An Iterative Weighting Method to apply the ISR correction in e^+e^- Hadronic Cross Section Measurements at BESIII

Wenyu Sun (孙文玉)[a,b], Tong Liu (刘桐)[b], Liangliang Wang (王亮亮)[b] *, Bin Zhong (钟彬)[c] † and Weimin Song(宋维民)[a] ‡

(a) College of Physics, Jilin University,Changchun 130012, China

(b) Institute of High Energy Physics, Chinese Academy of Sciences, Beijing 100049, China

(c) Department of Physics, Nanjing Normal University, Nanjing, Jiangsu 210023, China

Initial State Radiation (ISR) plays an important role in e^+e^- collision experiments like BESIII. To correct the ISR effects in measurements of hadronic cross sections of e^+e^- annihilation, an iterative method which weights the simulated ISR events is proposed to evaluate the event selection efficiency and the ISR correction factor for the observed cross section. In this method, the production of the simulated ISR events is performed only once and the obtained cross section line shape is used iteratively to weight the same simulated ISR events to evaluate efficiency and corrections until results converge. Comparing to the method of producing ISR events iteratively, this weighting method can provide consistent results and can dramatically save computing time and disk space to speed up the e^+e^- hadronic cross section measurements.

Keywords: BESIII, initial state radiation, iteration, Monto-Carlo weighting

I. INTRODUCTION

In e^+e^- collision experiment, measurements of the hadronic cross sections of e^+e^- annihilation are important to reveal the reaction mechanisms and to search for new resonances. The observed hadronic events are not only contributions from the Born level but also from initial state radiation (ISR), vacuum polarization (VP) and so on. Initial state radiation (ISR) is an universal process that one or more photons are emitted by an electron or positron before they annihilate into possible final states in e^+e^- collision. The emitted photon or photons take part of energy from the electron or positron and reduce the center-of-mass (c.m.) energy of annihilation. So the observed hadronic cross section of e^+e^- collision involves the Born cross section line shape from the production threshold up to the nominal collision c.m. energy via ISR which needs to be considered in e^+e^- hadronic cross section measurement.

Awareness of the ISR correction has a long history. The calculation of ISR contribution is an application of the Feynman rules for Quantum Electrodynamics (QED). There is a large number of published studies that describe the ISR corrections in theory in different ways in the past decades [1–6]. These studies led to the development of dedicated high precision Monte Carlo (MC) generators such as BabaYaga, MCGPJ, PHOKHARA, et al. which are used in the e^+e^- collision experiments in recent years.

BESIII is one such e^+e^- collision experiment. The BESIII detector is a magnetic spectrometer located at the Beijing Electron Positron Collider (BEPCII)[7]. The advanced design of BESIII allows it to take advantage of the high luminosity delivered by BEPCII and to collect large data samples at the τ-charm energy region which is between the perturbative and non-perturbative regimes of Quantum Chromodynamics (QCD) from 2 GeV to 4.7 GeV. At BESIII, KKMC + BesEvtGen is the most commonly used generator framework and MC events generation for a process e^+ e^- annihilation into final states X with intermediate states (like vector charmonia or vector-charmonum-like states) can be described by the Figure 1. KKMC is used to generate the intermediate states considering ISR and the beam energy spread. BesEvtGen is developed from the generator EvtGen and is used to generate final states of the intermediate states decays with final state radiation (FSR)[8]. The responds of BESIII detector to the generated events are simulated by a Geant4 based algorithm which consists of the detector description, digitization, backgrounds mixing and so on [9]. The simulated ISR events are reconstructed and selected as data to estimate the efficiency, corrections and so on which are needed in the measurement.

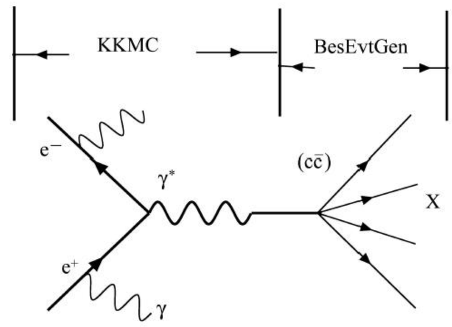

FIG. 1: Illustration of BESIII generator framework.

