ANNOUNCEMENTS

- Homework 1 is out on Canvas:
  - Due date: April 17 (midnight) on Canvas
    - should be submitted as a pdf file

- Quiz 1: Last 10 minutes of the class on Thursday April 9
  - Topics:
  - "Overture" Chapter from Logicomix
  - A couple of conceptual questions about what we covered in Lectures 2 and 3.

- Discussion Session information posted on Canvas and Piazza
- Reading for today's class: ZyBook 1.2-1.7

SET OPERATIONS

Some slides courtesy of Tracy Larrabee and

Rosen, Discrete Math and its Applications (Wiley)

SUBSETS

Definition: The set A is a *subset* of B, if and only if every element of A is also an element of B.

- The notation $A \subseteq B$ is used to indicate that A is a subset of the set B.
- $A \subseteq B$ holds if and only if $\forall x(x \in A \to x \in B)$ is true.
- ZyBook Exercises 1.1

SHOWING A SET IS OR IS NOT A SUBSET OF ANOTHER SET

- **Showing that A is a Subset of B**: To show that $A \subseteq B$, show that if x belongs to A, then **x** also belongs to B:

$$\forall x(x \in A \to x \in B)$$

- **Showing that A is not a Subset of B**: $A \nsubseteq B$

$$\neg\forall x(x \in A \to x \in B)$$

$$\exists x\neg(x \in A \to x \in B)$$

$$\exists x(x \in A \wedge x \notin B)$$

- To show that A is not a subset of B, $A \nsubseteq B$, find an element $x \in A$ with $x \notin B$. (Such an x is a counterexample to the claim that $x \in A$ implies $x \in B$.)

PROVING SETS ARE SUBSETS OF OTHERS

$$\forall x(x \in A \rightarrow x \in B)$$

- {1,5,5,5,3,3,1} ⊆ {1,3,5} ?
- Definition of subset

$$\forall x(x \in A \rightarrow x \in B)$$

  - for x in A:
    - Show x belongs to B

PROVING SETS ARE SUBSETS OF OTHERS

$$\forall x(x \in A \to x \in B)$$

- $S \subseteq S$, for every set S?
- Because $a \in S \to a \in S$,
- $\varnothing \subseteq S$

PROVING SETS ARE SUBSETS OF OTHERS

- Is the set of integers with squares less than 100 a subset of ℕ?
- No! We are now going to prove the first set is not a subset of the -2 belongs to the first set, but not to ℕ
- Proof:
  - -2 belongs to the set of integers with squares less than 100
  - -2 does not belong to ℕ
- The above proof is usually called a counterexample

$$\neg \forall x (x \in A \rightarrow x \in B)$$

Is equivalent to

$$\exists x (x \in A \wedge x \notin B)$$

ANOTHER LOOK AT EQUALITY OF SETS

- Recall that two sets A and B are *equal*, denoted by $A = B$, iff

$$\forall x(x \in A \leftrightarrow x \in B)$$

- Using logical equivalences we have that $A = B$ iff

$$\forall x[(x \in A \rightarrow x \in B) \wedge (x \in B \rightarrow x \in A)]$$

- This is equivalent to

$A \subseteq B$ and $B \subseteq A$

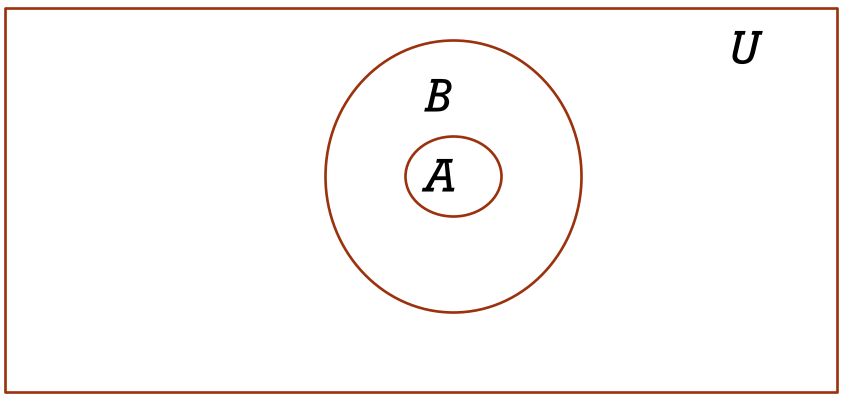
PROPER SUBSETS

Definition: If $A \subseteq B$, but $A \neq B$, then we say A is a *proper subset* of B, denoted by $A \subset B$. i.e.,:

$A \subset B$ ***iff***

$$\forall x(x \in A \to x \in B) \land \exists x(x \in B \land x \notin A)$$

Venn Diagram



TRUE OR FALSE?

