

A two phases three dimensional volcanic plume model with SPH Method

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Abstract

- **First Law Based Model** Employ a three dimensional, two phases, transient model based on basic physics laws.
- **More Suitable Numerical Tools** SPH is more suitable for geophysics flow: Automatically capture boundary of free boundary flow, treating of multiple phase flow is trivial for SPH, adding of new physics requires much less coding effort.
- **Utilize the power of parallel computing** timeliness, more comprehensive model and finer resolution are desired for prediction capacity of simulation models, power of parallel computing will help to achieve this goal.
- **Goal** This model is targetted at but not limited to providing source terms for VATDMs (Volcanic Ash Transport and Dispersal Models).

Physics Model [1]

The following assumptions are made:

- Neglect molecular viscosity.
- Assume erupted material are well mixed and behave like a single phase fluid all the time.
- Assume immediate thermodynamics equilibrium and dynamics equilibrium between two phases.
- Ignore micro-physics process (like phase change of H_2O , aggregation, decomposition) and chemical reaction

. The governing equations are:

$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \mathbf{v}) = 0 \quad (1)$$

$$\frac{\partial \rho \xi}{\partial t} + \nabla \cdot (\rho \xi \mathbf{v}) = 0 \quad (2)$$

$$\frac{\partial \rho \mathbf{v}}{\partial t} + \nabla \cdot (\rho \mathbf{v} \mathbf{v} + p \mathbf{I}) = \rho \mathbf{g} \quad (3)$$

$$\frac{\partial \rho E}{\partial t} + \nabla \cdot [(\rho E + p) \mathbf{v}] = \rho \mathbf{g} \cdot \mathbf{v} \quad (4)$$

With an additional equation of state to close system of equations.

