CS143: Normalization Theory

Book Chapters

(4th) Chapters 7.1-6
(5th) Chapters 7.1-5
(6th) Chapters 8.1-5
(7th) Chapters 7.1-5

Introduction

Main question

- How do we design "good" tables for a relational database?
  - Typically we start with ER and convert it into tables
  - Still, different people come up with different ER, and thus different tables. Which one is better? What design should we choose?
- Relational design theory
  - A theory on how to identify and create a good table design or a "normal form"
  - Several definitions of "normal forms" exist
  - We learn the most popular normal form, Boyce-Codd Normal Form (BCNF)

Warning

- The most difficult and theoretical part of the course. Pay attention!

Motivation & Intuition

⟨StudentClass(sid, name, addr, dept, cnum, title, unit) slide⟩

- **Q:** Is it a good table design?

- REDUNDANCY: The same information mentioned multiple times. Redundancy leads to potential anomaly.
  1. UPDATE ANOMALY: Only some information may be updated

- **Q:** What if a student changes the address?

2. INSERTION ANOMALY: Some information cannot be represented
   - **Q:** What if a student does not take any class?

3. DELETION ANOMALY: Deletion of some information may delete others
   - **Q:** What if the only class that a student takes is cancelled?

- **Q:** Is there a better design? What tables would you use?

- **Q:** Any way to arrive at such table design more systematically?
  - **Q:** Where is the redundancy from?
    ⟨ Slide on "guessing" missing info ⟩

  - FUNCTIONAL DEPENDENCY: Some attributes are "determined" by other attrs
    * e.g., sid → (name, addr), (dept, cnum) → (title, unit)
    * When there is a functional dependency, we may have redundancy.
      · e.g., (301, James, 11 West) is stored redundantly. So is (CS, 143, database, 04).

  - DECOMPOSITION: When there is a FD, no need to store multiple instances of this relationship. Store it once in a separate table
    * ⟨Intuitive normalization of StudentClass table⟩
      StudentClass(sid, name, addr, dept, cnum, title, unit)
      FDs: sid→(name, addr), (dept, cnum)→(title, unit)
      1. sid → (name, addr): no need to store it multiple time. separate it out

2. (dept, cnum) $\rightarrow$ (title, unit). separate it out

- Basic idea of table "normalization"
  - Whenever there is a FD, the table may be "bad" (not in normal form)
  - We use FDs to "split" or "decompose" table and remove redundancy
  - We learn FUNCTIONAL DEPENDENCY and DECOMPOSITION to formalize this.

Functional Dependency

Overview

- The fundamental tool for normalization theory
- May seem dry and irrelevant, but bear with me. Extremely useful
- Things to learn
  - FD, trivial FD, logical implication, closure, FD and key, projected FD

Functional dependency $X \rightarrow Y$

- Notation: $u[X]$ - values for the attributes X of tuple u
  e.g, Assuming u = (sid: 100, name: James, addr: Wilshire), u[sid, name] = (100, James)
- FUNCTIONAL DEPENDENCY $X \rightarrow Y$
  - For any $u_1, u_2 \in R$, if $u_1[X] = u_2[X]$, then $u_1[Y] = u_2[Y]$
  - More informally, $X \rightarrow Y$ means that "no two tuples in R can have the same X values but different Y values"

    ⟨e.g., StudentClass(sid, name, addr, dept, cnum, title, unit)⟩
    - **Q:** sid $\rightarrow$ name?
    - **Q:** dept, cnum $\rightarrow$ title, unit?
    - **Q:** dept, cnum $\rightarrow$ sid?

– Whether a FD is true or not depends on real-world semantics

⟨examples⟩

A	B	C
a_1	b_1	c_1
a_1	b_2	c_2
a_2	b_1	c_3

Q: AB $\rightarrow$ C. Is this okay?

Replace c_3 to c_1.

A	B	C
a_1	b_1	c_1
a_1	b_2	c_2
a_2	b_1	c_1

Q: AB $\rightarrow$ C. Is this okay?

NOTE: AB $\rightarrow$ C does not mean no duplicate C values.

Replace b_2 to b_1

A	B	C
a_1	b_1	c_1
a_1	b_1	c_2
a_2	b_1	c_3

Q: AB $\rightarrow$ C. Is this okay?

- TRIVIAL functional dependency: $X \rightarrow Y$ when $Y \subset X$
  - It is always true regardless of real world semantics
    (diagram)

- NON-TRIVIAL FD: $X \rightarrow Y$ when $Y \not\subset X$
  (diagram)

- COMPLETELY NON-TRIVIAL FD: $X \rightarrow Y$ with no overlap between X and Y
  (diagram)

We will focus on completely non-trivial functional dependency.

Implication and Closure

- LOGICAL IMPLICATION

ex) $R(A, B, C, G, H, I)$
F: $A \to B$, $A \to C$, $CG \to H$, $CG \to I$, $B \to H$ (set of functional dependencies)

- **Q:** Is $A \to H$ true under F?

  F LOGICALLY IMPLIES $A \to H$

  ⟨canonical database method to prove $A \to H$⟩

A	B	C	G	H	I
a_1	b_1	c_1	g_1	h_1	i_1
a_1				?	

  If ? $= h1$, then $A \to H$

  * **Q:** $AG \to I$?

- CLOSURE OF FD F: F$^+$

  F$^+$: the set of all FD's that are logically implied by F.

- CLOSURE OF ATTRIBUTE SET X: X$^+$

  X$^+$: the set of all attrs that are functionally determined by X

  - **Q:** What attribute values do we know given (sid, dept, cnum)?

- CLOSURE X^+ COMPUTATION ALGORITHM

  ⟨X$^+$ computation algorithm slide⟩
  Start with $X^+ = X$
  Repeat until no change in X^+
  If there is $Y \to Z$ and $Y \subset X^+$, add Z to X^+

  ⟨example⟩
  $R(A, B, C, G, H, I)$ and $A \to B, A \to C, CG \to H, CG \to I, B \to H$

- **Q:** $\{A\}^+$?

- **Q:** $\{A, G\}^+$?

- FUNCTIONAL DEPENDENCY AND KEY
  - Key determines a tuple and functional dependency determines other attributes. Any formal relationship?
  - **Q:** In previous example, is (A, B) a key of R?
    $R(A, B, C, G, H, I)$ and $A \to B, A \to C, CG \to H, CG \to I, B \to H$
  - X is a KEY of R if and only if
    1. $X \to$ all attributes of R (i.e., $X^+ = R$)
    2. No subset of X satisfies 1 (i.e., X is minimal)
- PROJECTING FD
  $R(A, B, C, D) : A \to B, B \to A, A \to C$
  - **Q:** What FDs hold for $R'(B, C, D)$ which is a projection of R?
  - In order to find FD's after projection, we first need to compute F^+ and pick the FDs from F^+ with only the attributes in the projection.

Decomposition

- ⟨Remind the decomposition idea of StudentClass table⟩
- Splitting table $R(A_1, \ldots, A_n)$ into two tables, $R_1(A_1, \ldots, A_i)$ and $R_2(A_j, \ldots, A_n)$
  - $\{A_1, \ldots, A_n\} = \{A_1, \ldots, A_i\} \cup \{A_j, \ldots, A_n\}$
  - ⟨Conceptual diagram for $R(X, Y, Z) \to R_1(X, Y)$ and $R_2(Y, Z)$⟩

- **Q:** When we decompose, what should we watch out for?

LOSSLESS-JOIN DECOMPOSITION

- $R = R_1 \bowtie R_2$
- Intuitively, we should not lose any information by decomposing R
- Can reconstruct the original table from the decomposed tables
- **Q:** When is decomposition lossless?

⟨example⟩

cnum	sid	name
143	1	James
143	2	Elaine
325	3	Susan

  - **Q:** Decompose into S_1(cnum, sid), S_2(cnum, name). Lossless?

  - **Q:** Decompose into S_1(cnum, sid), S_2(sid, name). Lossless?

- DECOMPOSITION $R(X, Y, Z) \Rightarrow R_1(X, Y), R_2(X, Z)$ IS LOSSLESS IF $X \rightarrow Y$ OR $X \rightarrow Z$
  - That is, the shared attributes are the key of one of the decomposed tables
  - We can use FDs to check whether a decomposition is lossless

  Example: StudentClass(sid, name, addr, dept, cnum, title, unit) sid $\rightarrow$ (name,addr), (dept,cnum) $\rightarrow$ (title,unit)

  * **Q:** Decomposition into R_1(sid, name, addr), R_2(sid, dept, cnum, title, unit). Lossless?

Boyce-Codd Normal Form (BCNF)

FD, key & redundancy

- **Example:** StudentClass(sid, name, addr, dept, cnum, title, unit)
  - **Q:** sid $\rightarrow$ (name,addr). Does it cause redundancy?
  - After decomposition, Student(sid, name, addr)
    * **Q:** sid $\rightarrow$ (name,addr). Does it still cause redundancy?
    * **Q:** Why does the same FD cause redundancy in one case, but not in the other?
- In general, FD $X \rightarrow Y$ leads to redundancy if X DOES NOT CONTAIN A KEY.

BCNF definition

- R is in BCNF with regard to F, iff for every non-trivial $X \rightarrow Y$, X contains a key
- "Good" table design (no redudancy due to FD)
- **Q:** Class(dept, cnum, title, unit). dept,cnum$\rightarrow$title,unit.
  - **Q:** Intuitively, is it a good table design? Any redundancy? Any better design?
  - **Q:** Is it in BCNF?
- **Q:** Employee(name, dept, manager). name$\rightarrow$dept, dept$\rightarrow$manager.

– **Q:** What is the English interpretation of the two dependencies?

– **Q:** Intuitively, is it a good table design? Any redundancy? Better design?

– **Q:** Is it in BCNF?

- **Remarks:** Most times, BCNF tells us when a design is "bad" (due to redundancy from functional dependency.

BCNF normalization algorithm

- Decomposing tables until all tables are in BCNF
  – For each FD $X \rightarrow Y$ that violates the condition, separate those attributes into another table to remove redundancy.
  – We also have to make sure that this decomposition is lossless.

- **Algorithm**

  For any R in the schema
  If non-trivial $X \rightarrow Y$ holds on R, and if X does not have a key

  1. Compute X^+ (X^+: closure of X)
  2. Decompose R into $R_1(X^+)$ and $R_2(X, Z)$ // X is common attributes where Z is all attributes in R except X^+

  Repeat until no more decomposition

- **Example:** ClassInstructor(dept, cnum, title, unit, instructor, office, fax)
  instructor $\rightarrow$ office, office $\rightarrow$ fax
  (dept, cnum) $\rightarrow$ (title, unit), (dept, cnum) $\rightarrow$ instructor.

  – **Q:** What is the English interpretation of the two dependencies?

  – **Q:** Intuitively, is it a good table design? Any redundancy? Better design?

  – **Q:** Is it in BCNF?

– **Q:** Normalize it into BCNF using the algorithm.

NOTE: The algorithm guarantees lossless join decomposition, because after the decomposition based on $X \to Y$, X becomes the key of one of the decomposed table

- **Example:** $R(A, B, C, G, H, I)$, $A \to B, A \to C, G \to I, B \to H$. Convert to BCNF.

- **Q:** Does the algorithm lead to a unique set of relations?

  ⟨e.g., $R(A, B, C), A \to C, B \to C$⟩
  Q: What if we start with $A \to C$?

  Q: What if we start with $B \to C$?

- **Q:** $R_1(A, B), R_2(B, C, D)$ with $A \to B, B \to A, A \to C$. Are R_1 and R_2 in BCNF?

  NOTE: We have to check all implied FD's for BCNF, not just the given ones.

Good Table Design in Practice

- Normalization splits tables to reduce redundancy.
- However, splitting tables has negative performance implication

  Example: Instructor: name, office, phone, fax
  name → office, office → (phone,fax)

  (design 1) Instructor(name, office, phone, fax)
  (design 2) Instructor(name, office), Office(offce, phone, fax)

Q: Retrieve (name, office, phone) from Instructor. Which design is better?

- As a rule of thumb, start with normalized tables and merge them if performance is not good enough

Things to Remember

- Functional dependency $X \rightarrow Y$
  - Trivial functional dependency
  - Logical implication
  - Closure
- Decomposition
  - Lossless join decomposition
- Boyce-Codd Normal Form (BCNF)
- BCNF decomposition algorithm