Problem 1

(a). HH HTH HTT

THH TTH THT TTT 7 outcomes

(b). $X = 0 \quad 1 \quad 2$

(c). $P(X=0) = \frac{1}{8}$

$P(X=1) = \frac{1}{8} + \frac{1}{8} + \frac{1}{8} = \frac{3}{8}$

$P(X=2) = \frac{1}{4} + \frac{1}{8} + \frac{1}{8} = \frac{4}{8}$

(d). $F(0 \leq X < 1) = \frac{1}{8}$

$F(1 \leq X < 2) = \frac{3}{8} + \frac{1}{8} = \frac{4}{8}$

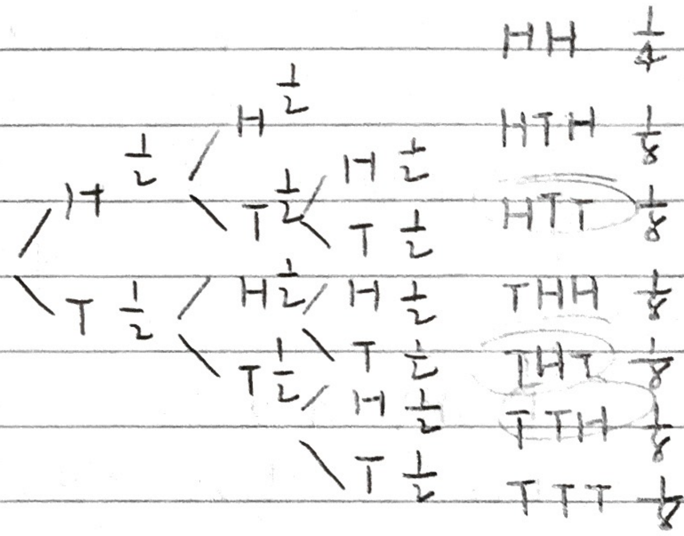
$F(X \geq 2) = \frac{3}{8} + \frac{4}{8} + \frac{1}{8} = 1$

Problem 2

(a). $F(x) = \begin{cases} \frac{1}{b-a} & a \leq x \leq b \\ 0 & \text{otherwise.} \end{cases}$

$[a,b] = [0,20]$

$F(x) = \begin{cases} \frac{1}{20-0} = \frac{1}{20} & 0 \leq x \leq 20 \\ 0 & \text{otherwise.} \end{cases}$



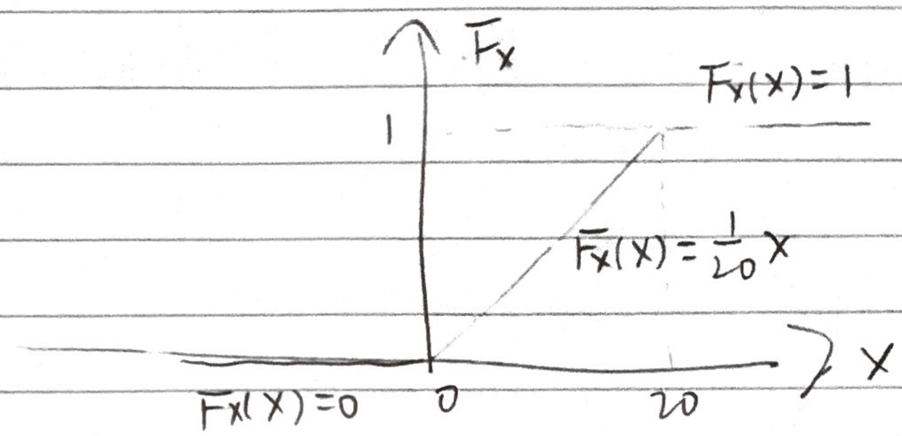
$F_X(x) = P(X \leq x) = \int_{-\infty}^{x} f(t)\,dt$

when $x < 0$, $F_X(x) = \int_{-\infty}^{x} 0\,dt = 0$

when $0 \leq x \leq 20$, $F_X(x) = \int_{-\infty}^{0} 0\,dt + \int_{0}^{x} \frac{1}{20}\,dt = 0 + \frac{x}{20} = \frac{x}{20}$.

