1.

(a) Feb has 30 days is false, so p is false. In p—>q, if p is false, then the statement is always true, so the statement “If February has 30 days, then 7 is an odd number” is **true**.

(b) Jan has 31 days is true. In p—>q, if p is true, when q is true, then the statement will be true, but when q is false, the statement will be false. 7 is an odd number, so “7 is an even number” is false. Therefore, the statement “If January has 31 days, then 7 is an even number” is **false**.

(c) 7 is an odd number, so p is true in p—>q. Feb has 28 0r 29 days, so the statement q “February does not have 30 days” is true too. When p and q are both true, the whole statement is true. Therefore the statement “If 7 is an odd number, then February does not have 30 days” is **true**.

(d) 7 is an odd number, not an even number, so the statement “7 is an even number” is false. Therefore, p is false. In p—>q, if p is false, then the statement is always true, so the statement “If 7 is an even number, then January has exactly 28 days” is **true**.

2.

(a) **inverse**: If she does not finish her homework, then she didn’t go to the party
contrapositive: If she didn’t go to the party, then she did not finish her homework
converse: If she went to the party, then she finished her homework

(b) **inverse**: If he didn’t train for the race, then he didn’t finish the race
contrapositive: If he didn’t finish the race, then he didn’t train for the race
converse: If he finished the race, then he trained for the race

(c) **inverse**: If the patience didn’t take the medicine, then she didn’t have side effect
contrapositive: If the patient didn’t have side effects, then she didn’t take the medicine
converse: If the patient had side effects, then she took the medicine

(d) **inverse**: If it was not sunny, then the game was not held
contrapositive: If the game was not held, then it was not sunny
converse: If the game was held, the it was sunny day

(e) **inverse**: If it didn’t snow last night, then school will not be cancelled
contrapositive: If the school will not be cancelled, then is didn’t snow last night
converse: If the school will be cancelled, then it snowed last night

3.

(a) **inverse**: If 3 is not a prime number, then 5 is not an even number
In p—>q, p is false, so the statement is always **true**.
contrapositive: If 5 is not an even number, then 3 is not a prime number

In p—>q, p is true and q is false, so the statement is **false**.

converse: If 5 is an even number, then 3 is a prime number

In p—>q, p is false, so the statement is always **true**.

(b) **inverse**: If 7 is not <5, then 5 is not <3

In p—>q, p is true and q is true, so the statement is **true**.

contrapositive: If 5 is not <3, then 7 is not <5

In p—>q, p is true and q is true, so the statement is **true**.

converse: If 5<3, then 7<5

In p—>q, p is false, so the statement is always **true**.

(c) **inverse**: If 5 is not a negative number, then 3 is not a positive number

In p—>q, p is true and q is false, so the statement is **false**.

contrapositive: If 3 is not a positive number, then 5 is not a negative number

In p—>q, p is false, so the statement is always **true**.

converse: If 3 is a positive, then 5 is a negative number

In p—>q, p is true and q is false, so the statement is **false**.

4.

(a)

p	q	¬p	¬p∧q	(¬p∧q)→p
T	T	F	F	T
T	F	F	F	T
F	T	T	T	F
F	F	T	F	T

(b)

p	q	p→q	q→p	(p→q)→(q→p)
T	T	T	T	T
T	F	F	T	T
F	T	T	F	F
F	F	T	T	T

(c)

p	q	¬p	p∨q	q→¬p	(p∨q)↔(q→¬p)

T	T	F	T	F	F
T	F	F	T	T	T
F	T	T	T	T	T
F	F	T	F	T	F

(d)

p	q	¬q	p↔q	p↔¬q	(p↔q)⊕(p↔¬q)
T	T	F	T	F	T
T	F	T	F	T	T
F	T	F	F	T	T
F	F	T	T	F	T

(e)

p	q	p∨q	q∧p	(p∨q)↔(q∧p)
T	T	T	T	T
T	F	T	F	F
F	T	T	F	F
F	F	F	F	T

5.