$$p = (\gamma_m - 1) \rho e \quad (5)$$

Shocktube Problems

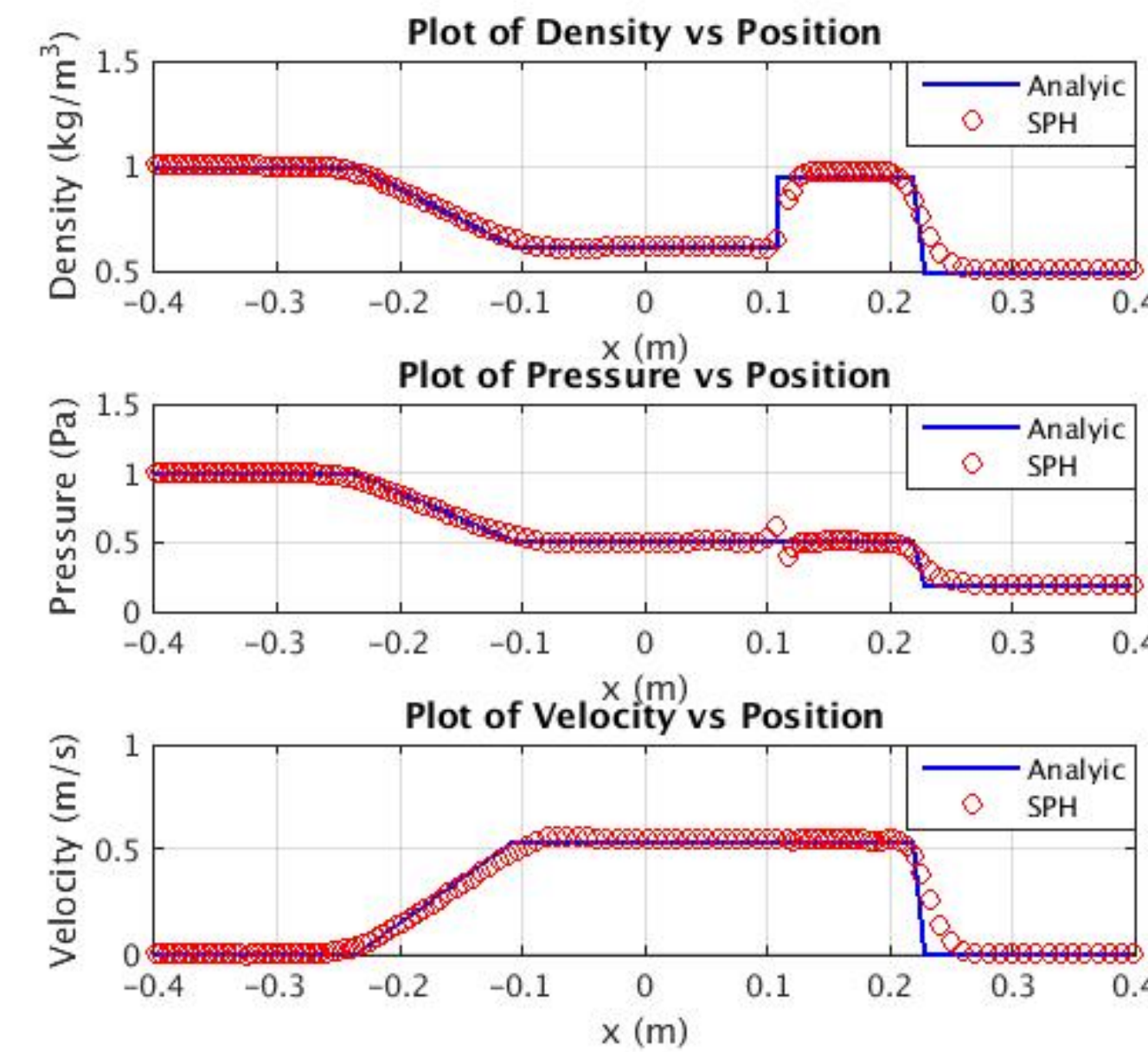


Figure 1: Solving of a 1D benchmark problem (shock tube) with SPH method, results are consistent with analytic solution

SPH – ϵ Turbulence Model [2]

Using the ideas associated with the Lagrangian averaged Navier Stokes equations(LANS), a smoothed velocity $\hat{\mathbf{v}}$ defined in terms of the unsmoothed velocity \mathbf{v} by:

$$\hat{\mathbf{v}}(\mathbf{r}) = \int \mathbf{v}(\mathbf{r}') G(|\mathbf{r}' - \mathbf{r}|, l) d\mathbf{r}' \quad (6)$$

the discretized momentum equation with $SPH - \epsilon$ turbulence model in our simulation will be:

$$\frac{d\mathbf{v}_a}{dt} = - \sum_b \left[m_b \left(\frac{p_b}{\rho_b^2} + \frac{p_a}{\rho_a^2} \right) \nabla_a w_{ab}(h_a) \right] + R_t \quad (7)$$

The stresses induced by the smoothing:

$$R_t = \sum_b m_b \frac{\epsilon \mathbf{v}_{ab} \cdot \mathbf{v}_{ab}}{2 \rho_b} \nabla_a G_{ab}(l_a) \quad (8)$$

