上海科技大学
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CS283: **Robotics Spring 2025: SLAM I**

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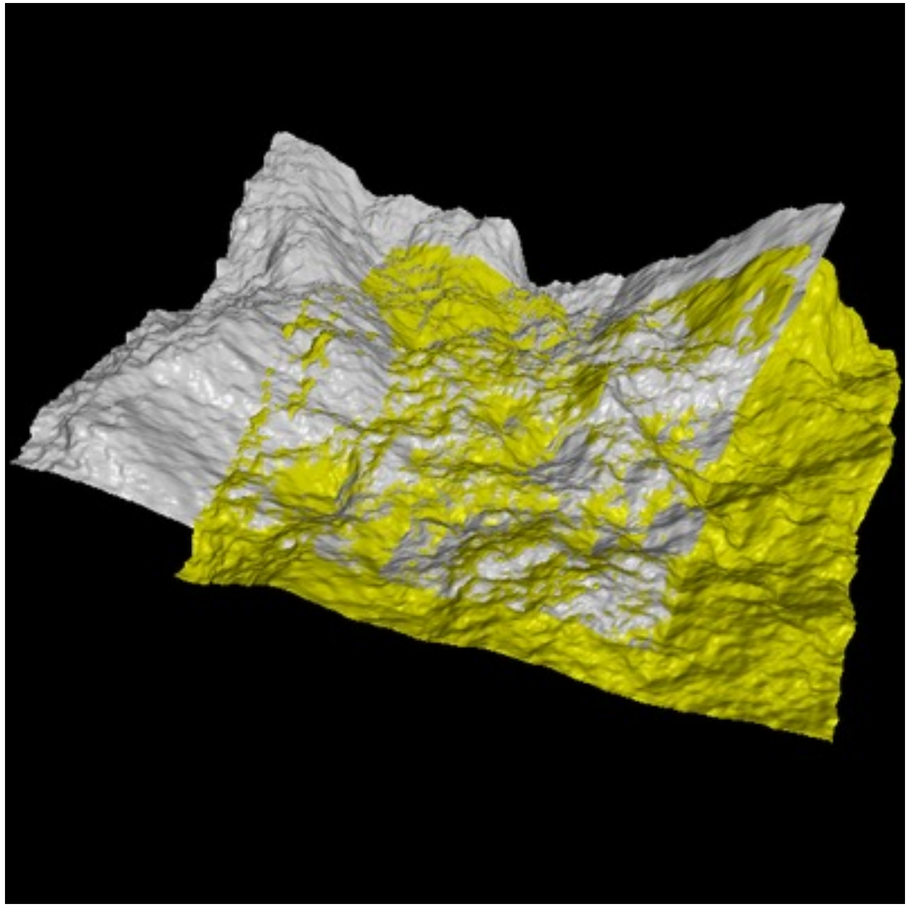
ICP

ICP: Iterative Closest Points Algorithm

- Align two partially-overlapping point sets (2D or 3D)
- Given initial guess for relative transform
- Warning: Using 3D ICP for 2D data may mirror the data (e.g. 180 degree roll)!
  - Use 2D ICP!
- ROS: Point Cloud Library (PCL)

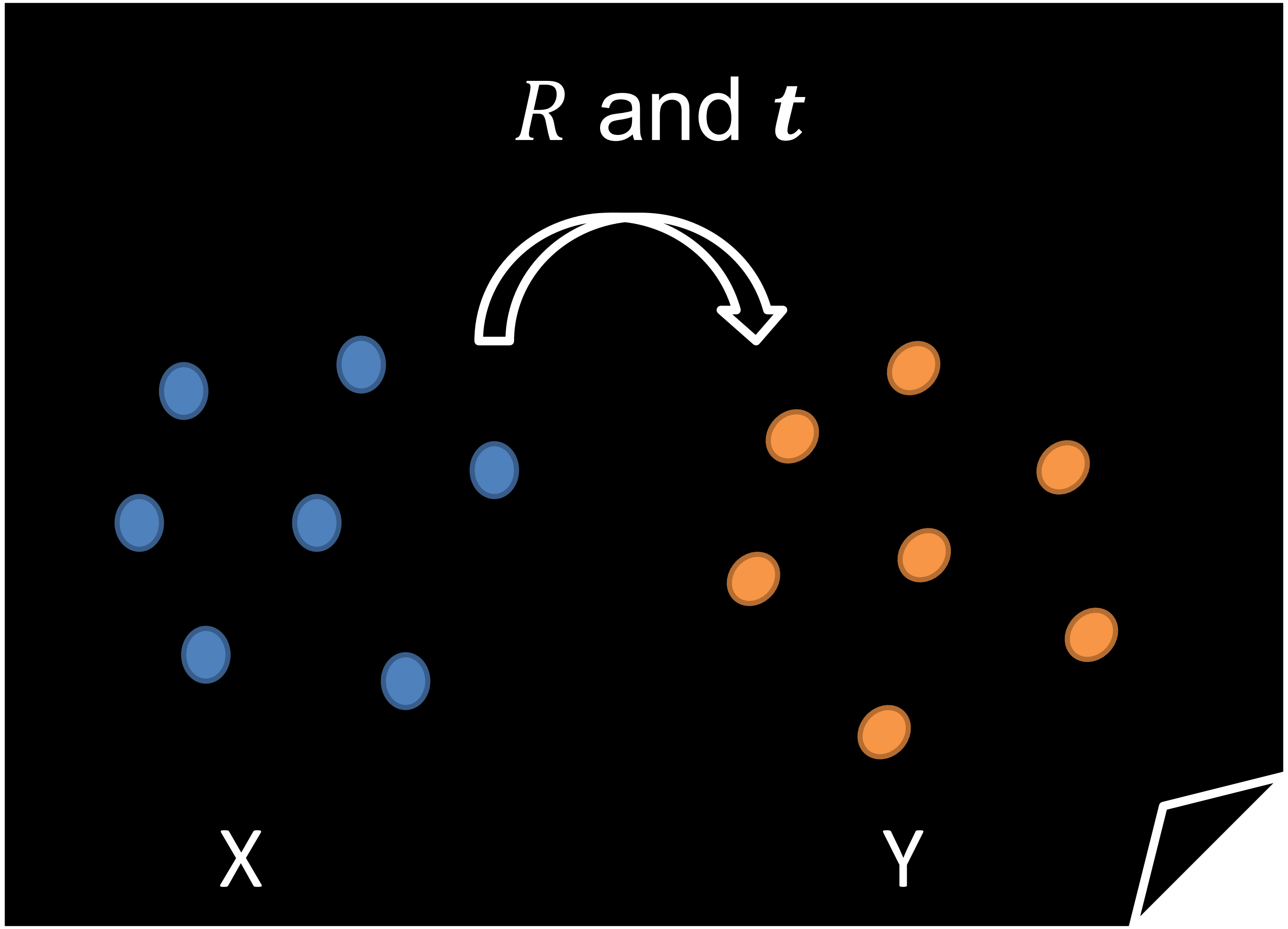


Material derived from Ronen Gvili : *www.cs.tau.ac.il/~dcor/Graphics/adv-slides/**ICP**.ppt*

Horn's method

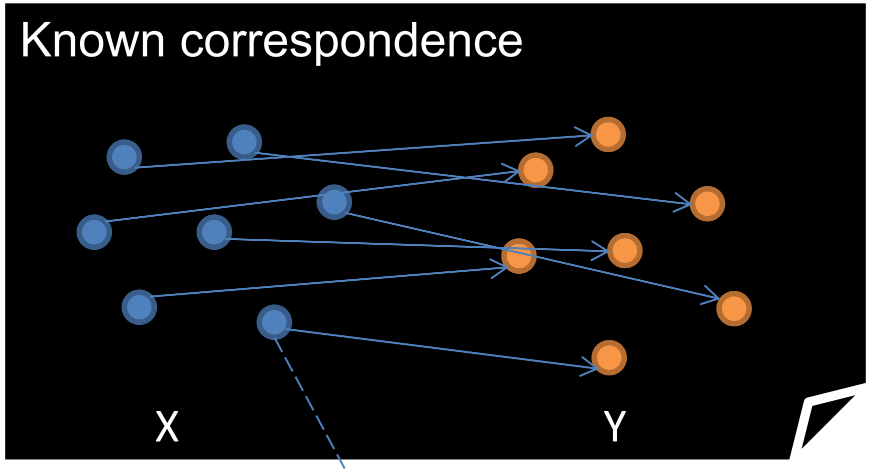
Material by Toru Tamaki, Miho Abe, Bisser Raytchev, Kazufumi Kaneda

- Input
  - Two point sets: X and Y
- Output
  - Rotation matrix R
  - Translation vector $\boldsymbol{t}$
  - Fitting error

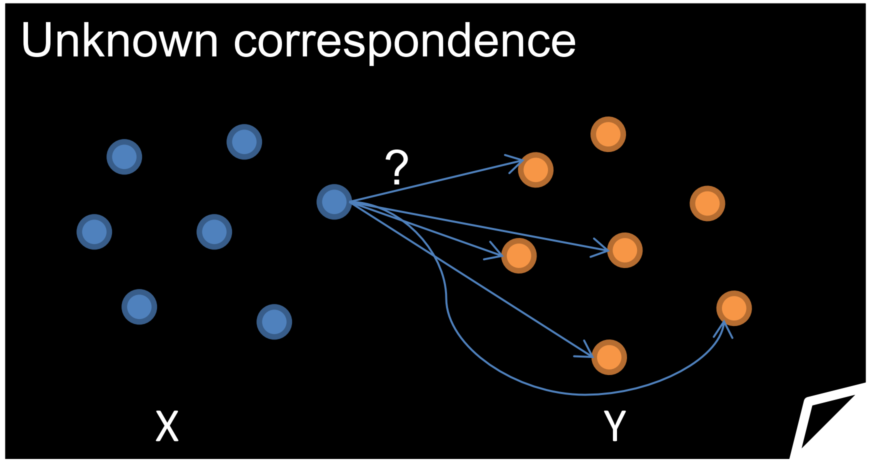


Horn’s method: correspondence is known.

$$X = \begin{bmatrix} \boldsymbol{x}_1^T \\ \boldsymbol{x}_2^T \\ \vdots \end{bmatrix} \qquad Y = \begin{bmatrix} \boldsymbol{y}_1^T \\ \boldsymbol{y}_2^T \\ \vdots \end{bmatrix}$$



$$\boldsymbol{x}_1 = (x_{1x}, x_{1y}, x_{1z})^T$$



Horn's method: correspondence is known.

