ANNOUNCEMENTS

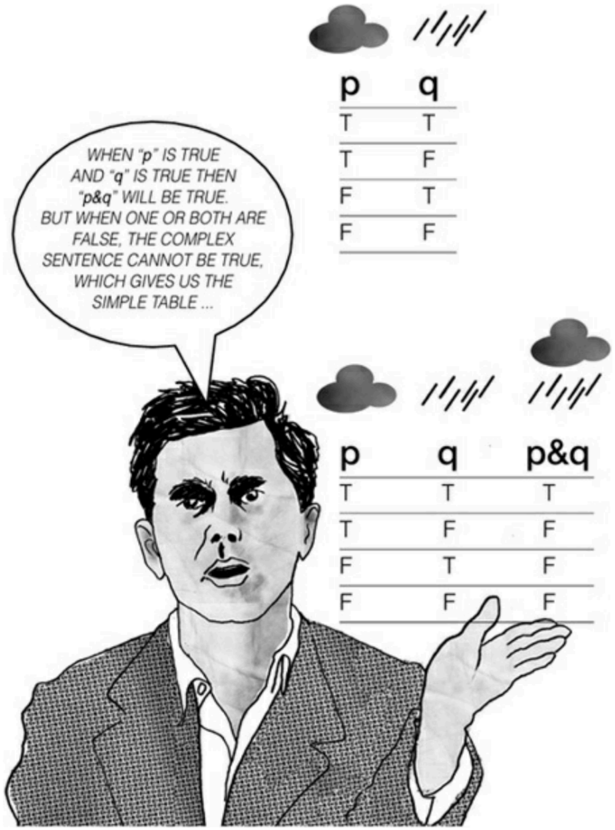
- Homework 1 is out on Canvas:
  - Due date: April 17 (midnight) on Canvas
    - Should be submitted as a pdf file
    - Show how you got an answer (just giving an answer without explanation will cost you points). You do not need to give a "formal proof" to each point (or use only math notation), but you need to provide enough details so the grader can see your logical reasoning.
    - Venn diagrams are NOT good arguments for justifying answers.
- Quiz 2: Today, last 10 minutes of class.
  - Material: "Pembroke Lodge" chapter from Logicomix and lectures 4 and 5 (today's lecture is lecture 6).
  - You will get 2 attempts.
- Exam 1: May 5. Canvas quiz during class time. Covering all month of April
- All videos are in the YouTube playlist (see Canvas announcement).

Wittgenstein’s Table of Logical Connectives

Wittgenstein invented a method of representing logical connectives as a simple table, and so saved everyone the bother of using Frege’s verbose machinery.

Suppose we represent “**The sky is grey**” as “**p**” and “**It is raining**” as “**q**”. Each of them may be either true or false, so altogether we have four possibilities, which may be represented as follows.

We can extend this table to show the way that the connective “**&**” works in the sentence “**p&q**”.



TRUTH TABLES

- Last class we ended with the truth tables for and, or, and not.
- Truth tables come from Wittgenstein, a student of Russell
- ZyBook activities 2.1.2, 2.1.3, and 2.1.7
- ZyBook example 2.2.3 and 2.2.6 (truth tables) and activity 2.2.2
- In the next slide we turn to the “most” important connective

PASSING DISCRETE MATH

- Assume the following premise is true:
  - If I study hard, I will pass Discrete Math
- Can we conclude then that
  - If I passed Discrete Math, then I studied hard?

GOING TO JAIL

- If Jones was convicted of murdering Smith, then he will go to jail.
  - Jones will go to jail
  - Therefore, Jones was convicted of murdering Smith

IMPLICATION $P \rightarrow Q$

- If it is Sunday, then I do not have to go to work
- If I am elected president, I will fund science
- If p Then q
  - $p \rightarrow q$
- If p denotes "I am at home." and q denotes "It is raining." then $p \rightarrow q$ denotes "If I am at home then it is raining."
- If p and q are propositions, then $p \rightarrow q$ is a *conditional statement* or *implication* which is read as "if p, then q"
- In $p \rightarrow q$, p is the *hypothesis* (*antecedent* or *premise*) and q is the *conclusion* (or *consequence*).

TRUTH TABLE FOR IMPLICATION

p	q	$p \to q$
T	T	T
T	F	F
F	T	?
F	F	?

- **Example**: If p denotes "I am elected president." and q denotes "I'll fund science." then $p \to q$ denotes "If I am elected president then I'll fund science."
  - If I am elected president and I fund science, then $p \to q$ *is True*
  - If I am elected president and I do not fund science, then $p \to q$ *isFalse*
  - *What if I am not elected president?*
  - *What is the truth value of $p \to q$ when I am not president and I will not fund science?*

WHAT ARE THE REMAINING VALUES?

- If $x > 2$ then $x^2 > 4$
  - $P(x) \rightarrow Q(x)$
- The above seems true for any value of x that we pick
- What if $x=1$. Then $P(x)$ is false and $Q(x)$ is false.
- What if $x=-5$. Then $P(x)$ is false and $Q(x)$ is true.
- The point of saying that this conditional statement is always true is simply to say that you will never find a value of x such that $x>2$ and $x^2<4$

IMPLICATION DOES NOT MEAN THERE IS A CAUSAL LINK BETWEEN THE TWO

- If I will it to be true, then 1+1=2
- Intuitively, this statement is false:
  - 1+1=2 is true because of the laws of math, not because of what I want
- Mathematically the statement is true:
  - Our focus is to prove the conclusion is true. Whether or not we use the premise is irrelevant in proofs.
  - Therefore If I will it to be true, then 1+1=2 is always True!