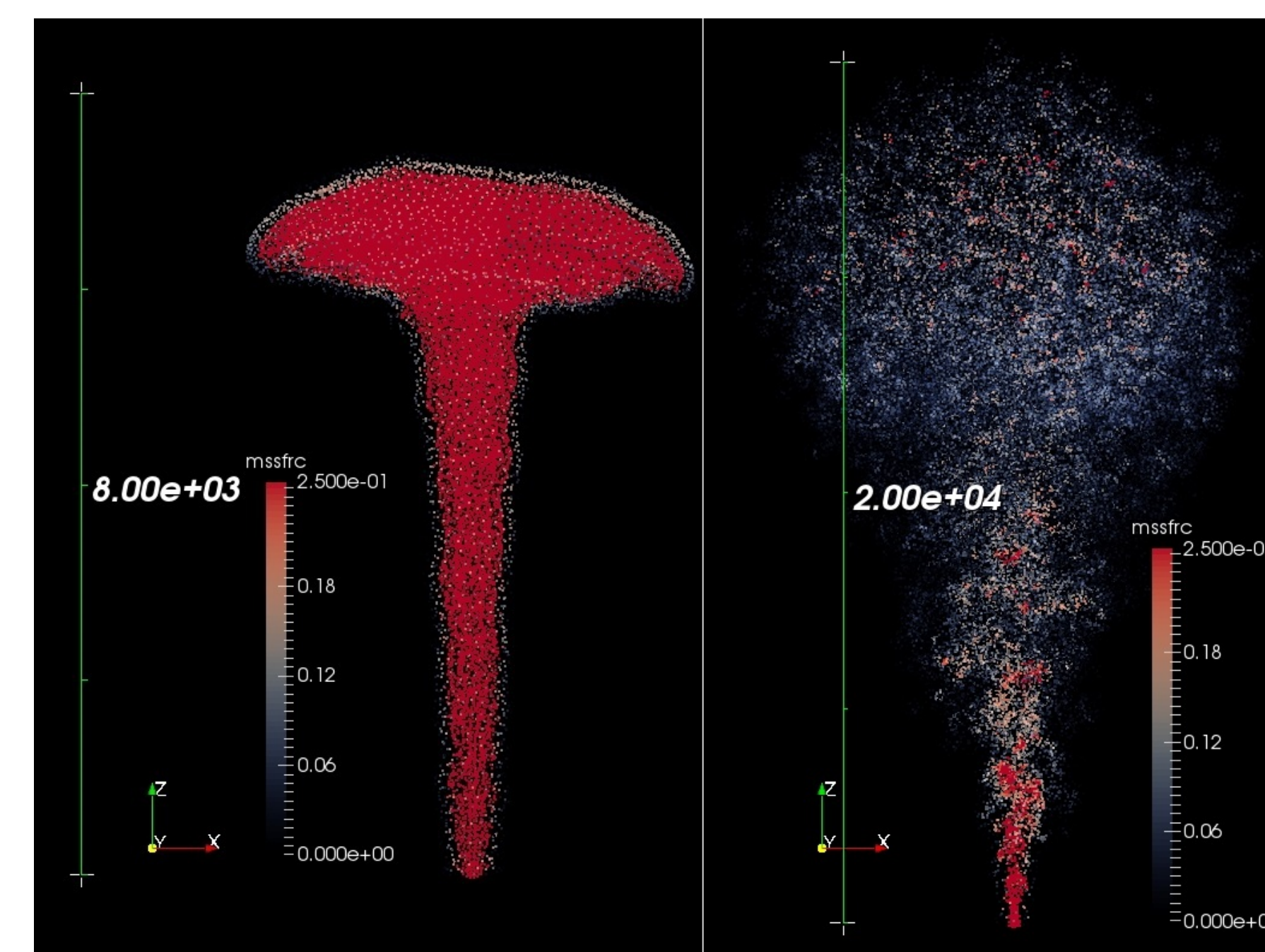


Figure 2: Simulation results at t=200. When turbulence model is not included, the entrainment is too less that plume will stop raising up after initial momentum exhausted.

