

# A Numerical Simulation of Volcanic Plume Using SPH Method

Z. Cao, A. Patra, M. I. Bursik and E. B. Pitman

University at Buffalo, SUNY

## Abstract

- **Physics Model** Employs a three dimensional, two phases, transient model based on basic physics laws.
- **Numerical Tools** SPH is suitable for geophysical flows for several reasons: 1) SPH can capture boundary of free boundary flow automatically, 2) treating of multiple phase flow is trivial for SPH, 3) adding of new physics requires much less coding effort.
- **Parallel Computing** fast solution and a more comprehensive model at finer resolution are conflicting demands for prediction capability of numerical simulation, distributed memory parallel computing is adopted to help.
- **Goal** This model is targeting at but not limited to providing source terms for VATDMs (Volcanic Ash Transport and Dispersal Models).

## Physics Model [1]

The following assumptions are made:

- Neglect molecular viscosity.
- Assume erupted material are well mixed and behave like a single phase fluid all the time.
- Assume immediate thermodynamics equilibrium and dynamic equilibrium between two phases.
- Ignore micro-physics process (like phase change of  $H_2O$ , aggregation, decomposition) and chemical reaction

. The governing equations are:

$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \mathbf{v}) = 0 \quad (1)$$

$$\frac{\partial \rho \xi}{\partial t} + \nabla \cdot (\rho \xi \mathbf{v}) = 0 \quad (2)$$

$$\frac{\partial \rho \mathbf{v}}{\partial t} + \nabla \cdot (\rho \mathbf{v} \mathbf{v} + p \mathbf{I}) = \rho \mathbf{g} \quad (3)$$

$$\frac{\partial \rho E}{\partial t} + \nabla \cdot [(\rho E + p) \mathbf{v}] = \rho \mathbf{g} \cdot \mathbf{v} \quad (4)$$

With an additional equation of state to close system of equations.