$\mathbb{N} \subset \mathbb{R}$

$\mathbb{Z} \subseteq \mathbb{N}$

$-3 \subseteq \mathbb{R}$

$\{1,2\} \notin \mathbb{Z}^{+}$

$\varnothing \subseteq \varnothing$

$\varnothing \subset \varnothing$

$\{x\} \subseteq \{x\}$

$\{x\} \in \{x,\{x\}\}$

$\{x\} \subseteq \{x,\{x\}\}$

$\{x\} \in \{x\}$

POWER SETS

Definition: The set of all subsets of a set A, denoted $\mathcal{P}(A)$, is called the *power set* of A.

Example: If $A = \{a,b\}$ then

$$\mathcal{P}(A) = \{\emptyset, \{a\},\{b\},\{a,b\}\}$$

- If a set has n elements, then the cardinality of the power set is 2^n.
- ZyBook Exercises 1.2

TUPLES

- The *ordered n-tuple* $(a_1,a_2,.....,a_n)$ is the ordered collection that has a_1 as its first element and a_2 as its second element and so on until a_n as its last element.
- Two n-tuples are equal if and only if their corresponding elements are equal.
- 2-tuples are called *ordered pairs*.
- The ordered pairs (a,b) and (c,d) are equal if and only if $a = c$ and $b = d$.
  - If a=c and b=d then (a,b) and (c,d)
  - And
  - If (a,b)=(c,d) then a=c and d=d

N-TUPLES

(11, 12) — ***ordered pair***

(, , ) — a 3-tuple

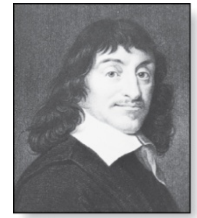
(, , , 11, Leo) — a 5-tuple

- As opposed to sets, repetition and ordering do matter with *n*-tuples.
  - (11, 11, 11, 12, 13) $\neq$ (11, 12, 13)
  - (, , ) $\neq$ (, , )

WHAT IS $\mathbb{R}^2$

- What about $\mathbb{R}^3$

CARTESIAN PRODUCT



René Descartes
(1596-1650)

Definition: The *Cartesian Product* of two sets A and B, denoted by $A \times B$ is the set of ordered pairs (a,b) where $a \in A$ and $b \in B$.

$$A \times B = \{(a, b) | a \in A \land b \in B\}$$

Example:

$A = \{a,b\}$ $B = \{1,2,3\}$

$A \times B = \{(a,1),(a,2),(a,3), (b,1),(b,2),(b,3)\}$

- **Definition**: A subset R of the Cartesian product $A \times B$ is called a *relation* from the set A to the set B. (Relations will be covered in depth in ZyBook Section 6.)

CARTESIAN PRODUCT

Definition: The cartesian products of the sets $A_1, A_2, \ldots\ldots, A_n$, denoted by $A_1 \times A_2 \times \ldots\ldots \times A_n$, is the set of ordered n-tuples $(a_1, a_2, \ldots\ldots, a_n)$ where a_i belongs to A_i for $i = 1, \ldots n$.

$$A_1 \times A_2 \times \cdots \times A_n = \{(a_1, a_2, \ldots, a_n) | a_i \in A_i \text{ for } i = 1, 2, \ldots n\}$$

Example: What is $A \times B \times C$ where $A = \{0,1\}$, $B = \{1,2\}$ and $C = \{0,1,2\}$

Solution: $A \times B \times C = \{(0,1,0), (0,1,1), (0,1,2), (0,2,0), (0,2,1), (0,2,2), (1,1,0), (1,1,1), (1,1,2), (1,2,0), (1,2,1), (1,2,2)\}$

ZyBook Exercise 1.3

WHAT IS $\mathbb{R}^2$

- What about $\mathbb{R}^3$

- $\mathbb{R}^2 = \mathbb{R} \times \mathbb{R}$
- $\mathbb{R}^3 = \mathbb{R} \times \mathbb{R} \times \mathbb{R}$

- In general, for an arbitrary set A,
  - $A^n = A \times A \times A \dots \times A = \{(x_1, x_2, \dots, x_n): x_1, x_2, \dots, x_n \in A\}$