1 Compute centers

$$X = \begin{pmatrix} \boldsymbol{x}_1^T \\ \boldsymbol{x}_2^T \\ \vdots \end{pmatrix} \Rightarrow \bar{\boldsymbol{x}} \qquad Y = \begin{pmatrix} \boldsymbol{y}_1^T \\ \boldsymbol{y}_2^T \\ \vdots \end{pmatrix} \Rightarrow \bar{\boldsymbol{y}}$$

2 Centering

$$\hat{X} = X - \bar{\boldsymbol{x}} \qquad \hat{Y} = Y - \bar{\boldsymbol{y}}$$

3

$$S = \hat{X}\,\hat{Y}$$

$$\begin{pmatrix} S_{xx} & S_{xy} & S_{xz} \\ S_{yx} & S_{yy} & S_{yz} \\ S_{zx} & S_{zy} & S_{zz} \end{pmatrix}$$

4

$$K = \begin{pmatrix} S_{xx}+S_{yy}+S_{zz} & S_{yz}-S_{zy} & S_{zx}-S_{xz} & S_{xy}-S_{yx} \\ S_{yz}-S_{zy} & S_{xx}-S_{yy}-S_{zz} & S_{xy}+S_{yx} & S_{zx}+S_{xz} \\ S_{zx}-S_{xz} & S_{xy}+S_{yx} & S_{yy}-S_{xx}-S_{zz} & S_{yz}+S_{zy} \\ S_{xy}-S_{yx} & S_{zx}+S_{xz} & S_{yz}+S_{zy} & S_{zz}-S_{xx}-S_{yy} \end{pmatrix}$$

5

Compute 1st Eigenvector $\boldsymbol{q}$

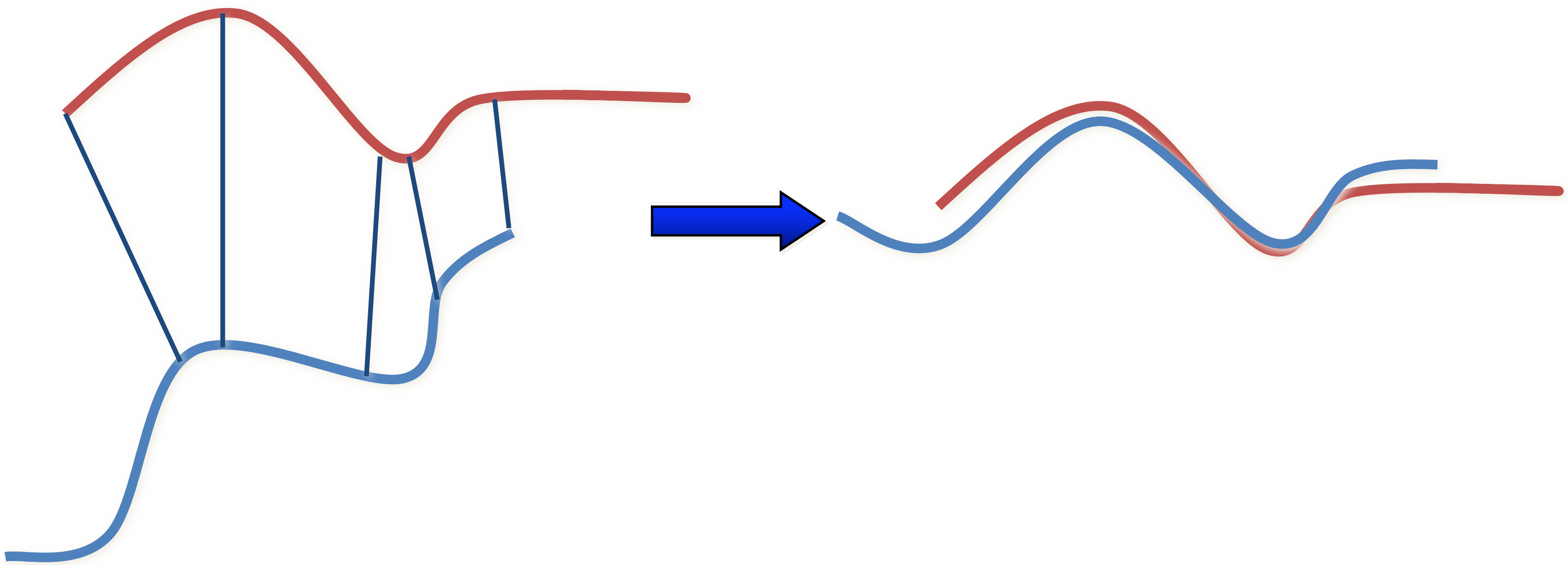
: quaternion q

Convert q to R

$\boldsymbol{t} = \bar{\boldsymbol{x}} - R\bar{\boldsymbol{y}}$

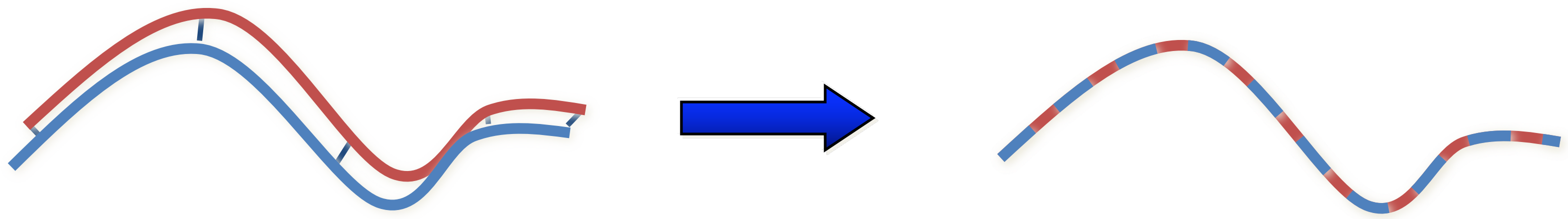
Aligning 3D Data

- How to find correspondences: User input? Feature detection? Signatures?
- Alternative: assume closest points correspond

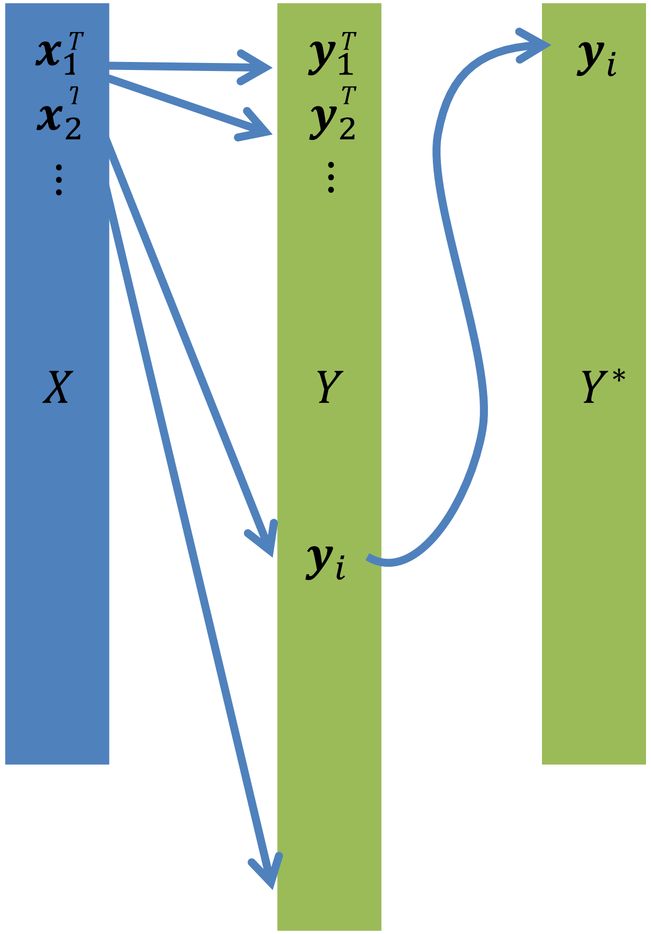


Aligning 3D Data

- Converges if starting position “close enough“



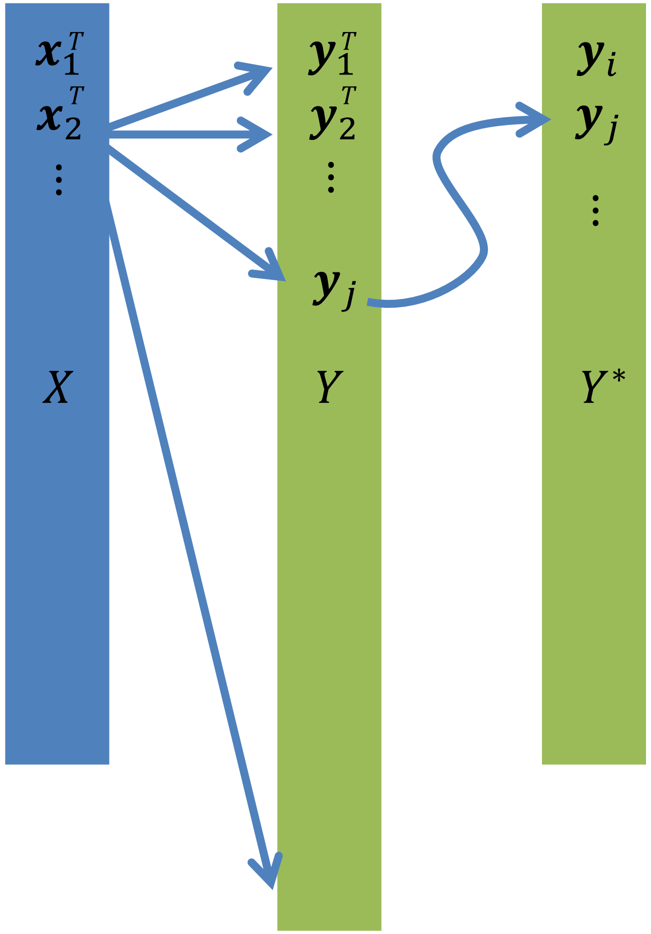
ICP: correspondence is unknown.



Find closest (nearest) point to $\boldsymbol{x}_1$ in Y

Put the point to Y^*

ICP: correspondence is unknown.



