A Numerical Simulation of Volcanic Plume Using SPH Method

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Abstract

- **Physics Model** Employs a three dimensional, two phases, transient model based on basic physics laws.
- **Numerical Tools** SPH is suitable for geophysical flows for several reasons: 1) SPH can capture boundary of free boundary flow automatically, 2) treating of multiple phase flow is trival for SPH, 3) adding of new physics requires much less coding effort.
- **Parallel Computing** fast solution and a more comprehensive model at finer resolution are conflicting demands for prediction capability of numerical simulation, distributed memory parallel computing is adopted to help.
- **Goal** This model is targeting at but not limited to providing source terms for VATDMs (Volcanic Ash Transport and Dispersal Models).

Physics Model [?]

The following assumptions are made:

- Neglect molecular viscosity.
- Assume erupted material are well mixed and behave like a single phase fluid all the time.
- Assume immediate thermodynamics equilibrium and dynamic equilibrium between two phases.
- Ignore micro-physics process (like phase change of H_2O, aggregation, decomposition) and chemical reaction

. The governing equations are:

$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \mathbf{v}) = 0 \quad (1)$$

$$\frac{\partial \rho \xi}{\partial t} + \nabla \cdot (\rho \xi \mathbf{v}) = 0 \quad (2)$$

$$\frac{\partial \rho \mathbf{v}}{\partial t} + \nabla \cdot (\rho \mathbf{v}\mathbf{v} + p\mathbf{I}) = \rho \mathbf{g} \quad (3)$$

$$\frac{\partial \rho E}{\partial t} + \nabla \cdot [(\rho E + p)\mathbf{v}] = \rho \mathbf{g} \cdot \mathbf{v} \quad (4)$$

With an additional equation of state to close system of equations.

$$p = (\gamma_m - 1)\rho e \quad (5)$$

Shocktube Problems

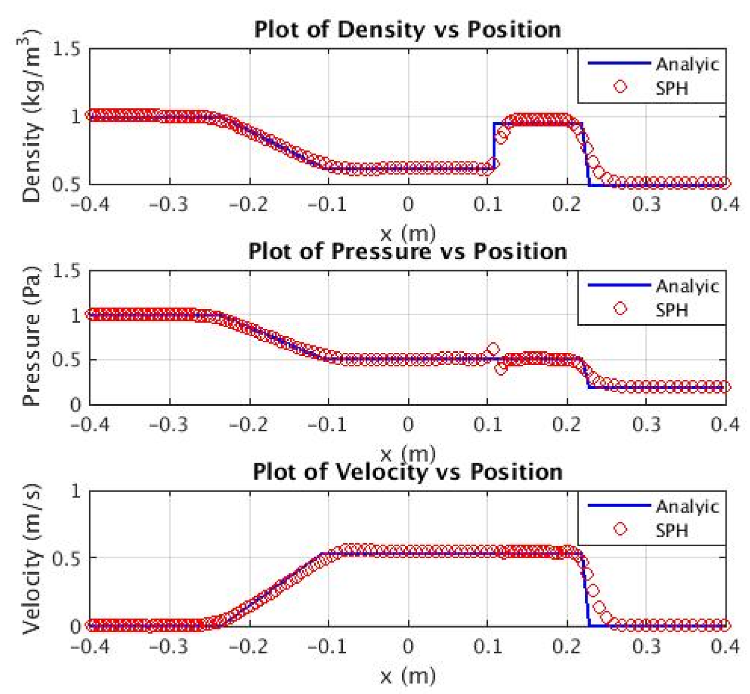

Figure 1: Solving a 1D benchmark problem (shock tube) with SPH method, results are consistent with analytic solution

$SPH - \varepsilon$ Turbulence Model [?]

