Problem 1

(a).

$$R = \begin{cases} x=1 & \{(WLLL), (LWLL), (LLWL), (LLLW)\} \\ x=2 & \{(WWLL), (WLWL), (WLLW), (LWWL), (LWLW), (LLWW) \\ x=3 & \{(WWWL), (WWLW), (WLWW), (LWWW)\} \\ x=4 & \{(WWWW)\} \\ x=0 & \{(LLLL)\} \end{cases}$$

(b). PDF $f_R = \binom{4}{x}(\frac{1}{2})^x(\frac{1}{2})^{4-x}$

$$f_R = \begin{cases} x=1 & \binom{4}{1}(\frac{1}{2})^1(\frac{1}{2})^3 = 0.25 \\ x=2 & \binom{4}{2}(\frac{1}{2})^2(\frac{1}{2})^2 = 0.375 \\ x=3 & \binom{4}{3}(\frac{1}{2})^3(\frac{1}{2}) = 0.25 \\ x=4 & \binom{4}{4}(\frac{1}{2})^4(\frac{1}{2})^0 = 0.0625 \\ x=0 & \binom{4}{0}(\frac{1}{2})^0(\frac{1}{2})^4 = 0.0625 \end{cases}$$

(c).

$$\text{CDF } F_R = \begin{cases} x<0 & 0 \\ 0 \le x < 1 & 0.0625 \\ 1 \le x < 2 & 0.0625 + 0.25 = 0.3125 \\ 2 \le x < 3 & 0.0625 + 0.25 + 0.375 = 0.6875 \\ 3 \le x < 4 & 0.0625 + 0.25 + 0.375 + 0.25 = 0.9375 \\ x \geq 4 & 0.0625 + 0.25 + 0.375 + 0.25 + 0.0625 = 1 \end{cases}$$

(d) $F_R(10)$

$\because x = 10 \quad \therefore x \geq 4 \quad \therefore F_R(10) = 1$

Because when $x=10$, x belongs to $x \geq 4$ which is the case that $F_R = 1$

Problem 2

(a). {(HHH), (HHT), (HTH), (THH), (HTT), (THT), (TTH), (TTT)}

First toss has head.

= {(HHH), (HHT), (HTH), (HTT)}

Two head occurs

$\therefore Pr = \frac{2}{4} = \frac{1}{2}$

(b). ∵ first roll was less than 5

∵ sum is greater than 7

first roll	1	2	3	4
	1 2 3 4 5 6	1 2 3 4 5	1 2 3 4	1 2 3
second roll	\	6	5 6	4 5 6

$\therefore Pr = \dfrac{6}{4(6)} = \dfrac{1}{4}$

first $\in \{1, 2, 3, 4\}$ second $\in \{1, 2, 3, 4, 5, 6\}$

(c).

first roll	1	2	3	4	5	6
	2 3 4	1 3	1 2	1	2 3 4	1 2 3 4 5
second roll	5	4	$\cancel{3}$	2	1	\
	6	5 6	4 5 6	3 5 6	6	

$\therefore Pr = \dfrac{4}{6(5)} = \dfrac{4}{30} = \dfrac{2}{15}$

first $\in \{1, 2, 3, 4, 5, 6\}$ second $= 6 - \boxed{1} = 5$ same as first

(d). $x^2 - x + \frac{2}{9} < 0$

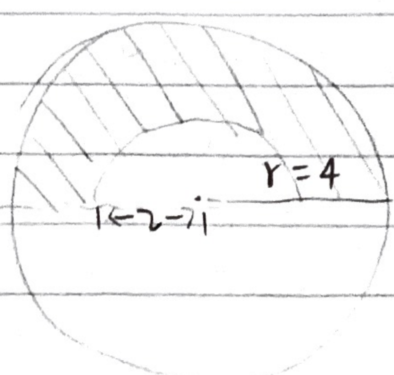
$9x^2 - 9x + 2 < 0$

$(3x-1)(3x-2) < 0$

$\therefore \frac{1}{3} < x < \frac{2}{3}$ $\because x > \frac{1}{2}$ $\therefore P(x > \frac{1}{2}) \cap P(\frac{1}{3} < x \frac{2}{3}) = \frac{2}{3} - \frac{1}{2}$

$Pr = \dfrac{\frac{2}{3} - \frac{1}{2}}{\frac{2}{3} - \frac{1}{3}} = \dfrac{\frac{1}{6}}{\frac{1}{3}} = \dfrac{1}{2}$

(e).



$A(\text{upper half}) = \frac{1}{2}\pi r^2 = \frac{1}{2}\pi(4^2) = 8\pi$

$A(\text{greater than } 2 \cap \text{upper half})$

$= 8\pi - \frac{1}{2}\pi(2^2) = 8\pi - 2\pi = 6\pi$

$\therefore Pr = \dfrac{6\pi}{8\pi} = \dfrac{6}{8} = \dfrac{3}{4}$

Problem 3

x for number of correct answered questions

$Pr(X=6) = \binom{8}{6}(0.75)^6(1-0.75)^{8-6} = \binom{8}{6}(0.75)^6(0.25)^2 = 0.3115$

$Pr(X=6 \cap X=3 \text{ for each jury}) =$

$= \binom{4}{3}(0.75)^3(1-0.75)^{4-3} \cdot \binom{4}{3}(0.75)^3(1-0.75)^{4-3}$ second jury

first jury $= [\binom{4}{3}(0.75)^3(0.25)]^2 = 0.1780$

$\therefore Pr = \frac{0.1780}{0.3115} = 0.5714$

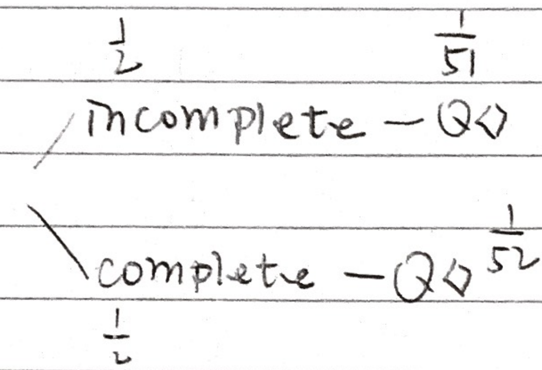
Problem 4

(a). $Pr(\text{complete}) = Pr(\text{incomplete}) = \frac{1}{2}$

$Pr(Q\diamondsuit \text{ from complete}) = \frac{1}{2}(\frac{1}{52})$

$Pr(Q\diamondsuit \text{ from incomplete}) = \frac{1}{2}(\frac{1}{51})$

$\therefore Pr = \frac{\frac{1}{2}(\frac{1}{52})}{\frac{1}{2}(\frac{1}{52}) + \frac{1}{2}(\frac{1}{51})} = 0.495$

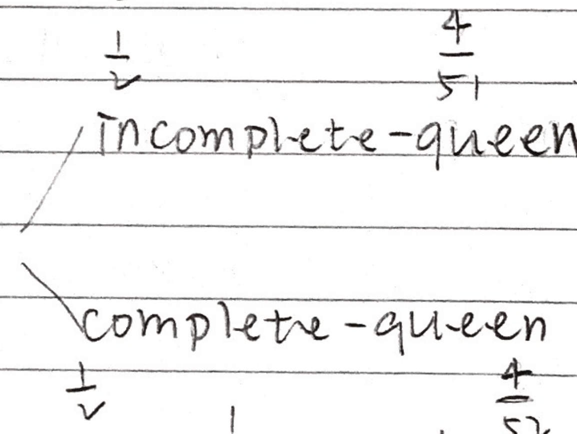


(b). $Pr(\text{complete}) = Pr(\text{incomplete}) = \frac{1}{2}$

$Pr(\text{queen from complete}) = \frac{1}{2}(\frac{13}{52})$

$Pr(\text{queen from incomplete}) = \frac{1}{2}(\frac{13}{51})$

$\therefore Pr = \frac{\frac{1}{2}(\frac{4}{52})}{\frac{1}{2}(\frac{4}{52}) + \frac{1}{2}(\frac{4}{51})} = 0.495$

