

Factor Based Investment Strategy

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1. Abstract

This report analyzes six U.S. equity factors (MKT, SMB, HML, RMW, CMA, and MOM) using monthly data while emphasizing their heterogeneous volatility and time-varying risk. Standard data cleaning and timing conventions are applied to minimize look-ahead bias and reduce the influence of extreme observations on results. Factor exposures are translated into investable strategies by comparing five portfolio construction methodologies—a market benchmark, a fixed-weight multi-factor, an equal-weight, a risk parity, and a mean-variance optimization—and then assessing performance from a risk-adjusted perspective. Fama–French-type factors are replicated from CRSP–Compustat characteristics, and replication quality is checked through factor-on-factor regressions. Since factors are heteroskedastic and serially correlated, inference relies on Newey–West HAC standard errors, which permit better conclusions on factor exposure and portfolio outcome.

2. Introduction

2.1 Background and Motivation

Factor models have been extensively employed in both asset pricing and portfolio construction, where a few systematic "factors" could encapsulate vital patterns in stock returns and thus yield investment building blocks that are easy to understand. The present paper investigates six US equity factors—the MKT, SMB, HML, RMW, CMA, and MOM—and focuses on three issues arising in practice: First, factors are characterized by heterogeneous volatility and time-varying risk; diversification among them, therefore, may enhance risk-adjusted returns. Second, factor performance is highly sensitive to data alignment and cleaning, and hence, standard choices in implementation must be made to avoid look-ahead bias. Third, time series of factor returns commonly exhibit heteroskedasticity and serial dependence, so reliable inference necessitates robust methods, including Newey-West HAC standard errors.

2.2 Objectives and Research Questions

This report will: (1) document the definitions and empirical features of six factors, (2) evaluate five factor-based portfolio constructions relative to a market benchmark, (3) replicate Fama–French-style factors from CRSP–Compustat characteristics using standard sorting conventions, and (4) test exposures and replication accuracy using regressions with robust (Newey–West HAC) inference.

We address three guiding questions: What are the key return characteristics of the six factors? How do alternative portfolio rules affect performance, drawdowns, and risk-adjusted metrics? How closely can the factors be replicated from raw CRSP–Compustat data, and how sensitive are the regression conclusions under robust standard errors?

2.3 Contributions

The current report thus makes three contributions in practice: it offers a unified pipeline—it connects factor definitions, data alignment choices, and portfolio construction using one monthly dataset; it explores several allocation rules—fixed-weight, equal-weight, risk parity, and mean-variance—to underline how implementation choices affect risk-adjusted outcomes and drawdowns; and it replicates the Fama-French-style factors from CRSP-Compustat characteristics and validates the replication through factor-on-factor regressions with Newey-West HAC inference, enhancing the veracity of statistical conclusions under realistic return dynamics.

3. Six Factors: Definitions and Data Features

3.1 Factor Definitions

The six standard US equity factors, which are broadly common in empirical asset pricing, thus include: MKT, the excess return earned on the broad value-weighted market portfolio; and then, long–short spreads intended to capture particular dimensions of return—SMB stands for the size effect, high minus low signifies the value–growth spread on book-to-market, RMW brings forth the evidence of the profitability premium, CMA identifies differences in corporate investment, and MOM—momentum based on past returns (typically using a 12–2 formation window). These

represent systematic "styles" and have different risk profiles; hence, factor analysis is performed before combining the separate factors into multi-factor portfolios.

3.2 Data and Sample Construction

We rely on the tidy.finance SQLite database, which combines CRSP monthly, Compustat, and Fama–French benchmark factor series into a single, consistent dataset. CRSP provides returns and market equity variables for portfolio formation, while Compustat provides fundamentals (book equity, operating profitability, and investment) for characteristic construction. The Fama–French series serves as the benchmark for evaluating our replicated factors. Our monthly sample covers 1960–2024.

Our analysis uses monthly U.S. equity data and follows standard conventions to ensure that factor measurement is not contaminated by look-ahead bias or extreme data issues. CRSP monthly returns provide stock-level returns and market equity (market capitalization) necessary for size measurement and value-weighted portfolio formation. Compustat accounting variables provide firm fundamentals required to measure book equity (for value), operating profitability (for profitability), and investment (for investment). Momentum is computed using historical CRSP returns under a conventional formation window (e.g., 12–2 momentum).

