1.

(a) Feb has 30 days is false, so p is false. In p—>q, if p is false, then the statement is always true, so the statement "If February has 30 days, then 7 is an odd number" is **true**.

- (b) Jan has 31 days is true. In p—>q, if p is true, when q is true, then the statement will be true, but when q is false, the statement will be false. 7 is an odd number, so "7 is an even number" is false. Therefore, the statement "If January has 31 days, then 7 is an even number" is **false**.
- (c) 7 is an odd number, so p is true in p—>q. Feb has 28 0r 29 days, so the statement q "February does not have 30 days" is true too. When p and q are both true, the whole statement is true. Therefore the statement "If 7 is an odd number, then February does not have 30 days" is **true**.
- (d) 7 is an odd number, not an even number, so the statement "7 is an even number" is false. Therefore, p is false. In p—>q, if p is false, then the statement is always true, so the statement "If 7 is an even number, then January has exactly 28 days" is **true**.

2.

- (a) inverse: If she does not finish her homework, then she didn't go to the party contrapositive: If she didn't go to the party, then she did not finish her homework converse: If she went to the party, then she finished her homework
- (b) inverse: If he didn't train for the race, then he didn't finish the race contrapositive: If he didn't finish the race, then he didn't train for the race converse: If he finished the race, then he trained for the race
- (c) inverse: If the patience didn't take the medicine, then she didn't have side effect contrapositive: If the patient didn't have side effects, then she didn't take the medicine converse: If the patient had side effects, then she took the medicine
- (d) inverse: If it was not sunny, then the game was not held contrapositive: If the game was not held, then it was not sunny converse: If the game was held, the it was sunny day
- (e) inverse: If it didn't snow last night, then school will not be cancelled contrapositive: If the school will not be cancelled, then is didn't snow last night converse: If the school will be cancelled, then it snowed last night

3.

(a) inverse: If 3 is not a prime number, then 5 is not an even number
 In p—>q, p is false, so the statement is always true.
 contrapositive: If 5 is not an even number, then 3 is not a prime number

In p->q, p is true and q is false, so the statement is **false**.

converse: If 5 is an even number, then 3 is a prime number

In p->q, p is false, so the statement is always **true**.

(b) **inverse**: If 7 is not <5, then 5 is not <3

In p->q, p is true and q is true, so the statement is **true**.

contrapositive: If 5 is not <3, then 7 is not <5

In p->q, p is true and q is true, so the statement is **true**.

converse: If 5<3, then 7<5

In p—>q, p is false, so the statement is always **true**.

(c) **inverse**: If 5 is not a negative number, then 3 is not a positive number

In p->q, p is true and q is false, so the statement is **false**.

contrapositive: If 3 is not a positive number, then 5 is not a negative number

In p->q, p is false, so the statement is always **true**.

converse: If 3 is a positive, then 5 is a negative number

In p->q, p is true and q is false, so the statement is **false**.

4.

(a)

p	q	¬р	¬p∧q	(¬p∧q)→p
Т	Т	F	F	Т
Т	F	F	F	Т
F	Т	Т	Т	F
F	F	Т	F	Т

(b)

р	q	p→q	q→p	$(p \rightarrow q) \rightarrow (q \rightarrow p)$
Т	Т	Т	Т	Т
Т	F	F	Т	Т
F	Т	Т	F	F
F	F	Т	Т	Т

(c)

р	q	¬р	p∨q	q→¬p	$(p \lor q) \leftrightarrow (q \rightarrow \neg p)$

Т	Т	F	Т	F	F
Т	F	F	Т	Т	Т
F	Т	Т	Т	Т	Т
F	F	Т	F	Т	F

(d)

р	q	¬q	p↔q	p⇔¬q	$(p \leftrightarrow q) \oplus (p \leftrightarrow \neg q)$
Т	Т	F	Т	F	Т
Т	F	Т	F	Т	Т
F	Т	F	F	Т	Т
F	F	Т	Т	F	Т

(e)

р	q	p∨q	q∧p	(p∨q)↔(q∧p)
Т	Т	Т	Т	Т
Т	F	Т	F	F
F	Т	Т	F	F
F	F	F	F	Т

5.

- (a) ¬j−>c
- (b) $c \rightarrow \neg j$
- (c) ¬j∧¬c
- (d) $c \rightarrow \neg j$
- (e) ¬c
- (f) j∧c

- (a) If Joe maintains a B average, then he is eligible for the honors program
- (b) If Joe is eligible for the honors program, then he maintains a B average
- (c) If Rajiv can go on the roller coaster, then he is at least four feet tall
- (d) If Rajiv is at least four feet tall, then he can go on the roller coaster 7.
- (a) $p \rightarrow (s \land y)$
- (b) $(s \lor y) -> p$

- (c) $p \rightarrow y$
- (d) $p \rightarrow s \wedge y$
- **(e)** p−>s∨y p<−>s∨y

8.

