Introduction to linear algebra 书4.1：想：两个在四维空间垂直的向量，应该如何可视化？

2. 4.2：projection，如果要visualize 4d object，直接使用投影矩阵行不行？还是使用perspective projection or stereographic projection

垂直投影

透视投影

立体投影

To visualize a 4D object in 3D (or 2D), projection is indeed one of the main techniques used, but the choice of the type of projection depends on the effect you want and the information you want to preserve.

**Types of Projections:**

1. **Projection Matrix (Orthographic Projection)**:
   * This is a **direct projection** where we map a 4D object onto a 3D space by ignoring one or more of its coordinates. The simplest projection matrix eliminates a dimension, for example, discarding the fourth coordinate www to map a 4D object to 3D space. While computationally straightforward, orthographic projections can lose depth perception or distort some structural relationships because it doesn't capture perspective or curvature.

**Pros**:

* + Simple to compute.
  + Preserves distances (no perspective distortion).

**Cons**:

* + Doesn't convey depth or a true sense of "dimensionality."
  + Can result in misleading shapes because of the lack of perspective.

1. **Perspective Projection**:
   * This mimics how the human eye perceives objects, projecting the 4D object onto a 3D space in a way that nearby objects appear larger, while distant objects appear smaller. A 4D perspective projection maps points based on their distance from a chosen "viewpoint," providing a more intuitive sense of depth.

**Pros**:

* + Provides a more realistic visualization.
  + Captures depth and dimensionality better.

**Cons**:

* + Requires more complex computation.
  + Possible distortion, particularly near the edges of the projection.

1. **Stereographic Projection**:
   * This is a type of **conformal projection** where angles are preserved, and it projects a 4D object onto 3D space from a specific point (often a point "at infinity" in 4D). It's commonly used in visualizing higher-dimensional spheres (like a 3-sphere in 4D space). Stereographic projection helps preserve the local structure of objects while embedding them in lower dimensions.

**Pros**:

* + Preserves the geometric relationships and angles.
  + Can be used to visualize certain features of curved objects, like hyperspheres.

**Cons**:

* + More complex to implement.
  + Can distort the overall shape, especially at the edges.