$$p = (\gamma_m - 1) \rho e \quad (5)$$

## Shocktube Problems

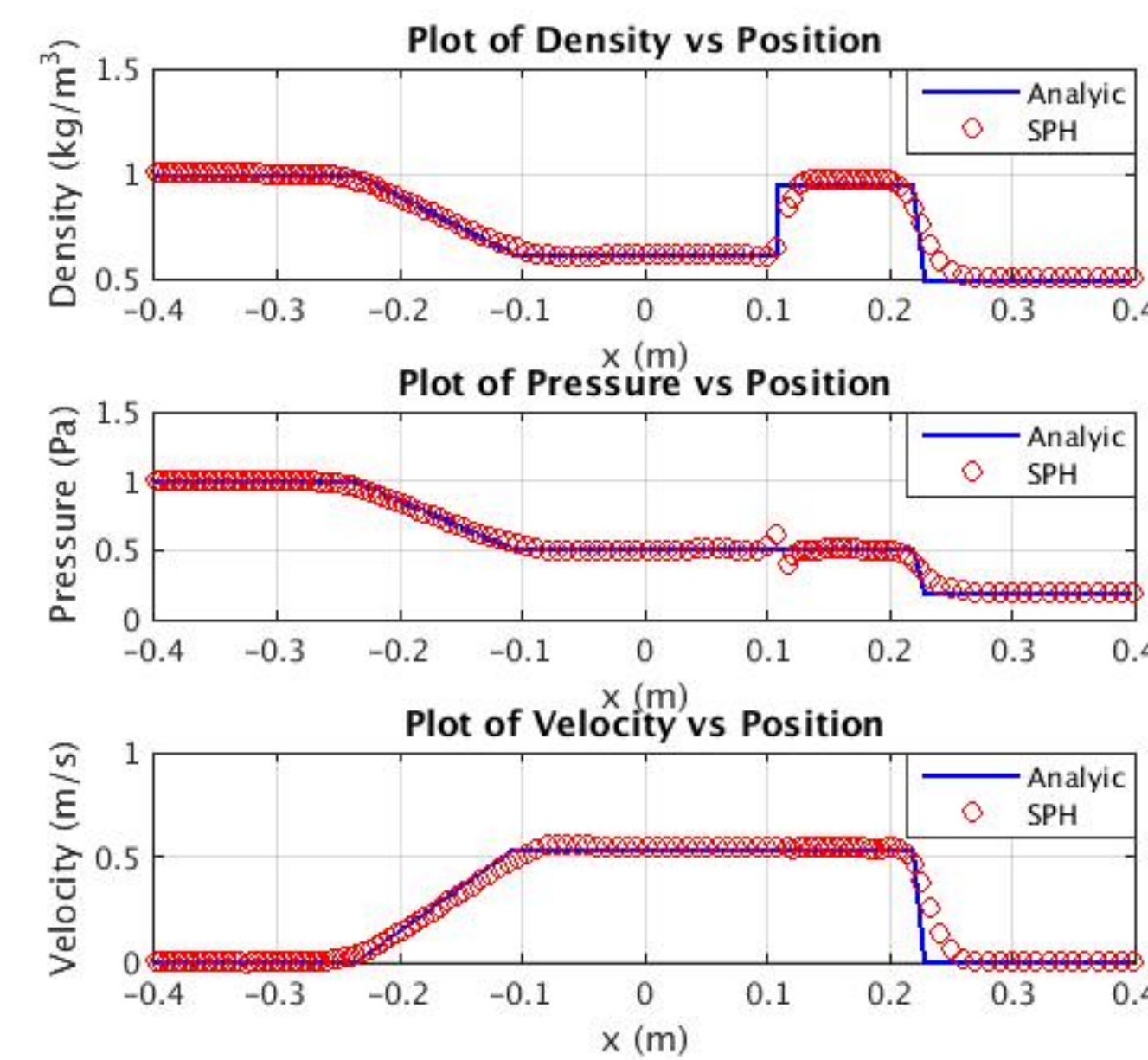


Figure 1: Solving a 1D benchmark problem (shock tube) with SPH method, results are consistent with analytic solution

## SPH – $\varepsilon$ Turbulence Model [2]

Using the ideas associated with the Lagrangian averaged Navier Stokes equations(LANS), a smoothed velocity  $\hat{\mathbf{v}}$  defined in terms of the unsmoothed velocity  $\mathbf{v}$  by:

$$\hat{\mathbf{v}}(\mathbf{r}) = \int \mathbf{v}(\mathbf{r}') G(|\mathbf{r}' - \mathbf{r}|, l) d\mathbf{r}' \quad (6)$$

the discretized momentum equation with  $SPH - \varepsilon$  turbulence model in our simulation will be:

$$\frac{d\mathbf{v}_a}{dt} = - \sum_b \left[ m_b \left( \frac{p_b}{\rho_b^2} + \frac{p_a}{\rho_a^2} \right) \nabla_a w_{ab}(h_a) \right] + R_t \quad (7)$$

The stresses induced by the smoothing:

$$R_t = \sum_b m_b \frac{\varepsilon \mathbf{v}_{ab} \cdot \mathbf{v}_{ab}}{2 \rho_b} \nabla_a G_{ab}(l_a) \quad (8)$$

