ANNOUNCEMENTS

- Homework 3 will be out on Monday/Tuesday next week:

The worst homework grade will be removed for the final score.

- Quiz 5: Today, last 10 minutes of class.
  - Material: "Paradoxes" chapter from Logicomix
  - You will get 2 attempts, but locked questions after answering.

PROOF BY CONTRADICTION

Meta – Theorem Let Γ be a set of premises

$$\Gamma, \neg\alpha \vdash \beta, \neg\beta \implies \Gamma \vdash \alpha$$

Meta – Corollary

$$(a) \Gamma, \alpha \vdash \beta, \neg\beta \implies \Gamma \vdash \neg\alpha$$

$$(b) \Gamma, \neg\alpha \vdash \alpha \implies \Gamma \vdash \alpha$$

USING PROOF BY CONTRADICTION

Theorem 6 $\alpha \to \beta, \neg\beta \vdash \neg\alpha$(Modus Tollens)

Proof Assume $\alpha \to \beta, \neg\beta, \neg\neg\alpha$

$1. \alpha \to \beta$	P
$2. \neg\beta$	P
$3. \neg\neg\alpha$	P
$4. \alpha$	$Thm 4, 3$
$5. \beta$	$MP 1, 4$

You get a contradiction with lines 2 and 5

USING PROOF BY CONTRADICTION (2)

Theorem 7 $\alpha \rightarrow \beta, \neg\alpha \rightarrow \beta \vdash \beta$

Proof Assume $\alpha \rightarrow \beta, \neg\alpha \rightarrow \beta, \neg\beta$

$1. \alpha \rightarrow \beta$	P
$2. \neg\beta$	P
$3. \neg\alpha \rightarrow \beta$	P
$4. \neg\alpha$	$Thm6,\ 1, 2$
$5. \beta$	$MP\ 3, 4$

You get a contradiction with lines 2 and 5

USING PROOF BY CONTRADICTION (3)

Theorem 8 $\alpha, \neg\beta \vdash \neg(\alpha \rightarrow \beta)$

Proof Assume $\alpha, \neg\beta, \alpha \rightarrow \beta$

$1. \alpha$	P
$2. \alpha \rightarrow \beta$	P
$4. \beta$	$MP\ 1, 2$
$5. \neg\beta$	P

You get a contradiction with lines 4 and 5

Notation for Tautology

$\models \alpha$ is the notation indicating α is a tautology

EXAMPLE OF TAUTOLOGY NOTATION

$$\models p \to p$$

p	$p \to p$
0	1
1	1

TAUTOLOGICAL CONSEQUENCE

A proposition q is a tautological consequence of propositions $p_1, p_2, dots, p_n$ if and only if every row of a joint truth table that assigns "T" to all propositions $p_1, p_2, \ldots, p_n$ also assigns "T" to q OR there is no row that assigns "T" to all propositions p_i

- Notation

$$p_1, p_2, \ldots, p_n \models q$$

SOUNDNESS

Everything that we prove is true

$$\vdash \alpha \implies \models \alpha$$

$$p_1, p_2, \ldots, p_n \vdash \alpha \implies p_1, p_2, \ldots, p_n \models \alpha$$

The axioms A1, A2, A3 are tautologies and Modus Ponens is closed under tautologies (i.e., the derivation is also a tautology)

COMPLETENESS OF PROPOSITIONAL LOGIC (NOT TO BE CONFUSED WITH INCOMPLETENESS THEOREM OF GÖDEL: GÖDEL'S RESULT IS FOR ARITHMETIC, HERE WE TALK ONLY ABOUT PROPOSITIONAL LOGIC)

Completeness: Everything that is true, can be proven

$$\models \alpha \implies \vdash \alpha$$

$$p_1, p_2, \ldots, p_n \models \alpha \implies p_1, p_2, \ldots, p_n \vdash \alpha$$