when $x > 20$, $F_X(x) = \int_{-\infty}^{x} 0\,dt + \int_{0}^{20} \frac{1}{20}\,dt + \int_{20}^{\infty} 0\,dt = 0 + \frac{20}{20} + 0 = 1$

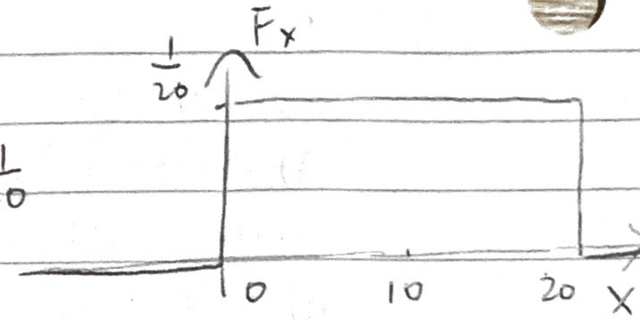
$F_X(x) = \begin{cases} 0 & x < 0 \\ \frac{x}{20} & 0 \leq x \leq 20 \\ 1 & x > 20 \end{cases}$



(b). when $x<0$, $\frac{d}{dx}(0)=0$ $\quad P(x)=0$

when $0\le x\le 20$, $\frac{d}{dx}(\frac{x}{20})=\frac{1}{20}$ $\quad P(x)=\frac{1}{20}$

when $x>20$, $\frac{d}{dx}(1)=0$ $\quad P(x)=0$

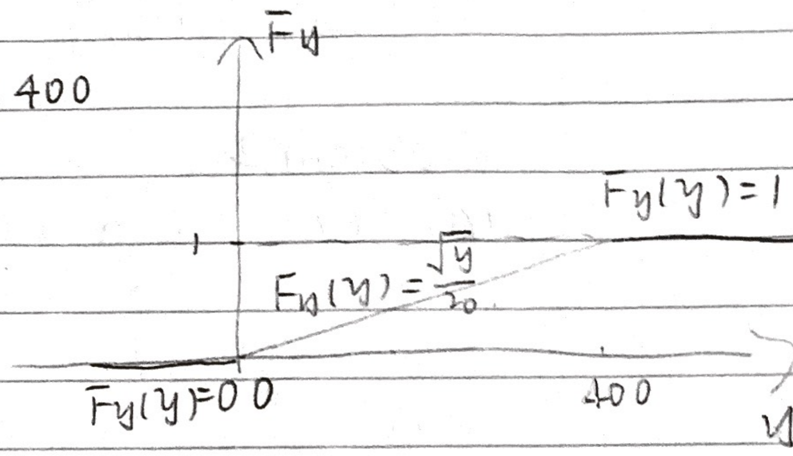


(c). $Y=X^2$

$F(y)=P(Y\le y)=P(X^2\le y)=P(X\le\sqrt{y})=F_X(\sqrt{y})$

$$F_X(\sqrt{y})=\begin{cases}0 & \sqrt{y}<0 & y<0\\ \frac{\sqrt{y}}{20} & 0\le\sqrt{y}\le 20 & 0\le y\le 400\\ 1 & \sqrt{y}>20 & y>400\end{cases}$$

$$\therefore F_Y(y)=\begin{cases}0 & y<0\\ \frac{\sqrt{y}}{20} & 0\le y\le 400\\ 1 & y>400\end{cases}$$



Problem 3

(a). X is discrete, because x is either \$1, \$50.50, \$10.10, or \$5.05 which is a finite set of possible values.