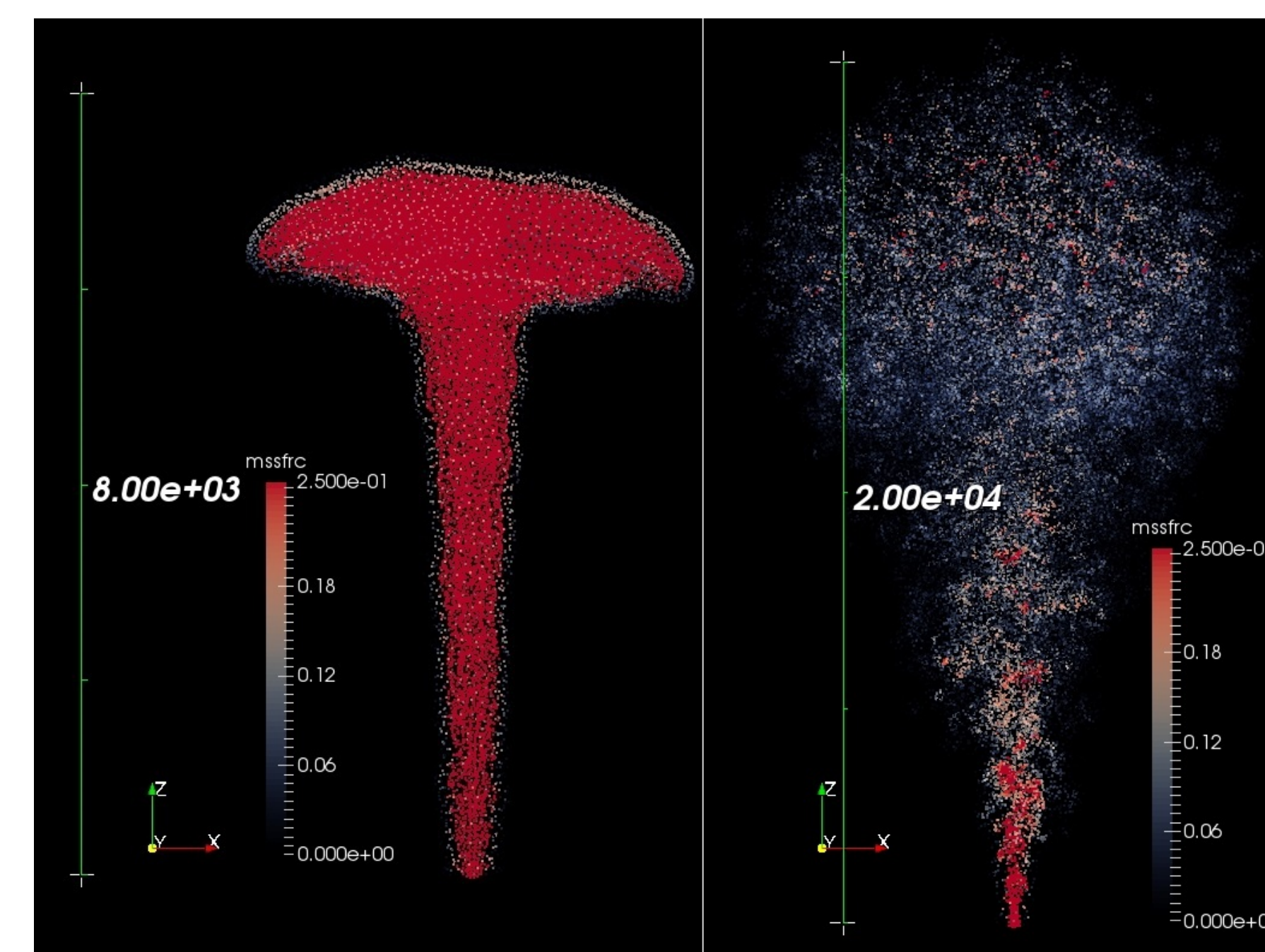


Figure 2: Simulation results at t=200. When turbulence model is not included (left figure), the entrainment of air is too less and plume will stop rising up after its initial momentum is exhausted.