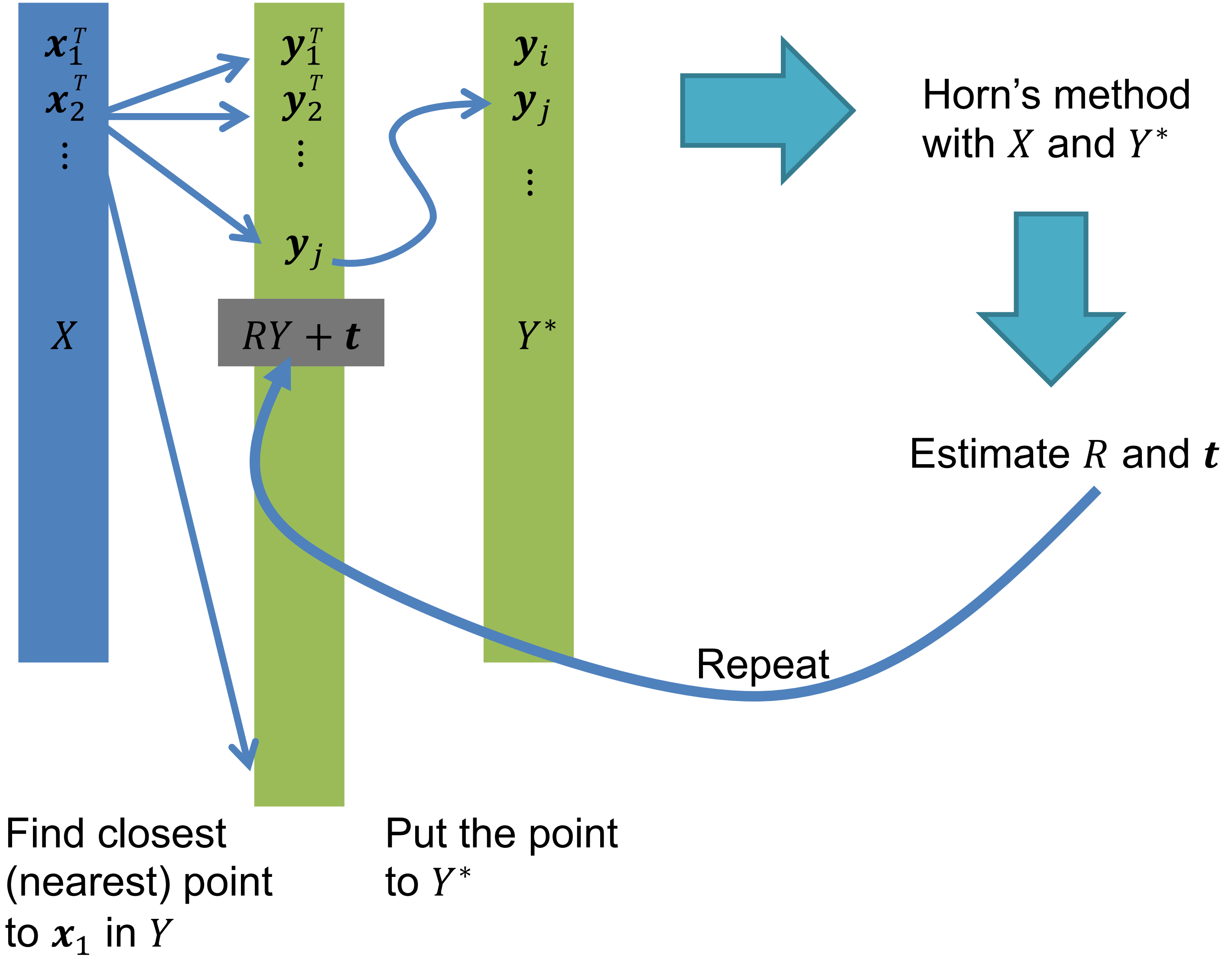
Find closest (nearest) point to $\boldsymbol{x}_1$ in Y

Put the point to Y^*

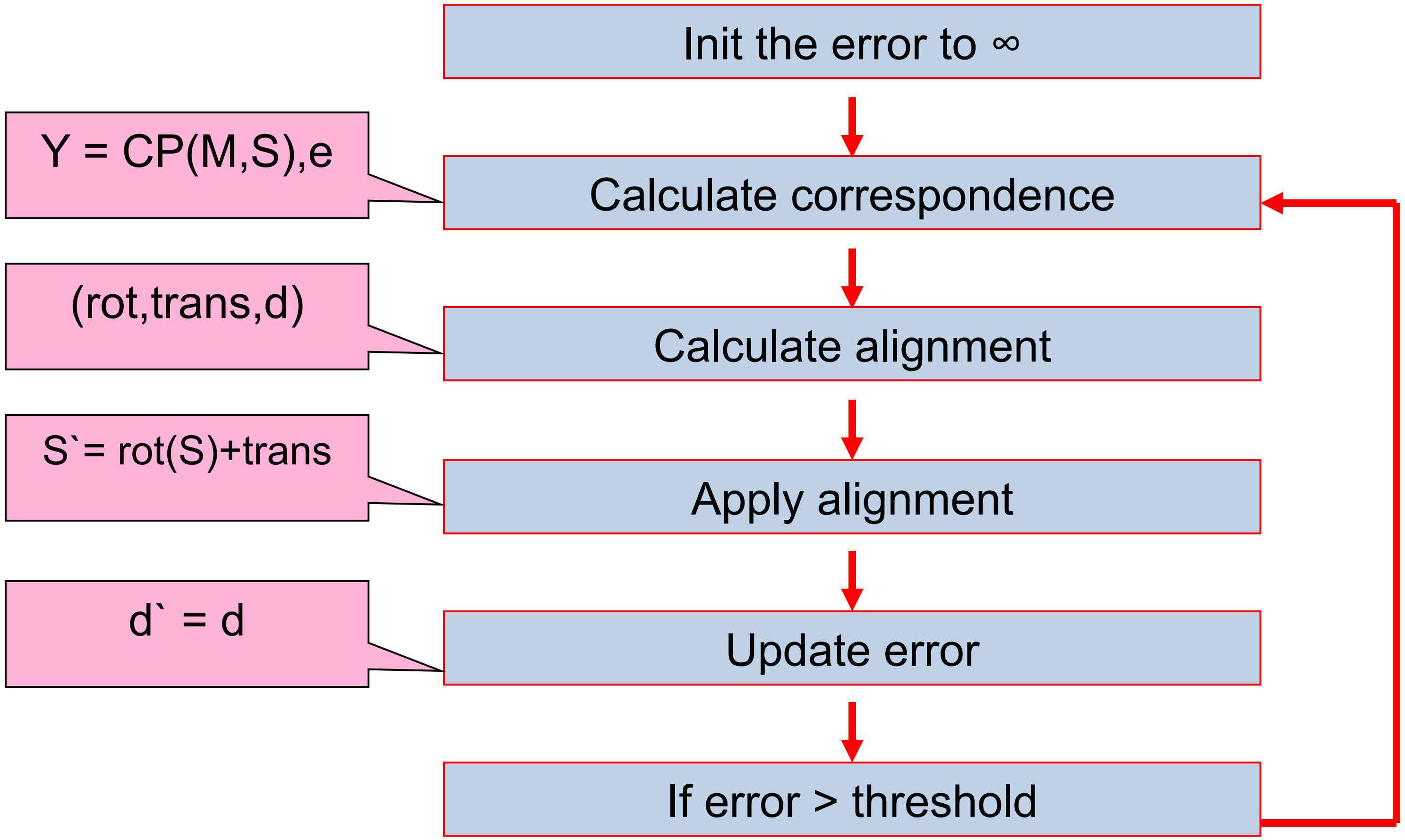
Horn’s method with X and Y^*

Estimate R and $\boldsymbol{t}$

ICP: correspondence is unknown.



The Algorithm



The Algorithm

```
function ICP(Scene,Model)
begin
E` ← + ∞;
(Rot,Trans) ← In Initialize-Alignment(Scene,Model);
repeat
        E ← E`;
        Aligned-Scene ← Apply-Alignment(Scene,Rot,Trans);
        Pairs ← Return-Closest-Pairs(Aligned-Scene,Model);
        (Rot,Trans,E`) ← Update-Alignment(Scene,Model,Pairs,Rot,Trans);
Until |E`- E|  < Threshold
return (Rot,Trans);
end
```

Convergence Theorem

- The ICP algorithm always converges monotonically to a local minimum with respect to the MSE distance objective function.

Time analysis

Each iteration includes 3 main steps

A. Finding the closest points :

$O(N_M)$ per each point

$O(N_M{*}N_S)$ total.

B. Calculating the alignment: $O(N_S)$

C. Updating the scene: $O(N_S)$

Optimizing the Algorithm

The best match/nearest neighbor problem :

Given **N** records each described by **K** real values (attributes), and a dissimilarity measure **D**, find the **m** records closest to a query record.

Optimizing the Algorithm

- K-D Tree :

  Construction time: $O(kn \log n)$

  Space: $O(n)$

  Region Query : $O(n^{1-1/k}+k)$

Time analysis

Each iteration includes 3 main steps

A. Finding the closest points :

$O(N_M)$ per each point

$O(N_M \log N_S)$ total.

B. Calculating the alignment: $O(N_S)$

C. Updating the scene: $O(N_S)$

Further optimization: Approximate k-d tree search

ICP Variants

- Variants on the following stages of ICP have been proposed:

1. Selecting sample points (from one or both point clouds)
2. Matching to points to a plane or mesh
3. Weighting the correspondences
4. Rejecting certain (outlier) point pairs
5. Assigning an error metric to the current transform
6. Minimizing the error metric w.r.t. transformation

- Can analyze various aspects of performance:
  - Speed
  - Stability
  - Tolerance to noise and/or outliers
  - Maximum initial misalignment

Rejecting Pairs

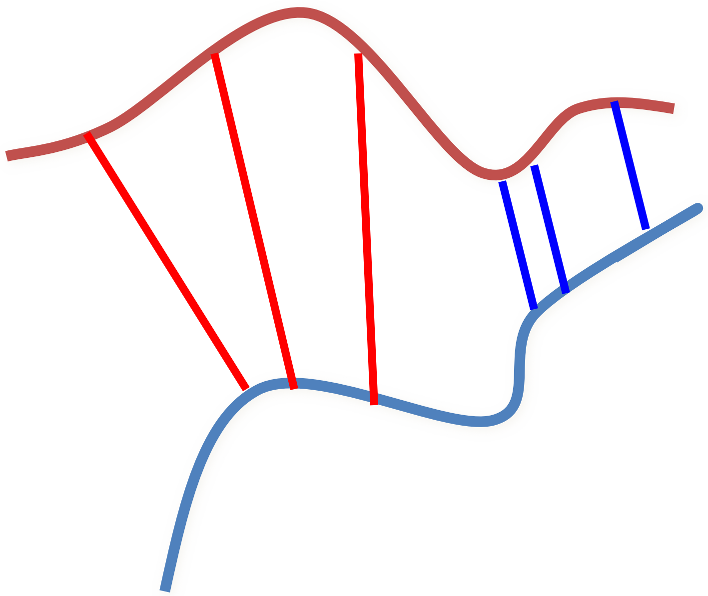
- Corresponding points with point to point distance higher than a given threshold.
- Rejection of worst n% pairs based on some metric.
- Pairs containing points on end vertices.
- Rejection of pairs whose point to point distance is higher than n*σ.
- Rejection of pairs that are not consistent with their neighboring pairs [Dorai 98] :

