MATH 0450: HOMEWORK 5

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Problem 1. Show that the function $f : \mathbb{N} \times \mathbb{N} \to \mathbb{N}$, defined by $f(a, b) = (a + b)(a + b + 1)/2 + b$ is bijective.

Proof. Write your solution here.

□

Problem 2. (Ex. 1.5.1) Finish the proof for Theorem 1.5.7: If $A \subseteq B$ and B is countable, then A is either countable or finite.

Proof. Write your solution here.

□

Problem 3. (Ex. 1.5.2) Use the following outline (as specified in the textbook) to supply proofs for the statements in Theorem 1.5.8.

Proof. Write your solution here.

□

Problem 4. (Ex. 1.5.4)

(a) Show $(a, b) \sim \mathbb{R}$ for any interval (a, b).
(b) Show that an unbounded interval like $(a, \infty) = \{x \ : \ x > a\}$ has the same cardinality as $\mathbb{R}$ as well.
(c) Using open intervals makes it more convenient to produce the required 1–1, onto functions, but it is not really necessary. Show that $[0, 1) \sim (0, 1)$ by exhibiting a 1–1 onto function between the two sets.

Proof. Write your solution here.

□

Problem 5. (Ex. 1.5.6 (b)) Give an example of an uncountable collection of disjoint open intervals, or argue that no such collection exists.

Proof. Write your solution here.

□

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