## Simulation of JPUE

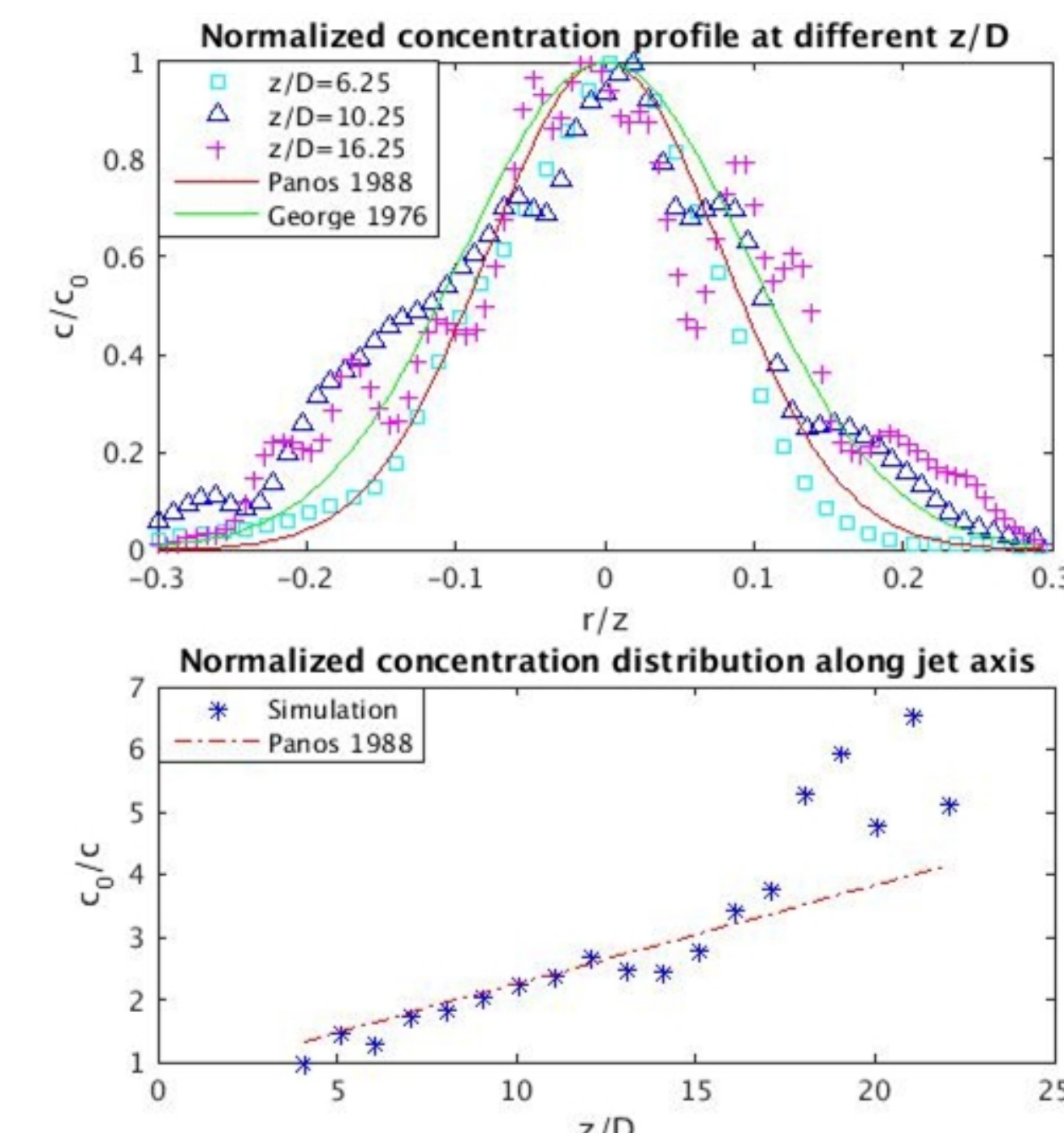


Figure 3: Jet or plume which is ejected from a nozzle into a uniform environment(JPUE) can be viewed as a simplified volcanic plume. Simulation results of JPUE is compared with experimental results [3, 4] for verification purpose. Concentration distribution across the cross-section is fit into a Gaussian profile (solid line) even though there is no priori reason. D is the diameter of vent,  $c_0$  is concentration at the vent.

## Simulation of Volcano Plume

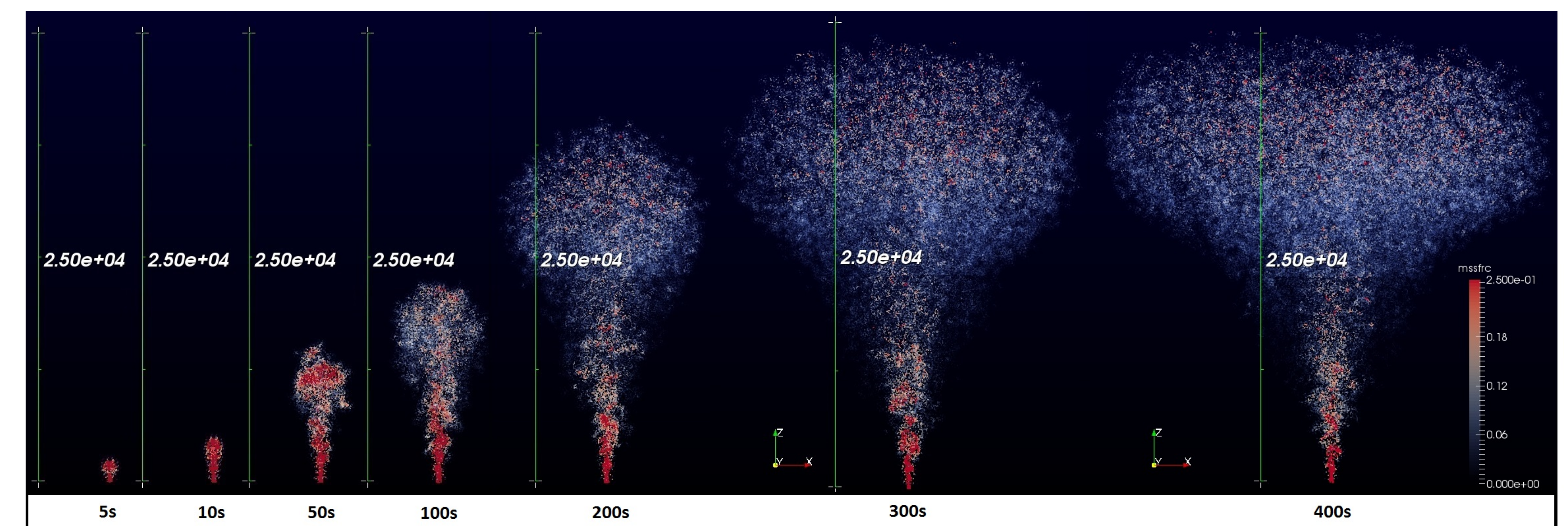


Figure 4: Evolution of plume with time. After around 300 seconds, the plume reaches its top height and starts expanding

