}{\frac{3}{64} + \frac{4}{27}} = \frac{81}{337}$$

Problem 5.



$$B \to D = \frac{B \to E}{D \to E} = \frac{\frac{3}{4}}{\frac{25}{28}} = \frac{3}{4}\left(\frac{28}{25}\right) \frac{21}{25}$$

$$E \to Z = \frac{C \to Z}{C \to E} = \frac{\frac{49}{100}}{\frac{7}{10}} = \frac{49}{100}\left(\frac{10}{7}\right) = \frac{7}{10}$$

$\therefore A \to Z = (A \to D)(D \to E)(E \to Z) = (\frac{4}{5})(\frac{25}{28})(\frac{7}{10}) = \frac{1}{2}$

$B \to Z = (B \to D)(D \to E)(E \to Z) = (\frac{21}{25})(\frac{25}{28})(\frac{7}{10}) = \frac{21}{40}$

$C \to Z = (C \to E)(E \to Z) = (\frac{7}{10})(\frac{7}{10}) = \frac{49}{100}$

$\because \frac{49}{100} < \frac{1}{2} < \frac{21}{40}$ $\quad \therefore$ choose $B \to Z$