- (a) If the roads were wet, then traffic was heavy
- (b) The roads were wet and there was an accident
- (c) There was not an accident and traffic was not heavy neither
- (d) If traffic was heavy, then there was an accident or the roads were wet
- (e) The roads were wet and traffic was not heavy

9.

- (a) ¬y∧p
- (b) $(y \lor p) -> c$
- (c) $c \rightarrow p$
- (d) $c \rightarrow p$

10.

(a) false

Because p is true, only both q and r are true can make the statement true, but q is false, so $q \land r$ is false. In $p \rightarrow q$, if p is true and q is false, the statement is **false**.

(b) unknown

Because p is true, $p \lor r$ is true whether r is true or false. However, if r is true, the statement will be true; if r is false, the statement will be false. Therefore, the truth value of this statement is **unknown**.

(c) false

Because p is true, $p \lor r$ is true whether r is true or false. q is false, so $q \land r$ is false. Therefore, the statement is **false**.

(d) unknown

Because q is false, $q \wedge r$ is false, but $p \wedge r$ is unknown, so the truth value of this statement is **unknown**.

(e) unknown

p is true, but $r \lor q$ is unknown. If r is true, then the statement will be true; if r is false, then the statement will be false. Therefore, the truth value of this statement is **unknown**.

(f) true

Because p is true and q is false, $p \land q$ is false. Therefore, whether r is true or not, the statement is **true**.

(a) p∧s

negation: ¬(p∧s)

applying De Morgan's law: ¬p∨¬s

english: The applicant has not written permission from his parents or is not at least 16 years

old

(b) p∨e

negation: ¬(p∨e)

applying De Morgan's law: ¬p∧¬e

english: The applicant has not written permission from his parents and is not at least 18 years

old

12.

(a) Conditional identities

Commutative laws

Distributive laws

Commutative laws

Complement laws

Identity laws

(b) Conditional identities

De Morgan's laws

Double negation law

Distributive laws

Complement laws

Identity laws

(c) Conditional identities

Double negation law

Associative laws

Idempotent laws

13.
(a). ¬p->¬q = q->p
77pv7a conditional identities
pv7q double negation law
79 vp commutative laws
9-7P conditional identities
 (b). p1(7p-)q) =p
PA(17PVq) conditional identities
PA(pvq) double negation law
p absorption laws the congression
(C). $(p \rightarrow q) \Lambda(p \rightarrow r) \equiv p \rightarrow (q \Lambda r)$
(7pvq)1(7pVr) conditional identities
7pv(qnr) distributive laws
77P->(qar) conditional identities
p-zigar) double negation law
(d) 7p7(Q7r) = Q-)(pvr)
77pV(q-7r) conditional identities
pv(7qvr) conditional identities
(pv7q)vr associative laws
(79VP) Vr commutative Jaws
7qv(pvr) associative laws
a-Jipvr) conditional identities
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pΛ7q identity laws) (PΛQΛ7r) V (PΛ7QΛ7r) ≡ PΛ7r Λ((QΛ7r) V (7QΛ7r)) clistributive laws Λ((7rΛQ) V (7rΛ7Q)) communative laws Λ(7rΛ(QV7Q)) complement laws Λ(7rΛ1) complement laws	
$(p\Lambda q \Lambda 7r) V (p\Lambda 7q \Lambda 7r) \equiv p\Lambda 7r$ $\Lambda ((q\Lambda 7r) V (7q\Lambda 7r))$ custributive laws $\Lambda ((7r\Lambda q) V (7r\Lambda 7q))$ commutative laws $\Lambda (7r\Lambda (qV 7q))$ distributive laws $\Lambda (7r\Lambda 1)$ complement laws	$(q) (P \wedge q \wedge 7r) \vee (P \wedge 7q \wedge 7r) \equiv P \wedge 7r$ $P \wedge ((q \wedge 7r) \vee (7q \wedge 7r)) \qquad \text{cust n'butive laws}$ $P \wedge ((7r \wedge q) \vee (7r \wedge 7q)) \qquad \text{commutative laws}$ $P \wedge (7r \wedge (q \vee 7q)) \qquad \text{cust n'butive laws}$ $P \wedge (7r \wedge 1) \qquad \text{complement laws}$
Λ((qΛ7r)ν(ηqΛ7r)) clistributive laws Λ((ηrΛq)ν(ηrΛ7q)) commuative laws Λ(ηrΛ(qνηα)) distributive laws Λ(ηrΛ) complement laws	pΛ((qΛ7r)ν(ηqΛ7r)) clistributive laws pΛ((ηrΛq)ν(ηrΛη)) commuative laws pΛ(ηrΛ(qνηα)) distributive laws pΛ(ηrΛ) complement laws
Λ((7rΛq)ν(7rΛ7q)) commuative laws Λ(7rΛ(qν7q)) distributive laws Λ(7rΛ) complement laws	pΛ((¬rΛq)ν(¬rΛ¬q)) commuative laws pΛ(¬rΛ(qν¬q)) distributive laws pΛ(¬rΛĪ) complement laws
Λ(¬rΛ(qv¬q)) distributive laws Λ(¬rΛΙ) complement laws	pΛ(¬rΛ(qν¬q)) distributive laws pΛ(¬rΛĪ) complement laws
(7-1) complement laws	PN (7 r N T) complement laws
17r domination laws	PAT domination laws