In this paper an iterative method of weighting the sim-

*Email:llwang@ihep.ac.cn

†Email:zhongb@njnu.edu.cn

‡Email:weiminsong@jlu.edu.cn

ulated ISR events to apply the ISR correction is proposed. The procedure is explained in detail and three different line shapes are used as examples to validate the method.

II. ISR CORRECTION AT BESIII

The observed cross section $\sigma^{\rm obs}$ of a process $e^+e^- \to$ hadrons at BESIII can be obtained experimentally by

$$\sigma^{\rm obs} = \frac{N^{\rm obs}}{\epsilon \cdot \mathcal{L}}, \tag{1}$$

where $N^{\rm obs}$ is the number of observed signals, ϵ is the efficiency, $\mathcal{L}$ is the integrated luminosity of the data sample.

As mentioned previously, the observed cross section includes effects from ISR and VP. We focus on the ISR correction in this article. The observed cross section can be corrected to the cross section without ISR contribution which is called "dressed cross section"

$$\sigma^{\rm dressed} = \frac{\sigma^{\rm obs}}{(1+\delta)} = \frac{N^{\rm obs}}{\epsilon \cdot \mathcal{L} \cdot (1+\delta)}, \tag{2}$$

where $1+\delta$ is the ISR correction factor.

The efficiency ϵ and the ISR correction factor $1+\delta$ can be evaluated by simulated ISR events which is usually generated by KKMC at BESIII. While to generate these events, a line shape of the dressed cross section as a function of energy is an essential input. For some $e^+e^- \to$hadrons cross section measurements in practice, there is no or very little knowledge known about the cross section line shape before the measurement to be performed. So some assumed cross section line shape $\sigma_0^{\rm dressed}(E)$ (like a Breit-Wigner function, a flat line and so on) is used in KKMC to generate ISR Monto-Carlo (MC) events as a starting point to roughly evaluate the efficiency and the ISR correction factor at each energy point, where E is the c.m. energy of e^+e^- annihilation. Then the dressed cross section can be obtained roughly at each energy point using Equation 2. The line shape of the rough dressed cross section can be used to re-valuate the efficiency, the ISR correction factor and so on. The procedure needs to be repeated several times until all the results converge. In other words, the final dressed cross sections are measured in an iterative way.

A. Iterative MC-Generating Method

To evaluate the efficiency ϵ and the ISR correction factor $1+\delta$ with a different line shape, a nature way is to re-generate the ISR MC events with KKMC taking the new line shape as input. The iterative procedure by re-generating ISR MC events is presented in Figure 2.