We implement common sample restrictions and data handling steps. First, we limit the universe to U.S. common stocks and remove obvious data errors and extreme penny-stock observations that can dominate long–short spreads. Second, we align the timing of variables carefully. Market equity used for weighting is lagged so that portfolio weights are based on information available prior to the return realization. Accounting variables are lagged according to typical reporting conventions to reflect publication delays. Third, to reduce sensitivity to extreme realizations, we winsorize or cap unusually large returns. These choices are important because factor estimates—especially long–short factors—can be unstable if they load heavily on microcaps or if accounting variables are misaligned.

3.3 Summary Statistics

Instead of relying on a summary table, we summarize factor behavior using visual and descriptive evidence. The monthly returns of the six factors, discussed in Figure 1, are immediately indicative of risks involved with the different profiles.

Factor	Mean	SD
MKT	0.00587	0.04476
SMB	0.00193	0.03036
HML	0.00282	0.02971
RMW	0.00282	0.02218
CMA	0.00251	0.02057
MOM	0.00623	0.04123

Table. Summary table

The market and momentum factors show larger variances, reflecting higher volatility, while profitability and investment present tighter bands with steadier dynamics. Generally, at least across factors, the returns are mostly centered around zero, with large shocks to either side only in those few months, suggesting fat tails and time-varying volatility. Indeed, at a high level, the six factors are rather similar in exhibiting time series features like volatility clustering and exceptional observations from time to time. However, they differ significantly in the degree of variability, which calls for diversification and risk-aware weighting in constructing multi-factor portfolios.

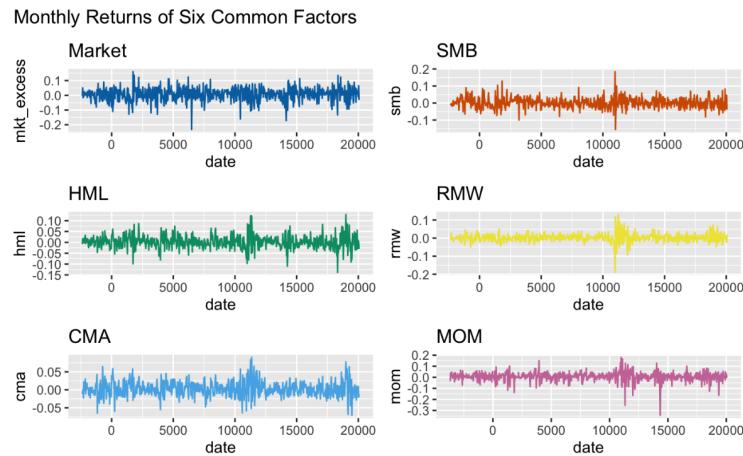


Figure 1. Factor Return Characteristics

3.4 Return Dynamics and CAPM vs. Multi-Factor Evidence

While summary averages are useful, the time series behavior of factor returns provides further evidence about risk. Monthly factor returns are not i.i.d. and show very clear volatility clustering—periods of relatively mild fluctuations followed by episodes of large shocks, consistent with time-varying risk in financial markets. This is evidenced in Figure 1 (Factor Return Characteristics). The degree of fluctuations varies considerably between factors: MKT and MOM show wider swings, identifying greater exposure to volatility, while RMW and CMA are relatively stable, showing narrower bands of returns. The presence of sporadic extreme observations on the upside and the downside indicates "fat tails" and highlights the fact that tail risk remains important at the monthly frequency. Of course, these stylized facts give direct motivation for robust inference later in the report, given that heteroskedasticity and serial correlation may cause conventional OLS standard errors to be biased.

Beyond individual factor dynamics, we also compare cumulative wealth paths under increasingly rich factor models. Figure 2 (CAPM vs. Multi-Factor) plots cumulative returns of the market benchmark against portfolios that progressively add factors (2-factor through 6-factor). Two patterns stand out. First, adding factors changes the trajectory of cumulative wealth; thus, multi-factor portfolios can have huge tracking error concerning the market—even when they remain highly correlated with broad market cycles.

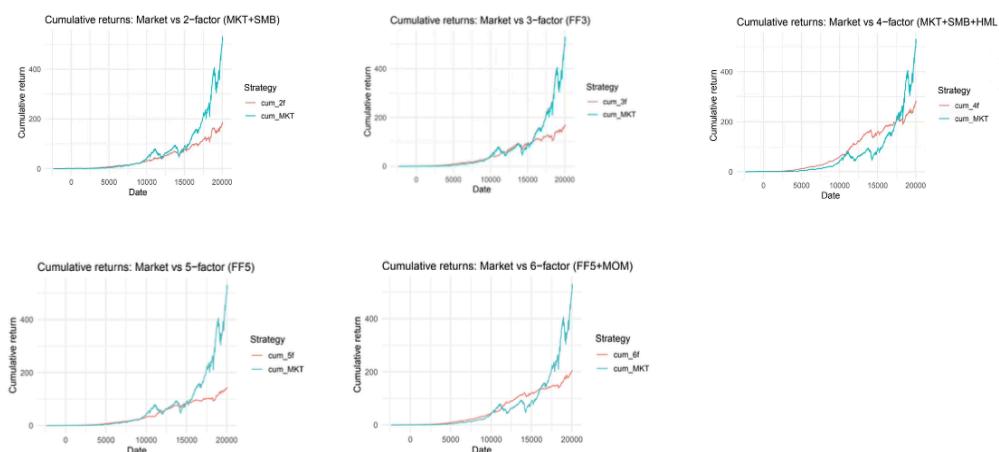


Figure 2 (CAPM vs. Multi-Factor) plots cumulative returns

Second, in this sample, the market benchmark often reaches a higher terminal level than the simple factor combinations, indicating that adding factors does not guarantee higher cumulative wealth in any given period. A natural interpretation is that factor premia are regime-dependent: long–short factor spreads can underperform during extended market rallies or when the dominant market trend overwhelms cross-sectional effects, even though diversification may reduce exposure to any single return driver.

Therefore, Figures 1 and 2 suggest that multi-factor strategies should be evaluated primarily using risk-adjusted metrics and downside risk—such as volatility, Sharpe ratios, and drawdowns—rather than terminal cumulative wealth alone, since the cumulative performance of different factor combinations can vary substantially across market regimes and horizons. Indeed, the figures demonstrate the extent to which the cumulative return path and drawdown profiles may be materially different in going from a market-only benchmark to ever richer combinations of factors: adding further factors may diversify return drivers and smooth performance but does not guarantee higher terminal wealth across all sample periods. Such observations provide motivation for the portfolio construction section that follows, where alternative allocation rules based on volatility, Sharpe ratios, and drawdowns are compared, and robust inferences—for instance, Newey–West HAC standard errors—provide more credible conclusions regarding factor exposures.

4. Portfolio Construction

In this section, we describe the portfolio construction approaches used to combine the replicated factor returns. Our objective is to compare a spectrum of portfolio construction methods, ranging from simple rule-based allocations to risk-based and optimization-based strategies. This allows us to evaluate how different levels of model complexity affect portfolio risk and return characteristics.

4.1 Market Portfolio (Benchmark)

We begin with the market portfolio, which serves as the baseline benchmark. This portfolio consists solely of the market excess return and represents the traditional CAPM exposure.

The market portfolio is expected to exhibit both the highest return and the highest volatility among all strategies, reflecting its full exposure to systematic equity risk. As such, it provides a natural point of comparison for assessing whether factor diversification can improve risk-adjusted performance relative to holding the market alone.

4.2 Fixed-Weight Multi-Factor Portfolio

Our primary multi-factor strategy follows the framework proposed by MSCI and FTSE Russell, where factors are treated not as short-term trading signals but as long-term compensated risk premia.

The portfolio is constructed by combining the market factor with five style factors—SMB, HML, RMW, CMA, and Momentum—using fixed weights over time. Specifically, we allocate 25% to the market factor and 15% to each of the five style factors.

This allocation reflects two key considerations. First, the market factor serves as the macro-consistent core of any equity portfolio, delivering the largest and most stable long-term premium. Second, equal weighting across the style factors avoids excessive concentration in any single factor while capturing diversification benefits across multiple independent sources of return.

Importantly, the weights are held constant through time. This design choice avoids factor timing and fragile optimization procedures, resulting in a portfolio that is simple, transparent, and robust. Consistent with FTSE Russell’s perspective, the portfolio’s risk and return are driven primarily by its factor exposure vector rather than by security selection or dynamic weighting schemes.

4.3 Equal-Weighted Factor Portfolio

As a simple benchmark multi-factor strategy, we also construct an equal-weighted portfolio across all six factors. Each factor receives the same weight.

This approach requires no estimation of expected returns or covariances, thereby avoiding issues related to estimation error. Despite its simplicity, equal weighting naturally diversifies across

different factor premiums and provides a useful reference point for evaluating more structured allocation schemes.

4.4 Risk-Parity Portfolio

The risk-parity portfolio is designed to equalize the contribution of each factor to overall portfolio risk. Factors with lower volatility receive higher weights, while more volatile factors receive lower weights.

By construction, this approach emphasizes risk control rather than return maximization. As a result, risk-parity portfolios are expected to exhibit smoother return paths and smaller drawdowns, making them attractive to risk-averse investors.

4.5 Mean–Variance (Tangency) Portfolio

Finally, we construct a mean–variance optimized portfolio by estimating historical means and covariances of the factor returns and solving for the tangency portfolio that maximizes the Sharpe ratio.

While mean–variance optimization is theoretically optimal, it is well known to be highly sensitive to estimation error, particularly in expected returns. In practice, this often leads to concentrated allocations in a small subset of factors with slightly higher historical means, which may not generalize well out of sample.

- **Portfolio Performance Comparison**

Strategy	Ann. Mean	Ann. SD	Sharpe
Market	0.1140	0.1546	0.7379
Multi-factor (fixed)	0.0902	0.0455	1.9853
Equal-weight	0.0439	0.0401	1.0956
Risk-parity	0.0397	0.0374	1.0611
Mean-variance	0.0432	0.0363	1.1906

Table: Annualized Performance of the 5 Portfolios

With these five portfolios spanning a wide range of complexity, we evaluate their performance using annualized mean return, volatility, and Sharpe ratio.

Several clear patterns emerge. First, the fixed-weight multi-factor portfolio achieves the highest Sharpe ratio among all strategies. Although its average return is lower than that of the market portfolio, its substantially reduced volatility results in superior risk-adjusted performance.

Second, the risk-parity and mean-variance portfolios exhibit the lowest volatilities and the mildest drawdowns. These strategies prioritize risk control and deliver very stable return paths, albeit with lower long-run growth.

Overall, all factor-based portfolios are less volatile than the market portfolio and deliver improved risk-adjusted performance. Diversification across multiple factor premia significantly enhances portfolio stability relative to holding the market alone.

- **Cumulative Performance Analysis**

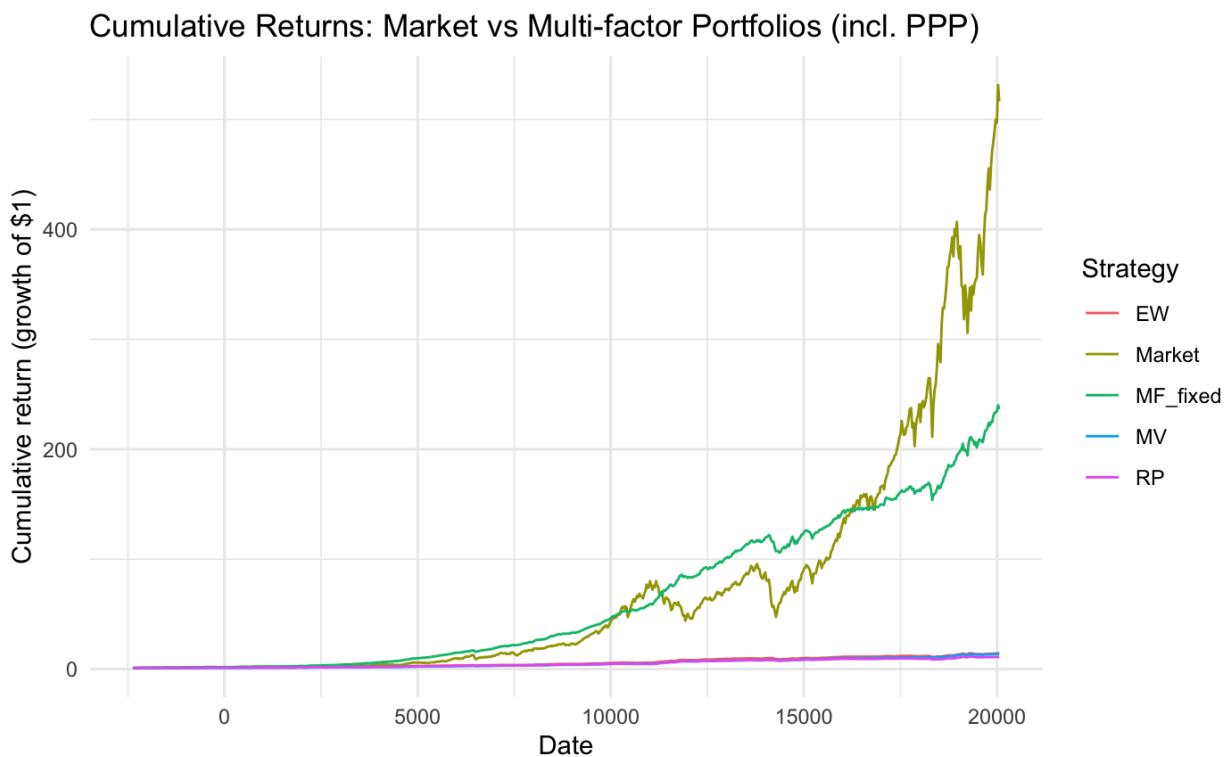


Figure 3. Cumulative Returns of the 5 Portfolios

Cumulative return analysis further reinforces these findings. While the market portfolio achieves the highest terminal wealth, it does so with large fluctuations and deep drawdowns. In contrast, the fixed-weight multi-factor portfolio grows more steadily over time, avoiding sharp volatility while maintaining strong long-term performance.