Using the ideas associated with the Lagrangian averaged Navier Stokes equations(LANS), a smoothed velocity $\hat{\mathbf{v}}$ defined in terms of the unsmoothed velocity $\mathbf{v}$ by:

$$\hat{\mathbf{v}}(\mathbf{r}) = \int \mathbf{v}(\mathbf{r}\prime) G(|\mathbf{r}\prime - \mathbf{r}|, l) d\mathbf{r}\prime \quad (6)$$

the discretized momentum equation with $SPH - \varepsilon$ turbulence model in our simulation will be:

$$\frac{d\mathbf{v}_a}{dt} = -\sum_b [m_b(\frac{p_b}{\rho_b^2} + \frac{p_a}{\rho_a^2}) \nabla_a w_{ab}(h_a)] + R_t \quad (7)$$

The stresses induced by the smoothing:

$$R_t = \sum_b m_b \frac{\varepsilon \mathbf{v}_{ab} \cdot \mathbf{v}_{ab}}{2 \rho_b} \nabla_a G_{ab}(l_a) \quad (8)$$

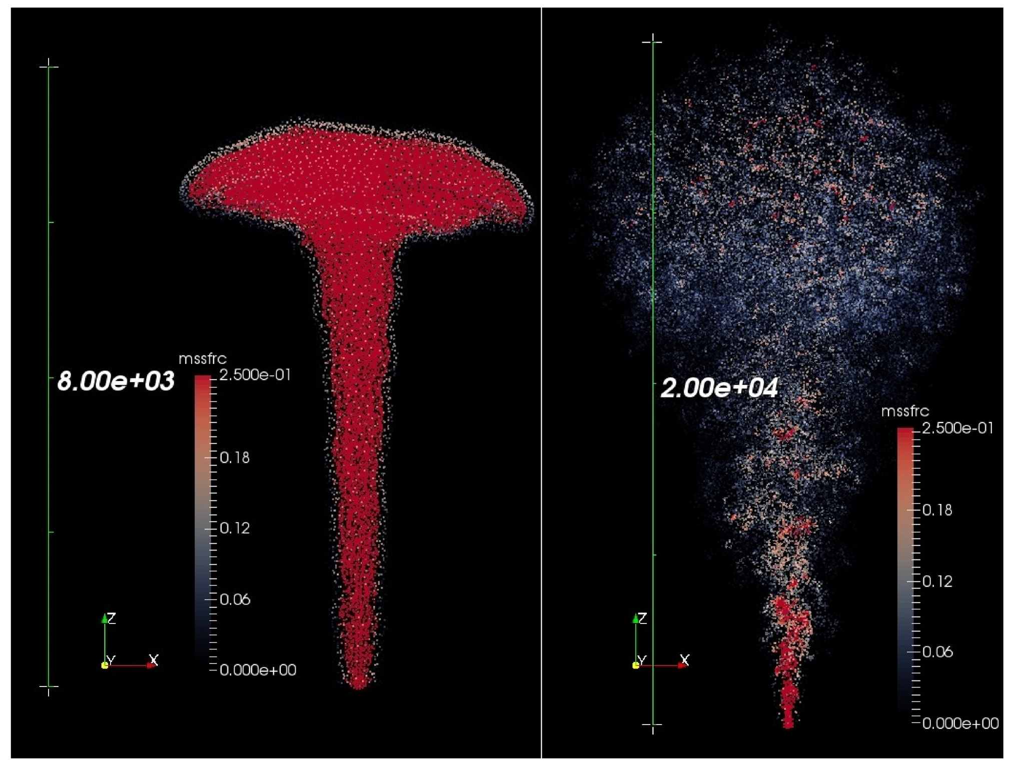

Figure 2: Simulation results at t=200. When turbulence model is not included (left figure), the entrainment of air is too less and plume will stop rising up after its initial momentum is exhausted.

