Union-find Forest

Diagram, shape

Description automatically generated

Find(x): find the root or cluster-id of x

Union(x, y): merge two clusters

*Check whether two elements are in the same set or not in O(1)\*.*

Find: O(a(n))\*~=O(1)

Union: O(a(n))\*~=O(1)

Space: O(n)

Without optimization: Find: O(n)

Two key optimizations:

Path compression: make tree flat

Union by rank: merge low rank tree to high rank one

\*: amortized

a(.) inverse Ackermann function

Optimization 1: Path compression, happened during Find

Diagram

Description automatically generated

Optimization 2: Union by rank, merge low rank tree into high rank on one.

If two sub-tree has the same rank, break tie arbitrarily and increase the merged tree’s rank by 1

Reduce path compression overhead.

Diagram

Description automatically generated