(a) ¬j—>c

(b) c—>¬j

(c) ¬j∧¬c

(d) c—>¬j

(e) ¬c

(f) j∧c

6.

(a) If Joe maintains a B average, then he is eligible for the honors program

(b) If Joe is eligible for the honors program, then he maintains a B average

(c) If Rajiv can go on the roller coaster, then he is at least four feet tall

(d) If Rajiv is at least four feet tall, then he can go on the roller coaster

7.

(a) p—>(s∧y)

(b) (s∨y)—>p

(c) p—>y

(d) p—>s∧y

(e) p—>s∨y p<—>s∨y

8.

(a) If the roads were wet, then traffic was heavy

(b) The roads were wet and there was an accident

(c) There was not an accident and traffic was not heavy neither

(d) If traffic was heavy, then there was an accident or the roads were wet

(e) The roads were wet and traffic was not heavy

9.

(a) ¬y∧p

(b) (y∨p)—>c

(c) c—>p

(d) c—>p

10.

(a) false

Because p is true, only both q and r are true can make the statement true, but q is false, so q∧r is false. In p—>q, if p is true and q is false, the statement is **false**.

(b) unknown

Because p is true, p∨r is true whether r is true or false. However, if r is true, the statement will be true; if r is false, the statement will be false. Therefore, the truth value of this statement is **unknown**.

(c) false

Because p is true, p∨r is true whether r is true or false. q is false, so q∧r is false. Therefore, the statement is **false**.

(d) unknown

Because q is false, q∧r is false, but p∧r is unknown, so the truth value of this statement is **unknown**.

(e) unknown

p is true, but r∨q is unknown. If r is true, then the statement will be true; if r is false, then the statement will be false. Therefore, the truth value of this statement is **unknown**.

(f) true

Because p is true and q is false, p∧q is false. Therefore, whether r is true or not, the statement is **true**.

11.

(a) $p \wedge s$

negation: $\neg(p \wedge s)$

applying De Morgan's law: $\neg p \vee \neg s$

english: The applicant has not written permission from his parents or is not at least 16 years old

(b) $p \vee e$

negation: $\neg(p \vee e)$

applying De Morgan's law: $\neg p \wedge \neg e$

english: The applicant has not written permission from his parents and is not at least 18 years old

12.

(a) Conditional identities
    Commutative laws
    Distributive laws
    Commutative laws
    Complement laws
    Identity laws

(b) Conditional identities
    De Morgan's laws
    Double negation law
    Distributive laws
    Complement laws
    Identity laws

(c) Conditional identities
    Double negation law
    Associative laws
    Idempotent laws

13.

13.

(a). $\neg p \to \neg q \equiv q \to p$

$\neg\neg p \vee \neg q$	conditional identities
$p \vee \neg q$	double negation law
$\neg q \vee p$	commutative laws
$q \to p$	conditional identities

(b). $p \wedge (\neg p \to q) \equiv p$

$p \wedge (\neg\neg p \vee q)$	conditional identities
$p \wedge (p \vee q)$	double negation law
p	absorption laws

(c). $(p \to q) \wedge (p \to r) \equiv p \to (q \wedge r)$

$(\neg p \vee q) \wedge (\neg p \vee r)$	conditional identities
$\neg p \vee (q \wedge r)$	distributive laws
$\neg\neg p \to (q \wedge r)$	conditional identities
$p \to (q \wedge r)$	double negation law

(d) $\neg p \to (q \to r) \equiv q \to (p \vee r)$

$\neg\neg p \vee (q \to r)$	conditional identities
$p \vee (\neg q \vee r)$	conditional identities
$(p \vee \neg q) \vee r$	associative laws
$(\neg q \vee p) \vee r$	commutative laws
$\neg q \vee (p \vee r)$	associative laws
$q \to (p \vee r)$	conditional identities