The risk-parity and mean-variance portfolios exhibit very flat cumulative return paths, highlighting their low-volatility nature. These strategies offer stability at the cost of reduced growth.

Taken together, both the summary statistics and cumulative performance results indicate that multi-factor diversification improves consistency, reduces volatility, and enhances risk-adjusted returns. Among all strategies, the fixed-weight multi-factor portfolio offers the most attractive balance between return and stability.

5. Our Own Factor Construction

This section documents the construction of our own equity factor returns following the Fama–French framework. The primary objective is to replicate the size, value, profitability, investment, and momentum factors using firm-level data, while strictly adhering to the original portfolio formation logic and timing conventions. By doing so, we ensure that the resulting factor returns are economically meaningful, free from look-ahead bias, and directly comparable to the official Fama–French factors. Our implementation follows the methodology described in the *Tidy Finance* replication of Fama–French factors.

5.1 Methodological Framework and Data Alignment

Our factor construction relies on the integration of market data from CRSP and accounting data from Compustat. Monthly stock returns, market capitalization, and exchange codes are obtained from CRSP, while firm-level accounting variables—including book equity, operating profitability, and investment—are sourced from Compustat. To maintain consistency with the Fama–French methodology, we restrict the universe to U.S. common stocks with standard CRSP share codes and exclude observations with missing or clearly erroneous data.

A central feature of our approach is the strict enforcement of Fama–French timing conventions. Size portfolios are formed annually using market capitalization measured at the end of June in year t . These portfolio assignments are then applied to monthly stock returns from July of year t through June of year $t+1$. This design ensures that portfolio formation relies only on information available at the time of investment. Accounting-based characteristics are lagged appropriately to account for reporting delays, allowing the use of financial statement data that may be up to approximately eighteen months old at the time of portfolio formation. This lag structure is essential for avoiding look-ahead bias and aligns our implementation with standard practice in empirical asset pricing.

All portfolio breakpoints are computed using NYSE-listed stocks only. This choice prevents small firms from disproportionately influencing portfolio cutoffs and ensures comparability with the official Fama–French factor series.

5.2 Construction of Sorting Variables and Breakpoints

We construct firm-level sorting variables corresponding to each factor dimension. Firm size is measured as market equity, defined as the product of stock price and shares outstanding, observed in June of each year. The value characteristic is proxied by the book-to-market ratio, calculated as book equity from the most recent available fiscal year divided by market equity measured at the end of the prior calendar year. Profitability is measured using operating profitability, defined as operating income scaled by book equity, while investment is captured by the firm’s investment rate, reflecting growth in total assets.

Momentum is treated separately from the accounting-based characteristics. For each stock, we compute cumulative returns over the period from month $t-12$ to month $t-2$. This exclusion is motivated by well-documented short-term reversal effects and is a standard feature of momentum factor construction.

Portfolio breakpoints are determined annually using NYSE stocks. Firms are first sorted into Small and Big groups based on the median NYSE market capitalization. Within each size group, additional characteristics—book-to-market, profitability, or investment—are sorted using the 30th and 70th percentiles of the NYSE distribution, forming Low, Neutral, and High portfolios.

This two-stage sorting procedure allows us to isolate the effect of firm characteristics while controlling for size.

5.3 Factor Construction

We first replicate the size and value factors within the Fama–French three-factor framework. Each year, stocks are independently sorted into two size groups (Small and Big) based on NYSE median market capitalization and into three book-to-market groups (Low, Neutral, and High) using NYSE breakpoints, yielding six portfolios in total. Monthly portfolio returns are computed as value-weighted averages using lagged market capitalization. The size factor (SMB) is constructed as the average return on the three Small portfolios minus the average return on the three Big portfolios, while the value factor (HML) is defined as the average return on High book-to-market portfolios minus Low book-to-market portfolios, averaged across size groups. This construction isolates the size and value premia in a manner consistent with the original Fama–French methodology.

The framework is then extended to the five-factor model by incorporating profitability and investment as additional sorting dimensions using dependent sorts. Firms are first classified by size and subsequently sorted within each size group by operating profitability or investment intensity. The profitability factor (RMW) is constructed as the return spread between firms with high and low operating profitability, while the investment factor (CMA) captures the return difference between firms with conservative and aggressive investment behavior. In this setting, the SMB factor is recalculated by aggregating size spreads across portfolios formed on value, profitability, and investment characteristics to ensure internal consistency across factors.