Simulation of JPUE

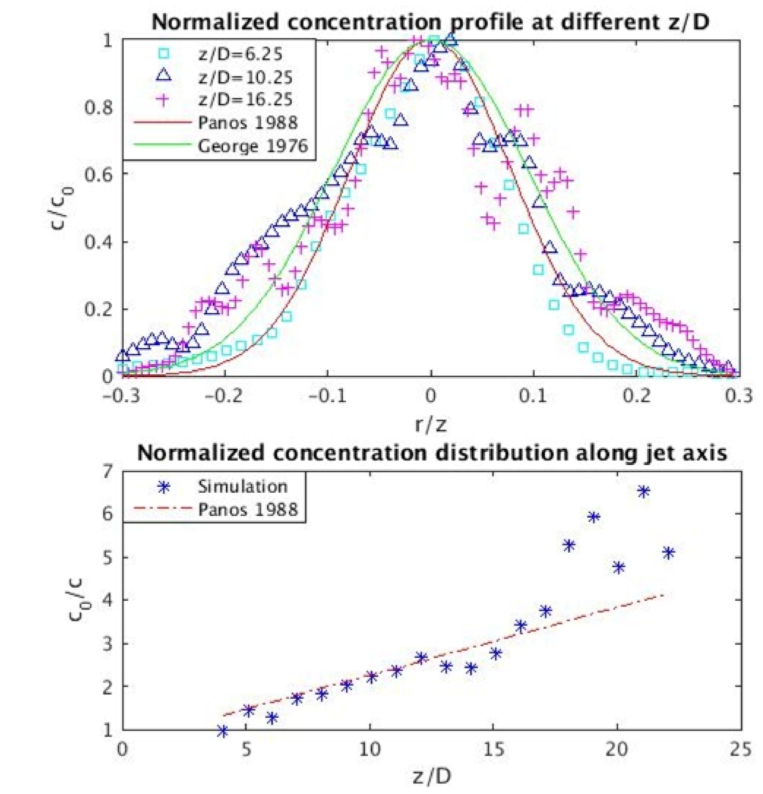

Figure 3: Jet or plume which is ejected from a nozzle into a uniform environment(JPUE) can be viewed as a simplified volcanic plume. Simulation results of JPUE is compared with experimental resutls [?, ?] for verification purpose. Concentration distribution across the cross-section is fit into a Gaussian profile (solid line) even though there is no priori reason. D is the diameter of vent, c_0 is concetration at the vent.

