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Labs in Precise GNSS

Exercise 2: **Positioning with Pseudoranges**

For this exercise C/A code observations on L1 and P code observations on L2 for the SAPOS station Garmisch (GARM) and station KIT from Karlsruhe Institute of Technology covering 1 hour as well as the positions and velocities of the observed satellites in the geocentric Earth-fixed frame and satellite clock corrections and differential code biases are given in separate files. For details see the `readme.txt` file of the zip archive. Atmospheric effects are neglected in Part A of this exercise while they are considered in Part B.

2.1 L_1 Pseudorange Solution - Part A

Compute epoch-wise L1 coordinate and receiver clock solutions for station GARM using C1 tracking data. Start with a priori station coordinates given below and receiver clock correction equal to zero. Due to numerical reasons, express the receiver clock correction in meters (i.e. use derivative 1 instead of c). Estimate for each epoch the station position and receiver clock correction from L_1 code observations, linearizing the observation equation and using the method of least squares. I.e., write a MATLAB script that performs more or less the following:

1. Read the satellite positions and observations you need
2. Loop over all observation epochs
3. For a given epoch:
   (a) Correct satellite positions for (1) light travel time and (2) Earth rotation during light travel time, and (3) apply periodic relativistic satellite clock corrections
   (b) Compute the derivatives of the observation equation w.r.t. the estimated parameters
   (c) Set up the grand design matrix **A** and the observation vector **y** for all observations of the epoch
   (d) Compute the normal equation matrix and solve the normal equation
   (e) Compute the formal errors of the parameters
   (f) Store the results and the formal errors and go to the next epoch

Constants

$c = 299\,792\,458\,\frac{\text{m}}{\text{s}}$ vacuum speed of light

$\omega_E = 7.292\,115\,146\,7 \cdot 10^{-5}\,\frac{\text{rad}}{\text{s}}$ Earth rotation rate

A priori coordinates

Garmisch:	$x_r = 4235957.180$ m,	$y_r = 834342.537$ m,	$z_r = 4681540.901$ m
KIT:	$x_r = 4239065.798$ m,	$y_r = 828841.745$ m,	$z_r = 4678313.623$ m

2.2 Interpretation of the results

1. Plot the time series of station coordinates and receiver clock corrections together with their formal errors. Give numerical values for station coordinates and receiver clock corrections of the first epoch in your report.

2. Assuming that the a priori coordinates are the true position: which accuracy and which precision have you achieved with this simple positioning algorithm, depending on the corrections (1) - (3) applied?

3. What is the order of magnitude of the impact of the different corrections on the station coordinates?

2.3 Atmospheric Refraction - Part B

Refine your positioning program to account for atmospheric delays.

- Account for tropospheric refraction by using the simple model

$$T_i^k(z_i^k) = \frac{T_0}{\cos z_i^k} \tag{1}$$

  with

$T_0 = 2.3$ m	tropospheric zenith path delay
z_i^k	topocentric zenith angle

- Account for ionospheric refraction by using the ionosphere-free linear combination of C1 and P2 observations.

- (Optional) Account for satellite differential code biases (DCBs).

Compare with your initial coordinate results and the a priori coordinates (assumed to be free of errors).

2.4 Differential Positioning

1. Repeat the positioning for the second station KIT.

2. Form differences between the estimated coordinates of GARM and KIT and compare them with the differences between the a priori coordinates.

3. Why have the results improved?

Submission deadline for joint lab report on part A and B: January 11, 2021