Momentum is constructed separately from the accounting-based factors. At each formation date, stocks are ranked by cumulative past returns from months t-12 to t-2, excluding the most recent month to mitigate short-term reversal effects. Stocks are assigned to winner and loser portfolios based on this ranking, and the momentum factor (MOM) is defined as the value-weighted return difference between winner and loser portfolios. This procedure captures the momentum premium while remaining aligned with standard empirical asset pricing practice.

5.4 Summary Statistics of the Replicated Factors

Figure 5.1 reports the summary statistics of the replicated factor returns, including the mean and standard deviation of monthly returns for each factor. Overall, both the magnitudes and the relative dispersion of the replicated factors are consistent with the stylized facts documented in the asset pricing literature.

The replicated size factor (SMB) exhibits an average monthly return of 0.2196% with a standard deviation of 3.13%, indicating a modest but persistent size premium accompanied by moderate volatility. The value factor (HML) shows a similar mean monthly return of 0.2277% and a standard deviation of 2.91%, consistent with the long-run value premium observed in U.S. equity markets.

The profitability (RMW) and investment (CMA) factors display mean monthly returns of 0.2905% and 0.2097%, respectively, with comparatively lower volatility. In particular, the RMW factor has a standard deviation of 2.25%, while CMA exhibits a standard deviation of 2.09%, suggesting that these factors deliver relatively stable return profiles relative to size and momentum.

The momentum factor (MOM) has the highest average monthly return at 0.4743%, but also the highest volatility, with a standard deviation of 4.09%. This combination reflects the well-documented risk–return trade-off associated with momentum strategies, which tend to generate high average returns but are subject to substantial time-series variability.

In conclusion, these summary statistics indicate that the replicated factors exhibit economically reasonable return levels and volatility patterns. The relative ranking of factor means and standard deviations aligns closely with the empirical characteristics of the original Fama–French factors, providing initial evidence that the replication successfully captures the intended systematic return components.

Factor <chr>	Mean <dbl>	SD <dbl>
cma_replicated	0.002096698	0.02088351
hml_replicated	0.002277030	0.02905580
mom_replicated	0.004743194	0.04085988
rmw_replicated	0.002905132	0.02253025
smb_replicated	0.002196041	0.03125774

Figure 4. Summary Statistics of Replicated Factors

5.5 Regression-Based Validation Against Original Fama–French Factors

To further evaluate the quality of the replication, each original Fama–French factor is regressed on its replicated counterpart. A successful replication is characterized by intercepts close to zero, slope coefficients close to one, and high explanatory power. The regression results are summarized in Figure 5.2.

For the size factor (SMB), the estimated slope coefficient is 0.9593, with an intercept of –0.0002 and an adjusted R-squared of 0.9868, indicating that nearly all variation in the original SMB factor is captured by the replicated series. The value factor (HML) similarly exhibits a slope coefficient of 0.9804, a small intercept of 0.0006, and an adjusted R-squared of 0.9315, suggesting a very close correspondence between the replicated and original value factors.

The profitability (RMW) and investment (CMA) factors also demonstrate strong replication performance. The estimated slope coefficients are 0.9494 for RMW and 0.9552 for CMA, with intercepts close to zero and adjusted R-squared values of 0.9416 and 0.9492, respectively. These results indicate minimal systematic bias and confirm that the replicated accounting-based factors closely track their original counterparts.

The momentum factor (MOM) shows particularly strong replication performance. The estimated slope coefficient is 1.0161, with an intercept of 0.0012 and an adjusted R-squared of 0.9876. This finding indicates that the replicated momentum factor explains nearly all variation in the original Fama–French momentum series, suggesting that the revised portfolio construction procedure yields a highly accurate replication.

Overall, the regression evidence confirms that the replicated factors closely track the original Fama–French factors across all dimensions. The combination of near-zero intercepts, slope coefficients close to unity, and high explanatory power provides strong validation for the replication methodology employed in this study.