IMPLICATION TRUTH TABLE

p	q	$p \rightarrow q$
T	T	T
T	F	F
F	T	T
F	F	T

IF THE PREMISE IS FALSE, THEN THE CONSEQUENCE IS TRUE

- This is how you prove
- $\varnothing \subseteq S$
  - For an arbitrary set S
  - Recall that the definition of $A \subseteq B$ is:

$$\forall x(x \in A \to x \in B)$$

THINK OF THE TRUTH TABLE OF $P \to Q$ AS A PROMISE

- One way to view the logical conditional is to think of an obligation or contract.
  - "If I am elected, then I will fund science."
- If the politician is elected and does not fund science, then the voters can say that she has broken the campaign pledge. This corresponds to the case where p is true and q is false.
- If the politician does not win the election, voters cannot say she broke the promise

UNDERSTANDING IMPLICATION

- In $p \to q$ there does not need to be any connection between the antecedent or the consequent. The "meaning" of $p \to q$ depends only on the truth values of p and q.
- These implications are perfectly fine, but would not be used in ordinary English.
  - "If the moon is made of green cheese, then I have more money than Bill Gates. "
  - "If the moon is made of green cheese then I'm on welfare."
  - "If 1 + 1 = 3, then your grandma wears combat boots."

DIFFERENT WAYS OF EXPRESSING

$$P \rightarrow Q$$

- **if** p**, then** q
- **if** p, q
- q **unless** $\neg p$
- q **if** p
- q **whenever** p
- q **follows from** p
- p **implies** q
- p **only if** q
- q **when** p
- p **is sufficient for** q
- q **is necessary for** p
- **a necessary condition for** p **is** q
  a sufficient condition for q **is** p
- *See also page 44 of Book of Proof and Table 2.3.2 in ZyBook*

ZyBook Challenge Activity 2.3.1

EQUIVALENT STATEMENTS FOR $P \to Q$

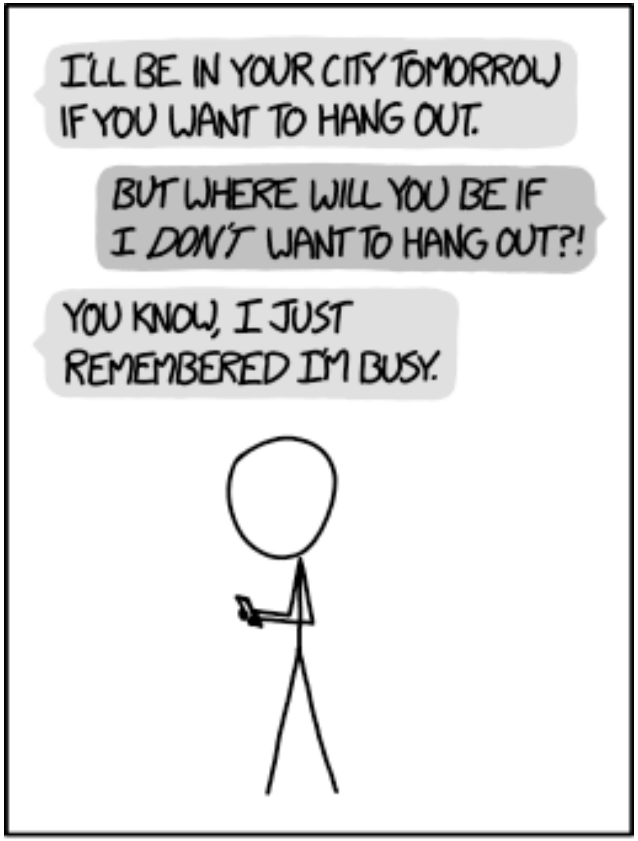
- If $x > 2$ then $x^2>4$
- $x^2>4$ if $x >2$
- $x > 2$ only if $x^2>4$
- $x > 2$ is sufficient for $x^2>4$
- $x^2>4$ is necessary for $x>2$

- You will find any of the above statements in many math books when introducing theorems
  - You need to memorize the above equivalent statements to understand what the theorem is saying.

PROVING $P \rightarrow Q$ IS FALSE: $P \wedge \neg Q$

- Sometimes you want to prove a $p \rightarrow q$ assertion is false
- Recall this is a promise that if p is true, then q is true
- So I need to find an example where p is true and q is false:
  - $p \wedge \neg q$
- If $x^2>4$ then $x>2$?
  - False!
    - Consider $x = -3$.
    - $x^2=9$ so p is true, and $x<2$ so q is false.

COMMON MISTAKE FOR $P \to Q$



WHY I TRY NOT TO BE PEDANTIC ABOUT CONDITIONALS.

- If x > 2 then $x^2>4$ $(p \to q)$
  - Does this mean that If $x^2>4$ then x>2? i.e., $(q \to p)$?
- If you are a bird, you have wings $p \to q$
  - In class someone said the above is false because bats have wings and they are not birds
    - This assumes I said if you have wings, you are a bird $q \to p$
      - *But that is now what I said!*
- If you make this mistake, you will fail this class

Xkcd1652:
Cueball does not assume the converse