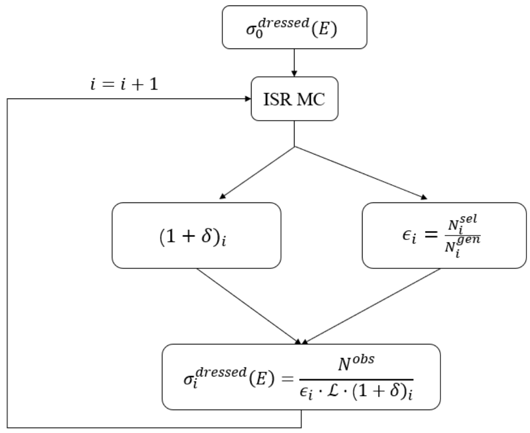

FIG. 2: Flow chart of the iterative MC-generating method

As mentioned already, an assumed cross section line shape $\sigma_0^{\rm dressed}(E)$ is used to generate a signal ISR MC sample by KKMC to start the iteration. In the i-th iteration ($i = 1, 2, 3, ...$), the ISR correction factor $(1+\delta)_i$ is obtained directly from KKMC, after the reconstruction and event selection of the MC sample, the efficiency is obtained by $\epsilon_i = N_i^{\rm sel}/N_i^{\rm gen}$ where $N_i^{\rm sel}$ is the number of events after selection and $N_i^{\rm gen}$ is the generated number of events. Then the dressed cross section after i-th iteration is $\sigma_i^{\rm dressed} = \frac{N^{\rm obs}}{\epsilon_i \cdot \mathcal{L} \cdot (1+\delta)_i}$. These operation is performed for all the energy points to obtain the line shape of the dressed cross section after i-th iteration $\sigma_i^{\rm dressed}(E)$ which can be used as input for KKMC to generate signal ISR MC samples in the next (i.e. $(i+1)$-th) iteration. The iterations can stop if the results converge (for example, if the relative difference in $\sigma^{\rm dressed}(E)$ between one iteration and the last is smaller than a certain quantity).

This method is already widely used in many cross section measurement at BESIII such as $e^+e^- \to \pi^+\pi^- h_c$, $e^+e^- \to \pi^+\pi^- J/\psi$, $e^+e^- \to \pi^+\pi^-\psi(3686)$ and so on [10–16].

B. Iterative MC-weighting Method

The iterative MC-Generating method works properly, but it takes much computing time and disk space to perform multi-rounds of ISR MC events generation, Geant4 simulation of the BESIII detector, reconstruction and event selection and so on, especially when a large amount of energy points are involved. And it slows down further the measurements if the related systematic uncertainty study is considered. So a more efficient iterative method is proposed in which the ISR MC events are generated, reconstructed and selected only once at each energy point. The start point is similar: from these ISR M-C events generated by KKMC using the assumed cross section line shape $\sigma_0^{\rm dressed}(E)$, the rough ISR correction factor $(1+\delta)_1$ and efficiency $\epsilon_1 = N^{\rm sel}/N^{\rm gen}$ are obtained, where $N^{\rm sel}$ is the number of events after selection and $N^{\rm gen}$ is the generated number of events. Then the rough cross section is obtained by $\sigma_1^{\rm dressed} = \frac{N^{\rm obs}}{\epsilon_1\cdot\mathcal{L}\cdot(1+\delta)_1}$. While in the next iterations, the same ISR MC events are weighted according to the cross section line shape from last iteration to calculate the ISR correction factor, the event selection efficiency and subsequently the cross section at each energy point. The flow chart of this new iteration MC-weighting method is demonstrated in Figure 3.

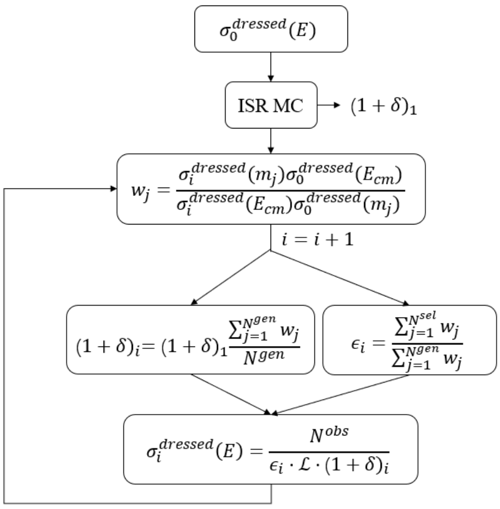

FIG. 3: Flow chart of the iterative MC-weighting method

For a specific ISR MC event j generated at a center-of-mass collision energy $E_{\rm cm}$, the corresponding differential cross section is proportional to $F(m_j, E_{\rm cm})\sigma_0^{\rm dressed}(m_j)$, where $F(m_j, E_{\rm cm})$ is the radiation function and m_j is the invariant mass of the hadrons in this event. Similarly, if this event is generated according to the cross section line shape $\sigma_i^{\rm dressed}(E)$ instead, the corresponding differential cross section should be proportional to $F(m_j, E_{\rm cm})\sigma_i^{\rm dressed}(m_j)$. So the ratio of the two differential cross sections is

$$r_j = \frac{F(m_j, E_{\rm cm})\sigma_i^{\rm dressed}(m_j)}{F(m_j, E_{\rm cm})\sigma_0^{\rm dressed}(m_j)} = \frac{\sigma_i^{\rm dressed}(m_j)}{\sigma_0^{\rm dressed}(m_j)}. \quad (3)$$