(e) $(p \to r) \vee (q \to r) \equiv (p \wedge q) \to r$

$(\neg p \vee r) \vee (\neg q \vee r)$ conditional identities

$(\neg p \vee r) \vee (r \vee \neg q)$ commutative laws

$\neg p \vee (r \vee r) \vee \neg q$ associative laws

$\neg p \vee r \vee \neg q$

$(\neg p \vee \neg q) \vee r$ commutative laws

$\neg(p \wedge q) \vee r$ De morgan's laws

$(p \wedge q) \to r$ conditional identities

(f) $\neg(p \vee (\neg p \wedge q)) \equiv \neg p \wedge \neg q$

$\neg p \wedge \neg(\neg p \wedge q)$ distributive laws

$\neg p \wedge (p \vee q)$ De Morgan's laws

$(\neg p \wedge p) \vee (\neg p \wedge \neg q)$ distributive laws

$(p \wedge \neg p) \vee (\neg p \wedge \neg q)$ commuative laws

$F \vee (\neg p \wedge \neg q)$ complement laws

$(\neg p \wedge \neg q) \vee F$ commuative laws

$\neg p \wedge \neg q$ identity laws

(g) $(p \wedge q \wedge \neg r) \vee (p \wedge \neg q \wedge \neg r) \equiv p \wedge \neg r$

$p \wedge ((q \wedge \neg r) \vee (\neg q \wedge \neg r))$ distributive laws

$p \wedge ((\neg r \wedge q) \vee (\neg r \wedge \neg q))$ commuative laws

$p \wedge (\neg r \wedge (q \vee \neg q))$ distributive laws

$p \wedge (\neg r \wedge T)$ complement laws

$p \wedge \neg r$ domination laws

(h) $p \leftrightarrow (p \wedge r) \equiv \neg p \vee r$

$(p \to (p \wedge r)) \wedge ((p \wedge r) \to p)$ conditional identities

$(\neg p \vee (p \wedge r)) \wedge (\neg (p \wedge r) \vee p)$

$((\neg p \vee p) \wedge (\neg p \vee r)) \wedge ((\neg p \vee \neg r) \vee p)$ distributive laws

$((p \vee \neg p) \wedge (\neg p \vee r)) \wedge ((\neg r \vee \neg p) \vee p)$ commutative laws

$(T \wedge (\neg p \vee r)) \wedge (\neg r \vee (\neg p \vee p))$ complement laws

$((\neg p \vee r) \wedge T) \wedge (\neg r \vee (p \vee \neg p))$ commutative laws

$((\neg p \vee r) \wedge T) \wedge (\neg r \vee T)$ complement laws

$(\neg p \vee r) \wedge T$ identity laws & domination laws

(i) $(p \wedge q) \to r \equiv (p \wedge \neg r) \to \neg q$

$\neg (p \wedge q) \vee r$ conditional identities

$(\neg p \vee \neg q) \vee r$ De morgan's laws

$\neg p \vee (\neg q \vee r)$ associative laws

$\neg p \vee (r \vee \neg q)$ commutative laws

$(\neg p \vee r) \vee \neg q$ associative laws

$\neg (p \wedge \neg r) \vee \neg q$ De Morgan's laws

$(p \wedge \neg r) \to \neg q$ conditional identities

14.

14.

(a). $p \to q \equiv \neg p \vee q$

$q \to p \equiv \neg q \vee p \equiv p \vee \neg q$.

$\neg p \vee q \not\equiv p \vee \neg q$

(b). $p \to q \equiv \neg p \vee q$

$\neg p \to \neg q \equiv \neg\neg p \vee \neg q \equiv p \vee \neg q$

$\neg p \vee q \not\equiv p \vee \neg q$

(c). $p \to q \equiv \neg p \vee q$

$\neg q \to \neg p \equiv \neg\neg q \vee \neg p \equiv q \vee \neg p \equiv \neg p \vee q$

$\neg p \vee q \equiv \neg p \vee q$

(d). $q \to p \equiv \neg q \vee p \equiv p \vee \neg q$

$\neg p \to \neg q \equiv \neg\neg p \vee \neg q \equiv p \vee \neg q$

$p \vee \neg q \equiv p \vee \neg q$

15.

