ANNOUNCEMENTS

- Homework 1 is out on Canvas:
  - Due date: April 17 (midnight) on Canvas
    - Should be submitted as a pdf file
    - Show how you got an answer (just giving an answer without explanation will cost you points). You do not need to give a "formal proof" to each point (or use only math notation), but you need to provide enough details so the grader can see your logical reasoning.
    - Venn diagrams are NOT proofs.
- Quiz 1: 4:45pm-4:55pm in today's class.
  - 5 questions: time yourself, do not spend more than 2 mins per question.
  - Topics:
  - "Overture" Chapter from Logicomix
  - A couple of conceptual questions about what we covered in Lectures 2 and 3.
  - Quiz 2: Same format and time, Thursday April 16. "Pembroke Lodge" chapter and lectures 4 and 5.
- Exam 1: May 5. Canvas quiz during class time. Covering all month of April
- DRC students getting appropriate time on quizzes (there is a feature on Canvas allowing specific students more time after I close the quiz or exam)
- Slides posted on Canvas after class, videos uploaded to YouTube playlist one or two days after class.
- Emails This is a class of 400 students with 10% of you emailing me questions every day. I cannot answer all emails. Try first Piazza, Tutors, Tas. If that doesn't work, then email me explaining why the above didn't work.

LECTURE SUMMARY

- Review of Quantifiers
- Cartesian Product
- Set Operations
  - Union
  - Intersection
  - Complementation
  - Difference
- More on Set Cardinality
- Set Identities
- Proving Identities

QUANTIFIERS

- We need *quantifiers* to express the meaning of English words including *all* and *some*:
  - "All men are Mortal."
  - "Some cats do not have fur."
- The two most important quantifiers are:
  - *Universal Quantifier,* **"**For all," symbol: $\forall$
  - *Existential Quantifier*, "There exists," symbol: $\exists$
- We write as in $\forall x\, P(x)$ and $\exists x\, P(x)$.
- $\forall x\, P(x)$ asserts $P(x)$ is true for <u>every</u> x in the *domain*.
- $\exists x\, P(x)$ asserts $P(x)$ is true for <u>some</u> x in the *domain*.
- The quantifiers are said to bind the variable x in these expressions.

UNIVERSAL QUANTIFIER

- $\forall x\, P(x)$ is read as "For all x, P(x)" or "For every x, P(x)"

  Examples:

  1) If $P(x)$ denotes "$x > 0$" and U is the integers, then $\forall x\, P(x)$ is false.
  2) If $P(x)$ denotes "$x > 0$" and U is the positive integers, then $\forall x\, P(x)$ is true.
  3) If $P(x)$ denotes "x is even" and U is the integers, then $\forall\, x\, P(x)$ is false.

EXISTENTIAL QUANTIFIER

- $\exists x\, P(x)$ is read as "For some x, P(x)", or as "There is an x such that P(x)," or "For at least one x, P(x)."

Examples:

1. If $P(x)$ denotes "$x > 0$" and U is the integers, then $\exists x\, P(x)$ is true. It is also true if U is the positive integers.
2. If $P(x)$ denotes "$x < 0$" and U is the positive integers, then $\exists x\, P(x)$ is false.
3. If $P(x)$ denotes "x is even" and U is the integers, then $\exists x\, P(x)$ is true.

TUPLES

- The *ordered n-tuple* $(a_1,a_2,\ldots\ldots,a_n)$ is the ordered collection that has a_1 as its first element and a_2 as its second element and so on until a_n as its last element.
- Two n-tuples are equal if and only if their corresponding elements are equal.
- 2-tuples are called *ordered pairs*.
- The ordered pairs (a,b) and (c,d) are equal if and only if $a = c$ and $b = d$.
  - If a=c and b=d then (a,b) and (c,d)
  - And
  - If (a,b)=(c,d) then a=c and d=d

N-TUPLES

(11, 12) ***ordered pair***

( ,  , ) a 3-tuple

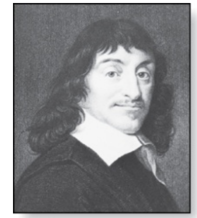
( ,  ,  , 11, Leo) a 5-tuple

- As opposed to sets, repetition and ordering do matter with *n*-tuples.
  - (11, 11, 11, 12, 13) $\neq$ (11, 12, 13)
  - ( ,  , ) $\neq$ ( ,  , )

WHAT IS $\mathbb{R}^2$

- What about $\mathbb{R}^3$

CARTESIAN PRODUCT



René Descartes (1596-1650)

Definition: The *Cartesian Product* of two sets A and B, denoted by $A \times B$ is the set of ordered pairs (a,b) where $a \in A$ and $b \in B$.

$$A \times B = \{(a, b) | a \in A \wedge b \in B\}$$

Example:

$A = \{a,b\}$ $B = \{1,2,3\}$

$A \times B = \{(a,1),(a,2),(a,3), (b,1),(b,2),(b,3)\}$

- **Definition**: A subset R of the Cartesian product $A \times B$ is called a *relation* from the set A to the set B. (Relations will be covered in depth in ZyBook Section 6.)