Simulation of JPUE

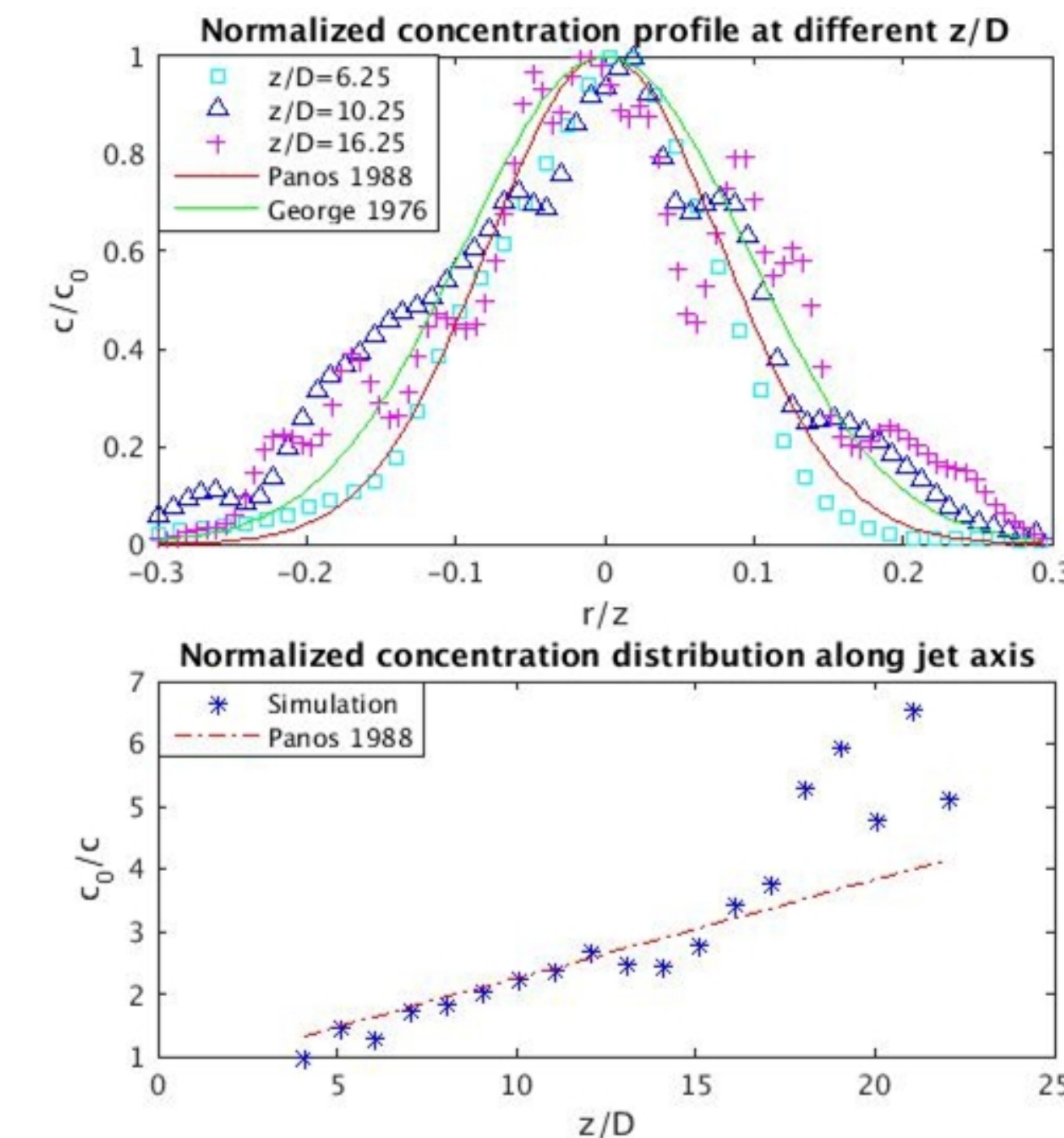


Figure 3: Jet or plume which is ejected from a nozzle into a uniform environment(JPUE) can be viewed as a simplified volcanic plume. Simulation results of JPUE is compared with experimental results [3, 4] in this figure. Concentration distribution across the cross-section is fit into a Gaussian profile (solid line) even though there is no priori reason. D is the diameter of vent, c_0 is concentration at the vent.

Simulation of Volcano Plume

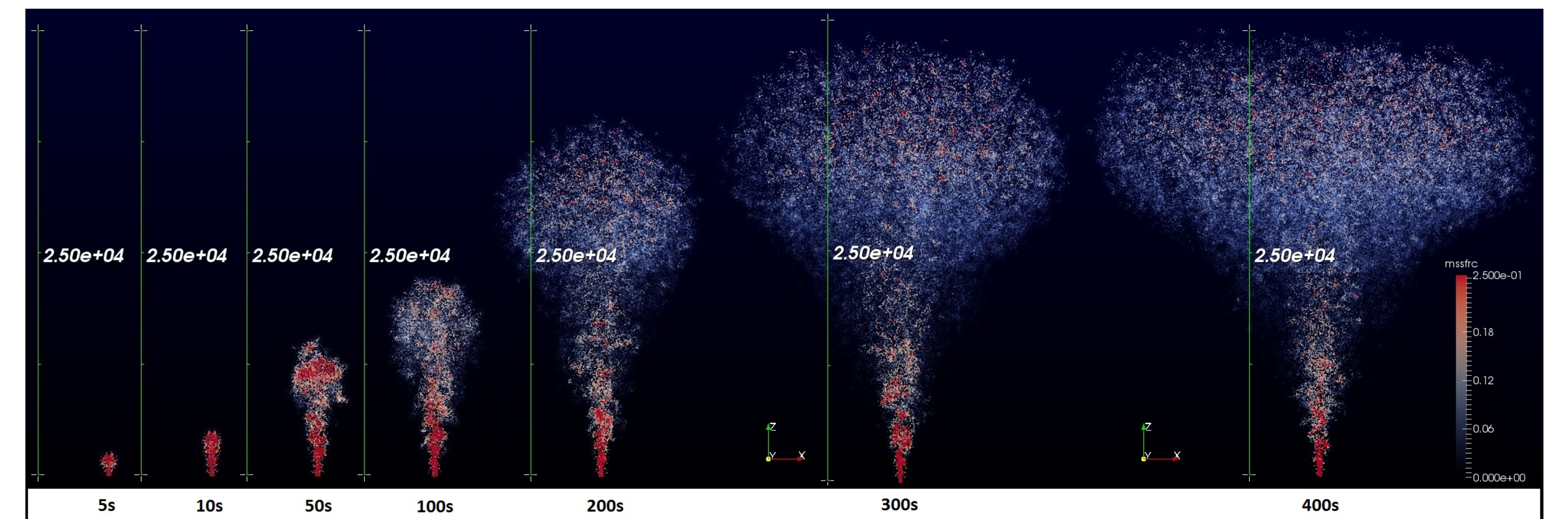


Figure 4: Evolution of plume with time. After around 300 seconds, this plume reaches its top height and start expanding