(b). Pr(\$50.50 = Ace ∩ heart) = Pr(Ace) + Pr(heart) − Pr(Ace ∪ heart)

$=\frac{4}{52}+\frac{13}{52}-\frac{1}{52}=\frac{16}{52}$

Pr(\$10.10 = face ∩ $\overline{\text{heart}}$) = $Pr(J\cap\overline{\heartsuit})+Pr(Q\cap\overline{\heartsuit})+Pr(K\cap\overline{\heartsuit})$

$=\frac{4-1}{52}+\frac{4-1}{52}+\frac{4-1}{52}=\frac{9}{52}$

Pr(\$5.05 = even ∩ $\overline{\text{heart}}$) = $Pr(2\cap\overline{\heartsuit})+Pr(4\cap\overline{\heartsuit})+Pr(6\cap\overline{\heartsuit})+Pr(8\cap\overline{\heartsuit})+Pr(10\cap\overline{\heartsuit})$

$=\frac{4-1}{52}+\frac{4-1}{52}+\frac{4-1}{52}+\frac{4-1}{52}+\frac{4-1}{52}=\frac{15}{52}$

Pr(\$1 = otherwise) $=1-\frac{16}{52}-\frac{9}{52}-\frac{15}{52}=\frac{12}{52}$

(c). $F_X(X<1)=0$

$F_X(1\le X<5.05)=0+\frac{12}{52}=\frac{12}{52}$

$F_X(5.05\le X<10.10)=0+\frac{12}{52}+\frac{15}{52}=\frac{27}{52}$

$F_X(10.10\le X<50.50)=0+\frac{12}{52}+\frac{15}{52}+\frac{9}{52}=\frac{36}{52}$

$F_X(50.50\le X)=0+\frac{12}{52}+\frac{15}{52}+\frac{9}{52}+\frac{36}{52}=1$

Problem 4

(a). $\because \int_{-\infty}^{\infty} f(x)\,dx = 1$

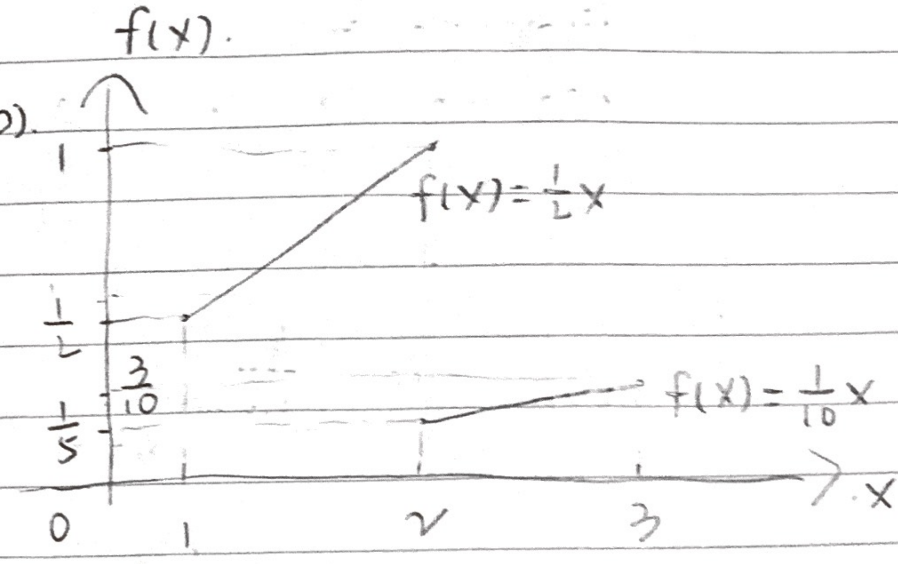
$\therefore \int_1^2 \frac{1}{2}x\,dx + \int_2^3 kx\,dx = 1$