Corresponding to the same MC luminosity $\mathcal{L}_{\rm MC} = N^{\rm gen}/\sigma_0^{\rm obs}(E_{\rm cm})$ but with a different cross section line shape $\sigma_i^{\rm dressed}(E)$, the same event j should be generated r_j times instead of 1 time, where $\sigma_0^{\rm obs}(E_{\rm cm})$ is the observed cross section at $E_{\rm cm}$ for the dressed cross section line shape $\sigma_0^{\rm dressed}(E)$. Then ratio of the total observed cross sections with the two cross section line shapes can be calculated using these MC events in terms of r_j:

$$\frac{\sigma_i^{\rm obs}(E_{\rm cm})}{\sigma_0^{\rm obs}(E_{\rm cm})} = \frac{\mathcal{L}_{\rm MC}\cdot\sigma_i^{\rm obs}(E_{\rm cm})}{\mathcal{L}_{\rm MC}\cdot\sigma_0^{\rm obs}(E_{\rm cm})} = \frac{\sum_{j=1}^{N^{gen}} r_j}{\sum_{j=1}^{N^{gen}} 1} = \frac{\sum_{j=1}^{N^{gen}} r_j}{N^{gen}}. \quad (4)$$

Then starting from the ISR correction factor definition and using the Equation 4, the ISR correction factor $(1+\delta)_{i+1}$ for the cross section line shape $\sigma_i^{\rm dressed}(E)$ can be related to the original ISR correction factor $(1+\delta)_1$ via the same ISR MC events like this:

$$\begin{aligned}(1+\delta)_{i+1} &= \frac{\sigma_i^{\rm obs}(E_{\rm cm})}{\sigma_i^{\rm dressed}(E_{\rm cm})} \\ &= \frac{\sigma_0^{\rm obs}(E_{\rm cm})\sum_{j=1}^{N^{\rm gen}} r_j/N^{\rm gen}}{\sigma_i^{\rm dressed}(E_{\rm cm})} \\ &= \frac{\sigma_0^{\rm obs}(E_{\rm cm})\sum_{j=1}^{N^{\rm gen}} \frac{\sigma_i^{\rm dressed}(m_j)}{\sigma_0^{\rm dressed}(m_j)}}{\sigma_i^{\rm dressed}(E_{\rm cm})N^{\rm gen}} \\ &= \frac{\sigma_0^{\rm obs}(E_{\rm cm})}{\sigma_0^{\rm dressed}(E_{\rm cm})}\frac{\sum_{j=1}^{N^{\rm gen}} \frac{\sigma_i^{\rm dressed}(m_j)\sigma_0^{\rm dressed}(E_{\rm cm})}{\sigma_i^{\rm dressed}(E_{\rm cm})\sigma_0^{\rm dressed}(m_j)}}{N^{\rm gen}} \\ &= (1+\delta)_1\frac{\sum_{j=1}^{N^{\rm gen}} \frac{\sigma_i^{\rm dressed}(m_j)\sigma_0^{\rm dressed}(E_{\rm cm})}{\sigma_i^{\rm dressed}(E_{\rm cm})\sigma_0^{\rm dressed}(m_j)}}{N^{\rm gen}}.\end{aligned} \quad (5)$$

So if a weight

$$w_j \equiv \frac{\sigma_i^{\rm dressed}(m_j)\sigma_0^{\rm dressed}(E_{\rm cm})}{\sigma_i^{\rm dressed}(E_{\rm cm})\sigma_0^{\rm dressed}(m_j)} \quad (6)$$

is calculated for MC event j, the ratio of the sum of the weights over $N^{\rm gen}$ can be used to convert $(1+\delta)_1$ to $(1+\delta)_{i+1}$. If the original cross section line shape used in the ISR MC generation is flat, i.e. $\sigma_0^{\rm dressed}(E)$ is a constant, the weight can be simplified as $w_j = \frac{\sigma_i^{\rm dressed}(m_j)}{\sigma_i^{\rm dressed}(E_{cm})}$.