  (p_1,q_1) , (p_2,q_2) are inconsistent iff

  $$\left|Dist(p_1, p_2) - Dist(q_1, q_2)\right| > threshold$$

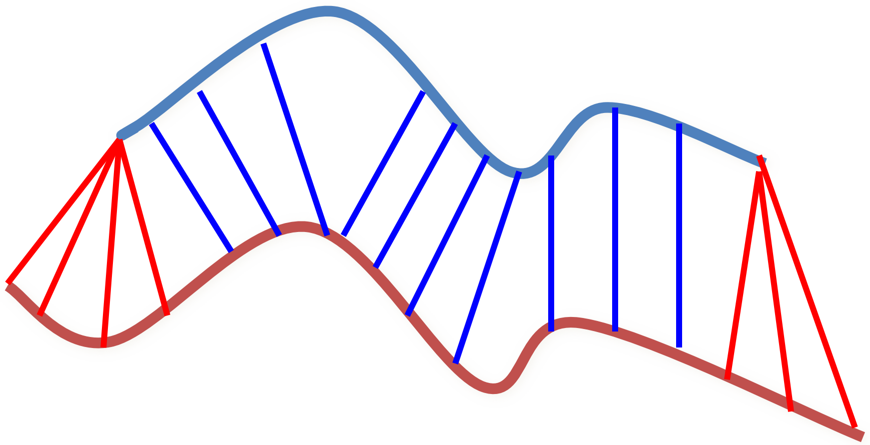
Rejecting Pairs

Distance thresholding



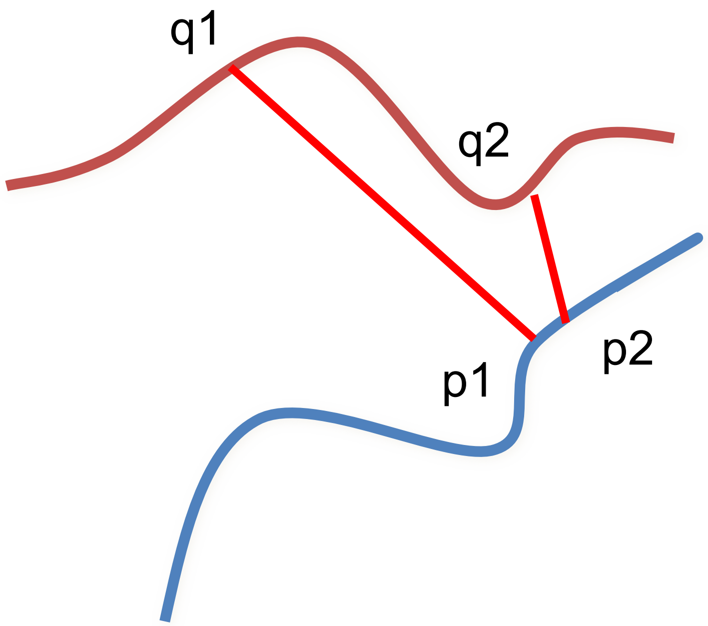
Rejecting Pairs

Points on end vertices

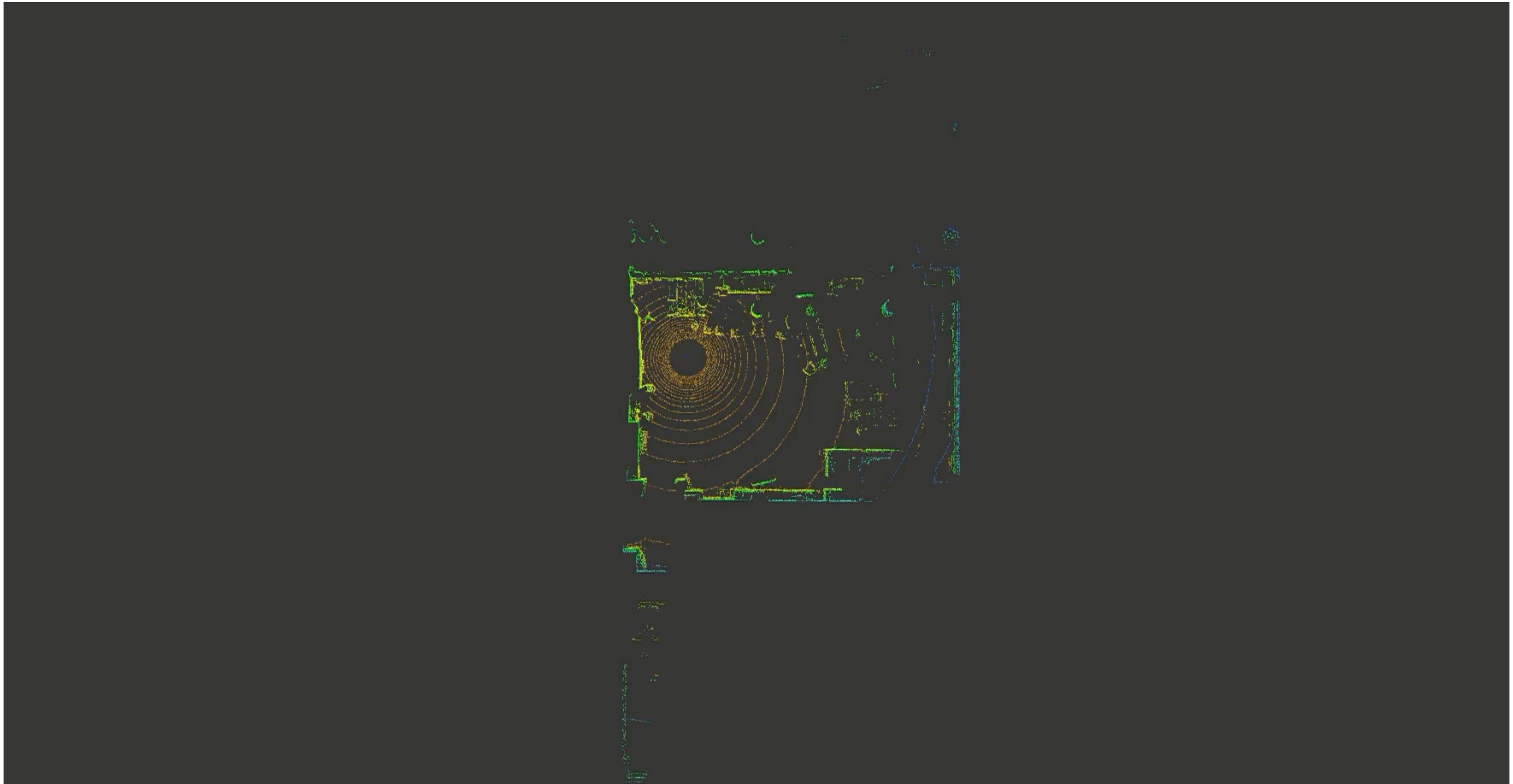


Rejecting Pairs

Inconsistent Pairs



BLAM: ICP in action



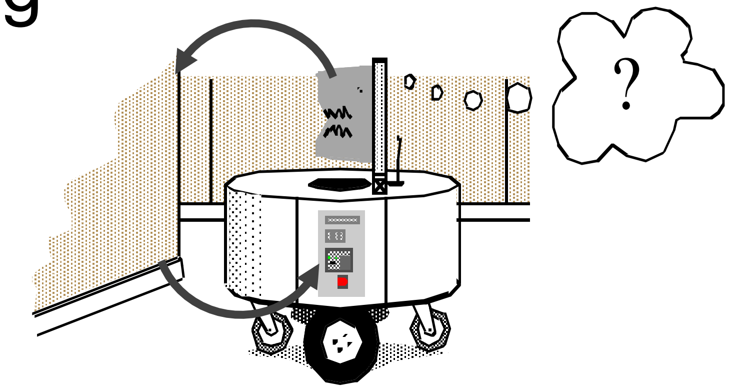

PrimeSense

DEFINITION OF SLAM

What is SLAM?

- **Localization**: inferring location given a map
- **Mapping**: inferring a map given locations
- **SLAM**: learning a map and locating the robot simultaneously
- SLAM has long been regarded as a chicken-and-egg problem:

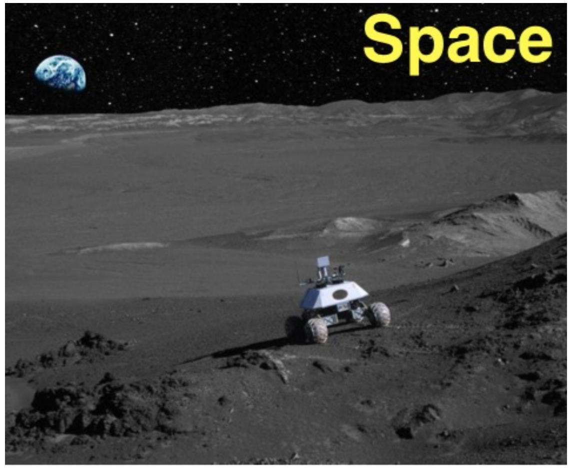
  - a map is needed for localization and
  - a pose estimate is needed for mapping

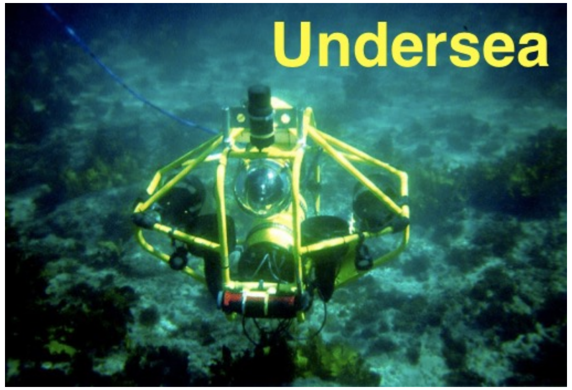


Material derived from Wolfram Burgard:
http://ais.informatik.uni-freiburg.de/teaching/ss20/robotics/slides/13-slam.pdf

