H3.

- The assignment is due at Gradescope on Monday, September 27, at 10pm. Submit early and often.
- Read and sign the collaboration and honesty policy. Submit the signed policy to Gradescope before submitting any work.
- Unless otherwise specified, you can leave your answer in closed form (e.g. $1 - \binom{120}{7}(0.1)^{200}$).
- *Show your work.* Answers without justification will be given little credit. Justify each step in your solutions e.g. by stating that the step follows from an axiom of probability, a definition, algebra, etc.; for example, your answer could include a line like this:

$$\Pr(X \cap Y \cap Z) \cdot \Pr(A \cup B) = \Pr(X \cap Y \cap Z) \cdot (\Pr(A) + \Pr(B)) \qquad (A \text{ and } B \text{ are disjoint})$$

- The syllabus has some pointers on using LaTeX and Python.

PROBLEM 1. If A, B, and C are any three events, show that

$$\Pr(A \cup B \cup C) = \Pr(A) + \Pr(B) + \Pr(C) - \Pr(A \cap B) - \Pr(B \cap C) - \Pr(C \cap A) + \Pr(A \cap B \cap C)$$

Solution: Your solution here.

PROBLEM 2 (Problem 17.3 in the textbook). The New York Yankees and the Boston Red Sox are playing a two-out-of-three series. In other words, they play until one team has won two games. Then that team is declared the overall winner and the series ends. Assume that the Red Sox win each game with probability 3/5, regardless of the outcomes of previous games. Use the tree diagram method to answer the following questions:

(a) What is the probability that a total of 3 games are played?

(b) What is the probability that the winner of the series loses the first game?

(c) What is the probability that the Red Sox win the series?

Solution: Your solution here.

PROBLEM 3. Seto Kaiba is very intent on obtaining the ultra-rare Yu-gi-oh card "Blue Eyes White Dragon." Using his vast fortune from his multi-national company Kaiba Corp., he regularly purchases cards in batches of 100 packs of cards. There is probability $p = \frac{1}{9001}$ of Blue Eyes being in any given pack, independently of other packs, and there will never be more than 1 in a single pack. Calculate:

(a) The probability of drawing at least 1 Blue Eyes from a batch of 100 packs.

(b) The probability of drawing exactly 2 Blue Eyes from 100 packs.

(c) Write a probability function for the probability of drawing $b \in \{0, 1, ..., 100\}$ Blue Eyes from 100 packs.

Solution: Your solution here.

PROBLEM 4. The national weather service is tracking a new tropical storm. They forecast that the probability that the storm hits New York City is p and the probability that the storm hits Boston is q. Given this information, they would like to find the tightest possible range for the probability of each of the following two events:

- E_1 = "the storm hits both cities";
- E_2 = "the storm hits at least one of the two cities".

(a) Suppose that $p = 0.32$ and $q = 0.4$. For each of the two events E_1 and E_2, bound its probability from below and above. More precisely, find the largest numbers a, c and the smallest numbers b, d such that

$$a \leq \Pr(E_1) \leq b \quad \text{and} \quad c \leq \Pr(E_2) \leq d$$

(b) Suppose that $p = 0.75$ and $q = 0.4$. As in the previous part, for each of the two events above, bound its probability from below and above.

Solution: Your solution here.

PROBLEM 5. Let Ω be a finite sample space. As we have seen in class, a probability function is a function f that assigns a value $f(A)$ to every event $A \subseteq \Omega$. In this problem, we consider the following function f. Let $\omega_0 \in \Omega$ be one of the outcomes in the sample space. For every event $A \subseteq \Omega$, we define

$$f(A) = \begin{cases} 1 & \text{if } \omega_0 \in A \\ 0 & \text{otherwise} \end{cases}$$

i.e., for any event A, its probability is equal to 1 if the event contains our chosen outcome ω_0 and it is equal to 0 otherwise.

(a) Consider the sample space $\Omega = \{1, 2, 3\}$ and suppose we choose $\omega_0 = 2$. Write down what the function f is in this case, i.e., write down all of the events and their probabilities.

(b) Consider an arbitrary finite sample space Ω and outcome $\omega_0 \in \Omega$. Show that the function f defined above is a valid probability function, i.e., f satisfies the axioms of probability.

Solution: Your solution here.

PROBLEM 6 (Programming exercises). Download this Jupyter notebook. Complete all the exercises in the notebook. Submit the Jupyter notebook with your solutions to the Homework 3 Programming assignment on Gradescope. Your submission should be a single .ipynb file.