The event selection efficiency ϵ_{i+1} for the dressed cross section line shape $\sigma_i^{\rm dressed}(E)$ can also be estimated with the same ISR MC events in terms of weights by

$$\epsilon_{i+1} = \frac{\sum_{j=0}^{N^{\rm sel}} w_j}{\sum_{j=1}^{N_{\rm gen}} w_j}. \quad (7)$$

Except the production of ISR MC events at the beginning, in next iterations this MC-weighting method only involves a loop of the ISR MC events to calculate weights, and then the weights are summed up to calculate the ISR correction factor, efficiency and subsequently the dressed cross sections. So this method only needs very limited computing time and disk space during iterations, which can significantly speed up the whole procedure.

III. VALIDATION OF THE MC-WEIGHTING METHOD

To validate the MC-weighting method, one can compare the obtained ISR correction factor and efficiency with those using the MC-generating method.

Three line shapes are used as examples to test the method: 1) one-Breit-Wigner, 2) two-Breit-Wigner and 3) two-Breit-Wigner plus a phase space function. The three line shapes are shown in Figure 4.

ISR MC samples with a flat line shape are produced (including simulation, reconstruction, event selection), and then following the MC-weighting method the three different line shapes are used to calculate the corresponding weights, ISR correction factors and event selection efficiencies. The three line shapes are also used to produce directly the corresponding ISR MC samples (i.e. with so called "MC-generating method") to get ISR correction factors and event selection efficiencies. These results as functions of energy with the two methods are consistent with each other as shown in Figure 5 for the line shape of one-Breit-Wigner, Figure 6 for the line shape of two-Breit-Wigner and Figure 7 for the line shape of two-Breit-Wigner plus a phase space.

In practical applications, there is some uncertainty due to statistics and other effects. In the tests performed here, one million ISR MC events are generated at each energy point with each line shape. The ratios of ISR correction factors, event selection efficiencies and the products of the two with the two methods are shown in the Figure 8 for the line shape of one-Breit-Wigner, Figure 9 for the line shape of two-Breit-Wigner and Figure 10 for the line shape of two-Breit-Wigner plus a phase space. These ratios are around 1 but with some fluctuations almost within 2%. So in the cases shown here, a conservative relative quantity like 2% can be assigned to account for the possible uncertainty from the MC-weighting method.

IV. CONCLUSION

An iterative MC-weighting method is proposed to evaluate ISR correction factors and event selection efficiencies in e^+e^- hadronic cross section measurements at e^+e^- collision experiments like BESIII. As the ISR MC samples are produced only once in this procedure, the iterative MC-weighting method can save the computing time and disk space significantly comparing to the iterative MC-generating method. Three different line shapes are taken as examples to validate the MC-weighting method and the results are consistent with these using the MC-generation method.

Acknowledgments

This work is supported by the National Natural Science Foundation of China under Grant No. U1732105, No. U1632106, and by the Research Foundation for Advanced Talents of Nanjing Normal University under Grant No. 2014102XGQ0085. We also appreciate the valuable discussions with Dr. Kai ZHU, Dr. Yuping GUO and Prof. Changzheng YUAN.

