1.1 $m=11$ $d_1=3$ $d_2=3$ $d_3=4$ $d_4=4$ $d_5=4$ $d_6=2$
$d_7=2$

A: $\sum_{i,j\in P_1}\left(A_{ij}-\frac{d_id_j}{2m}\right)=\left(A_{12}-\frac{d_1d_2}{2m}\right)+\left(A_{13}-\frac{d_1d_3}{2m}\right)+\left(A_{14}-\frac{d_1d_4}{2m}\right)+$

$\left(A_{23}-\frac{d_2d_3}{2m}\right)+\left(A_{24}-\frac{d_2d_4}{2m}\right)+\left(A_{34}-\frac{d_3d_4}{2m}\right)$

$=\left(1-\frac{9}{22}\right)+\left(1-\frac{12}{22}\right)+\left(1-\frac{12}{22}\right)+\left(1-\frac{12}{22}\right)+\left(1-\frac{12}{22}\right)$

$+\left(1-\frac{16}{22}\right)$

$=\frac{132}{22}-\frac{73}{22}=\frac{59}{22}$

$\sum_{i,j\in P_2}\left(A_{ij}-\frac{d_id_j}{2m}\right)=\left(A_{56}-\frac{d_5d_6}{2m}\right)+\left(A_{57}-\frac{d_5d_7}{2m}\right)+\left(A_{67}-\frac{d_6d_7}{2m}\right)=$

$\left(1-\frac{8}{22}\right)+\left(1-\frac{8}{22}\right)+\left(1-\frac{4}{22}\right)=\frac{46}{22}$

$Q_A=\frac{1}{22}\left(\frac{59}{22}+\frac{46}{22}\right)\approx 0.217.$

B: $\sum_{i,j\in P_1}\left(A_{ij}-\frac{d_id_j}{2m}\right)=\left(A_{12}-\frac{d_1d_2}{2m}\right)=\left(1-\frac{9}{22}\right)=\frac{13}{22}$

$\sum_{i,j\in P_2}\left(A_{ij}-\frac{d_id_j}{2m}\right)=\left(A_{34}-\frac{d_3d_4}{2m}\right)+\left(A_{35}-\frac{d_3d_5}{2m}\right)+\left(A_{36}-\frac{d_3d_6}{2m}\right)$

$+\left(A_{37}-\frac{d_3d_7}{2m}\right)+\left(A_{45}-\frac{d_4d_5}{2m}\right)+\left(A_{46}-\frac{d_4d_6}{2m}\right)+\left(A_{47}-\frac{d_4d_7}{2m}\right)$

$+\left(A_{56}-\frac{d_5d_6}{2m}\right)+\left(A_{57}-\frac{d_5d_7}{2m}\right)+\left(A_{67}-\frac{d_6d_7}{2m}\right)$

$=\left(1-\frac{16}{22}\right)+\left(1-\frac{16}{22}\right)+\left(0-\frac{8}{22}\right)+\left(0-\frac{8}{22}\right)+\left(1-\frac{16}{22}\right)$

$+\left(0-\frac{8}{22}\right)+\left(0-\frac{8}{22}\right)+\left(1-\frac{8}{22}\right)+\left(1-\frac{8}{22}\right)+\left(1-\frac{4}{22}\right)$

$=\frac{6}{22}+\frac{6}{22}-\frac{8}{22}-\frac{8}{22}+\frac{6}{22}-\frac{8}{22}-\frac{8}{22}+\frac{14}{22}+\frac{14}{22}+\frac{18}{22}$

$=\frac{32}{22}$

$QB = \frac{1}{22}\left(\frac{13}{22} + \frac{32}{22}\right) \approx 0.093$

$\therefore$ A is better because its modularity ≈ 0.217 which is great than B's modularity ≈ 0.093. The larger modularity is, the more structured the communities are $\rightarrow$ the better community.

1.2.

Node ID	Class
1	0
2	0
3	0
4	0
5	1
6	1
7	1

2. Precision: $\frac{TP}{TP+FP}$ Recall: $\frac{TP}{TP+FN}$ F: $2 \cdot \frac{P \cdot R}{P+R}$

For community 1:

Green $= \binom{3}{2} = \frac{3\times2}{2\times1} = 3$ pairs

Blue $= \binom{7}{2} = \frac{7\times6}{2\times1} = 21$ pairs

For community 2

Green $= \binom{8}{2} = \frac{8\times7}{2\times1} = 28$ pairs

Blue $= \binom{2}{2} = 1$ pairs

$\therefore TP = 3+21+28+1 = 53$

$FP = 21+16 = 37$

$FN = 24+14 = 38$

Precision $= \frac{53}{53+37} \approx 0.589$

Recall $= \frac{53}{53+38} \approx 0.582$

F-measure $= 2 \cdot \frac{(0.589)(0.582)}{(0.589)+(0.582)} \approx 0.585$

Purity $= \frac{7+8}{20} = 0.75$

3. At time 0, node 5 is activated, and it has three outgoing links to node 4, 2, 1

$|5-4| \equiv 1 \pmod 3$ $|5-2| \not\equiv 1 \pmod 3$ $|5-1| \equiv 1 \pmod 3$

and node 5 can go either node 4 or node 1.

