Problem 1

$Pr(A \cup B \cup C)$

assume $X = A \cup B$

$Pr(X \cup C) = Pr(X) + Pr(C) - Pr(X \cap C)$

$Pr(A \cup B \cup C) = Pr(A \cup B) + Pr(C) - Pr((A \cup B) \cap C)$ distributive law

$= Pr(A) + Pr(B) - Pr(A \cap B) + Pr(C) - Pr((A \cap C) \cup (B \cap C))$

$= Pr(A) + Pr(B) - Pr(A \cap B) + Pr(C) - (Pr(A \cap C) + Pr(B \cap C) - Pr((A \cap C) \cap (B \cap C)))$

assume $X = A \cap C$ $Y = B \cap C$

$Pr(A \cup B \cup C) = Pr(A) + Pr(B) + Pr(C) - Pr(A \cap B) - Pr(B \cap C) - Pr(A \cap C) + Pr(A \cap B \cap C)$

Problem 2

RS $\frac{3}{5}$ — RS $\frac{3}{5}$ — ~~RS $\frac{3}{5}$~~, ~~Y $\frac{2}{5}$~~ $\rightarrow$ RS win on second round $\rightarrow \frac{3}{5}(\frac{3}{5}) = \frac{9}{25}$

RS $\frac{3}{5}$ — Y $\frac{2}{5}$ — RS $\frac{3}{5}$ $\rightarrow \frac{3}{5}(\frac{2}{5})(\frac{3}{5}) = \frac{18}{125}$ RS win

RS $\frac{3}{5}$ — Y $\frac{2}{5}$ — Y $\frac{2}{5}$ $\rightarrow \frac{3}{5}(\frac{2}{5})(\frac{2}{5}) = \frac{12}{125}$ Y win

Y $\frac{2}{5}$ — RS $\frac{3}{5}$ — RS $\frac{3}{5}$ $\rightarrow \frac{2}{5}(\frac{3}{5})(\frac{3}{5}) = \frac{18}{125}$ RS win

Y $\frac{2}{5}$ — RS $\frac{3}{5}$ — Y $\frac{2}{5}$ $\rightarrow \frac{2}{5}(\frac{3}{5}(\frac{2}{5}) = \frac{12}{125}$ Y win

Y $\frac{2}{5}$ — Y $\frac{2}{5}$ — ~~RS $\frac{3}{5}$~~, ~~Y $\frac{2}{5}$~~ $\rightarrow$ Y win on second round $\rightarrow \frac{2}{5}(\frac{2}{5}) = \frac{4}{25}$

(a). $Pr = \frac{18}{125} + \frac{12}{125} + \frac{18}{125} + \frac{12}{125} = \frac{60}{125} = \frac{12}{25}$

(b). $Pr = \frac{12}{125} + \frac{18}{125} = \frac{30}{125} = \frac{6}{25}$

(c). $Pr = \frac{9}{25} + \frac{18}{125} + \frac{18}{125} = \frac{81}{125}$

Problem 3

(a). $Pr = 1 - (\frac{9000}{9001})^{100} \approx 0.011$

(b). $Pr = \binom{100}{2}(\frac{1}{9001})^2(\frac{9000}{9001})^{98} \approx 0.0006$

(c). $Pr = \binom{100}{b}(\frac{1}{9001})^b(\frac{9000}{9001})^{100-b}$

Problem 4

$Pr(\text{hits NY}) = 0.32$

$Pr(\text{hits BOS}) = 0.4$

(a). $\because 0.32 < 0.4$

$\therefore b = 0.32$

$\because 0.32 + 0.4 = 0.72 < 1$

$\therefore a = 0$

$\because 0.32 + 0.4 = 0.72$

$\therefore d = 0.72$

$\because 0.32 < 0.4$

$\therefore c = 0.4$

$\therefore \boxed{\begin{array}{l} 0 \le Pr(E_1) \le 0.32 \\ 0.4 \le Pr(E_2) \le 0.72 \end{array}}$

(b). $a = 0.75 + 0.4 - 1 = 0.15$

$\because 0.75 > 0.4$

$\therefore b = 0.4$

$\because 0.75 + 0.4 > 1$

$\therefore d = 1$

$\because 0.75 > 0.4$

$\therefore c = 0.75$

$\therefore \boxed{\begin{array}{l} 0.15 \le Pr(E_1) \le 0.4 \\ 0.75 \le Pr(E_2) \le 1 \end{array}}$

Problem

(a).

$$f(A) = \begin{cases} A = \emptyset & 0 \\ A = \{1\} & 0 \\ A = \{2\} & 1 \\ A = \{3\} & 0 \\ A = \{1,2\} & 1 \\ A = \{2,3\} & 1 \\ A = \{1,3\} & 0 \\ A = \{1,2,3\} = \Omega & 1 \end{cases}$$

(b).

$$f(A) = \begin{cases} 1 & w_0 \in A \\ 0 & \text{otherwise } (w_0 \notin A) \end{cases}$$

for $A \subseteq \Omega$, $f(A) = 0$ or 1 $\therefore f(A) \geq 0$

$\therefore$ satisfies Non-negativity axiom.

$w_0 \in \Omega$, $f(\Omega) = 1$

$\therefore$ satisfies Normalization axiom

for $w_0 \notin A$, $f(A_1 \cup A_2 \cup \ldots) = 0 = 0 + 0 + \ldots = f(A_1) + f(A_2) + \ldots$ $f(A) = 0$

for $w_0 \in A$, since w_0 is definetly belong to one of A,

$f(A_1) + f(A_2) + \ldots = 0 + 0 + \ldots + 0 + 1 + 0 + \ldots = 1$

$\therefore f(A_1 \cup A_2 \cup \ldots) = 1 = f(A_1) + f(A_2) + \ldots$

$f(A) = 1$

$\therefore$ satisfies Additivity axiom Hence, proved.