H1.

- The assignment is due at Gradescope on Monday September 13, 2021 at 10pm. Submit early and often.
- Read and sign the collaboration and honesty policy. Submit the signed policy to Gradescope before submitting any work.
- Unless otherwise specified, you can leave your answer in closed form (e.g. $1 - \binom{120}{7}(0.1)^{200}$).
- *Show your work.* Answers without justification will be given little credit. Justify each step in your solutions e.g. by stating that the step follows from an axiom of probability, a definition, algebra, etc.; for example, your answer could include a line like this:

$$\Pr(X \cap Y \cap Z) \cdot \Pr(A \cup B) = \Pr(X \cap Y \cap Z) \cdot (\Pr(A) + \Pr(B)) \qquad (A \text{ and } B \text{ are disjoint})$$

- The syllabus has some pointers on using LaTeX and Python.

This homework reviews material from the prerequisites for the class: discrete mathematics and proofs (CS 131), calculus (MA 123 or equivalent).

PROBLEM 1 (Combinations and Permutations). (a) There are four different bike routes between Brookline and Cambridge. In how many distinct ways can you make a round-trip from Brookline to Cambridge and back? In how many ways can you make this trip if you take a different route on the way back?

(b) You want to arrange 4 different flower vases in a row on your shelf. How many distinct arrangements can you come up with?

(c) You have meal prepped three types of main dishes and four types of appetizers. How many distinct meals can you have during the week, if a meal = main dish + appetizer.

Solution: Your solution here.

PROBLEM 2 (Counting). A standard deck of cards has 52 cards. Each card has a rank and a suit. There are 13 ranks: A (Ace), 2, 3, 4, 5, 6, 7, 8, 9, 10, J (Jack), Q (Queen), K (King). There are 4 suits: clubs ($\clubsuit$), diamonds ($\diamondsuit$), hearts ($\heartsuit$), and spades ($\spadesuit$).

We draw 3 cards from a standard deck without replacement. How many possibilities are there if:

(a) the cards have the same rank;

(b) the cards have different ranks;

(c) two of the cards have the same rank and the third has a different rank.

Solution: Your solution here.

PROBLEM 3 (Induction). Show by induction on n that

$$1 + r + r^2 + \cdots + r^n = \frac{r^{n+1} - 1}{r - 1},$$

for all $n \in \mathbb{N}$ and $r \neq 1$. ($\mathbb{N}$ denotes the set of all natural numbers. In this class, we adopt the convention that $\mathbb{N}$ includes 0.)

Solution: Your solution here.

PROBLEM 4 (Proofs by contradiction). Prove by contradiction that if a^2 is even then a is even.

Solution: Your solution here.

PROBLEM 5 (Set Operations). Let A, B, C be sets. Show that

$$(A \setminus B) \setminus C = (A \setminus C) \setminus (B \setminus C) .$$

Solution: Your solution here.

PROBLEM 6 (Calculus). Consider the following function $f\colon \mathbb{R} \to \mathbb{R}$:

$$f(x) = \begin{cases} x^2 - 2x - 1 & \text{for } x \in [0,5] \\ 3 & \text{for } x \in [7,10] \\ 0 & \text{otherwise} \end{cases}$$

(a) Sketch the graph of the function.

(b) For each of the following intervals $[a, b]$, evaluate the integral $\int_a^b f(x)dx$: $[-1, 3]$, $[1, 8]$, $[6, 12]$.

(c) Find the derivative of f wherever it exists, and clearly identify at which points the derivative does not exist.

(d) Let $g\colon \mathbb{R} \to \mathbb{R}$ be a differentiable function and let g' be its derivative. Let h be the composition of g with f, i.e., the function $h\colon \mathbb{R} \to \mathbb{R}$ such that $h(x) = g(f(x))$ for all $x \in \mathbb{R}$. Find the derivative of h wherever it exists, expressed in terms of g'.

Solution: Your solution here.