CARTESIAN PRODUCT

Definition: The cartesian products of the sets $A_1, A_2, \ldots\ldots, A_n$, denoted by $A_1 \times A_2 \times \ldots\ldots \times A_n$, is the set of ordered n-tuples $(a_1, a_2, \ldots\ldots, a_n)$ where a_i belongs to A_i for $i = 1, \ldots n$.

$$A_1 \times A_2 \times \cdots \times A_n = \{(a_1, a_2, \ldots, a_n) | a_i \in A_i \text{ for } i = 1, 2, \ldots n\}$$

Example: What is $A \times B \times$ C where $A = \{0,1\}$, $B = \{1,2\}$ and $C = \{0,1,2\}$

Solution: $A \times B \times$ C = {(0,1,0), (0,1,1), (0,1,2),(0,2,0), (0,2,1), (0,2,2),(1,1,0), (1,1,1), (1,1,2), (1,2,0), (1,2,1), (1,2,2)}

ZyBook Exercises 1.3 (1.3.1-1.3.3)

WHAT IS $\mathbb{R}^2$

- What about $\mathbb{R}^3$

- $\mathbb{R}^2 = \mathbb{R} \times \mathbb{R}$
- $\mathbb{R}^3 = \mathbb{R} \times \mathbb{R} \times \mathbb{R}$

- In general, for an arbitrary set A,
  - $A^n = A \times A \times A \ldots \times A = \{(x_1, x_2, \ldots, x_n): x_1, x_2, \ldots, x_n \in A\}$

UNION

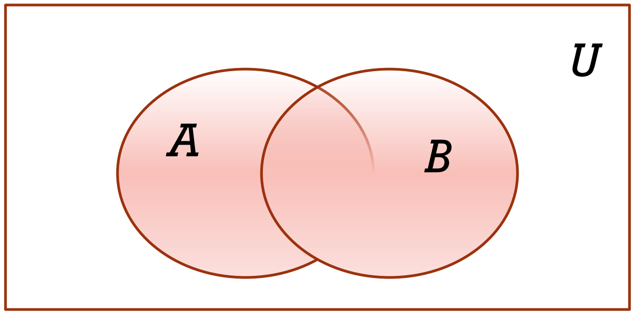
- **Definition**: Let A and B be sets. The *union* of the sets A and B, denoted by $A \cup B$, is the set:

$$\{x | x \in A \vee x \in B\}$$

- **Example**: What is $\{1,2,3\} \cup \{3, 4, 5\}$?

Solution: $\{1,2,3,4,5\}$

Venn Diagram for $A \cup B$



INTERSECTION

- **Definition**: The *intersection* of sets A and B, denoted by $A \cap B$, is

$$\{x | x \in A \land x \in B\}$$

- Note if the intersection is empty, then A and B are said to be *disjoint*.
- **Example**: What is $\{1,2,3\} \cap \{3,4,5\}$?

    Solution: $\{3\}$

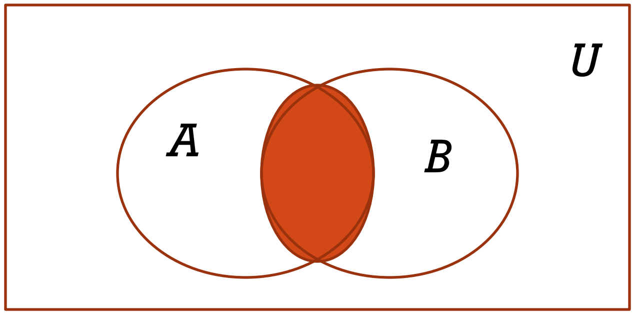
- **Example:** What is

    $\{1,2,3\} \cap \{4,5,6\}$?

    Solution: $\emptyset$

Zybook Exercises 1.4

Venn Diagram for $A \cap B$



COMPLEMENT

Definition: If A is a set, then the complement of the A (with respect to U), denoted by $\bar{A}$ is the set $U - A$

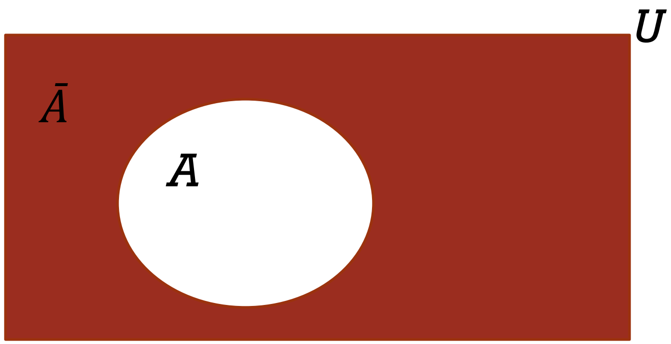
$$\bar{A} = \{x \in U \mid x \notin A\}$$

(The complement of A is sometimes denoted by A^c .)

Example: If U is the positive integers less than 100, what is the complement of $\{x \mid x > 70\}$

Solution: $\{x \mid x \leq 70\}$

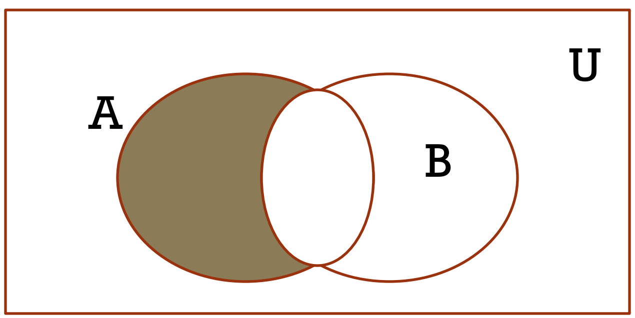
Venn Diagram for Complement



DIFFERENCE

- **Definition**: Let A and B be sets. The *difference* of A and B, denoted by $A - B$, is the set containing the elements of A that are not in B. The difference of A and B is also called the complement of B with respect to A.

$$A - B = \{x \mid x \in A \land x \notin B\} = A \cap \overline{B}$$



Venn Diagram for $A - B$