(h) p+>(p/r) = 7	pvr	s Ktir
(P→(P/r)) Λ ((P/	r)-7p)	conditional identities
$(\neg p \lor (p \land r)) \land (\neg ($	PATIVE) DEVE	71V302750
((¬pvp) Λ(¬pvr))) ((\pv\r)vp)	distributive laws
((PV7P)A(7PVr))) N((TrV7p)Vp)	commutative laws
(TA(7pVr)) A (7 r v (7 p v p)).	complement laws
((ZPVr)))T	$)\Lambda(7rV(pV7p))$	commutative laws
 ((7pvr) NT	(,,,,,,,,,,,,,,,,,,,,,,,,,,,,,,,,,,,,,,	complementlaws
(7pvr)) AT identity	1 laws & domination laws
(i)(PAQ)→r=(P	17r)-779	ANTO CONTRACT
 7(pag) Vr	conditional iden	tities of the state of the
(TPV7Q)Vr	De morgan's laws	or was a second
 7pV(7qvr)	associative law:	S TOWN THE TANK
 JPV(rV7g)	commutative la	WS
(7pvr)v7q 0	associative laws	
7(p/1r)/79 [De Morgan's law	S
(p/1r)-779 (conditional iden	tities

		5
	14. NOT THE THE	
	(a). p-79 =7pv9	
	$q \rightarrow p \equiv \gamma q \vee p \equiv p \vee \gamma q$	-
3. J.F.	7pvq≠pv7q	
البية تاليات	Lb) p>q=7pVq	
- 12.3 F	7p-77q = 71pv7q = pv7q	
No. 13	PVQ ≠ pV7Q	
2 - 11	cc).p-q=apvq	
Alles (110)	79-77p=77av7p=qv7p=7pvq	
	7pvq =7pvq	
	(d) Q-7p =7QVp = pv7q	
	7p->7a=77pV7a=pV7a	
	pv1q = pv1q	

15.

15.
P: Xi3 a rational number
79: yis an irrational number
7r: X-y is an irrational number
r: x-y is a rational number
q: yis a rational number
O. p 179-77 (7p/q) V7r O. p/r-7q
7(PA7a)V7r 7PV(QV7r) 7(PAr)Vq
$(7P\Lambda77Q)V7\Gamma$ $7PV(7\GammaVQ)$ $(7PV7\Gamma)VQ$

16.

(a) False there is no example, so false

(b) True ex. x=-1

(c) True there is no counterexample, so true

(d) False ex. x=0

(e) False ex. x=0

(f) Trueex. x=1 or any nonzero integers17.

x∈R

(a) $\exists x(x^3=2)$

(b) $\forall x(x^2>0)$

(c) $\exists x(x^2=x)$

(d) $\forall x(x \le x^2)$

18.

(a) $\exists x(S(x))$

- (b) $\forall x (\neg S(x) \land W(x))$
- (c) $\forall x(S(x) -> \neg W(x))$
- (d) $\exists x(S(X) \land W(x))$
- (e) $\forall x(\neg W(x) -> S(x))$
- (f) $\forall x (\neg W(x) -> (S(x) \lor V(x)))$
- (g) $\exists x (\neg W(x) \rightarrow \neg (S(x) \lor (V(x)))$
- (h) $\forall x (\neg W(x) -> (S(x) \lor V(x)))$
- (i) S(Ingrid)∧W(Ingrid)
- (j) $\exists x(S(x)\land(x\neq Ingrid))$
- (k) $\forall x(S(x) \land (x \neq Ingrid))$

19.