$\frac{x^2}{4}\Big|_1^2 + k\left(\frac{x^2}{2}\right)\Big|_2^3 = 1$

$1 - \frac{1}{4} + k\left(\frac{9}{2} - \frac{4}{2}\right) = 1$

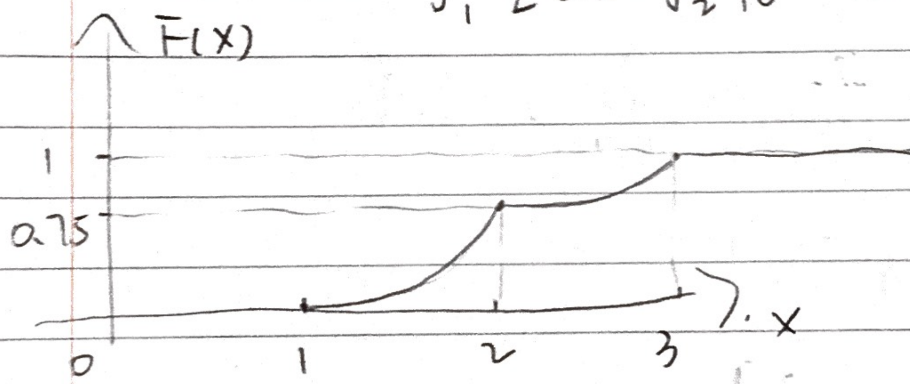
$\frac{3}{4} + \frac{5}{2}k = 1 \quad \therefore k = \frac{1}{10}$

(b).



(c). $F_X(1 \le x \le 2) = \int_1^x \frac{x}{2}\,dx = \frac{x^2}{4}\Big|_1^x = \frac{x^2}{4} - \frac{1}{4}$

$F_X(2 < x \le 3) = \int_1^2 \frac{x}{2}\,dx + \int_2^x \frac{1}{10}x\,dx = \frac{x^2}{4}\Big|_1^2 + \frac{x^2}{20}\Big|_2^x = 1 - \frac{1}{4} + \left(\frac{x^2}{20} - \frac{1}{5}\right) = \frac{11}{20} + \frac{x^2}{20}$



(d). $P_r(1.5 \le x \le 2.5) = F(2.5) - F(1.5) = \frac{2.5^2}{20} + \frac{11}{20} - \left(\frac{1.5^2}{4} - \frac{1}{4}\right) = \frac{11}{20}$

Problem 5

(a). $\because \int_{-\infty}^{\infty} f(x)\,dx = 1$

$\therefore \int_{-1}^{2} cx^2\,dx + 0 = 1$

$\int_{-1}^{2} cx^2\,dx = 1$

$c\left(\frac{x^3}{3}\right)\Big|_{-1}^{2} = 1$

$c\left(\frac{8}{3} + \frac{1}{3}\right) = 1 \quad \therefore c = \frac{1}{3}$

(b). $F(x) = \int_{-1}^{x} \frac{1}{3}x^2\,dx = \frac{x^3}{9}\Big|_{-1}^{x} = \frac{x^3}{9} + \frac{1}{9}$

(c). $Y = 2X - 1 \quad \therefore x = \frac{y+1}{2}$

$dY = 2dx \quad \therefore \frac{dx}{dY} = \frac{1}{2}$

$F(y) = \frac{1}{2}F(x) = \frac{1}{2}\left(\frac{1}{3}x^2\right) = \frac{1}{6}x^2$

$= \frac{1}{6}\left(\frac{y+1}{2}\right)^2 = \frac{(y+1)^2}{24}$

$2(-1) - 1 \le y \le 2(2) - 1 \quad -3 \le y \le 3$

Problem 6 $\frac{g}{g+v}$

(a). $g=1100 \quad v=1000$

$Pr(W=1)=\frac{1100}{1100+1000}=\frac{11}{21}$

$Pr(W=0)=1-\frac{11}{21}=\frac{10}{21}$

(b). $W=1 \quad \frac{g}{g+1000}$

$W=0$ revive $1-\frac{g}{g+1000}$

$W=1 \quad \frac{g}{g+1000}$

$W=0 \quad 1-\frac{g}{g+1000}$

$\therefore Pr(W=1)=\frac{g}{g+1000}+\left(1-\frac{g}{g+1000}\right)\left(\frac{g}{g+1000}\right)=0.9$

$\frac{g}{g+1000}+\frac{g}{g+1000}-\frac{g^2}{(g+1000)^2}=0.9$

$\frac{2g}{g+1000}-\frac{g^2}{(g+1000)^2}=0.9$

$\frac{2g(g+1000)-g^2}{g^2+2000g+1000^2}=0.9$

$g^2+2000g=0.9g^2+1800g+900000$

$0.1g^2+200g-900000=0$

$\because g>0 \quad \therefore g=1000(\sqrt{10}-1)\approx 2162$