Let's go node 1 first:

At time 1, node 1 is activated, and it has one outgoing link to node 6

$|1-6| \equiv 2 \pmod 3$

∴ node 1 is activated, but node 6 cannot be activate because it does not satisfy the condition $|i-j| \equiv 1 \pmod 3$. So, the propagation stops.

$|1-6| \equiv 2 \pmod 3$, which does not satisfy the condition

Then Let's go node 4:

At time 1, node 4 is activated. It has only one outgoing edge to node 2.

$|4-2| \equiv 2 \pmod 3$

∴ node 2 can not be activated by node 4 because it does not satisfy the condition $|i-j| \equiv 1 \pmod 3$. So, the propagation stops.

4. 1. To find the first node V, we compute the number of activated nodes for each node.

At time 0, node 1 is activated, and it activates node 5 because

$|1-6| \not\equiv 1 \pmod 3$ $|1-5| \equiv 1 \pmod 3$

At time 1, node 5 is activated, it activates node 4 because

$|5-4| \equiv 1 \pmod 3$ $|5-2| \not\equiv 1 \pmod 3$

At time 2, node 4 is activated, and the propagation stops because

$|4-2| \not\equiv 1 \pmod 3$.

∴ Node 1 → 1, 5, 4.

Let's try Node 2.

At time 0, Node 2 is activated, and it activates Node 3 because
$|2-3| \equiv 1 \pmod 3$
At time 1, Node 3 is activated, and it activates node 4 because
$|3-6| \not\equiv 1 \pmod 3$ $|3-4| \equiv 1 \pmod 3$

At time 2, Node 4 is activated, and the propagation stops
because Node 2 is already activated.
$\therefore$ Node $2 \to 2, 3, 4$
Let's try Node 3.
At time 0, Node 3 is activated, it activates either node 2 and
node 4 because
$|3-4| \equiv 1 \pmod 3$, $|3-2| \equiv 1 \pmod 3$, $|3-6| \not\equiv 1 \pmod 3$.
So, we have to do the calculation for both of them.
Node 2: At time 1, Node 2 is activated, the propagation stops because
Node 3 is already activated.
Node 4: At time 1, Node 4 is activated, the propagation stops because
$|4-2| \not\equiv 1 \pmod 3$
$\therefore$ Node $3 \to 3, 2, 4$
Let's try Node 4.
At time 0, Node 4 is activated, the propagation stops because
$|4-2| \not\equiv 1 \pmod 3$

$\therefore$ Node $4 \to 4$.
Let's try Node 5.
At time 0, Node 5 is activated, it activates either Node 4 or
Node 1 because
$|5-4| \equiv 1 \pmod 3$ $|5-1| \equiv 1 \pmod 3$
$\therefore$ we have to do the calculation for both of them.

Node 1: At time 0, Node 1 is activated and the propagation stops because $|1-6| \not\equiv 1 \pmod 3$.

Node 4: At time 0, Node 4 is activated, and the propagation stop because

$|4-2| \not\equiv 1 \pmod 3$

∴ Node 5 : 5, 1, 4.

Let's try Node 6.

At time 0, Node 6 is activated, and it activates Node 5 becau

$|6-5| \equiv 1 \pmod 3$.

$|6-1| \not\equiv 1 \pmod 3$ $|6-4| \not\equiv 1 \pmod 3$

At time 1, Node 5 is activated, and we just need to copy the calculation that we did for Node 5 to here.

∴ Node 6 → 6, 5, 1, 4.

1. To find the first node V, the number of activated nodes

Node 1 → 1, 5, 4

Node 2 → 2, 3, 4

Node 3 → 3, 2, 4

Node 4 → 4

Node 5 → 5, 1, 4

Node 6 → 6, 5, 1, 4.

2. Node 6 is the first node because it activates the most nodes (four). → 1, 4, 5, 6 activated, inactive nodes are 2, 3.

3. Choose either 2 or 3 because they have same number of nodes they can activate.

∴ Two nodes are {6, 2} or {6, 3}, which can in total activate all the 6 nodes.