Robotic Hand-Eye Calibration Report

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1 Abstract

This report presents the theory, implementation, and results of a robotic hand-eye calibration experiment. The calibration was performed using the Tsai-Lenz algorithm to determine the fixed transformation between a camera mounted on a robot's end-effector (eye-in-hand configuration) and the robot base. The mathematical foundations are derived in detail, followed by presentation of the experimental results and discussion.

2 Introduction

Hand-eye calibration is a fundamental problem in robotics that aims to determine the fixed transformation between a camera (eye) and a robot's end-effector (hand). This transformation is crucial for tasks that require visual feedback to guide robotic manipulation. The core equation governing this problem is:

$$A_i X = X B_i \tag{1}$$

where:

- A_i represents the transformation between robot base and end-effector for pose i
- B_i represents the transformation between camera and calibration object for pose i
- X is the unknown transformation between camera and end-effector (eye-in-hand) or camera and base (eye-to-hand)

3 Mathematical Foundations

3.1 Homogeneous Transformations

The transformations are represented as 4×4 homogeneous matrices:

$$H = \begin{bmatrix} R & t \\ 0 & 1 \end{bmatrix} \tag{2}$$

where R is a 3×3 rotation matrix and t is a 3×1 translation vector.

3.2 Rotation Representations

Rotations can be represented in several forms:

1. **Axis-angle representation**: A rotation of angle θ about unit axis k
2. **Rotation vector**: $r = \theta k$
3. **Rodrigues' rotation formula**:

$$R = I + \sin\theta \cdot k_x + (1 - \cos\theta) \cdot k_x^2 \tag{3}$$

   where k_x is the skew-symmetric matrix of k.

3.3 Modified Rodrigues Parameters (MRP)

$$P = 2\sin\left(\frac{\theta}{2}\right) k \tag{4}$$

4 Tsai-Lenz Algorithm Derivation

The Tsai-Lenz algorithm solves the hand-eye calibration problem in two steps: first for rotation, then for translation.

4.1 Solving for Rotation

From $A_i X = X B_i$, we extract the rotation part:

$$R_{A_i} R_x = R_x R_{B_i} \tag{5}$$

1. Convert R_{A_i} and R_{B_i} to rotation vectors r_{A_i} and r_{B_i} using Rodrigues' formula:

$$\theta = \cos^{-1}\left(\frac{\text{trace}(R) - 1}{2}\right) \tag{6}$$

$$k = \frac{1}{2\sin\theta} \begin{bmatrix} R_{32} - R_{23} \\ R_{13} - R_{31} \\ R_{21} - R_{12} \end{bmatrix} \tag{7}$$

2. Compute MRP for each rotation:

$$P_A = 2\sin\left(\frac{\theta_A}{2}\right) k_A \tag{8}$$

$$P_B = 2\sin\left(\frac{\theta_B}{2}\right) k_B \tag{9}$$

3. Solve for P'_X using:

$$\text{skew}(P_A + P_B) \cdot P'_X = P_B - P_A \tag{10}$$

   where skew(v) is the skew-symmetric matrix of vector v.

4. Compute final MRP:

$$P_x = \frac{2P'_x}{\sqrt{1 + |P'_x|}} \tag{11}$$

5. Convert back to rotation matrix R_x:

$$R_x = \left(1 - \frac{|P_x|^2}{2}\right) I + \frac{1}{2}\left(P_x P_x^T + \sqrt{4 - |P_x|^2}\,\text{Skew}(P_x)\right) \tag{12}$$

4.2 Solving for Translation

With R_x known, solve the translation part:

$$(I - R_{A_i})t_x = t_{A_i} - R_x t_{B_i} \tag{13}$$

This forms a linear system that can be solved using least squares when multiple measurements are available.

5 Experimental Results

The hand-eye calibration was performed using 19 valid image poses (out of 20 attempted). The camera intrinsics were provided as:

Camera matrix:

$$K = \begin{bmatrix} 594.0418 & 0 & 310.17813 \\ 0 & 593.92505 & 214.5701 \\ 0 & 0 & 1 \end{bmatrix} \tag{14}$$

Distortion coefficients:

$$[-0.4783, 0.3981, 0.000756, 0.0002098, -0.3071] \tag{15}$$

The calibration results provided the transformation from camera to base:

Rotation matrix:

$$R = \begin{bmatrix} -0.1768 & -0.6447 & -0.7437 \\ -0.8130 & 0.5215 & -0.2588 \\ 0.5547 & 0.5589 & -0.6163 \end{bmatrix} \tag{16}$$

Translation vector:

$$t = [0.0917, 0.2581, 1.6916]^T \tag{17}$$

This transformation can be represented as the homogeneous matrix:

$$X = \begin{bmatrix} -0.1768 & -0.6447 & -0.7437 & 0.0917 \\ -0.8130 & 0.5215 & -0.2588 & 0.2581 \\ 0.5547 & 0.5589 & -0.6163 & 1.6916 \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{18}$$

6 Discussion and Conclusion

The experiment successfully implemented the Tsai-Lenz algorithm for hand-eye calibration. The results show a reasonable transformation between the camera and robot base, with the z-component of translation (1.69m) suggesting the camera was mounted at some distance from the base.

Key observations:

- The algorithm successfully processed 19 out of 20 images, demonstrating robustness to occasional detection failures.
- The rotation matrix satisfies orthogonality conditions ($R^T R \approx I$) with minor numerical errors.
- The translation vector appears physically plausible for a robotic setup.

Potential improvements:

- Using more image poses could improve accuracy

This calibration provides the essential transformation needed for vision-guided robotic operations, enabling accurate mapping between camera coordinates and robot coordinates.

References

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