



Public-Key Infrastructure (P

Junghoo Cho
cho@cs.ucla.edu

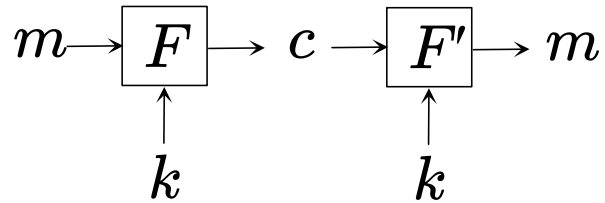
Four Security Guarantees

- Internet is an open and public forum where everyone talks to
else
 - Data packets can be intercepted and seen by anyone
 - No guarantee on the origin and integrity of data packet
- Q: Given this, what guarantees may we desire before we can
important transactions over the Internet?
 1. *Confidentiality*
 2. *Message/data integrity*
 3. *Authentication*
 4. *Authorization*

Confidentiality

- Q: How can we keep confidentiality of the messages?
 1. *Steganography*: “embed” true message within harmless-looking messages
 - Kathy is laughing loudly
 - Change the lowest bit of image pixels
 - “Security by obscurity”
 2. *Encryption*: “scramble” message with a secret key, so that it wouldn’t be understood by others unless they have the key
 - Example: bitwise XOR with k
 - 11110000 (message) XOR 10111001 (key) → 01001001 (ciphertext)
 - 01001001 (ciphertext) XOR 10111001 (key) → 11110000 (message)

Symmetric-Key Cipher



- $F(m, k)$: encryption function, e.g., $F(m, k) = m \text{ XOR } k$
 - m : plaintext (= message), k : secret key
 - c : ciphertext. transmitted over insecure channel
- $F'(c, k)$: decryption function, e.g., $F'(c, k) = c \text{ XOR } k$
 - Inverse of F : $F' \cdot F = I$
- The pair $[F(m, k), F'(c, k)]$ is called a *cipher*

Security of Cipher

- Q: What property should $F(m, k)$ have?
- A: Ideally, one should never be able to guess m from c alone
 - Ciphertext should not reveal any information about plaintext
- *Perfect secrecy* (= *Shannon secrecy*)
 - For all plaintext x and ciphertext y , $Pr(x|y) = Pr(x)$
- OTP (one time pad) encryption is proven to be perfectly secret due to practical limitation, cannot be used directly
 - Many encryption algorithms try to “mimic” OTP, e.g., RC4

Popular Ciphers

- AES (advanced encryption standard)
 - 128 bit block cipher
 - 128, 192, 256 bit keys
 - Adopted by NIST (national institute of standard and technology) as replacement of DES in 2000
- IDEA, A5 (used by GSM), ...

Challenges

- Q: Can A use the same key for communicating with B and C
- Q: If there are n parties, how many keys are needed?
- Q: How can two parties agree on a key “secretly” over the Int the first place?

Key Agreement Problem

- Q: Can two parties send and receive encrypted messages without agreeing on a shared secret key?
- A: *Asymmetric-key cipher*

Asymmetric-Key Cipher

- Two pairs of keys, not one!
 - e : encryption key
 - d : decryption key
- Q: How does this help?

Asymmetric-Key Cipher

- Everyone has their own (e, d) key pair
- Everyone shares their e with anyone: *public key*
 - Other users use the public key to encrypt a message to the user
- Users keep their d secret: *private key*
 - Users use their private key to decrypt message
- No need to send the private key over insecure channel
 - Private key *NEVER* leaves the owner of the key

Asymmetric-Key Cipher

- Idea first developed by Ellis, Cocks, and Williams (working for NSA)
 - In early 70's, but could not publish
- First public-key cryptosystem by Diffie and Hellman in 1976
- RSA (Rivest, Shamir and Adleman)
 - Most widely used asymmetric-key cipher
 - Used by many security protocols: SSL, PGP, CDPD, ...

RSA: Key Generation

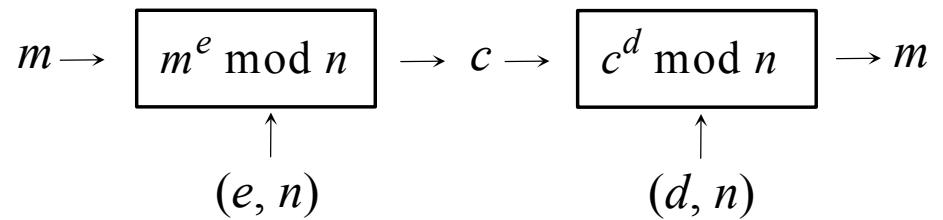
1. Pick two *random* prime numbers p and q .
2. Pick $e < (p - 1)(q - 1)$
 - e does not have to be random
 - Popular choice: $e = 65537 (= 2^{16} + 1)$, 3, 5, 35, ...
3. Find $d < (p - 1)(q - 1)$ such that $de \bmod (p - 1)(q - 1) = 1$
 - Using *extended-euclid algorithm*
4. (e, n) becomes public key, (d, n) becomes private key where
 - Throw away p and q

RSA Cipher

- Encryption and Decryption functions

- $F(m, (e, n)) = m^e \bmod n$

- $F'(c, (d, n)) = c^d \bmod n$



- Q: Does this work?

