ANNOUNCEMENTS

- Homework 3 is out on Canvas:
  - Due date: May 25 (midnight) on Canvas
    - Should be submitted as a pdf file
- Quiz 6: Today, last 10 minutes of class.
  - Material: "Entracte" chapter from Logicomix and lectures on 5/7 and 5/12
- Exam 2: June 2. Canvas quiz during class time. Covering all month of May.

Rule of inference	Name
p $p \to q$ $\therefore q$	Modus ponens
$\neg q$ $p \to q$ $\therefore \neg p$	Modus tollens
p $\therefore p \lor q$	Addition
$p \land q$ $\therefore p$	Simplification
p q $\therefore p \land q$	Conjunction
$p \to q$ $q \to r$ $\therefore p \to r$	Hypothetical syllogism
$p \lor q$ $\neg p$ $\therefore q$	Disjunctive syllogism
$p \lor q$ $\neg p \lor r$ $\therefore q \lor r$	Resolution

- Modus Ponens
  - $p, p \to q \vdash q$

Theorem 6 $\alpha \to \beta, \neg\beta \vdash \neg\alpha$

Rule 7 $\alpha \vdash \alpha \lor \beta$

Rule 8 $\beta \vdash \alpha \lor \beta$

Rule 4 $\alpha \land \beta \vdash \alpha$

Rule 5 $\alpha \land \beta \vdash \beta$

Theorem 10 $\alpha, \beta \vdash \alpha \land \beta$

Theorem 2 $\alpha \to \beta, \beta \to \gamma \vdash \alpha \to \gamma$

Theorem 11 $\alpha \lor \beta, \neg\alpha \vdash \beta$

Not in cheat sheet, but easily provable

SUMMARY

Table 2.13.1: Rules of inference for quantified statements.

Rule of Inference	Name	Example
c is an element (arbitrary or particular) $\underline{\forall x\ P(x)}$ $\therefore P(c)$	Universal instantiation	Sam is a student in the class. Every student in the class completed the assignment. Therefore, Sam completed his assignment.
c is an arbitrary element $\underline{P(c)\qquad}$ $\therefore \forall x\ P(x)$	Universal generalization	Let c be an arbitrary integer. $c \le c^2$ Therefore, every integer is less than or equal to its square.
$\underline{\exists x\ P(x)}$ $\therefore$ (c is a particular element) $\wedge$ P(c)	Existential instantiation*	There is an integer that is equal to its square. Therefore, $c^2 = c$, for some integer c.
c is an element (arbitrary or particular) $\underline{P(c)\qquad}$ $\therefore \exists x\ P(x)$	Existential generalization	Sam is a particular student in the class. Sam completed the assignment. Therefore, there is a student in the class who completed the assignment.

*Note: each use of Existential instantiation must define a new element with its own name (e.g., "c" or "d").

ZyBook 2.13.2

ALLOWED ASSUMPTIONS IN PROOFS

The rules of algebra.

For example if x, y, and z are real numbers and $x = y$, then $x+z = y+z$.

The set of integers is closed under addition, multiplication, and subtraction.

In other words, sums, products, and differences of integers are also integers.

Every integer is either even or odd.

This fact is proven elsewhere in the material.

If x is an integer, there is no integer between x and $x+1$.

In particular, there is no integer between 0 and 1.

The relative order of any two real numbers.

For example $1/2 < 1$ or $4.2 \geq 3.7$.

The square of any real number is greater than or equal to 0.

This fact is proven in a later exercise.

PREMISES AND PREVIOUS RESULTS IN PROOFS

By definition

A fact that is known because of a definition, can be started with the phrase "By definition". For example: "The integer m is even. By definition, m = 2k for some integer k."

By assumption

A fact that is known because of an assumption, can be started with the phrase "By assumption". For example: "By assumption, x is positive. Therefore x >0."

PROVING CONDITIONAL STATEMENTS: $P \rightarrow Q$

- *Direct Proof*: Assume that p is true. Use rules of inference, axioms, and logical equivalences to show that q must also be true.

Theorem: If n is an odd integer, then n^2 is odd.

Solution: Assume that n is odd. Then $n = 2k + 1$ for an integer k. Squaring both sides of the equation, we get:

$$n^2 = (2k + 1)^2 = 4k^2 + 4k + 1 = 2(2k^2 + 2k) + 1 = 2r + 1,$$

where $r = 2k^2 + 2k$, an integer.

(integers are closed under multiplication)

We have proved that if n is an odd integer, then n^2 is an odd integer.

◀

(◀ marks the end of the proof. Sometimes **QED** is used instead.)

PROVING CONDITIONAL STATEMENTS: $P \rightarrow Q$

Theorem: The sum of two rational numbers is rational.

Solution*:* Assume *r* and *s* are two rational numbers. Then there must be integers *p, q* and also *t, u* such that

$$r = p/q, \quad s = t/u, \quad u \neq 0, \; q \neq 0$$

$$r + s = \frac{p}{q} + \frac{t}{u} = \frac{pu+qt}{qu} = \frac{v}{w}$$

where $v = pu + qt$
$w = \text{qu} \neq 0$

Thus the sum is rational.

◀

ZyBook Participation Activities 4.2.1 up to 4.4.5