CE7453 Numerical Algorithms

Assignment 2 (Mini-Project)

This assignment gives you a chance to practise numerical techniques (such as least-squares approximation, cubic B-spline interpolation, trigonometric interpolation, and numerical integration) on a parametric curve.

Each of you is given an individual planar curve represented parametrically by $\boldsymbol{r}(u) = (x(u), y(u))$ where

$$\begin{cases} x(u) &= 1.5\left(e^{1.5\sin(6.2u-0.027h)}+0.1\right)\cos(12.2u) \\ y(u) &= \left(e^{\sin(6.2u-0.027h)}+0.1\right)\sin(12.2u) \end{cases} \qquad u \in [0,1]$$

and h is your index number in the class attendance list given in Appendix A.

You are required to do the following tasks:

T1. (10 marks) Plot curve $\boldsymbol{r}(u)$ using some software tools such as Matlab, the visualization tool provided in Appendix B, etc., which you prefer to use.

T2. (25 marks) Propose a least-squares method to fit a parametric cubic polynomial curve to curve $\boldsymbol{r}(u)$ (*hints: sample a few points on the curve and fit a cubic polynomial curve to the points*). Plot your cubic polynomial curve and discuss how to choose the sampling points.

T3. (25 marks) Using the cubic B-spline interpolation algorithm you implemented in Assignment 1, propose a method to approximate curve $\boldsymbol{r}(u)$ by a cubic B-spline curve (*hints: sample at least 10 points on the curve and find a cubic B-spline curve interpolating these points*). Plot your cubic B-spline curve and discuss what will affect the approximation result.

T4. (25 marks) Using discrete Fourier Transform, propose a method to approximate curve $\boldsymbol{r}(u)$ by a trigonometric interpolation curve that can be represented by the following basis functions:

$$\{1, \cos(2\pi u), \sin(2\pi u), \cos(4\pi u), \sin(4\pi u), \cos(6\pi u), \sin(6\pi u), \cos(8\pi u)\}.$$

Plot your trigonometric interpolation curve and discuss the result.

T5. (15 marks) Using composite Simpson's rule, compute $\int_0^1 \boldsymbol{x}(\boldsymbol{u})\boldsymbol{du}$ and discuss your result.

Note: The total 100 marks of this assignment will contribute 30 marks in the final grade.

Submission instruction:

- Write your **report** in the format given in Appendix C, and name it by "assignment2-yourName.pdf". Copy your report into sub-folder "assignment2".
- After you complete the two assignments at the end of the semester, zip your folder that contains two sub-folders "assignment1" and "assignment2". Go to the NTULearn course-site, and within the **Assignments** you will find **CE7453-Projects**, where you submit the zipped file.
- **Submission deadline: 18 April 2022 (Monday)**

Academic Integrity

Please pay attention to academic integrity, which we should take very seriously.

This assignment will be completed **individually**. Copying is not allowed and is considered "cheating". Students caught "cheating" will receive an F in the course.

- Students are allowed to read through the assignment together and discuss what is asked by the assignment, examples of input & output of provided programs, and possible high level strategies for solving the problems.
- Students are not allowed to work together in writing the answers and detailed methods/implementations.

Appendix A. Class Attendance List

1.;AZAM ABU BAKR
2.;BALSEBRE PASQUALE
3.;BIAN QINGTIAN
4.;BURTON-BARR JONATHAN WESTON
5.;CAO ZHIWEI
6.;CHEN HAILIN
7.;CHEN QI JING
8.;CHEN YUANYUAN
9.;CHEN ZHAOXI
10.;GOH YU CHEN
11.;HE XIAOSHENG
12.;HOU WENLONG
13.;HOU XINYU
14.;JU CE
15.;LAN TIANMING
16.;LI CHUNBO
17.;LI QUANZHOU
18.;LI SIYAO
19.;LI XINGRAN
20.;LI YANG
21.;LI YEWEN
22.;LIU FENGMING
23.;LUO JIAYUN
24.;MA YUBO
25.;PENG HONGYI
26.;QIU HAONAN
27.;RAO HAOCONG
28.;REN JIAWEI
29.;REN XINGYU
30.;SHAO YIDI

31.;SU ZHIXIANG
32.;SUEN HUNG YING SIDNEY
33.;SUN SHUO
34.;SUN WEN
35.;TAO SHUAILIN
36.;WANG HONGBO
37.;WANG JIANKUN
38.;WANG JIANYI
39.;WANG JING
40.;WANG YUHAN
41.;WILTON KIM
42.;WU SIZE
43.;WU TIANHAO
44.;WU YUFEI
45.;WU ZIQING
46.;XU SHIFENG
47.;YANG FAN
48.;YEO WEI JIE
49.;YU WEIPING
50.;ZHANG JIAHUI
51.;ZHANG LUO
52.;ZHANG MINGYUAN
53.;ZHANG YUANHAN
54.;ZHANG ZHAOQI
55.;ZHOU CHONG
56.;ZHOU JIANAN
57.;ZHU ZULUN