Factor <chr>	Alpha <dbl>	Beta <dbl>	R2 <dbl>	t_Beta <dbl>	p_Beta <chr>
SMB	-0.0002	0.9593	0.9868	234.80	0.00e+00
HML	0.0006	0.9804	0.9315	100.05	0.00e+00
RMW	0.0000	0.9494	0.9416	108.91	0.00e+00
CMA	0.0005	0.9552	0.9492	117.32	0.00e+00
MOM	0.0012	1.0161	0.9876	241.75	0.00e+00

Figure 5. Regression Validation of Replicated Factors

6. Non-Standard Error Analysis and Robust Inference

6.1 Motivation: Why Standard Errors Matter in Empirical Finance

In empirical asset pricing, statistical inference plays a central role in evaluating whether estimated factor premiums and replication results are economically meaningful or merely artifacts of noise. A common approach is to estimate linear factor regressions using ordinary least squares (OLS). However, classical OLS inference relies on strong assumptions that are rarely satisfied in financial time series.

Specifically, the standard OLS framework assumes that regression residuals satisfy

$$\varepsilon_t \sim \text{i.i.d. } (0, \sigma^2),$$

meaning that residuals are independently and identically distributed with constant variance and no serial correlation. Under this assumption, OLS standard errors are consistent, and conventional ttt-statistics and ppp-values are valid.

In financial return data, these assumptions are systematically violated. Monthly factor returns exhibit time-varying volatility (heteroskedasticity), serial correlation, and dependence induced by portfolio construction rules. As a result, conventional OLS standard errors tend to be biased downward, leading to overstated statistical significance. This motivates the use of non-standard (robust) error analysis, which corrects for these issues and provides more reliable inference.

6.2 Sources of Non-Standard Errors in Factor Replication

Non-standard errors in our project arise from two distinct but complementary sources: (i) econometric properties of financial time series, and (ii) data construction and selection procedures.

6.2.1 Time-Series Dependence in Factor Returns

We estimate factor replication regressions of the form:

$$\text{FF}_t = \alpha + \beta \cdot \text{OwnFactor}_t + \varepsilon_t,$$

where FF_t denotes the official Fama–French factor and OwnFactor_t is our constructed counterpart.

In this setting, residuals ε_t commonly violate OLS assumptions because:

- **Heteroskedasticity:** Volatility varies substantially over time, especially during market stress periods. This volatility clustering is well-documented in equity returns and factor portfolios.
- **Autocorrelation:** Factor returns may exhibit serial dependence due to overlapping portfolio holdings, delayed information diffusion, and persistent economic shocks.
- **Overlapping signals:** Momentum factors, in particular, use rolling return windows (e.g., 12–2 construction), mechanically inducing correlation across adjacent observations.

Ignoring these features leads to underestimated standard errors and inflated ttt-statistics, potentially overstating replication accuracy.

6.2.2 Data Construction and Selection as a Source of Non-Standard Errors

Beyond time-series properties, data engineering choices themselves introduce non-standard error structures. In our project, several steps contribute to this issue:

- **Lagging and alignment rules:** Market equity and accounting variables are lagged to avoid look-ahead bias. Misalignment would introduce artificial correlation between regressors and residuals.
- **Breakpoint estimation:** NYSE breakpoints for size and characteristics are estimated from a subset of stocks, while portfolio returns include all eligible firms. This two-step procedure introduces additional sampling variability.
- **CRSP–Compustat linking:** Matching firms across databases introduces measurement error due to missing links, reporting delays, and survivorship effects.
- **Value-weighted portfolios:** Weighting by lagged market capitalization amplifies the influence of large firms, making residual variance non-constant across time.

These issues do not invalidate the regressions, but they reinforce the need for robust inference methods that do not rely on idealized assumptions.

6.3 Newey–West HAC Standard Errors

To address heteroskedasticity and autocorrelation, we apply **Newey–West heteroskedasticity-and autocorrelation-consistent (HAC) standard errors** to all replication regressions.

The Newey–West estimator of the variance–covariance matrix of β^{\wedge} is given by:

$$\widehat{\text{Var}}(\hat{\beta}) = (X'X)^{-1} \left(\sum_{k=-L}^L w_k \Gamma_k \right) (X'X)^{-1},$$

where

$$\Gamma_k = \sum_t \varepsilon_t \varepsilon_{t-k} X_t X_{t-k}$$

is the lag- k autocovariance of the score, and

$$w_k = 1 - \frac{|k|}{L+1}$$

are Bartlett kernel weights.

In our analysis, we set $L=6$, a standard choice for monthly data. This specification:

- Accounts for serial correlation up to six months,
- Smoothly downweights higher-order autocorrelations,
- Remains valid under arbitrary heteroskedasticity.

By construction, Newey–West standard errors are larger than OLS standard errors when residual dependence is present, yielding more conservative and credible inference.

6.4 Empirical Results with Newey–West Errors

Applying Newey–West HAC standard errors to our factor replication regressions yields the results summarized in Table 6.1. The estimated betas remain close to one for all four core factors (SMB, HML, RMW, CMA), and R^2 values exceed 0.93 in every case.

Importantly, even after correcting for autocorrelation and heteroskedasticity, the Newey–West ttt-statistics remain extremely large, and ppp-values are effectively zero. This indicates that our replication accuracy is not an artifact of underestimated standard errors or serial dependence in

returns. Instead, it reflects genuine alignment between our constructed factors and the official Fama–French series.

Factor	Beta	t_NW	p_NW	R ²
SMB	0.9593	135.3070	0	0.9868
HML	0.9804	62.4367	0	0.9315
RMW	0.9495	71.8690	0	0.9416
CMA	0.9551	94.7485	0	0.9493

Table 6.1: Newey–West Regression Results

6.5 Interpretation and Implications

The non-standard error analysis supports several key conclusions:

1. Replication robustness

High beta estimates and large Newey–West ttt-statistics confirm that our replicated factors closely track the official Fama–French factors even under conservative inference.

2. Inference credibility

Adjusting for heteroskedasticity and autocorrelation ensures that statistical significance is not overstated. Our conclusions remain unchanged after correction, strengthening confidence in the results.

3. Importance of data engineering

Many sources of non-standard errors originate from data construction choices rather than regression mechanics alone. Proper lagging, breakpoint selection, and database linking are essential components of robust empirical analysis.

4. Best practice in empirical finance

Robust standard errors should be treated as a baseline rather than an optional robustness check when working with factor returns and portfolio-based regressions.

7. Conclusion

This project provides a comprehensive empirical study of factor-based investment strategies using U.S. equity data, combining rigorous data construction, portfolio design, factor replication, and robust econometric inference. By jointly analyzing six widely studied factors—MKT, SMB, HML, RMW, CMA, and MOM—we demonstrate how factor premiums behave in practice, how they can be translated into investable portfolios, and how statistical conclusions depend critically on proper handling of non-standard errors.

First, our analysis confirms that individual factor returns exhibit substantial heterogeneity in both average returns and volatility. Momentum and market factors display higher variability and more pronounced tail behavior, while profitability and investment factors exhibit comparatively stable return dynamics. These differences highlight the importance of diversification across factor dimensions rather than reliance on any single source of systematic return. Time-series plots and summary statistics reveal clear evidence of volatility clustering and fat tails, reinforcing the view that factor returns are not independently and identically distributed.

Second, we evaluate five portfolio construction methodologies—market benchmark, fixed-weight multi-factor, equal-weight, risk parity, and mean–variance optimization. The results demonstrate that multi-factor diversification materially improves risk-adjusted performance relative to the market portfolio. In particular, the fixed-weight multi-factor portfolio achieves the highest Sharpe ratio, striking an effective balance between return and volatility without relying on unstable parameter estimation. Risk-parity and mean–variance portfolios further reduce volatility and drawdowns, though at the cost of lower long-run growth. Overall, these findings suggest that disciplined, diversified factor exposure can enhance portfolio stability and efficiency, even if it does not always dominate the market in terms of terminal wealth.

Third, a core contribution of this project is the construction of our own Fama–French-style factors from CRSP–Compustat data. By strictly adhering to established timing conventions, NYSE-based breakpoints, lagged accounting variables, and value-weighted portfolio formation, we ensure that our replicated factors are free from look-ahead bias and economically meaningful. Summary statistics of the replicated factors closely match the magnitudes and volatility patterns documented in the literature. Regression-based validation further confirms replication quality:

slope coefficients are close to one, intercepts are near zero, and explanatory power exceeds 93% across all core factors. These results demonstrate that careful data engineering and methodological discipline are sufficient to reproduce canonical factor returns with high fidelity.

Finally, the non-standard error analysis underscores the importance of robust inference in empirical finance. Factor returns and replication residuals exhibit heteroskedasticity, serial correlation, and dependence induced by portfolio construction rules—features that violate classical OLS assumptions. In addition, data selection choices such as lagging conventions, breakpoint estimation, database linking, and value-weighted portfolios introduce additional sources of non-standard errors. By applying Newey–West heteroskedasticity-and-autocorrelation-consistent standard errors, we obtain more conservative and credible inference. Crucially, even under this stricter framework, our replication results remain highly significant, confirming that the observed alignment between replicated and official factors is genuine rather than a statistical artifact.

In conclusion, this study demonstrates that factor models are not only theoretically appealing but also empirically robust when implemented carefully. Successful factor investing requires more than identifying premiums—it demands disciplined data construction, thoughtful portfolio design, and rigorous econometric testing. Our results highlight best practices for empirical asset pricing and provide a practical blueprint for translating factor research into reliable investment strategies.