SLAM Applications

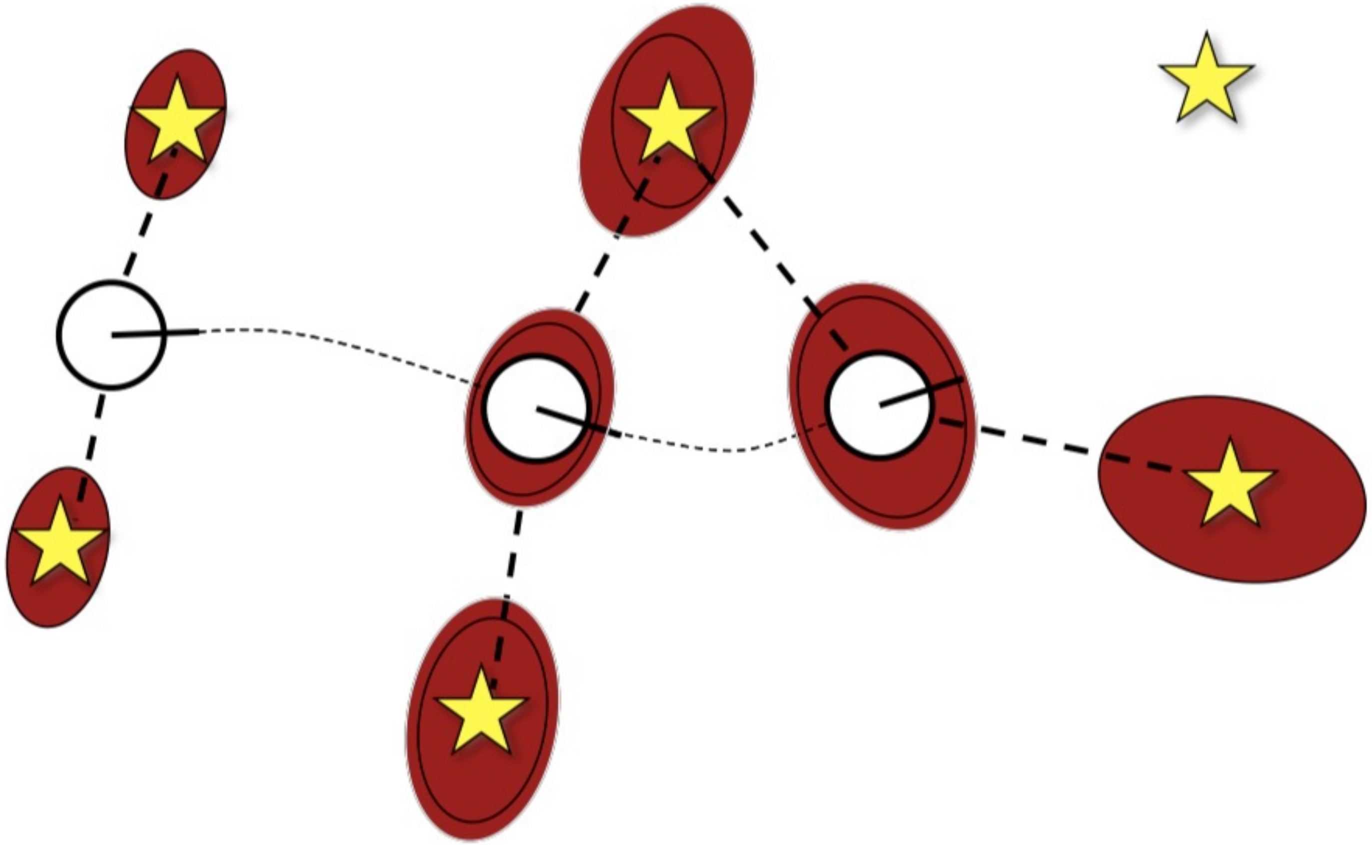
- At home: vacuum cleaner, lawn mower
- Air: surveillance with unmanned air vehicles
- Underwater: reef monitoring
- Underground: exploration of mines
- Space: terrain mapping for localization
- ……





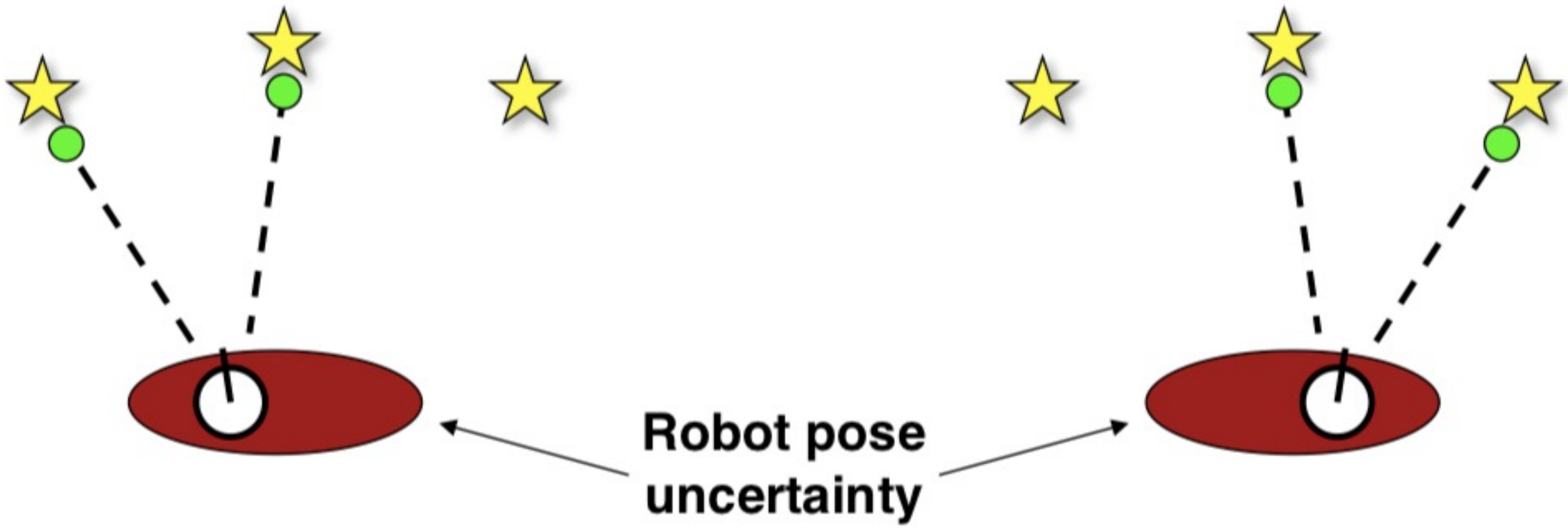
Why is SLAM a Hard Problem?

- Robot path and map are both unknown
- Errors in map and pose estimates correlated



Why is SLAM a Hard Problem?

- The mapping between observations and landmarks is unknown
- Picking wrong data associations can have catastrophic consequences (divergence)



Overview of SLAM Methods

- Camera
  - Feature-Based Methods
    - MonoSLAM
    - PTAM
    - ORB-SLAM
  - Direct Methods
    - DTAM
    - LSD-SLAM
    - DSO
  - Semi-Direct Methods
    - SVO
  - Others
    - PoseNet
    - CNN-SLAM
    - …
- Laser
  - Pose Graph
    - Cartographer
    - Karto-SLAM
    - Hector-SLAM
    - BLAM
    - LIO
  - Particle Filter
    - FastSLAM
    - Gmapping
  - Extended Kalman Filter
    - EKF-SLAM
    - LINS
  - Others
    - LOAM
    - IMLS-SLAM
    - …

SLAM Front-end & Back-end

- Front-end
  - calculate relative poses between several frames/ to map
    - scan matching
    - image registration
    - …
  - estimate absolute poses
  - construct the local map
- Back-end
  - optimize the absolute poses and mapping
  - only if a loop was closed

https://ww2.mathworks.cn/help/nav/ug/implement-simultaneous-localization-and-mapping-with-lidar-scans.html

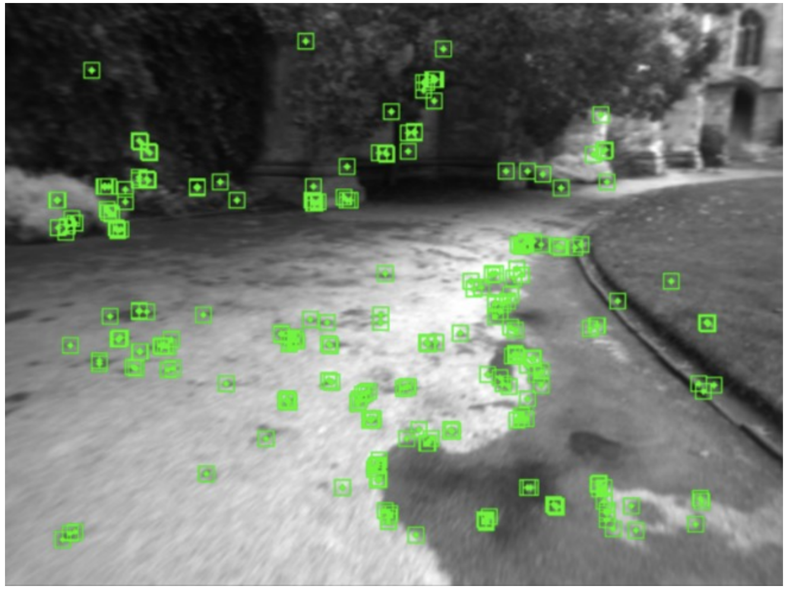
FRONT END – LASER - ICP

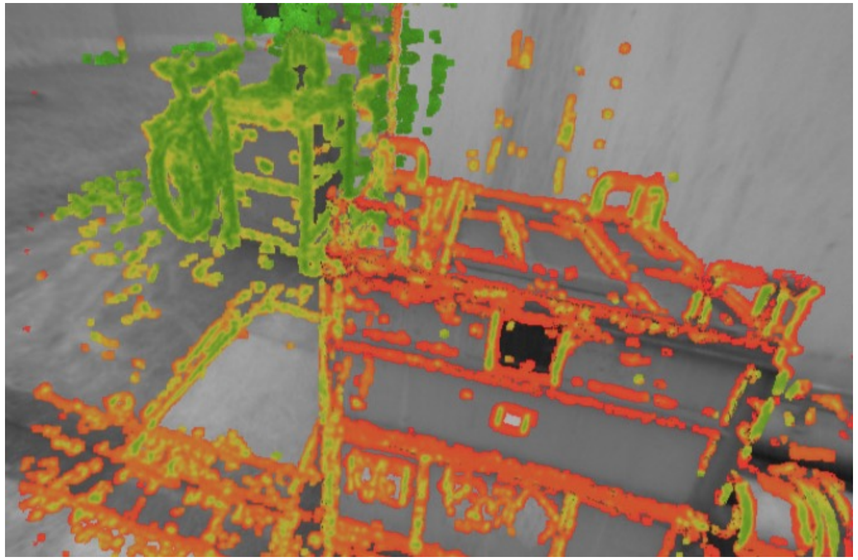
FRONT END - CAMERA

Methods

- **Feature-based Methods**
  - SIFT
  - ORB (ORB-SLAM)
  - BRISK
  - AKAZE
- **Direct Methods**
  - Optical Flow
  - Inverse Depth (LSD-SLAM)
  - Fourier-Mellin Transform
- **Semi-Direct Methods**
  - SVO

more details in the next lectures





Feature-based Methods

- Feature Extraction
  - Feature Detectors & Feature Descriptor; more in vision lectures

