1.a).

Set $W^{(0)}=0$, $W^{(t+1)}=\sum_{n=1}^{N}\eta C_n t_n \Phi_n$

$C_n \Rightarrow$ times that $t_n\Phi_n$ has been added from step 0 to step $t+1$

$\therefore W=\sum_{n=1}^{N}\alpha_n t_n \Phi_n$

$$\sum_{n=1}^{N}\alpha_n^{(t+1)} t_n\Phi_n = \sum_{n=1}^{N}\alpha_n^{(t)} t_n\Phi_n + \eta t_n \Phi_n$$

$$\Downarrow$$

$$\alpha_n^{(t+1)} = \alpha_n^{(t)} + \eta$$

$$\begin{aligned} y(x) &= f(W^T\Phi(x)) \\ \therefore\ &= f\left(\sum_{n=1}^{N}\alpha_n t_n \Phi_n^T \Phi(x)\right) \\ &= f\left(\sum_{n=1}^{N}\alpha_n t_n k(x_n, x)\right). \end{aligned}$$

2. $x_1 \in$ class one ; $x_2 \in$ class two.
$t_1=1$ $\qquad$ $t_2=-1$

$\because$ we only have two points

$\therefore t_i \cdot y(x_i)=1$

$$\min \frac{1}{2}\|W\|^2 \quad s.t. \begin{cases} W^T\Phi(x_1)+b=1 \\ W^T\Phi(x_2)+b=-1 \end{cases}$$

$$\Uparrow$$

convex optimization problem $\Leftarrow$ global optimal exists.

3. $\rho=\frac{1}{\|W\|} \Rightarrow \frac{1}{\rho^2}=\|W\|^2$

$L(w,b,a)=\frac{1}{2}\|W\|^2-\sum_{n=1}^{N}a_n\{t_n(W^T\Phi(x_n)+b-1\}$.

$W=\sum_{n=1}^{N}a_n t_n \Phi(x_n)$.

$\because a_n\{t_n y(x_n)-1\}=0$

$\therefore L(w,b,a)=\frac{1}{2}\|W\|^2$

$\tilde{L}(a)=L(w,b,a)$

$\sum a-\frac{1}{2}\|W\|^2=\frac{1}{2}\|W\|^2$

$\sum a=\|W\|^2=\frac{1}{\rho^2}$

(a).

4. $\frac{1}{\rho^2}=\|W\|^2$

$$\begin{aligned} \tilde{L}(a) &= \sum a-\frac{1}{2}\|W\|^2 \\ &= \|W\|^2-\frac{1}{2}\|W\|^2=\frac{1}{2}\|W\|^2 \end{aligned}$$

$\therefore \frac{1}{\rho^2}=2\tilde{L}(a)$.

(b).

$L(w,b,a)=\tilde{L}(a)=\frac{1}{2}\|W\|^2$

$\therefore \frac{1}{\rho^2}=2\tilde{L}(a)=2\left(\frac{1}{2}\|W\|^2\right)=\|W\|^2$