Simulation of Volcano Plume

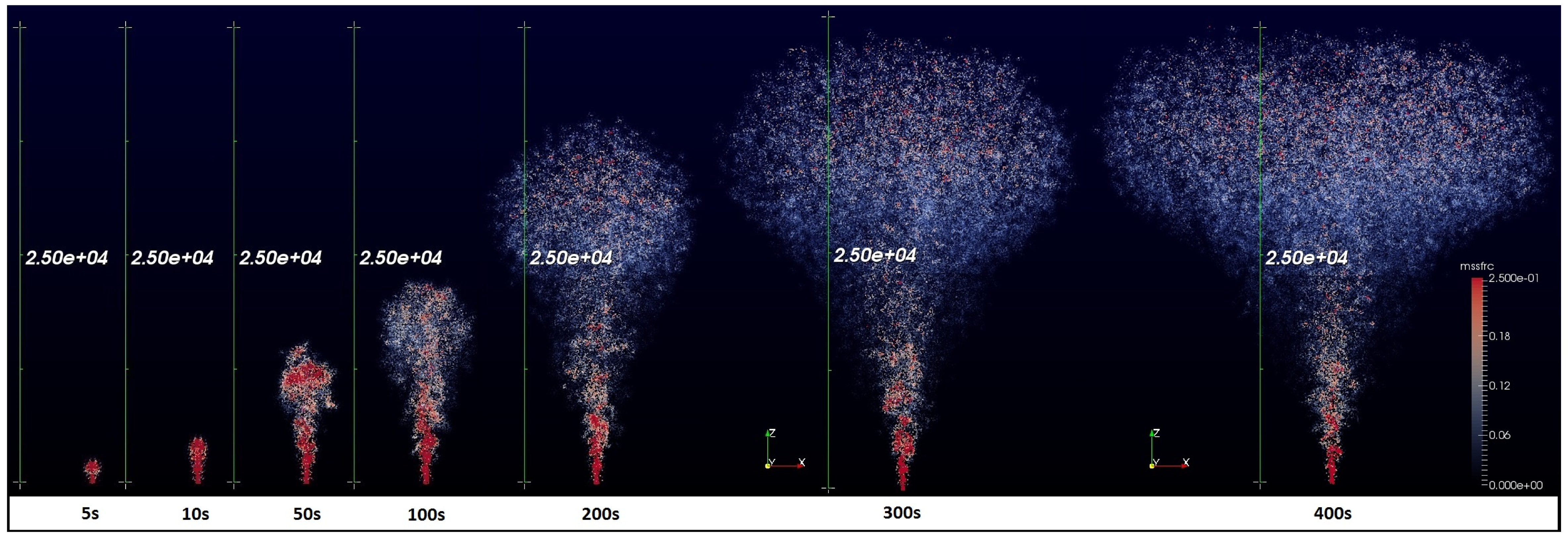

Figure 4: Evolution of plume with time. After around 300 seconds, the plume reaches its top height and starts expanding

Table 1: Input Parameters for Simulation

Radius	Speed	Temperature	Water%	Mass Flux
140m	150m/s	1000K	0.05	39810717kg/s

Atmosphere is determined according to:

$$T_a(z) = \begin{cases} T_{a0} - \mu_1 z & 0 \leq z < H_1 \\ T_{a0} - \mu_1 H1 & H_1 \leq z < H_2 \\ T_{a0} - \mu_1 H1 + \mu_2(z - H2) & z > H_2 \end{cases}$$

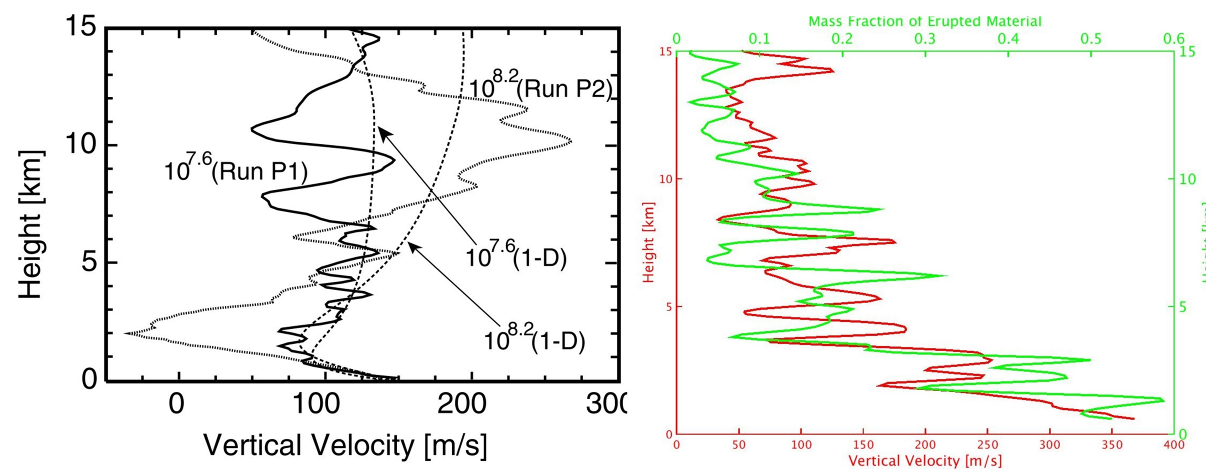

Figure 5: Distribution of vertical velocity and mass fraction along central axis of the plume. Input parameters and atmosphere are the same as "Run P1" of left figure[?].

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