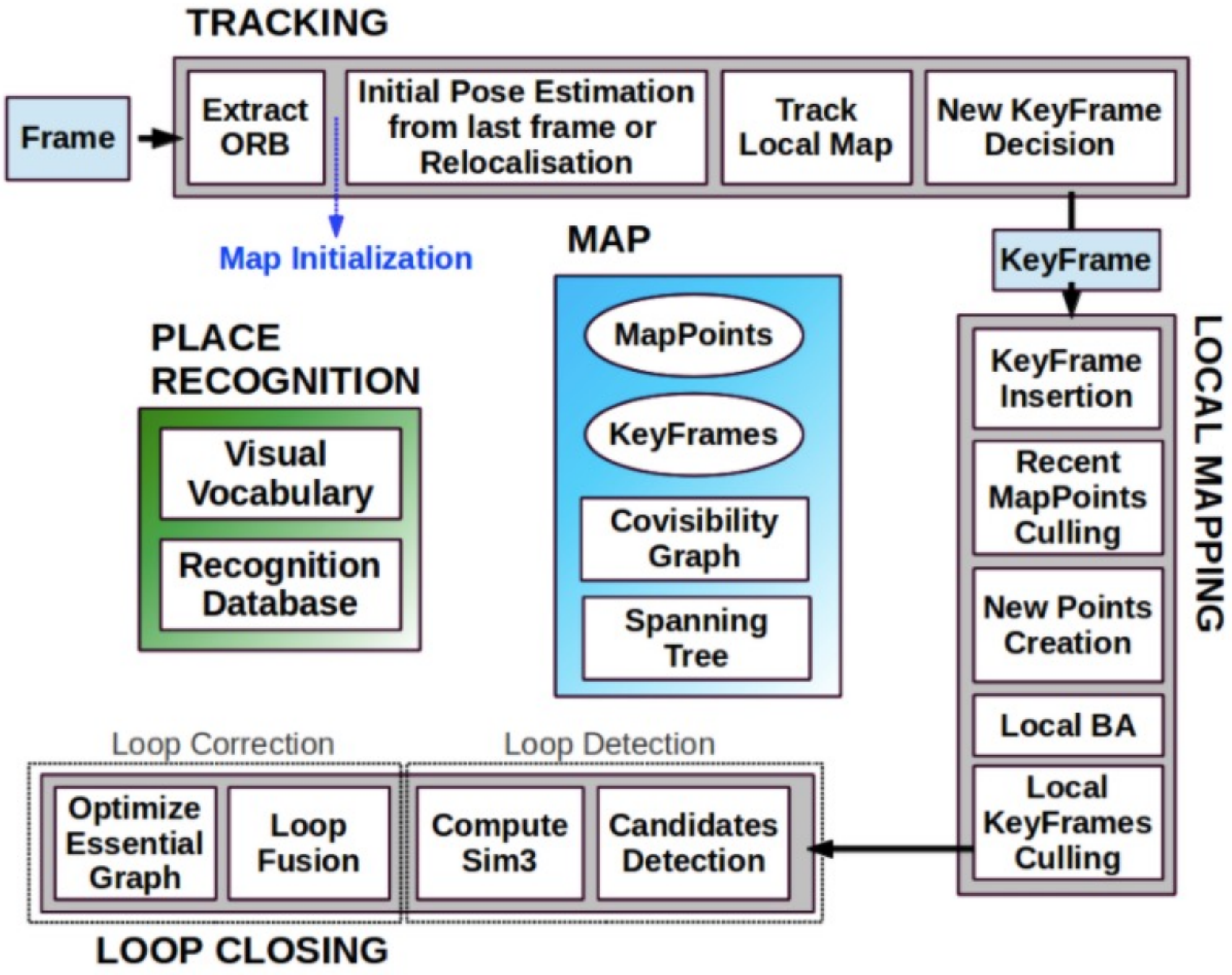
    - ORB, SIFT, AKAZE, BRISK, etc …
- Feature Matching

    - BFM, KNN, etc …
- Relative Pose Calculation

    - 5-pt, 7-pt, 8-pt, PnP, etc …

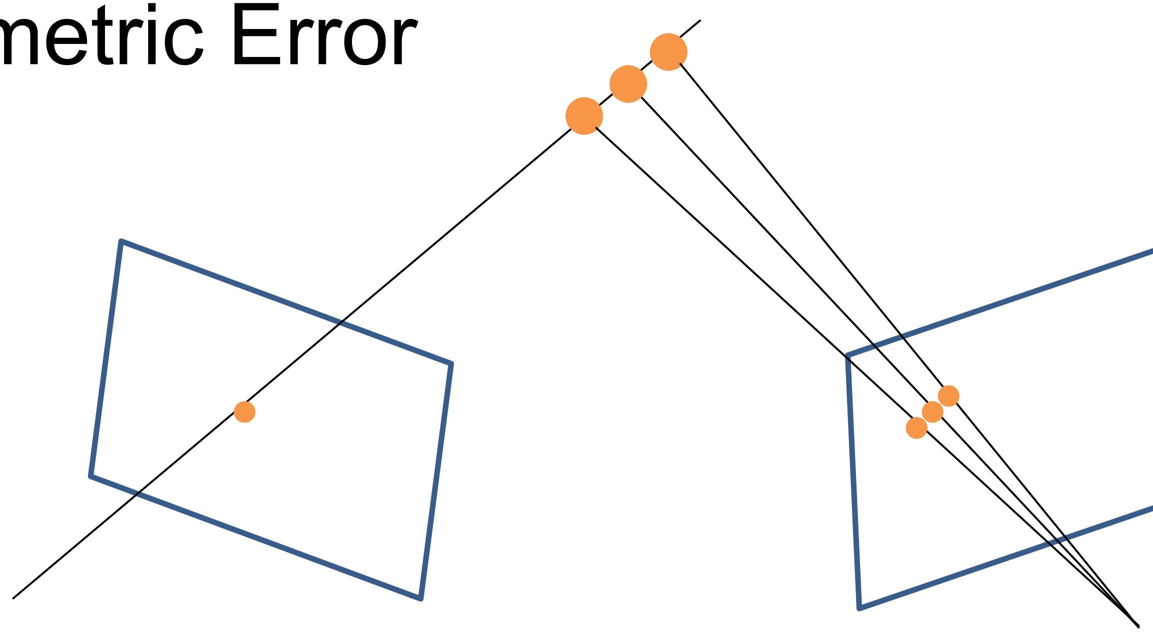
Feature-based Method: ORB-SLAM

Mur-Artal R, Montiel J M M, Tardos J D. ORB-SLAM: a versatile and accurate monocular SLAM system[J]. IEEE transactions on robotics, 2015, 31(5): 1147-1163.



Direct Method: LSD-SLAM

- Construct Photometric Error

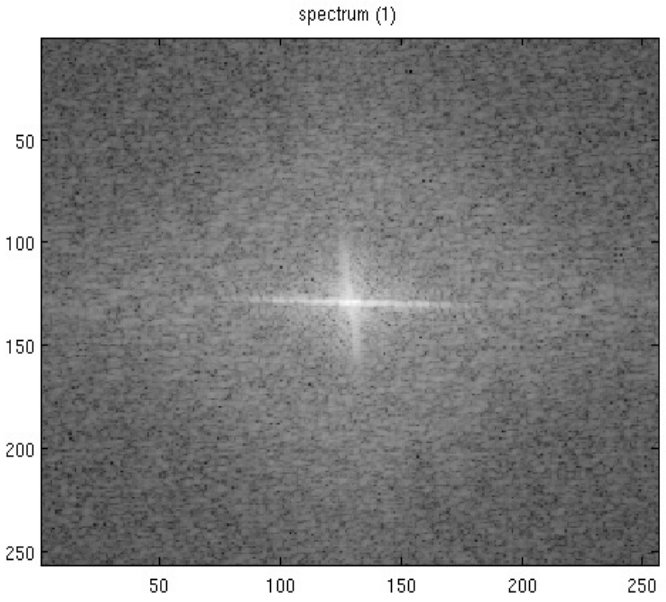


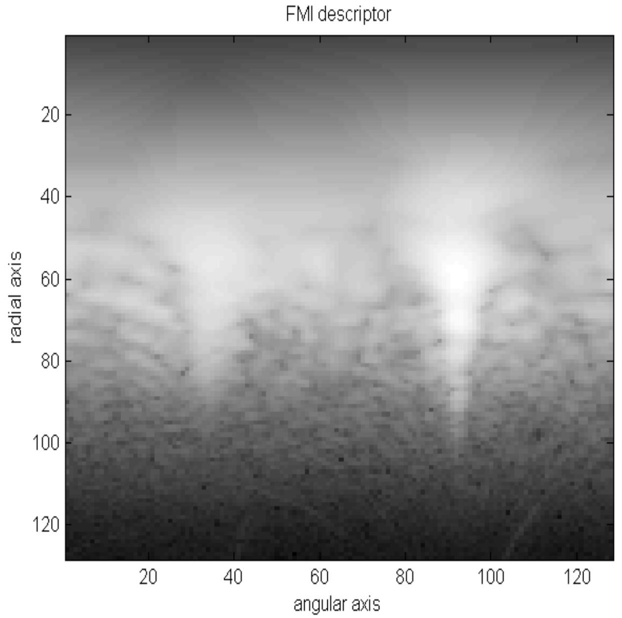
- Construct Depth Error
- Minimize Objective Error Function

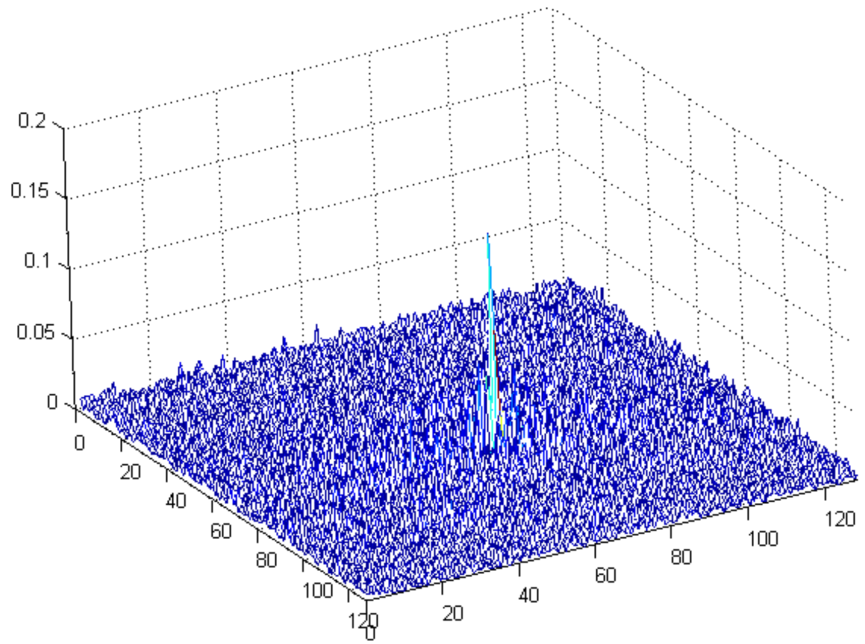
Engel J, Schöps T, Cremers D. LSD-SLAM: Large-scale direct monocular SLAM[C]//European conference on computer vision. Springer, Cham, 2014: 834-849.