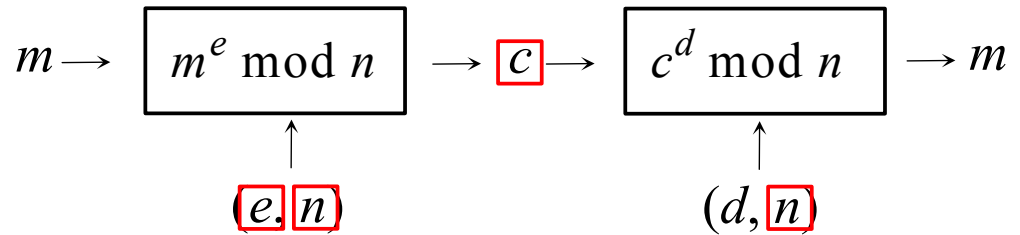
RSA: Two Important Theorems

- Q: Given a choice of e , can we always find d such that $de \bmod (p-1)(q-1) = 1$?
- A: Yes, there exists unique d if e is a *coprime* to $(p-1)(q-1)$
 - i.e., e does not share any factor with $(p-1)(q-1)$
- Q: Is $F'(c, (d, n))$ the inverse of $F(m, (e, n))$?
- A: Yes, $m = [(m^e \bmod n)^d \bmod n]$ for such e, d and $n = pq$
- RSA works!
 - But most asymmetric-key ciphers are 1000x slower than any symmetric cipher
- Q: Is it secure? What should we make sure for the security of

Security of Asymmetric-Key Cipher

- Q: What properties should F , F' , e , and d satisfy to make this secure?
- A: One should never guess m from c without d (\sim perfect secrecy)
- A: One should never guess d from e

Security of RSA (1)



- Q: Can a hacker “break RSA”?
- Q: What does the hacker know? m ? c ? (e, n) ? (d, n) ?
- Q: What other relationship does the hacker know?
- A: $de \bmod (p-1)(q-1) = 1$, $n = pq$, $c = m^e \bmod n$

Security of RSA (2)

$$de \bmod (p-1)(q-1) = 1, \quad n = pq, \quad c = m^e \bmod n$$

- Q: Can the hacker get m by solving $c = m^e \bmod n$?
- A: *RSA problem*. No efficient solution known.
- Q: Can the hacker get d by solving $de \bmod (p-1)(q-1) = 1$?
- Q: Can the hacker get p and q from $n = pq$?
- A: *Large-number factorization problem*. No efficient solution known.

Security of RSA (3)

- Security of RSA depends on the difficulty of two key problem
 - RSA problem: solve $c = m^e \bmod n$ for m
 - Large-number factorization problem: factorize $n = pq$ for large n , p

Application of Asymmetric-Key Cipher

- Q: How can we use an asymmetric-key cipher to keep message “confidential”?
- A:
 1. Use asymmetric-key cipher to establish a shared key
 2. Using the shared key, use symmetric-key cipher to encrypt message
 - Performance and complexity issue
- Q: How can we “authenticate” the other party?
- A: Challenge-Response
 - Challenge: generate random value r and send $c = F(r, e)$
 - Response: send back $F'(c, d) = r$
 - Only the one with d can send back r

Application of Asymmetric-Key Cipher

- Q: How can we check the message integrity? How can we make sure others did not temper with message?
- A: *Signature*
 - Main idea: $I = F' \cdot F = F \cdot F'$. That is, $F(F'(m, d), e) = m!$
 - In RSA, for example, $m = (m^e \bmod n)^d \bmod n = (m^d \bmod n)^e \bmod n$
 - “Private-key decrypted” checksum of message body
 - Given a message with signature, “encrypt” the signature using the public key of the author
 - Correct signature should have correct checksum

Public-Key Infrastructure

- Q: How do we know the public key for *A really* belongs to A?
- Q: In real world, how do we verify the identity of a person?
- Q: Why do we trust it?
- A: Public-Key Infrastructure (PKI)
 - *Certificate Authority (CA)*
 - Trusted entity that can issue trusted *certificates* to Web sites
 - Performs out-of-band identity verification before issuing a certificate
 - *Certificate*
 - Text (XXXX is the public key of A) signed by CA's secret key
 - Others can "trust" the public key if they trust CA

HTTPS: High-Level Description

1. When contacted by client, server presents its signed certificate
 - “XXX is the public key of amazon.com. This certificate is valid until ..
2. Client “authenticates” server through challenge/response using public key
3. Client/server agrees on a symmetric-key through a secure channel established through asymmetric-key cipher
4. Client/server communicate securely through symmetric-key

Multi-Factor Authentication

- Q: What if the user loses their secret password?
- *Multi-factor authentication*
 - To minimize possibility of compromised keys, systems authenticate on combinations of
 - What you have (e.g., physical key, id card)
 - What you know (e.g., password)
 - Who you are (e.g., fingerprint)
 - *2-factor authentication*

Popular Second Factor

- Smartphone
 - Send an SMS/push notification on a registered device
- USB key
 - e.g., FIDO U2F Security Key



- SmartCard
 - Temper-resistant security card



Popular Second Factor

- OTP (one time password) key
 - A physical card flashing a new security code, say, every minute
 - e.g. SecurID by RSA security
 - User provides the security code to log in



What We Learned

- Four security guarantees
 - Confidentiality, integrity, authentication, authorization
- Symmetric-key cipher: AES algorithm
- Asymmetric-key cipher: RSA algorithm
- Public-Key Infrastructure (PKI)
 - Certificate Authority (CA), certificate
- HTTPS
- Multi-factor authentication