15.

p: x is a rational number

¬q: y is an irrational number

¬r: x−y is an irrational number

r: x−y is a rational number

q: y is a rational number

①. $p \wedge \neg q \to \neg r$

$\neg(p \wedge \neg q) \vee \neg r$

$(\neg p \wedge \neg\neg q) \vee \neg r$

$(\neg p \wedge q) \vee \neg r$

$\neg p \vee (q \vee \neg r)$

$\neg p \vee (\neg r \vee q)$

$(\neg p \vee \neg r) \vee q$.

②. $p \wedge r \to q$

$\neg(p \wedge r) \vee q$

$(\neg p \vee \neg r) \vee q$

16.

(a) False

there is no example, so false

(b) True

ex. x=-1

(c) True

there is no counterexample, so true

(d) False

ex. x=0

(e) False

ex. x=0

(f) True

ex. x=1 or any nonzero integers

17.

x∈R

(a) $\exists x(x^3=2)$

(b) $\forall x(x^2>0)$

(c) $\exists x(x^2=x)$

(d) $\forall x(x \le x^2)$

18.

(a) $\exists x(S(x))$

(b) $\forall x(\neg S(x) \land W(x))$

(c) $\forall x(S(x) \rightarrow \neg W(x))$

(d) $\exists x(S(X) \land W(x))$

(e) $\forall x(\neg W(x) \rightarrow S(x))$

(f) $\forall x(\neg W(x) \rightarrow (S(x) \lor V(x)))$

(g) $\exists x(\neg W(x) \rightarrow \neg(S(x) \lor (V(x)))$

(h) $\forall x(\neg W(x) \rightarrow (S(x) \lor V(x)))$

(i) $S(Ingrid) \land W(Ingrid)$

(j) $\exists x(S(x) \land (x \neq Ingrid))$

(k) $\forall x(S(x) \land (x \neq Ingrid))$

19.

(a) Each person who is on the board of directors earns more than $100,000

(b) At least one person earns more than $100,000 and doesn't work more than 60 hours per week

(c) Everyone who works more than 60 hours per week earns more than $100,000

(d) Someone is not on the board of directors but earns more than $100,000

(e) Everyone who earns more than $100,000 is on the board of directors or works more than 60 hours per week

(f) At least one person is on the board of directors and works more than 60 hours per week, but doesn't earn more than $100,000

20.

(a) $\forall x(\neg P(x))$

(b) $\forall x(\neg P(x) \land \neg Q(x))$

(c) $\exists x(\neg P(x) \lor \neg Q(x))$

(d) $\exists x(\neg P(x) \lor \neg(Q(x) \lor R(x)))$

$\exists x(\neg P(x) \lor (\neg Q(x) \land \neg R(x)))$

21.

(a) $\forall x(D(x))$

negation: $\neg\forall x(D(x))$

applying De Morgan's law: $\exists x(\neg D(x))$

english: There is a patient who was not given the medication

(b) $\forall x(D(x) \lor P(x))$

negation: $\neg\forall x(D(x) \lor P(x))$

applying De Morgan's law: $\exists x(\neg D(x) \land \neg P(x))$

english: There is a patient who was not given the medication and was not given the placebo

(c) $\exists x(D(x) \wedge M(x))$

negation: $\neg\exists x(D(x) \wedge M(x))$

applying De Morgan's law: $\forall x(\neg D(x) \vee \neg M(x))$

english: Every patient was either not given the medication or didn't have migraines (or both)

(d) $\forall x(P(x) \text{—>} M(x))$

negation: $\neg\forall x(P(x) \text{—>} M(x))$

$\neg\forall x(\neg P(x) \vee M(x))$

applying De Morgan's law: $\exists x(P(x) \wedge \neg M(x))$

english: There is a patient was given the placebo and didn't have migraines

(e) $\exists x(M(x) \wedge P(x))$

negation: $\neg\exists x(M(x) \wedge P(x))$

applying De Morgan's law: $\forall x(\neg M(x) \vee \neg P(x))$

english: Every patient either didn't have migraines or was not given the placebo (or both)

22.