Table 1: Input Parameters for Simulation

Radius	Speed	Temperature	Water%	Mass Flux
140m	150m/s	1000K	0.05	39810717kg/s

Atmosphere is determined according to:

$$T_a(z) = \begin{cases} T_{a0} - \mu_1 z & 0 \leq z < H_1 \\ T_{a0} - \mu_1 H_1 & H_1 \leq z < H_2 \\ T_{a0} - \mu_1 H_1 + \mu_2 (z - H_2) & z > H_2 \end{cases}$$

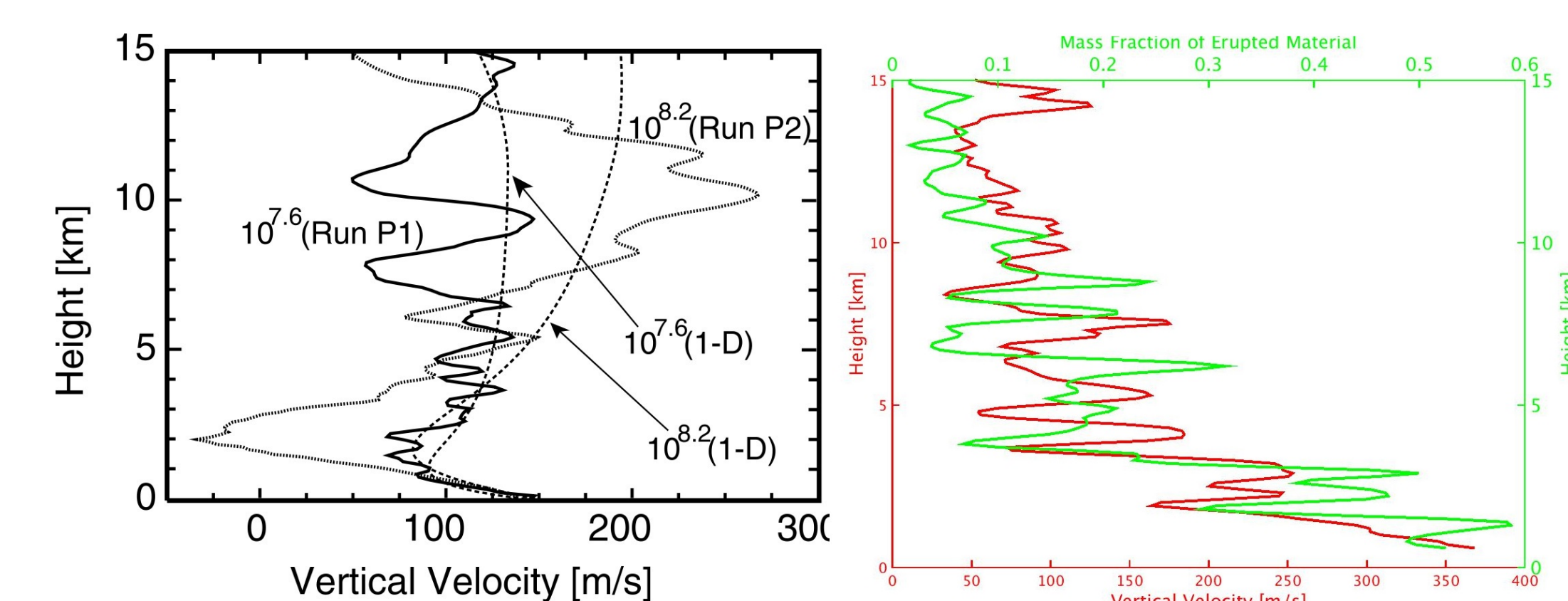


Figure 5: Distribution of vertical velocity and mass fraction along central axis of the plume. Input parameters and atmosphere are the same as "Run P1" of left figure[1].

References

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