- (a) Each person who is on the board of directors earns more than \$100,000
- (b) At least one person earns more than \$100,000 and doesn't work more than 60 hours per week
- (c) Everyone who works more than 60 hours per week earns more than \$100,000
- (d) Someone is not on the board of directors but earns more than \$100,000
- (e) Everyone who earns more than \$100,000 is on the board of directors or works more than 60 hours per week
- (f) At least one person is on the board of directors and works more than 60 hours per week, but doesn't earn more than \$100,000

20.

- (a) $\forall x(\neg P(x))$
- (b) $\forall x (\neg P(x) \land \neg Q(x))$
- (c) $\exists x (\neg P(x) \lor \neg Q(x))$
- (d) $\exists x (\neg P(x) \lor \neg (Q(x) \lor R(x)))$ $\exists x (\neg P(x) \lor (\neg Q(x) \land \neg R(x)))$

21.

(a) $\forall x(D(x))$

negation: $\neg \forall x(D(x))$

applying De Morgan's law: $\exists x(\neg D(x))$

english: There is a patient who was not given the medication

(b) $\forall x(D(x)\lor P(x))$

negation: $\neg \forall x (D(x) \lor P(x))$

applying De Morgan's law: $\exists x(\neg D(x) \land \neg P(x))$

english: There is a patient who was not given the medication and was not given the placebo (c) $\exists x(D(x) \land M(x))$ **negation**: $\neg \exists x (D(x) \land M(x))$ applying De Morgan's law: $\forall x (\neg D(x) \lor \neg M(x))$ english: Every patient was either not given the medication or didn't have migraines (or both) (d) $\forall x(P(x) -> M(x))$ **negation**: $\neg \forall x (P(x) -> M(x))$ $\neg \forall x (\neg P(x) \lor M(x))$ applying De Morgan's law: $\exists x (P(x) \land \neg M(x))$ english: There is a patient was given the placebo and didn't have migraines (e) $\exists x (M(x) \land P(x))$ **negation**: $\neg \exists x (M(x) \land P(x))$ applying De Morgan's law: $\forall x (\neg M(x) \lor \neg P(x))$ english: Every patient either didn't have migraines or was not given the placebo (or both) 22. (a) $\exists x(P(x))$ negation: $\neg \exists x (P(x))$ applying De Morgan's law: $\forall x(\neg P(x))$ english: Every student showed up without a pencil (b) $\forall x (P(x) \lor C(x))$ **negation**: $\neg \forall x (P(x) \lor C(x))$ applying De Morgan's law: $\exists x(\neg P(x) \land \neg C(x))$ english: Some student showed up without a pencil and a calculator (c) $\forall x(P(x) -> C(x))$ **negation**: $\neg \forall x (P(x) -> C(x))$ $\neg \forall x (\neg P(x) \lor C(x))$ applying De Morgan's law: $\exists x(P(x) \land \neg C(x))$ english: There is a student showed up with a pencil but without a calculator (d) $\exists x (P(x) \land C(x))$ **negation**: $\neg\exists x(P(x) \land C(x))$ applying De Morgan's law: $\forall x (\neg P(x) \lor \neg C(x))$ english: Every student showed up without either a pencil or a calculator (or both)

(e) $\exists x (P(x) \lor C(x))$

negation: $\neg \exists x (P(x) \lor C(x))$

applying De Morgan's law: $\forall x (\neg P(x) \land \neg C(x))$

english: Every student showed up without a pencil and a calculator

(f) $\forall x (P(x) \land C(x))$

negation: $\neg \forall x (P(x) \land C(x))$

applying De Morgan's law: $\exists x (\neg P(x) \lor \neg C(x))$

english: At least one of the students showed up without a pencil or a calculator (or both) 23.

(a) $\exists x \neg (P(x) \land \neg Q(x))$

 $\exists x (\neg P(x) \lor \neg \neg Q(x))$

 $\exists x (\neg P(x) \lor Q(x))$

(b) $\exists x \neg (\neg P(x) -> Q(x))$

$$\exists x \neg (\neg \neg P(x) \lor Q(x))$$

$$\exists x \neg (P(x) \lor Q(x))$$

$$\exists x (\neg P(x) \land \neg Q(x))$$

(c) $\forall x \neg (\neg P(x) \lor (Q(x) \land \neg R(x)))$

$$\forall x(\neg \neg P(x) \land \neg (Q(x) \land \neg R(x)))$$

$$\forall x (P(x) \land (\neg Q(x) \lor \neg \neg R(x)))$$

$$\forall x (P(x) \land (\neg Q(x) \lor R(x)))$$

24.