[1] G. Bonneau and F. Martin, Nucl. Phys. B, 27: 381-397 (1971)
[2] D. R. Yennie, Phys. Rev. Lett., 34: 239 (1975)
[3] V. N. Baier *et al.*, Phys. Rep., 78: 293 (1981)
[4] F. A. Berends and R. Kleiss, Nucl. Phys. B, 178: 141 (1981)
[5] E. A. Kuraev and V. S. Fadin, Sov. J. Nucl. Phys., 41: 466-472 (1985)
[6] O. Nicrosini and L. Trentadue, Phys. Lett. B, 196: 551 (1987)
[7] M. Ablikim *et al.*, Nucl. Instrum. Methods Phys. Res. A, 614: 345-399 (2010)
[8] R. G. Ping , Chin. Phys. C, 32: 599 (2008)
[9] Z. Deng, H. Liu, G. Cao, M. He, Y. Yuan, Z. You and Y. Liang, PoS ACAT (2007), 043
[10] M. Ablikim *et al.* (BES Collaboration), Phys. Lett. B, 660: 315-319 (2008)
[11] M. Ablikim *et al.* (BESIII Collaboration), Phys. Rev. D, 96: 032004 (2017)
[12] M. Ablikim *et al.* (BESIII Collaboration), Phys. Rev. Lett., 118: 092001 (2017)
[13] M. Ablikim *et al.* (BESIII Collaboration), Phys. Rev. Lett., 118: 092002 (2017).
[14] M. Ablikim *et al.* (BESIII Collaboration), Phys. Rev. Lett., 122: 102002 (2019)
[15] M. Ablikim *et al.* (BESIII Collaboration), Phys. Rev. Lett., 122: 232002 (2019)
[16] M. Ablikim *et al.* (BESIII Collaboration), Phys. Rev. D, 99: 091103 (2019)

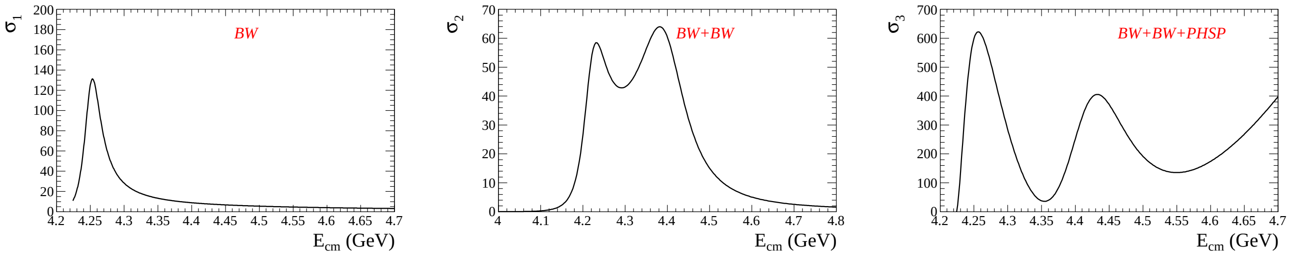

FIG. 4: Three different line shapes to validate the MC-weighting method. From left to right, they are one-Breit-Wigner function, two-Breit-Wigner functions and two-Breit-Wigner functions plus a phase space function.

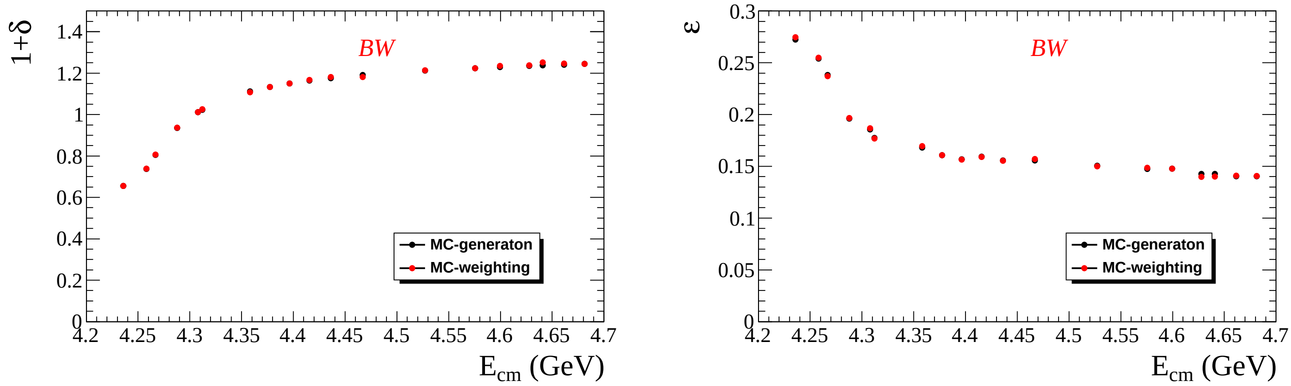

FIG. 5: The ISR correction factor (left) and event selection efficiency (right) as functions of energy for the line shape of one-Breit-Wigner with the MC-weighting method and the MC-generating method.

