

- The assignment is due at Gradescope on Monday, November 15, 2021 at 10pm. Late assignments will not be accepted. Submit early and often.
- Read and sign the [collaboration and honesty policy](#). Include the signed policy with the first homework that you submit.
- Unless otherwise specified, you can leave your answer in closed form (e.g.  $1 - \binom{120}{7}(0.1)^{200}$ ).
- *Show your work.* Answers without justification will be given little credit. Justify each step in your solutions e.g. by stating that the step follows from an axiom of probability, a definition, algebra, etc.; for example, your answer could include a line like this:

$$\Pr(X \cap Y \cap Z) \cdot \Pr(A \cup B) = \Pr(X \cap Y \cap Z) \cdot (\Pr(A) + \Pr(B)) \quad (A \text{ and } B \text{ are disjoint})$$

- The syllabus has some pointers on using LaTeX and Python.

**PROBLEM 1.** Let  $X$  be a random variable with expectation  $\mu$  and standard deviation  $\sigma$ . Find each of the following, expressed in terms of  $\mu$  and  $\sigma$ :

- (a)  $\text{Ex}(2X^2 - 3X + 4)$ ;
- (b)  $\text{Var}(-3X + 4)$ .

**Solution:**

**PROBLEM 2.** Let  $X$  be a continuous random variable with the following PDF,  $f_X(x)$ , where  $c$  is a constant:

$$f_X(x) = \begin{cases} cx(1-x)^2 & \text{if } x \in (1, 2) \\ 0 & \text{otherwise} \end{cases}$$

- (a) Find the value of  $c$ .
- (b) Find  $\text{Ex}(X)$ .
- (c) Find  $\text{Var}(X)$ .

**Solution:**

**PROBLEM 3.** Recall that a random variable with range  $\{0, 1\}$  is called an indicator random variable. Let  $X$  and  $Y$  be two independent indicator random variables, with  $\Pr(X = 1) = p$  and  $\Pr(Y = 1) = r$ . Find  $\text{Ex}((X - Y)^2)$  in terms of  $p$  and  $r$ .

**Solution:**

**PROBLEM 4.** There are  $n$  people on a friendship-matchmaking app. We assume that each pair of people will connect and become friends with probability 0.1, independently of other pairs.

- (a) You are one of the  $n$  people on the app. Let  $X$  be the random variable denoting the number of friends you will make on the app. Find the PDF of  $X$ .
- (b) What is the expected number of friends you will make on the app?

- (c) What is the probability that you will make at least 2 friends on the app?
- (d) What is the expected number of friendships formed on the app, i.e. what is the expected number of pairs that will connect and become friends?

**Solution:**

**PROBLEM 5 (Problem 19.33 in the textbook).** Alice has decided to volunteer in a bakery. She has taken CS 237 and she knows that it will be great fun if she uses the following strategy for deciding how many loaves of bread to bake. Each day, Alice first rolls a fair,  $k$ -sided die to get a number  $n \in \{1, 2, \dots, k\}$ , and then she tosses a fair coin. Alice bakes  $n$  loaves of bread if the coin is heads, and she bakes  $2n$  loaves if the coin is tails.

- (a) For any positive integer  $\ell \leq 2k$ , what is the probability that Alice bakes exactly  $\ell$  loaves of bread on any given day?  
*Hint: you can express your solution by cases.*
- (b) What is the expected number of loaves that Alice bakes on any given day?
- (c) Suppose Alice volunteers in the bakery every day for 30 days. What is the expected total number of loaves that Alice bakes?

You may find the following identity helpful:

$$\sum_{i=1}^n i = \frac{n(n+1)}{2}$$

**Solution:**

**PROBLEM 6 (Simulations).** Download the HW9 Jupyter notebook (notebook coming soon!). Complete all the exercises in the notebook. Submit the Jupyter notebook with your solutions to the Homework 9 Programming assignment on Gradescope. Your submission should be a single .ipynb file.