Direct Method: Fourier Mellin Transform

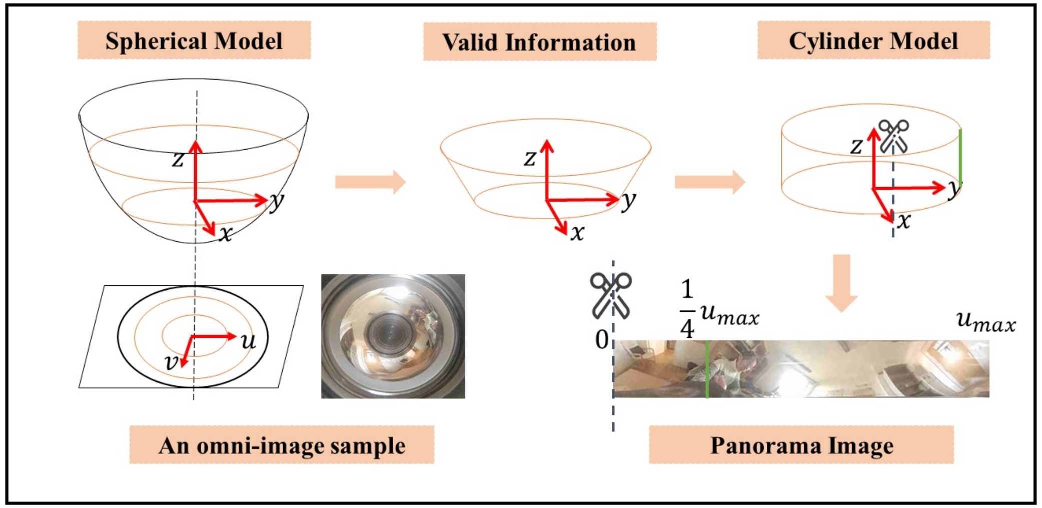
- Spectral based registration: detection of scaling, rotation and translation in 2 subsequent frames
- Processing spectrum magnitude decouples translation from affine transformations
  - Detection of signal shift between 2 signals by phase information
  - Resampling to polar coordinates → Rotation turns into signal shift !
  - Resampling the radial axis from linear to logarithmic presentation
    → Scaling turns into signal shift !
  - Calculate a Phase Only Match Filter (POMF) on the resampled magnitude spectra

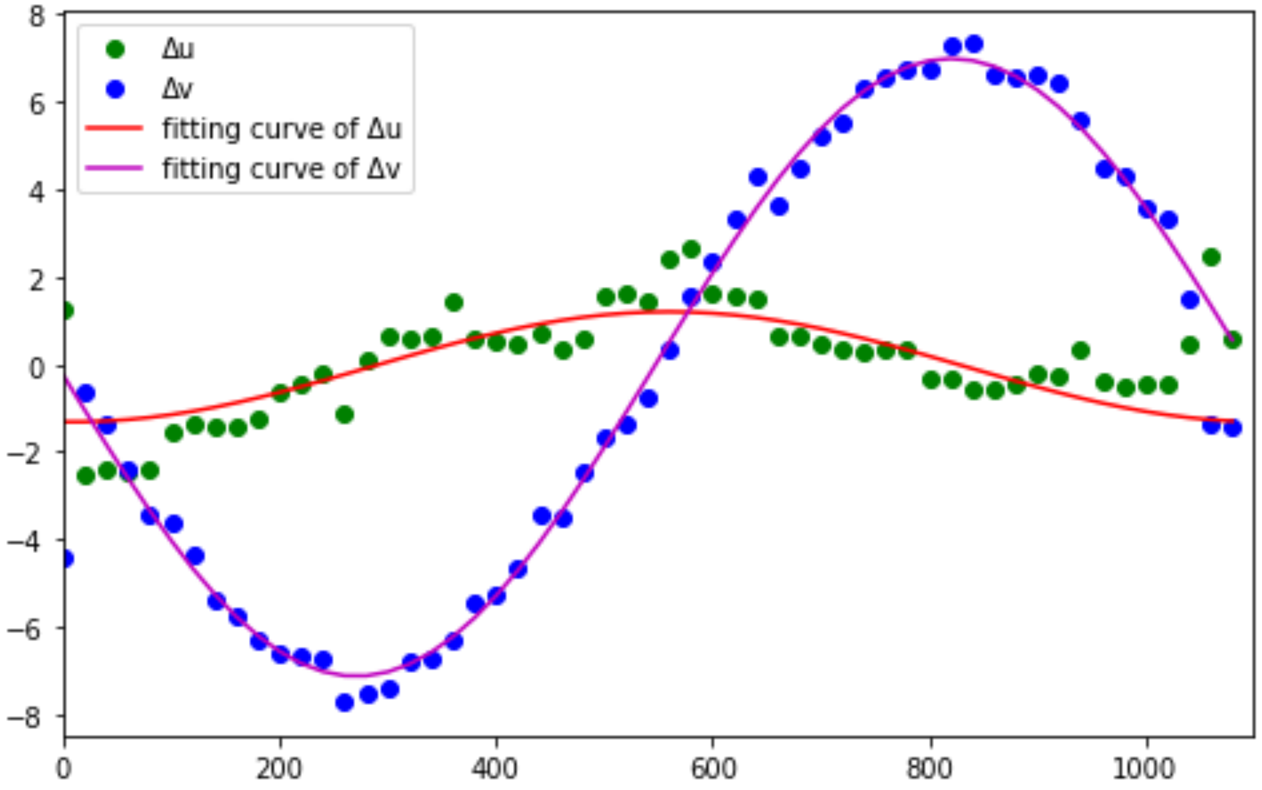






Pose Estimation for Omni-directional Cameras using Sinusoid Fitting





$$y = B + A\sin(\omega x + \phi) \qquad \text{(1a)}$$

$$\Delta v(u_p) = \lambda_p t_z + \gamma \left\| P_{xy}(R) \cdot \sin\left(\gamma u_p + \frac{P_{xy}(R)}{\|P_{xy}(R)\|}\right) \right\| \qquad \text{(1b)}$$

$$\Delta u(u_p) = \gamma P_z(R) + \lambda_p \|P_{xy}(t)\| \cdot \sin(\gamma u_p + \hat{t}_{xy}) \qquad \text{(1c)}$$

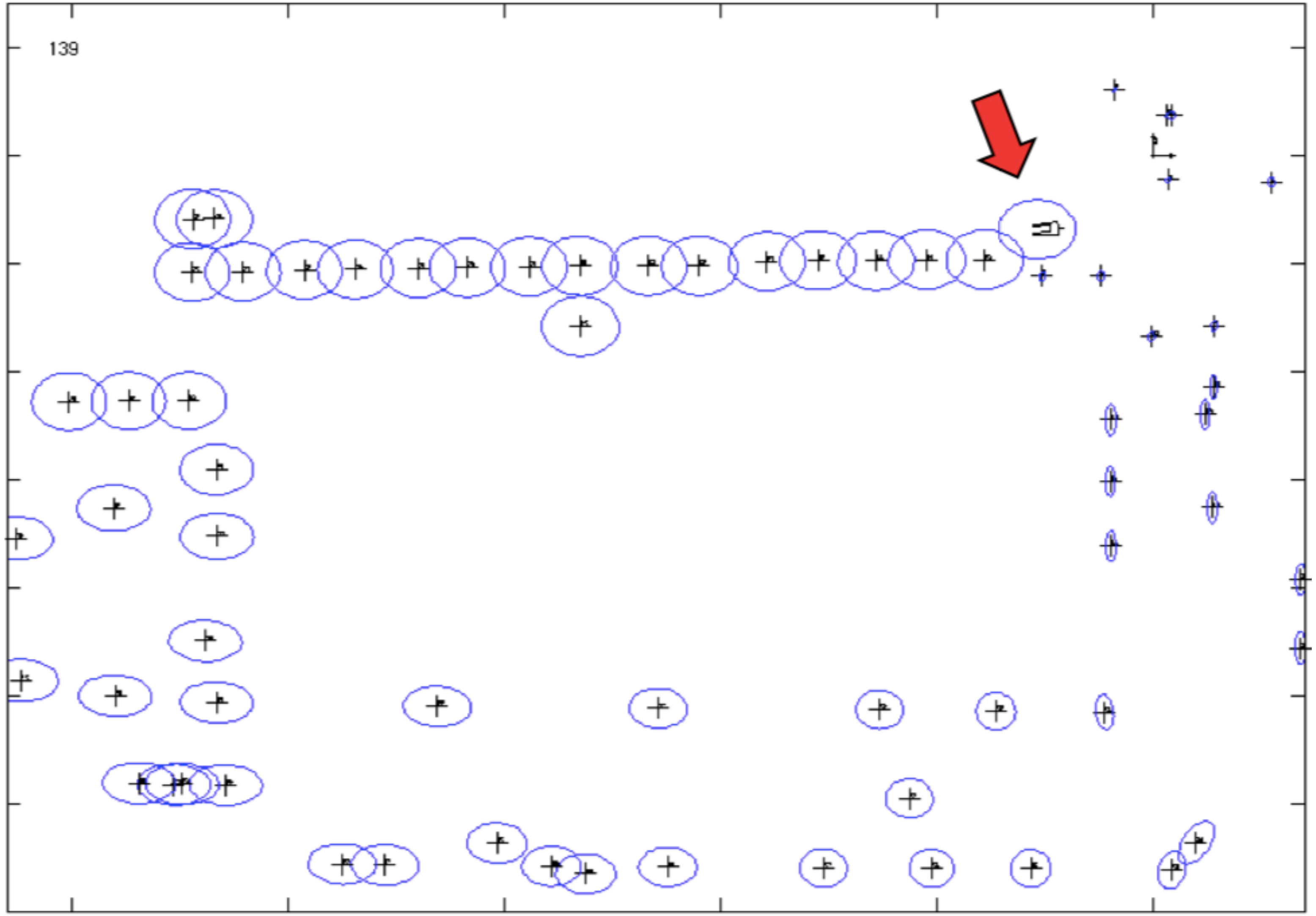
BACK END

Overview of Back-end

- Loop Detection
  - Find candidates of scan pairs/ scan with old map
  - E.g. based on global pose estimated (chain rule) OR image similarity (bag of words)
- Loop Closure
  - E.g. use scan matching to find the transform AND its uncertainty
- Optimization
  - Pose Graph optimization (e.g. minimize error of poses, based on uncertainty)
  - Bundle Adjustment
- Map Rendering
  - E.g. generate grid map based on optimized graph