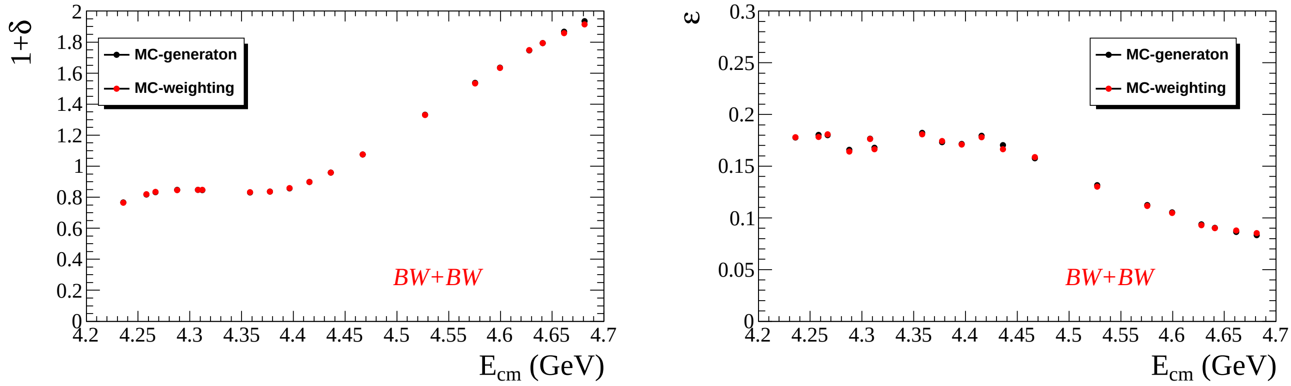

FIG. 6: The ISR correction factor (left) and event selection efficiency (right) as functions of energy for the line shape of two-Breit-Wigner with the MC-weighting method and the MC-generating method.

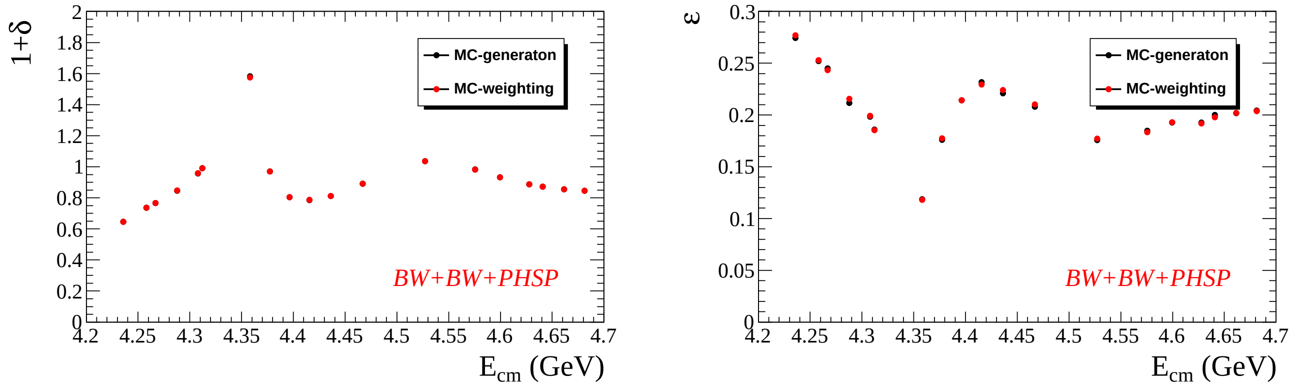

FIG. 7: The ISR correction factor (left) and event selection efficiency (right) as functions of energy for the line shape of two-Breit-Wigner plus a phase space with the MC-weighting method and the MC-generating method.

