Problem 1

(a). $P=\frac{1}{3}$

$Ex(X)=\frac{1}{P}=\frac{1}{\frac{1}{3}}=3$

(b). ∵ after getting a chocolate chunk, the promotion will stop

∴ $Ex(Y)=Ex(X)-1=3-1=2$

Problem 2

∵ after 6 days the stock has original price of \$10.

∴ 3 increase and 3 decrease

$\Downarrow\frac{1}{3}$ $\Downarrow\frac{2}{3}$

$Pr=(\frac{1}{3})^3({}^6C_3)(\frac{2}{3})^3\approx 0.219$

Problem 3 $P=0.75$

$Pr(X\geq 7)=Pr(X=7)+Pr(X=8)+Pr(X=9)+Pr(X=10)$

$=(0.75)^7({}^{10}C_7)(0.25)^3+(0.75)^8({}^{10}C_8)(0.25)^2+(0.75)^9({}^{10}C_9)(0.5)$

$+(0.25)^{10}({}^{10}C_{10})(0.25)^0$

$=0.776$

∴ Pr(Charles wins) $=0.776$

$P=0.5$

$Pr(X<7)=1-Pr(X\geq 7)=1-(Pr(X=7)+\ldots+Pr(X=10))$

$=1-((0.5)^7({}^{10}C_7)(0.5)^3+(0.5)^8({}^{10}C_8)(0.5)^2+(0.5)^9({}^{10}C_9)(0.5)$

$+(0.5)^{10}({}^{10}C_{10})(0.5)^0)$

$=1-0.172=0.828$

Problem 4

(a). $P(1\text{ days})=\frac{2}{3}$ $P(2\text{ days})=\frac{1}{3}$

$Ex(\text{complete one Problem Set})=1(\frac{2}{3})+2(\frac{1}{3})=\frac{4}{3}$

∴ $Ex(B)=3(\frac{4}{3})=4$ days.

(b). $Ex(\text{finish laundry})=\frac{1}{P}=\frac{1}{\frac{1}{6}}=6$ days

$Ex(R)=Ex(\text{finish laundry})-1=6-1=5$ days

include the last day of succeed in getting a 1,

previous (6-1) days failure on getting a 1.

(c). $Ex(\text{delay days if heads}) = \frac{1+2+3+4+5+6}{6} = \frac{7}{2}$ days

$Ex(\text{delay days if tails}) = 2(Ex(\text{delay days if heads}))$

$= 2(\frac{7}{2}) = 7$ days

$\because Pr(\text{heads}) = Pr(\text{tails}) = \frac{1}{2}$

$\therefore Ex(U) = \frac{1}{2}(\frac{7}{2}) + \frac{1}{2}(7) = \frac{21}{4} = 5.25$ days

(d). $Ex(D) = \frac{1}{2}(4) + \frac{1}{3}(5) + \frac{1}{6}(5.25) \simeq 4.54$ days

Problem 5

(a). $Pr(\text{no figure}) = Pr(\text{some figures}) = \frac{1}{2}$

$Pr(\text{obtain figure is a new type}) = \frac{4-k}{4}$

$P = \frac{1}{2}(\frac{4-k}{4}) = \frac{4-k}{8}$

$Ex = \frac{1}{P} = \frac{1}{\frac{4-k}{8}} = \frac{8}{4-k}$

(b). $k=0$

$Ex = \frac{8}{4-0} = \frac{8}{4} = 2$

$k=1$

$Ex = \frac{8}{4-1} = \frac{8}{3}$

$k=2$

$Ex = \frac{8}{4-2} = \frac{8}{2} = 4$

$k=3$

$Ex = \frac{8}{4-3} = \frac{8}{1} = 8$

$\therefore Ex = 2 + \frac{8}{3} + 4 + 8 = \frac{50}{3} \simeq 16.67$