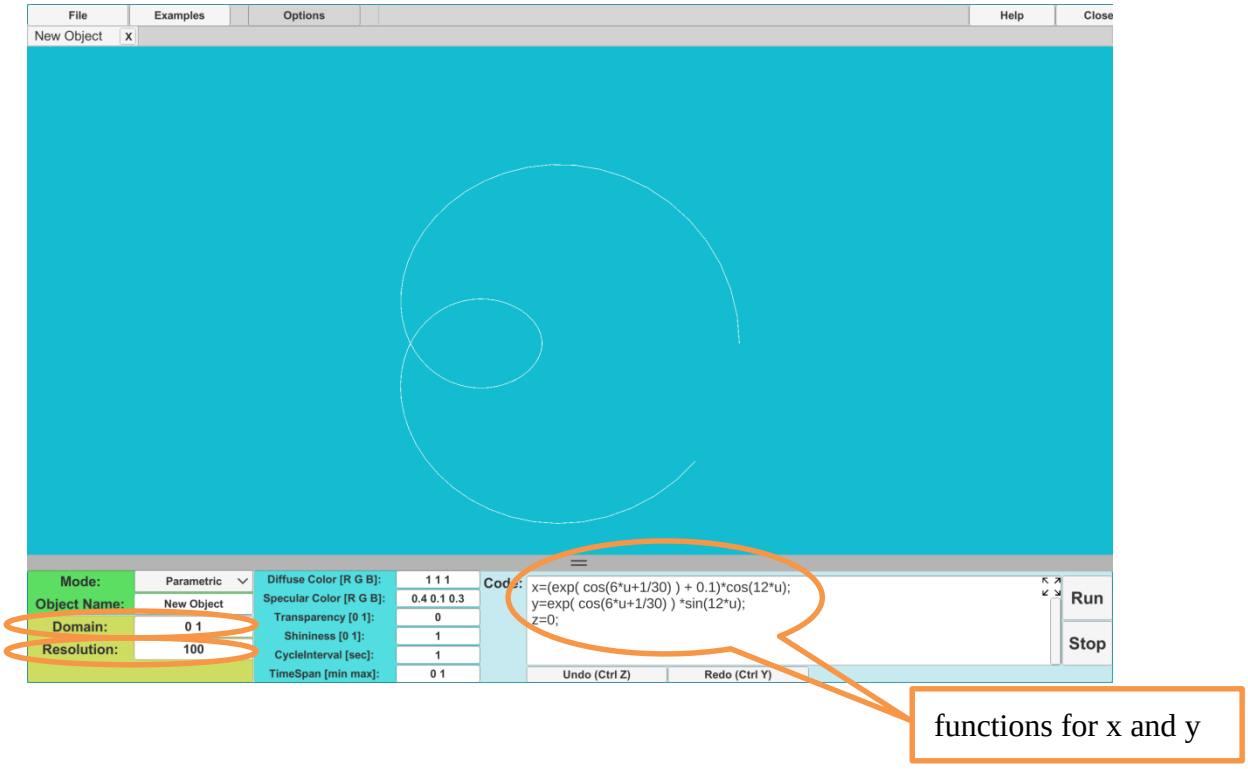
Appendix B. Visualization tool (developed by Prof Alexei Sourin)

This software tool has two versions: one for PC and the other for Mac. It allows the user to enter function definitions, domains of the input variables, and sampling resolution.

Example: to display the parametric curve

$$\begin{cases} x(u) = \left(e^{\cos\left(6u+\frac{1}{30}\right)}+0.1\right)\cos(12u) \\ y(u) = e^{\cos\left(6u+\frac{1}{30}\right)}\sin(12u) \end{cases} \qquad u \in [0,1]$$

we enter the functions x(u) and y(u), the domain [0,1] and the resolution 100 into the program as follows:



Syntax: Mathematical operations and functions that can be used in formulas are:

(,), +, - , *, /, ^, sin, cos, tan, exp, sqrt, fabs, asin, acos, atan, atan2, cosh, fmod, log, log10, sinh, tanh, ceil, floor, min, max

For example, x^3 stands for x*x*x.

Appendix C. Report format

Assignment 2 Report

Your name: XXX

Matric No.: YYY

T1.

The plot of the curve:

T2.

Description or setup of your least-squares method:

The expression (i.e., representation) of your final cubic polynomial curve:

The plot of your cubic polynomial curve:

Your discussion:

T3.

Description or setup of your cubic B-spline interpolation method:

The expression (i.e., the representation) of your final cubic B-spline curve:

The plot of your cubic B-spline curve:

Your discussion:

T4.

Description or setup of your method:

The expression (i.e., representation) of your final trigonometric interpolation curve:

The plot of your trigonometric interpolation curve:

Your discussion:

T5.

The result:

Your discussion: