

$$J(w, b) = -\frac{1}{m} \sum_i \mathcal{L}(a^i, y^i)$$

$$= -\frac{1}{m} \sum_i (y^i \log a^i + (1-y^i) \log(1-a^i))$$

$$\text{where } a = \hat{y} = \frac{1}{1 + e^{-w^T x}}$$

$$\frac{\partial a}{\partial w_i} = -\frac{1}{(1 + e^{-w^T x})^2} \cdot e^{-w^T x} \cdot (-x_i) = a^2 \cdot e^{-w^T x} \cdot x_i$$

$$\left| e^{-w^T x} = \frac{1-a}{a} \right| \Rightarrow = a^2 \frac{1-a}{a} x_i = a(1-a)x_i$$

$$\frac{\partial a}{\partial b} = a(1-a)$$

$$\frac{\partial J}{\partial w_i} = -\frac{1}{m} \sum \left[y \frac{1}{a} \frac{\partial a}{\partial w_i} + (1-y) \frac{1}{1-a} \left(-\frac{\partial a}{\partial w_i} \right) \right]$$

$$= -\frac{1}{m} \sum \left[y \frac{1}{a} (a(1-a)) x_i + (1-y) \frac{1}{1-a} (-a(1-a)x_i) \right]$$

$$= -\frac{1}{m} \sum [y(1-a)x_i - (1-y)a x_i]$$

$$= \frac{1}{m} \sum [-y x_i + y a x_i + a x_i - y a x_i]$$

$$= \frac{1}{m} \sum (a - y) x_i$$