(a) $\exists x(P(x))$

negation: $\neg\exists x(P(x))$

applying De Morgan's law: $\forall x(\neg P(x))$

english: Every student showed up without a pencil

(b) $\forall x(P(x) \vee C(x))$

negation: $\neg\forall x(P(x) \vee C(x))$

applying De Morgan's law: $\exists x(\neg P(x) \wedge \neg C(x))$

english: Some student showed up without a pencil and a calculator

(c) $\forall x(P(x) \text{—>} C(x))$

negation: $\neg\forall x(P(x) \text{—>} C(x))$

$\neg\forall x(\neg P(x) \vee C(x))$

applying De Morgan's law: $\exists x(P(x) \wedge \neg C(x))$

english: There is a student showed up with a pencil but without a calculator

(d) $\exists x(P(x) \wedge C(x))$

negation: $\neg\exists x(P(x) \wedge C(x))$

applying De Morgan's law: $\forall x(\neg P(x) \vee \neg C(x))$

english: Every student showed up without either a pencil or a calculator (or both)

(e) $\exists x(P(x) \vee C(x))$

negation: $\neg\exists x(P(x)\vee C(x))$

applying De Morgan's law: $\forall x(\neg P(x)\wedge\neg C(x))$

english: Every student showed up without a pencil and a calculator

(f) $\forall x(P(x)\wedge C(x))$

negation: $\neg\forall x(P(x)\wedge C(x))$

applying De Morgan's law: $\exists x(\neg P(x)\vee\neg C(x))$

english: At least one of the students showed up without a pencil or a calculator (or both)

23.

(a) $\exists x\neg(P(x)\wedge\neg Q(x))$

$\exists x(\neg P(x)\vee\neg\neg Q(x))$

$\boldsymbol{\exists x(\neg P(x)\vee Q(x))}$

(b) $\exists x\neg(\neg P(x) \rightarrow Q(x))$

$\exists x\neg(\neg\neg P(x)\vee Q(x))$

$\exists x\neg(P(x)\vee Q(x))$

$\boldsymbol{\exists x(\neg P(x)\wedge\neg Q(x))}$

(c) $\forall x\neg(\neg P(x)\vee(Q(x)\wedge\neg R(x)))$

$\forall x(\neg\neg P(x)\wedge\neg(Q(x)\wedge\neg R(x)))$

$\forall x(P(x)\wedge(\neg Q(x)\vee\neg\neg R(x)))$

$\boldsymbol{\forall x(P(x)\wedge(\neg Q(x)\vee R(x)))}$

24.

(a) False

x=0 is a counterexample, so **false**

(b) True

x=0 can make xy=0 for all y, so **true**

(c) True

there exists a real number z=(x-y)/3 for all x and all y, so **true**

(d) True

for all x there exists a y that satisfies the requirement, but there is not exists y z for all x that can satisfies the requirement, so false

there exists a real number y=x-3z for all x and all z, so **true**

(e) True

commutative for real numbers x and y, so **true**

(f) True

x=3, y=4, z=5 is an example, so **true**

(g) False

x=-1 or any number less than 0 can be a counterexample, so **false**

(h) True

$y=x^2>0$, $(x<0 \vee x>0) \longrightarrow \forall x$, so **true**

(i) True

$x=-1$, $y=1$ or any number that $|x|=|y|$ but $x \neq y$ can be a counterexample, so **false**

(j) False

$|x| \neq |y| \longrightarrow x^2 \neq y^2$, $x^2=y^2 \wedge x^2 \neq y^2$ doesn't exist, so **false**

(k) True

$|x|=|y| \longrightarrow x^2=y^2$, $(x^2 \neq y^2 \vee x^2=y^2) \longrightarrow \forall x \forall y$, so **true**

25.

25.