(a) False

x=0 is a counterexample, so false

(b) True

x=0 can make xy=0 for all y, so true

(c) True

there exists a real number z=(x-y)/3 for all x and all y, so **true**

for all x there exists a y that satisfies the requirement, but there is not exists y z for all x that can satisfies the requirement, so false

there exists a real number y=x-3z for all x and all z, so true

(e) True

commutative for real numbers x and y, so true

(f) True

x=3, y=4, z=5 is an example, so **true**

(a) False

x=-1 or any number less than 0 can be a counterexample, so false

(h) True

$$y=x^2>0$$
, $(x<0\lor x>0)->\forall x$, so **true**

(i) True

x=-1, y=1 or any number that |x|=|y| but $x\neq y$ can be a counterexample, so **false**

(j) False

$$|x| \neq |y| -> x^2 \neq y^2$$
, $x^2 = y^2 \wedge x^2 \neq y^2$ doesn't exist, so **false**

(k) True

$$|x|=|y|->x^2=y^2$$
, $(x^2 \neq y^2 \lor x^2=y^2)->\forall x \forall y$, so **true**

	ν5.
	(a). TYX=y=zP(y,x,z)
	IXYY YzP(y, x,z)
	(b).7 bx 3y (P(x,y)) \Q(x,y))
	3x4y7(P(x,y))0(x,y))
	3x)ty(7P(x,y)V7Q(x,y))
	(c). 73x 4y (P(x,y) -> Q(x,y))
	U(y,x))VQ(x,y))
	Ψx3y(77P(x,y)Λ7Q(x,y))
	∀x∃y(P(x,y))\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\
	(d). 73x4y (P(x,y) <7 P(x,y))
- ')	Yx∃y7(P(x,y)->P(y,x) ∧P(y,x) →P(x,y))
J	¥x∃y7 ((7 P(x,y) V P(y,x))∧(1P(y,x) V P(x,y)))
	\frac{1}{2} \left(\pi \left(\frac{1}{2} \reft(\frac{1}{2} \reft) \reft(\frac{1}{2} \reft(\frac{1} \reft
	∀x∃y (< <p>(<</p> (7) \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \
	YX3y((P(X,Y)))V(P(Y,X))/7P(X,Y)))
	(e). 7(3×3y P(x,y) N XX YyQ(x,y))
	(V,Y)DPAXALN(L,X)d DEXEL
	YxYy7P(X,y)V3X3y7Q(X,y)

26.

(a) $\forall x \forall y F(x, y)$

negation: $\neg \forall x \forall y F(x, y)$

applying De Morgan's law: $\exists x \exists y \neg F(x, y)$

english: Someone is an enemy of someone

(b) $\exists x \exists y F(x, y)$

negation: ¬∃x∃yF(x, y)

applying De Morgan's law: ∀x∀y¬F(x, y)

english: Everyone is an enemy of everyone

(c) $\exists x \forall y F(x, y)$

negation: $\neg \exists x \forall y F(x, y)$

applying De Morgan's law: $\forall x \exists y \neg F(x, y)$

english: Everyone is an enemy of someone

(d) $\forall x \exists y F(x, y)$

negation: $\neg \forall x \exists y F(x, y)$

applying De Morgan's law: $\exists x \forall y \neg F(x, y)$

english: Someone is an enemy of everyone

27.

(a) True

for all x, y=-x is a real number, so true

(b) False

there does not exist an x for all y that satisfies x=-y, so false

(c) True

there exists an x for all y that satisfies xy=y, for example x=1, so true

(d) True

there exists an x and a y that satisfies $(x^2 = y^2) \land (x \neq y)$, for example x=1 and y=-1, so **true**

(e) True

For all x and all y, z=(x + y)/2 is a real number, so **true**

(f) True

For any x, there extends the constraint of the

- (a) $\exists x \exists y (x/y > 1)$
- (b) $\forall x(x>0->1/x>0)$

- (c) $\exists x \exists y (x+y=xy)$
- (d) $\forall x \forall y (x>0 \land y>0 \longrightarrow x/y>0)$
- (e) $\forall x \forall y (x>0 \land x<1 ->1/x>1)$
- (f) ¬∃x∀y(x<y)
- (g) $\forall x \exists y (x \neq 0 \rightarrow y = 1/x)$
- (h) $\forall x \exists ! y(x \neq 0 \longrightarrow y=1/x)$