Problem 1

(a) $4^2 = \boxed{16}$

$\therefore$ 16 distinct ways can be made for a round-trip from Brookline and Cambridge

$4 \cdot 3 = \boxed{12}$

$\therefore$ 12 ways can be made if take a different route on the back

(b). $4 \cdot 3 \cdot 2 \cdot 1 = \boxed{24}$

(c). $3 \cdot 4 = \boxed{12}$

Problem 2

(a). $\frac{3}{51} \cdot \frac{2}{50} = \frac{1}{425}$

(b). $4(12) = 48$

$4(11) = 44$

$\frac{48}{51} \cdot \frac{44}{50} = \frac{352}{425}$

(c). $\frac{3}{51} \cdot \frac{48}{50} = \frac{24}{425}$

	1st	2nd	3rd
(a).	1	$\frac{3}{51}$	$\frac{2}{50}$
(b).	1	$\frac{48}{51}$	$\frac{44}{50}$
(c).	1	$\frac{3}{51}$	$\frac{48}{50}$

Problem 3

$$\frac{(1+r+r^2+\dots+r^n)(r-1)}{(r-1)}$$

$$= \frac{(r-1)+r(r-1)+r^2(r-1)+\dots+r^n(r-1)}{r-1}$$

$$= \frac{(\cancel{r}-1)+(\cancel{r^2}-\cancel{r})+(\cancel{r^3}-\cancel{r^2})+\dots+(r^{n+1}-\cancel{r^n})}{r-1}$$

$$= \frac{-1+r^{n+1}}{r-1} = \frac{r^{n+1}-1}{r-1}$$

Problem 4

$x \in \mathbb{R}$

assume a is not even (is odd)

$\therefore a = 2x+1$

$\therefore a^2 = (2x+1)^2 = 4x^2+4x+1$

$\because 4x^2$ and $4x$ is even $\therefore 4x^2+4x+1$ is odd

$\therefore$ the assumption is false

$\therefore$ proved.

Problem 5

$(A \setminus B) \setminus C$

$= (A \cap \bar{B}) \cap \bar{C}$

$= A \cap \bar{B} \cap \bar{C}$

$= A \cap \bar{C} \cap \bar{B}$

$(A \setminus C) \setminus (B \setminus C)$

$= (A \cap \bar{C}) \cap \overline{(B \cap \bar{C})}$

$= (A \cap \bar{C}) \cap (\bar{B} \cup C)$

$= ((A \cap \bar{C}) \cap \bar{B}) \cup ((A \cap \bar{C}) \cap C)$

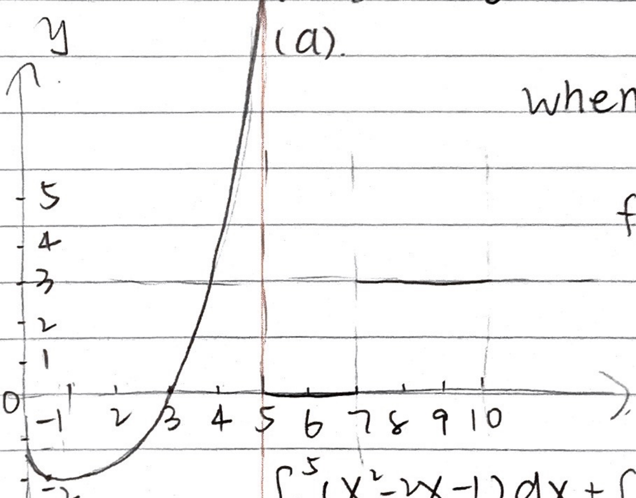
$= (A \cap \bar{C} \cap \bar{B}) \cup \varnothing$

$= A \cap \bar{C} \cap \bar{B}$

$\therefore (A \setminus B) \setminus C = (A \setminus C) \setminus (B \setminus C)$

Problem 6

(a)



when $x=0$, $f(x)=-1$

$x=\frac{1}{2}$, $f(x)=-2$

$f(x)=0$, $x=1\pm\sqrt{2}$ $\rightarrow$

$x=5$, $f(x)=14$

$x^2-2x+1=2$

$(x^2-1)=1$ $\quad x-1=\pm\sqrt{2}$

$x=1\pm\sqrt{2}$

(b) $\int_{-1}^{0} 0\,dx + \int_{0}^{3} (x^2-2x-1)\,dx$

$= 0 + \left(\frac{x^3}{3} - x^2 - x\right)\Big|_0^3 = 9-9-3 = -3$

$\int_{1}^{5} (x^2-2x-1)\,dx + \int_{5}^{7} 0\,dx + \int_{7}^{8} 3\,dx$

$= \left(\frac{x^3}{3} - x^2 - x\right)\Big|_1^5 + 0 + (3x)\Big|_7^8 = \left(\frac{125}{3} - 25 - 5\right) - \left(\frac{1}{3} - 1 - 1\right) + (24-21)$

$= \frac{124}{3} - 28 + 3 = \frac{49}{3}$

$\int_{6}^{7} 0\,dx + \int_{7}^{10} 3\,dx + \int_{10}^{12} 0\,dx = 0 + (30-21) + 0 = 7$

(c). $f'(x) = \begin{cases} 2x-2 & \text{for } x \in [0,5] \\ 0 & \text{for } x \in [7,10] \\ 0 & \text{otherwise} \end{cases}$

at $x = 0, 5, 7, 10$. $f'(x)$ DNE, because discontinuity.

(d). $h'(x) = g'(f(x))\, dg$

$= g'(f(x)) \frac{d}{dx}(f(x)) = g'(f(x)) f'(x)$

$h'(x) = \begin{cases} g'(x^2-2x-1)(2x-1) & \text{for } x \in [0,5] \\ g'(3)(0) = 0 & \text{for } x \in [7,10] \\ g'(0)(0) = 0 & \text{otherwise} \end{cases}$

$h'(x)$ DNE at $x = 0, 5, 7, 10$