CS237 - PROBABILITY IN COMPUTING - FALL 2021 — JOHN BYERS AND ALINA ENE
DUE MONDAY, SEPTEMBER 20, AT 10PM VIA GRADESCOPE

H2.

- The assignment is due at Gradescope on Monday, September 20, at 10pm. Submit early and often.

- Read and sign the collaboration and honesty policy. Submit the signed policy to Gradescope before submitting any work.

- Unless otherwise specified, you can leave your answer in closed form (e.g. $1 - \binom{120}{7}(0.1)^{200}$).

- *Show your work.* Answers without justification will be given little credit. Justify each step in your solutions e.g. by stating that the step follows from an axiom of probability, a definition, algebra, etc.; for example, your answer could include a line like this:

$$\Pr(X \cap Y \cap Z) \cdot \Pr(A \cup B) = \Pr(X \cap Y \cap Z) \cdot (\Pr(A) + \Pr(B)) \qquad (A \text{ and } B \text{ are disjoint})$$

- The syllabus has some pointers on using LaTeX and Python.

PROBLEM 1 (Census). Random Census, Inc. — a data science company — conducted a study of families that have 1, 2, or 3 children, all having different ages (i.e., no family surveyed had twins). The data scientists randomly sampled families and, for each sampled family, they wrote down the genders of the children in ascending order. For example, the outcome G indicates that the family has one child, who is a girl; the outcome GB indicates that the family has two children, the younger child is a girl, and the older child is a boy. They performed a statistical analysis of the data and concluded that every possible configuration of genders and numbers of children is equally likely.

(a) Write down the sample space of this experiment and the probability of each outcome.

(b) For each of the following events, write down the set of outcomes comprising the event and find the probability of the event.

$$A = \text{"families where the oldest child is a boy"}$$
$$B = \text{"families with exactly one girl and any number of boys"}$$
$$C = \text{"families with at least one child of each gender"}$$

Solution: Your solution here.

PROBLEM 2 (Random passwords). The passwords for a certain website are sequences comprised of 4 numbers and 5 letters, where each number is one of the 10 digits $\{0, 1, 2, \ldots, 9\}$ and each letter is one of the upper-case letters of the English alphabet $\{A, B, \ldots, Z\}$. Suppose we choose a password uniformly at random (each password is equally likely to be chosen). Find the probability of the event that the letters come before the numbers.

Solution: Your solution here.

PROBLEM 3 (Rolling dice). We have a fair 4-sided die with the numbers $1, 2, 3, 4$ written on its faces. Suppose we roll the die n times. Each outcome is equally likely. Find the probability of getting:

(a) All threes.

(b) No threes.

(c) Not all threes.

(d) At least one three.

Solution: Your solution here.

PROBLEM 4 (Combination of events). Let A, B, C be three events. Using set notation and operations, write down each of the following combinations of the events:

(a) A and C occur, but not B.

(b) One of B or C occurs, bot not both of them.

(c) Neither of the events occurs.

(d) At least one of the events occurs.

(e) All of the events occur.

(f) Only one of the events occurs.

Solution: Your solution here.

PROBLEM 5 (Pokemon Go). Alice and Bob take part in a special Pokemon Go event where each of them gets to win one of three prizes: a Pikachu, a Sinoh Stone, or a Technical Machine. The pair of prizes are randomly selected (not necessarily uniformly or independently of each other).

(a) Write a suitable sample space for this experiment.

(b) Suppose that:

  – the probability that Alice gets a Pikachu is 0.2;
  – the probability that Bob gets a Pikachu is 0.5;
  – the probability that at least one gets a Pikachu but neither gets a Sinoh Stone is 0.3.

  Find the probability that at least one gets a Sinoh Stone but neither gets a Technical Machine.

Solution: Your solution here.

PROBLEM 6 (Programming exercises). Download this Jupyter notebook. Complete all the exercises in the notebook. Submit the Jupyter notebook with your solutions to the Homework 2 Programming assignment on Gradescope. Your submission should be a single .ipynb file.