(a). $\neg\forall x \exists y \exists z\, P(y,x,z)$

$\exists x \forall y \forall z\, P(y,x,z)$

(b). $\neg\forall x \exists y (P(x,y) \wedge Q(x,y))$

$\exists x \forall y \neg(P(x,y) \wedge Q(x,y))$

$\exists x \forall y (\neg P(x,y) \vee \neg Q(x,y))$

(c). $\neg\exists x \forall y (P(x,y) \rightarrow Q(x,y))$

$\forall x \exists y \neg(\neg P(x,y) \vee Q(x,y))$

$\forall x \exists y (\neg\neg P(x,y) \wedge \neg Q(x,y))$

$\forall x \exists y (P(x,y) \wedge \neg Q(x,y))$

(d). $\neg\exists x \forall y (P(x,y) \leftrightarrow P(x,y))$

$\forall x \exists y \neg(P(x,y) \rightarrow P(y,x) \wedge P(y,x) \rightarrow P(x,y))$

$\forall x \exists y \neg((\neg P(x,y) \vee P(y,x)) \wedge (\neg P(y,x) \vee P(x,y)))$

$\forall x \exists y (\neg(\neg P(x,y) \vee P(y,x)) \wedge (\neg(\neg P(y,x) \vee P(x,y)))$

$\forall x \exists y ((\neg\neg P(x,y) \wedge P(y,x)) \wedge (\neg\neg P(y,x) \wedge \neg P(x,y)))$

$\forall x \exists y ((P(x,y) \wedge \neg P(y,x)) \vee (P(y,x) \wedge \neg P(x,y)))$

(e). $\neg(\exists x \exists y\, P(x,y) \wedge \forall x \forall y\, Q(x,y))$

$\neg\exists x \exists y\, P(x,y) \vee \neg\forall x \forall y\, Q(x,y)$

$\forall x \forall y \neg P(x,y) \vee \exists x \exists y \neg Q(x,y)$

26.

(a) $\forall x \forall y F(x, y)$

negation: $\neg \forall x \forall y F(x, y)$

applying De Morgan's law: $\exists x \exists y \neg F(x, y)$

english: Someone is an enemy of someone

(b) $\exists x \exists y F(x, y)$

negation: $\neg \exists x \exists y F(x, y)$

applying De Morgan's law: $\forall x \forall y \neg F(x, y)$

english: Everyone is an enemy of everyone

(c) $\exists x \forall y F(x, y)$

negation: $\neg \exists x \forall y F(x, y)$

applying De Morgan's law: $\forall x \exists y \neg F(x, y)$

english: Everyone is an enemy of someone

(d) $\forall x \exists y F(x, y)$

negation: $\neg \forall x \exists y F(x, y)$

applying De Morgan's law: $\exists x \forall y \neg F(x, y)$

english: Someone is an enemy of everyone

27.

(a) True

for all x, y=-x is a real number, so **true**

(b) False

there does not exist an x for all y that satisfies x=-y, so **false**

(c) True

there exists an x for all y that satisfies xy=y, for example x=1, so **true**

(d) True

there exists an x and a y that satisfies $(x^2 = y^2) \wedge (x \neq y)$, for example x=1 and y=-1, so **true**

(e) True

For all x and all y, z=(x + y)/2 is a real number, so **true**

(f) True

For any x, there exists a y for which any z, for example y=0, so **true**

Once the values for x and y are determined, the value for z can be selected to be not equal to (x+y)/2, which makes the statement false

28.

(a) $\exists x \exists y (x/y > 1)$

(b) $\forall x (x > 0 \longrightarrow 1/x > 0)$

(c) $\exists x \exists y (x+y=xy)$

(d) $\forall x \forall y (x>0 \wedge y>0 \rightarrow x/y>0)$

(e) $\forall x \forall y (x>0 \wedge x<1 \rightarrow 1/x>1)$

(f) $\neg \exists x \forall y (x<y)$

(g) $\forall x \exists y (x \neq 0 \rightarrow y=1/x)$

(h) $\forall x \exists ! y (x \neq 0 \rightarrow y=1/x)$