

- The assignment is due at Gradescope on Monday, December 6, 2021 at 10pm. Submit early and often.
- Read and sign the [collaboration and honesty policy](#). Submit the signed policy to Gradescope before submitting any work.
- Unless otherwise specified, you can leave your answer in closed form (e.g. $1 - \binom{120}{7}(0.1)^{200}$).
- *Show your work.* Answers without justification will be given little credit. Justify each step in your solutions e.g. by stating that the step follows from an axiom of probability, a definition, algebra, etc.; for example, your answer could include a line like this:

$$\Pr(X \cap Y \cap Z) \cdot \Pr(A \cup B) = \Pr(X \cap Y \cap Z) \cdot (\Pr(A) + \Pr(B)) \quad (A \text{ and } B \text{ are disjoint})$$

- The syllabus has some pointers on using LaTeX and Python.

PROBLEM 1. Suppose that the time (in hours) required to repair a car is an exponentially distributed random variable with parameter $\lambda = 1/2$. What is the probability that the repair time exceeds 4 hours? Given that it exceeds 4 hours, what is the probability that it exceeds 8 hours?

Solution:

PROBLEM 2. Suppose that in a population of individuals, their height is normally distributed with a mean of 68 inches and a standard deviation of 1.45 inches.

- What is the probability that a randomly-selected individual has a height less than 66 inches?
- What is the probability that a randomly-selected individual has a height more than 72 inches?
- What is the probability that a randomly-selected individual has a height between 66.5 and 71 inches?

Solution:

PROBLEM 3. An ambulance station, 30 miles from one end of a 100-mile road, services accidents along the whole road. Suppose an accident occurs at a location chosen uniformly at random along the road, i.e., the location lies in the continuous interval $[0, 100]$. An ambulance is dispatched immediately and it travels at a constant speed of 60 miles per hour. Let T be the time (in minutes) that the ambulance takes to arrive at the location of the accident.

- Find $\Pr(T > 30)$.
- Find $\Pr(T > t)$ as a function of t .
- Find the PDF of T .
- Find $\text{Ex}(T)$ and $\text{Var}(T)$.

Solution:

PROBLEM 4. Let T be a random variable that has the exponential distribution with parameter λ . Let $X = \lceil T \rceil$, where $\lceil T \rceil$ is the smallest integer that is greater than or equal to T . Show that X has a geometric distribution and determine its parameter (write the parameter in terms of λ).

Solution:

PROBLEM 5. A professor teaching a weed-out course at Challenging University is legendary for crafting the final exams so that, if we choose a student uniformly at random from the class, their exam score is normally distributed with parameters μ and σ^2 . Suppose that the professor assigns letter grades to the exam scores as shown in the following table:

$\mu + \sigma < x$	A
$\mu < x < \mu + \sigma$	B
$\mu - \sigma < x < \mu$	C
$\mu - 2\sigma < x < \mu - \sigma$	D
$x < \mu - 2\sigma$	F

What fraction of the class gets A, B, C, D, F?

Solution:

PROBLEM 6. Random Company Inc. has designed a randomized algorithm that seems to have a strange behavior. For example, if we run the algorithm more than once on the same data, it has different running times. However, one thing that we know for sure is that the *expected* running time of the algorithm is 10 seconds.

- (a) Upper bound the probability that the running time of the algorithm is at least 1 minute.
- (b) Suppose that we know that the variance of the running time of the algorithm is 25. Upper bound the probability that the running time of the algorithm is at least 1 minute.

Solution:

PROBLEM 7. In your role as Chief Revenue Officer for a major online retailer, you often need to evaluate new website features to see the impact on the company's bottom line. The design team has implemented a new shopping cart design that they believe will provide a better checkout experience for customers. You are tasked with testing the new cart design.

- (a) Your first task is to estimate the conversion rate of the new design, i.e., the fraction of customers that actually complete the purchase after putting items into the cart. Using the estimation by sampling approach from class, design a procedure for estimating the conversion rate of the new cart design. Determine how many customers you need to sample in order to estimate the rate within 1% with probability at least 95%.
- (b) Suppose you know the conversion rate p of the old cart design. Your second task is to determine whether the conversion rate q of the new cart design is significantly better than the conversion rate p of the old cart design. To this end, you use the estimation by sampling approach from part (a) to estimate q , and declare that the new rate is better if your estimate is at least $p + 0.01$. Determine how many customers you need to sample so that, if $q < p$, the probability that you declare that the new rate is better is at most 0.05.

Solution: