Suppose we have the causal model consisting of $U$, $A$, $X$ with domain $\mathbb{U}$, $\mathbb{X}$ and $\mathbb{A}$. The prior distribution of $U$ and $A$ are encoded in the probability density functions $p(U)$ and $p(A)$.

The joint distribution of $U, A, X$ can be written as

$$p(U, A, X) = p(U)p(A)\delta(X - f(A, U)) \tag{4}$$

From Bayes theorem, we know

$$p(U|A, X) = \frac{p(U, A, X)}{p(X, A)} = \frac{p(U)p(A)\delta(X - f(A, U))}{\int_{\mathbb{U}} p(U)p(A)\delta(X - f(A, U))dU} \tag{5}$$

With the same definition of $H$, we can know

$$p(H|X, A) = \delta\left(\int_{\mathbb{U}} \frac{p(U)p(A)\delta(X - f(U, A))}{\int_{\mathbb{U}} p(U)p(A)\delta(X - f(U, A))dU}[s(\cdots), U] - H\right) \tag{6}$$

The conditional distribution of $H$ on $A$ is

$$p(H|A) = \int_{\mathbb{X}} p(H|X, A)p(X|A)dX \tag{7}$$

And

$$p(X|A) = \frac{P(X, A)}{p(A)} = \frac{\int_{\mathbb{U}} p(A)p(U)\delta(X - f(A, U))dU}{\int_{\mathbb{U}}\int_{\mathbb{X}} p(A)p(U)\delta(X - f(A, U))dUdX} \tag{8}$$

which equals to

$$p(X|A) = \frac{P(X, A)}{p(A)} = \frac{\int_{\mathbb{U}} p(A)p(U)\delta(X - f(A, U))dU}{p(A)} \tag{9}$$

As a result,

$$p(H|A) = \int_{\mathbb{X}} \delta\left(\int_{\mathbb{U}} \frac{p(U)p(A)\delta(X - f(U, A))}{\int_{\mathbb{U}} p(U)p(A)\delta(X - f(U, A))dU}[s(\cdots), U] - H\right) \frac{\int_{\mathbb{U}} p(A)p(U)\delta(X - f(A, U))dU}{p(A)} dX \tag{10}$$

Because $U$ follows a uniform distribution, the probability density function can be simplified as

$$p(H|A) = \int_{\mathbb{X}} \delta\left(\int_{\mathbb{U}} \frac{\delta(X - f(U, A))}{\int_{\mathbb{U}} \delta(X - f(U, A))}[s[\cdots], U] - H\right)\left(\int_{\mathbb{U}} p(U)\delta(X - f(U, A))dU\right) dX \tag{11}$$

Again, since the condition 3, we can assume $\mathbb{U}$ can be divide into infinite sub-spaces $\mathbb{U}^1$, $\mathbb{U}^2$, $\cdots$. In the $k$-th sub-space, $f(U, A)$ has the same value for each specific value of $A$.

Then the conditional distribution can be written as

$$p(H|A) = \frac{1}{\int_{\mathbb{U}} dU} \int_{U} \delta([s(\cdots), U] - H)dU \tag{12}$$

which is irrelevant to $A$. That is to say $H \perp A$.

When $H$ is defined as

$$H = [s(f(U, a^{[1]}, ..., f(U, a^{[|\mathcal{A}|]})), U] \tag{13}$$

, we have

$$p(H|A) = \int_{\mathbb{U}} \delta \left( [s(f(U, a^{[1]}, ..., f(U, a^{[|\mathcal{A}|]})), U] - H \right) dU \tag{14}$$

it is irrelevant to $A$. So, $H \perp A$.