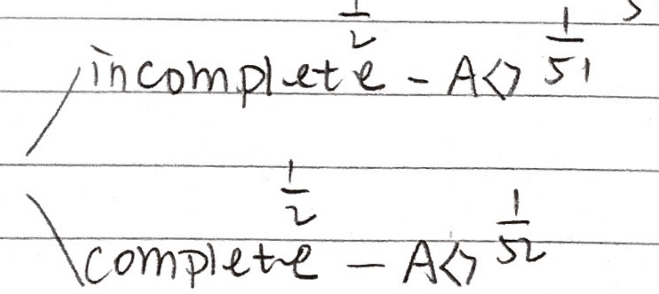


(c). $Pr(\text{complete}) = Pr(\text{incomplete}) = \frac{1}{2}$

$Pr(A\diamondsuit \text{ from complete}) = \frac{1}{2}(\frac{1}{52})$

$Pr(A\diamondsuit \text{ from incomplete}) = \frac{1}{2}(\frac{1}{51})$

$\therefore Pr = \frac{\frac{1}{2}(\frac{1}{52})}{\frac{1}{2}(\frac{1}{52}) + \frac{1}{2}(\frac{1}{51})} = 0.495$

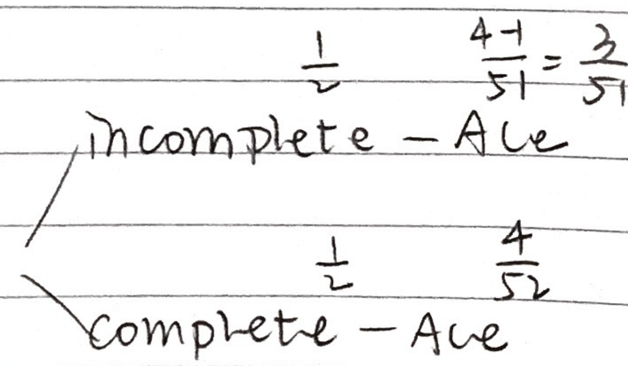


(d). $Pr(\text{complete}) = Pr(\text{incomplete}) = \frac{1}{2}$

$Pr(\text{ace from complete}) = \frac{1}{2}(\frac{4}{52})$

$Pr(\text{ace from incomplete}) = \frac{1}{2}(\frac{4-1}{51}) = \frac{1}{2}(\frac{3}{51})$

$\therefore Pr = \frac{\frac{1}{2}(\frac{4}{52})}{\frac{1}{2}(\frac{4}{52}) + \frac{1}{2}(\frac{3}{51})} = 0.567$



Problem 5

(a). $\because$ six-bullet cylinder and place two bullets

$\therefore Pr = \frac{2}{6} = \frac{1}{3}$

(b). Pr(don't get shoot for first time) $= \frac{6-2}{6} = \frac{4}{6} = \frac{2}{3}$

Pr(don't get shoot for first time $\cap$ get shoot for second time)

$= \frac{2}{3}\left(\frac{2}{6-1}\right) = \frac{2}{3}\left(\frac{2}{5}\right)$

$\therefore Pr = \dfrac{\frac{2}{3}\left(\frac{2}{5}\right)}{\frac{2}{3}} = \frac{2}{5}$

(c). For (a), it doesn't change the answer, because the order of bullets doesn't matter in this case.

For (b), don't get shoot for first time excluding the possibility of the position before the second bullet

Pr(don't get shoot for first time) remain unchanged

$\therefore$ Pr(don't get shoot for first time $\cap$ get shoot for second time

$= \frac{2}{3}\left(\frac{2-1}{6-1-1}\right) = \frac{2}{3}\left(\frac{1}{4}\right)$

$\therefore Pr = \dfrac{\frac{2}{3}\left(\frac{1}{4}\right)}{\frac{2}{3}} = \frac{1}{4}$