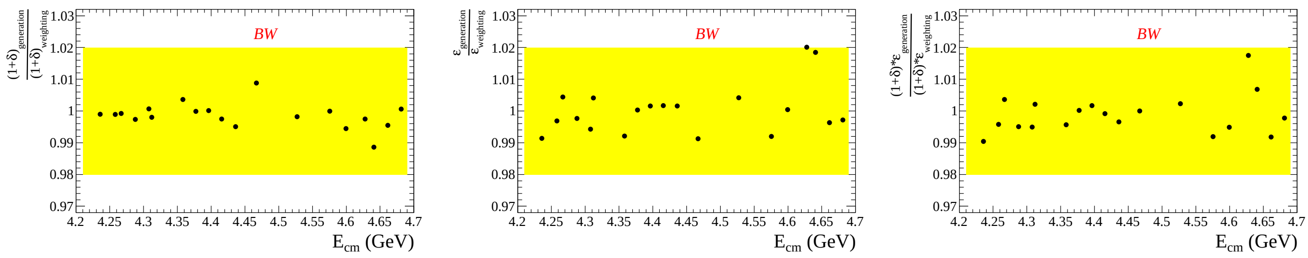

FIG. 8: The ratio of ISR correction factors (left), event selection efficiencies (middle) and the products of the two (right) with the two methods for the line shape of one-Breit-Wigner. The fluturations around 1 are within $\pm 2\%$ indicated by the yellow bands.

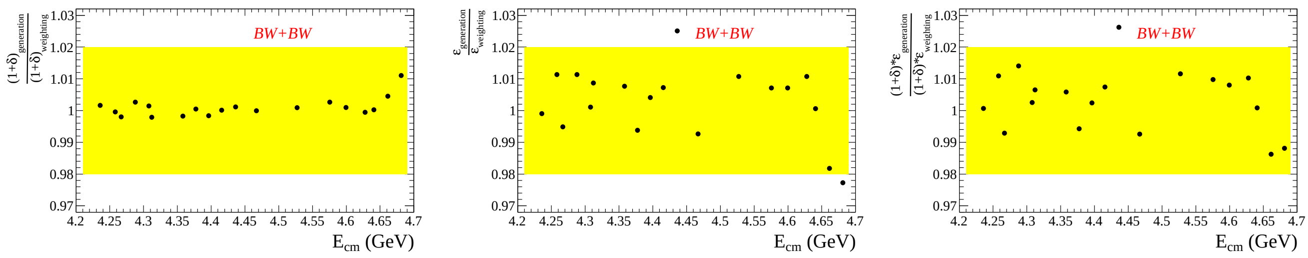

FIG. 9: The ratio of ISR correction factors (left), event selection efficiencies (middle) and the products of the two (right) with the two methods for the line shape of two-Breit-Wigner. The fluturations around 1 are within $\pm 2\%$ indicated by the yellow bands.

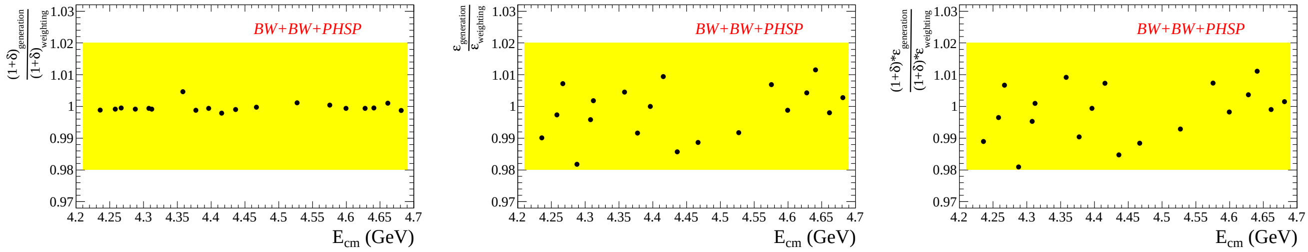

FIG. 10: The ratio of ISR correction factors (left), event selection efficiencies (middle) and the products of the two (right) with the two methods for the line shape of two-Breit-Wigner plus a phase space. The fluturations around 1 are within $\pm 2\%$ indicated by the yellow bands.