PHYS411 Homework 2

Due: Sept 29 at 2:30 pm

1. (15 pts) A fluid that flows through a (very long) pipe has zero velocity on the pipe wall and a maximum velocity along the centerline of the pipe. The velocity v(r) varies through the pipe cross section according to the following formula:

$$v(r) = \left(\frac{\beta}{2\mu_0}\right)^{1/n} \frac{n}{n+1}(R^{1+1/n} - r^{1+1/n})$$

where R is the radius of the pipe, β is the pressure gradient (the force that drives the flow through the pipe), μ_0 is a viscosity coefficient (small for air, larger for water and even larger for toothpaste), n is a real number reflecting the viscous properties of the fluid (n = 1 for water and air, n < 1 for many modern plastic materials), and r is a radial coordinate that measures the distance from the centerline (r = 0 is the centerline, r = R is the pipe wall). Make a function that evaluates v(r). Plot v(r) as a function of r $\in$ [0,R], with R = 1, β = 0.02, μ_0= 0.02, and n = 0.1. Thereafter, make an animation of how the v(r) curves varies as n goes from 1 and down to 0.01. Because the maximum value of v(r) decreases rapidly as n decreases, each curve can be normalized by its v(0) value such that the maximum value is always unity.

2. (10 pts) Write the code to use the trapezoid rule and the simpson rule to integrate the following function:

$$\int_0^{\pi} \frac{x}{x^2+1}\cos(10x^2)dx$$

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Produce a table to compare the results from both methods for the following number of intervals used: 2, 4, 8, 16, 32, 64, 128, 256, 512, 1024 and 2048.

3. (10 pts) Exercise 5.7
4. (10 pts) Exercise 5.9
5. (10 pts) Exercise 5.10
6. (10 pts) Exercise 5.13
7. (15 pts) Exercise 5.14