Table 1: Input Parameters for Simulation

Radius	Speed	Temperature	Water%	Mass Flux
140m	150m/s	1000K	0.05	39810717kg/s

Atmosphere is determined according to:

$$T_a(z) = \begin{cases} T_{a0} - \mu_1 z & 0 \leq z < H_1 \\ T_{a0} - \mu_1 H_1 & H_1 \leq z < H_2 \\ T_{a0} - \mu_1 H_1 + \mu_2 (z - H_2) & z > H_2 \end{cases}$$

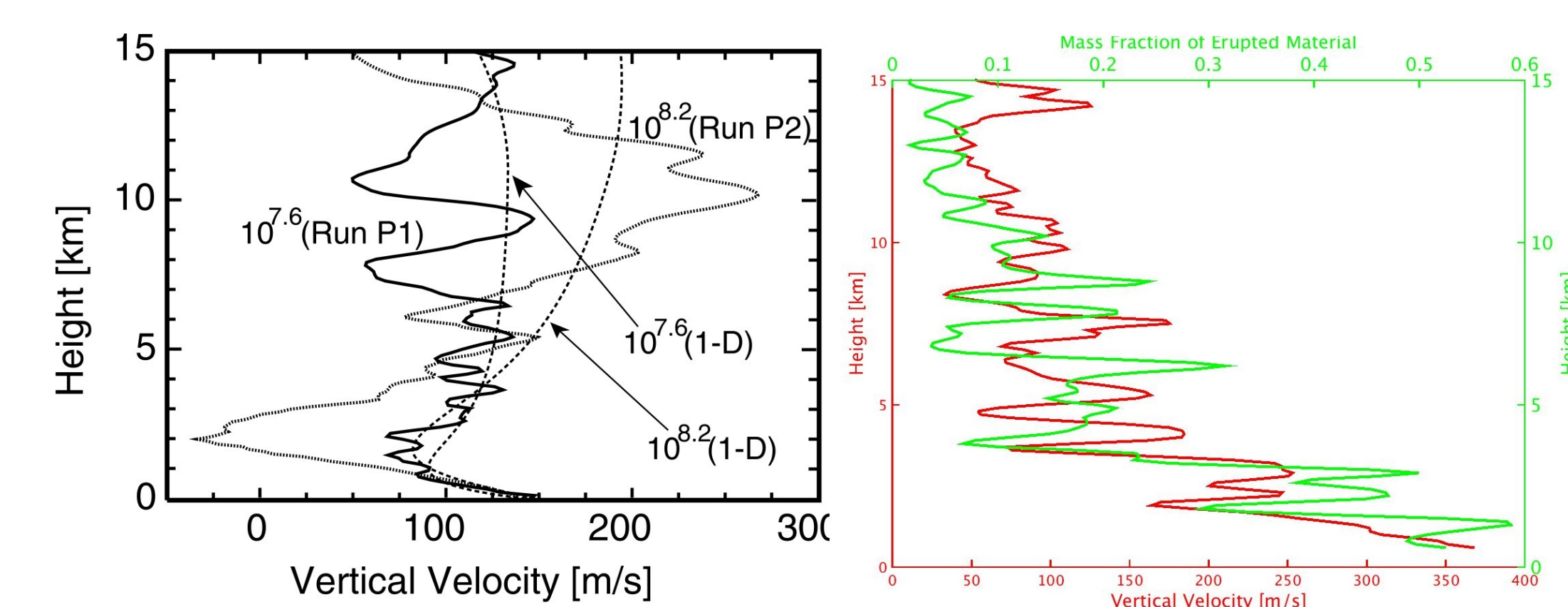


Figure 5: Distribution of vertical velocity and mass fraction along central axis of the plume. Input parameters and atmosphere are the same as "Run P1" of left figure[1].

## References

- [1] Yujiro J Suzuki, Takehiro Koyaguchi, Masaki Ogawa, and Izumi Hachisu.  
A numerical study of turbulent mixing in eruption clouds using a three-dimensional fluid dynamics model.  
*Journal of Geophysical Research: Solid Earth* (1978–2012), 110(B8), 2005.
- [2] Joe J Monaghan.  
A turbulence model for smoothed particle hydrodynamics.  
*European Journal of Mechanics-B/Fluids*, 30(4):360–370, 2011.
- [3] Panos N Papanicolaou and E John List.  
Investigations of round vertical turbulent buoyant jets.  
*Journal of Fluid Mechanics*, 195:341–391, 1988.
- [4] William K George, Ronald L Alpert, and Francesco Tamanini.  
Turbulence measurements in an axisymmetric buoyant plume.  
*International Journal of Heat and Mass Transfer*, 20(11):1145–1154, 1977.