Assignment 1

CSE 16

April 7, 2020

1 Part I

1. Use set builder notation to give a description of each of these sets.
   a. $\{0, 3, 6, 9, 12\}$
   b. $\{-3, -2, -1, 0, 1, 2, 3\}$
   c. $\{m, n, o, p\}$

2. Determine whether each of these pairs of sets are equal.
   a. $\{1, 3, 3, 3, 5, 5, 5, 5, 5\}, \{5, 3, 1\}$
   b. $\{\{1\}\}, \{1, \{1\}\}$
   c. $\varnothing, \{\varnothing\}$

3. Determine whether these statements are true or false:
   a. $\varnothing \in \{\varnothing\}$
   b. $\varnothing \in \{\varnothing, \{\varnothing\}\}$
   c. $\{\varnothing\} \in \{\varnothing\}$
   d. $\{\varnothing\} \in \{\{\varnothing\}\}$
   e. $\{\varnothing\} \subset \{\varnothing, \{\varnothing\}\}$
   f. $\{\{\varnothing\}\} \subset \{\varnothing, \{\varnothing\}\}$
   g. $\{\{\varnothing\}\} \subset \{\{\varnothing\}, \{\varnothing\}\}$

4. Determine whether these statements are true or false:
   a. $x \in \{x\}$
   b. $\{x\} \subseteq \{x\}$
   c. $\{x\} \in \{x\}$
   d. $\{x\} \in \{\{x\}\}$
   e. $\varnothing \subseteq \{x\}$
   f. $\varnothing \in \{x\}$

5. Find two sets A and B such that $A \in B$ and $A \subseteq B$. A=$\varnothing$, B=$\{\varnothing\}$

6. What is the cardinality of each of these sets?
   a. $\varnothing$

b. $\{\varnothing\}$
c. $\{\varnothing, \{\varnothing\}\}$
d. $\{\varnothing, \{\varnothing\}, \{\varnothing, \{\varnothing\}\}\}$

7. Find the power set of each of these sets, where a and b are distinct elements.
   a. $\{a\}$
   b. $\{a, b\}$
   c. $\{\varnothing, \{\varnothing\}\}$

2 Part II

1. Suppose that A, B, and C are sets such that $A \subseteq B$ and $B \subseteq C$, show that $A \subseteq C$
2. Prove that $\mathcal{P}(A) \subseteq \mathcal{P}(B)$ if and only if $A \subseteq B$
3. Show that if $A \subseteq C$ and $B \subseteq D$, then $A \times B \subseteq C \times D$
4. Let A be a set. Show that $\varnothing \times A = A \times \varnothing = \varnothing$.
5. This exercise presents **Russell's paradox**. Let S be the set that contains a set x if the set x does not belong to itself, so that $S = \{x \mid x \notin x\}$.

   a) Show the assumption that S is a member of S leads to a contradiction.
   b) Show the assumption that S is not a member of S leads to a contradiction.

   By parts (a) and (b) it follows that the set S cannot be defined as it was. This paradox can be avoided by restricting the types of elements that sets can have.

3 Part III

1. Let A and B be sets. Show that
   a. $(A \cap B) \subseteq A$
   b. $A \subseteq (A \cup B)$
   c. $A - B \subseteq A$
   d. $A \cap (B - A) = \varnothing$
   e. $A \cup (B - A) = A \cup B$

2. Show that if A and B are sets with $A \subseteq B$, then
   a. $A \cup B = B$
   b. $A \cap B = A$

3. Can you conclude that $A = B$ if A, B, and C are sets such that (prove it or find a counterexample)
   a. $A \cup C = B \cup C$?
   b. $A \cap C = B \cap C$?
   c. $A \cup C = B \cup C$ and $A \cap C = B \cap C$?

4. Show that if A and B are sets, then
   a. $A \oplus B = B \oplus A$
   b. $(A \oplus B) \oplus B = A$