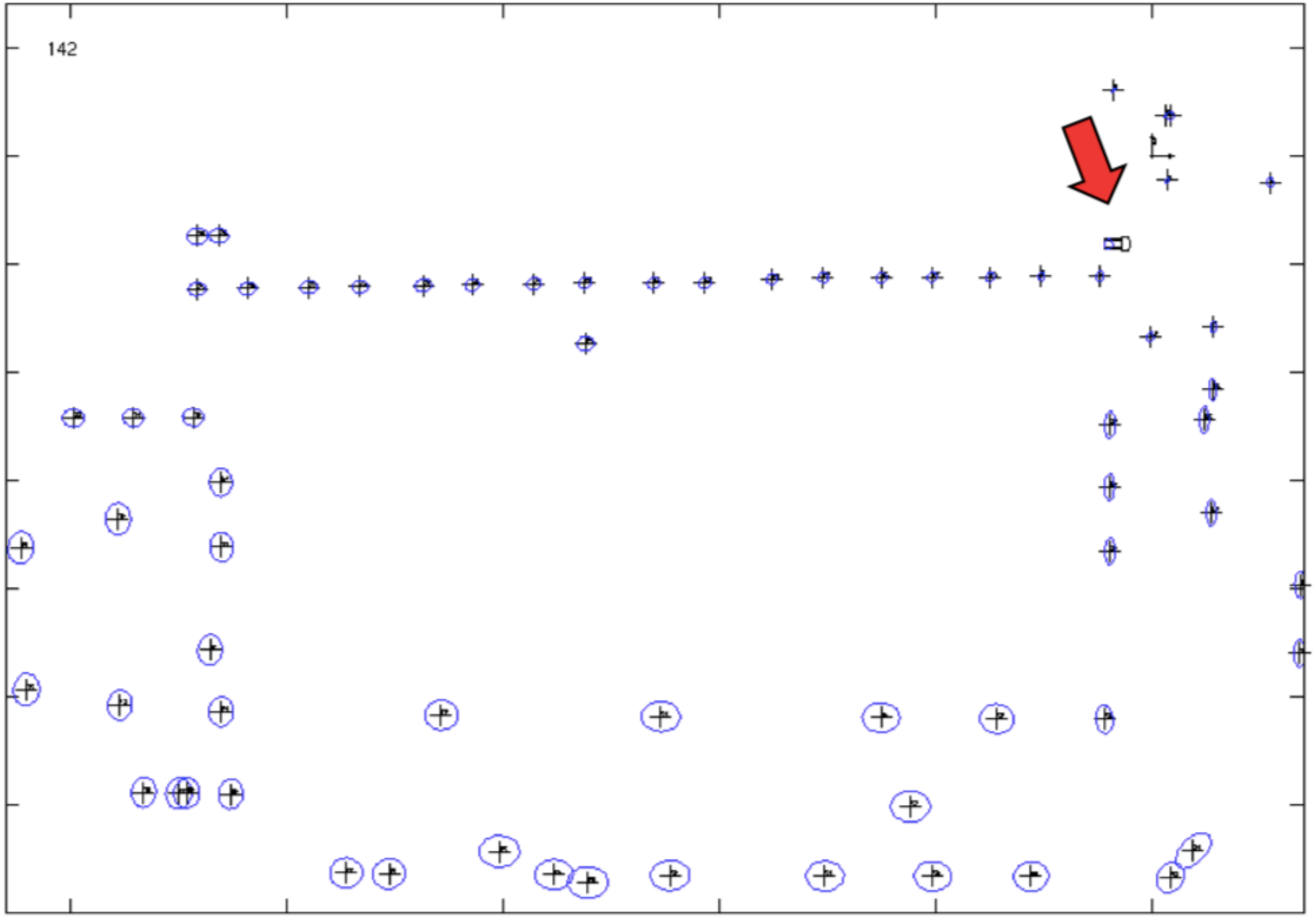
Loop Closure

- Before loop closure



Loop Closure

- After loop closure

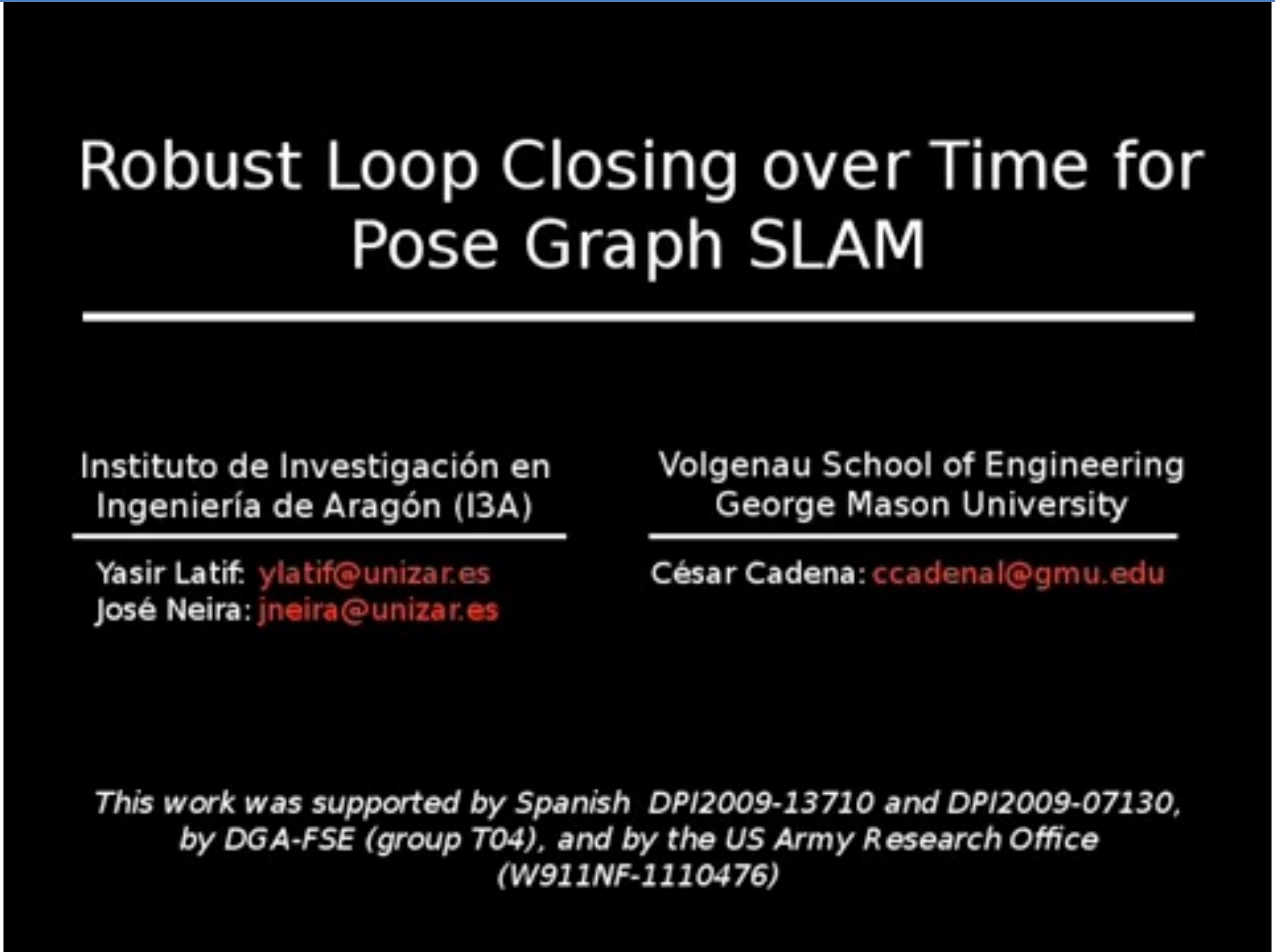


Loop Closure

- Recognizing an already mapped area, typically after a long exploration path (the robot “closes a loop”)
- Structurally identical to data association, but
  - high levels of ambiguity
  - possibly useless validation gates
  - environment symmetries
- Uncertainties collapse after a loop closure (whether the closure was correct or not)

Loop Closure

- By revisiting already mapped areas, uncertainties in robot and landmark estimates can be reduced
- This can be exploited when exploring an environment for the sake of better (e.g. more accurate) maps
- Exploration: the problem of where to acquire new information


Robust Loop Closing over Time for Pose Graph SLAM
Instituto de Investigación en Ingeniería de Aragón (I3A)
Volgenau School of Engineering George Mason University
Yasir Latif: ylatif@unizar.es
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This work was supported by Spanish DPI2009-13710 and DPI2009-07130, by DGA-FSE (group T04), and by the US Army Research Office (W911NF-1110476)

OVERVIEW:
THREE SLAM PARADIGMS

The Three SLAM Paradigms

- Most of the SLAM algorithms are based on the following three different approaches:
  - Extended Kalman Filter SLAM: (called EKF SLAM)
  - Particle Filter SLAM: (called FAST SLAM)
  - Graph-Based SLAM

EKF SLAM: overview

- **Extended state vector** $\boldsymbol{y_t}$: robot pose $\boldsymbol{x_t}$ + position of all the features $\boldsymbol{m_i}$ in the map:

$$y_t = [x_t, m_0, \ldots, m_{n-1}]^T$$

- Example: 2D line-landmarks, size of $\boldsymbol{y_t}$ = $3+2n$: three variables to represent the robot pose + $2n$ variables for the n line-landmarks having vector components

$$(\alpha_i, r_i)$$

$$y_t = [x_t, y_t, \theta_t, \alpha_0, r_0, \ldots, \alpha_{n-1}, r_{n-1}]^T$$

- As the robot moves and takes measurements, the state vector and covariance matrix are updated using the standard equations of the extended Kalman filter
- Drawback: EKF SLAM is computationally very expensive.