C3-Introduction to Active Filters

1. **Aims:**

This exercise aims to:

1. provide an introduction to active analogue filters
2. familiarise students with anti-aliasing filtering process
3. demonstrate some of the limitations of real operational amplifiers

Objectives:

By the end of the exercise you should be able to:

1. design, build, and test a fourth order Butterworth filter with Rausch filters
2. understand what the purpose of an anti-aliasing filter is
3. understand and predict the limitations of such filters

Please work through **Actions 1-4** at home as your preparation for the lab and work through **Actions 5-8** during the lab sessions.

2. **Introduction**

Analogue filters can be designed and implemented using purely passive components. Simple filter designs using one a resistor and a capacitor is almost trivial. For more demanding frequency characteristics, one or more inductors must be used, and/or the filter must have several stages. It is common to include amplification within filters, if only for the purpose of signal buffering and to avoid loading. Integrated Circuit Operational Amplifiers (IC op-amps) used as inverting or non-inverting amplifiers such as those encountered in Part I are ideal for these purposes.

Inductors are circuit elements which are not easy to use in practise in filter implementations with complex frequency characteristics. Op-Amps are therefore employed to realise high performance complex filters without any inductor. Such filters are known as active filters. *Rausch filters* are a type of active filter.

Rausch filters also known as *Infinite-Gain filters* are a type of active filters with two feedback paths from the operational amplifier to the passive filter. Such filters may therefore be referred to as *multiple-feedback filters.* Active filters can be cascaded with gain stages to achieve a variety of filters with various characteristics such as Butterworth. Figure (1) shows the frequency responses of different order Butterworth filters. As can be seen, the higher the order, the closer the filer response to the ideal low pass-filter response. The main feature of the Butterworth response is that the pass band is as flat as can be achieved without having any ripples or bumps.

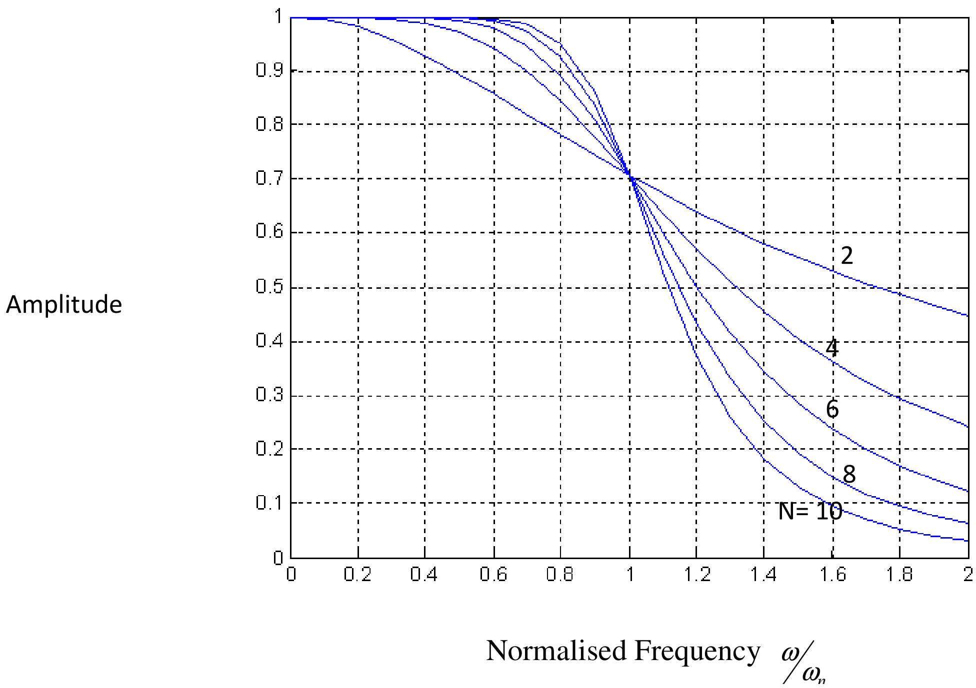

Figure 1: Frequency response of Butterworth filters with even order N up to 10 (Both the vertical and horizontal scales are linear)

The main objective of this exercise is to design, build and test a fourth order Butterworth filter with two cascaded Rausch filters to implement an anti-aliasing filter. An anti-aliasing filter is usually placed before an Analogue to Digital Converter (ADC) to remove any high frequency component which may cause an aliasing effect in the process of Analogue to Digital conversion.

3. Rausch Filers

Figure (2) shows a typical low pass Rausch filter (Infinite-Gain Multiple-Feedback low pass filter).

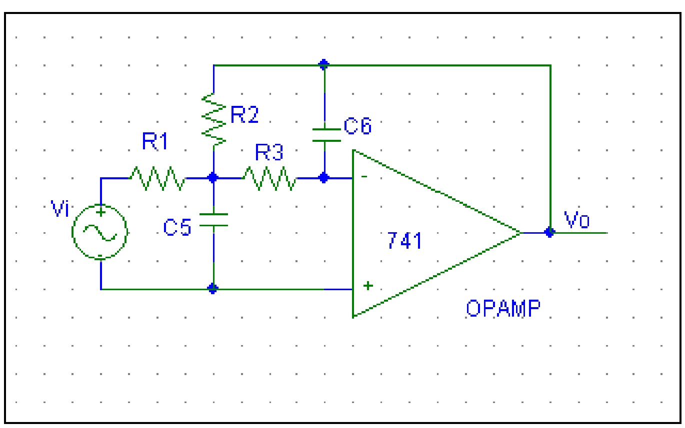

Figure (2): A low-pass infinite-gain multiple-feedback filter

Action 1: Show that the transfer function of the Rausch filter depicted in figure (2) can be written as:

$$H(s) = \frac{V_o(s)}{V_i(s)} = \frac{-1/R_1R_3C_5C_6}{s^2 + s\frac{1}{C_5}\left(\frac{1}{R_1}+\frac{1}{R_2}+\frac{1}{R_3}\right)+\frac{1}{R_2R_3C_5C_6}} \tag{1}$$

□

The transfer function (1) is a transfer function of a second order low pass filter. A general form of the transfer function of a second order low pass filter is written as:

$$H(s) = \frac{G\omega_n^2}{s^2 + \left(\frac{\omega_n}{Q}\right)s + \omega_n^2} \tag{2}$$

where G is the filter gain, ω_n is the natural un-damped frequency of the filter and Q is a parameter which varies the shape of frequency response. Transfer function (2) can be written with respect to frequency by letting $s = j\omega$ in (2). Figure (3) shows the form of magnitude and phase of transfer function (2) for various values of Q with respect to the normalised frequency $\frac{\omega}{\omega_n}$.

As can be seen from figure (3), to a first order approximation, frequencies below ω_n are allowed to be transmitted, while frequencies above ω_n are attenuated. High Q produces a peak in the magnitude response at ω_n. Notice that the frequency of the peak is below ω_n for moderate Q. Also there is no peak for $Q < 0.707$, since for $Q < 0.707$ the amplitude of the output is never greater than that of input. The phase effects span a larger band of frequencies and can be approximated as: more than a decade below ω_n the phase change is zero; more than a decade above ω_n the phase change is 180 degrees (i.e. the signal is inverted.); at ω_n the signal's phase is retarded by a quarter-cycle. Higher values of Q also make the transition from zero phase change to 180 degrees phase change more abrupt in the frequency domain. For a simple second order filter, a value of Q somewhere between 0.7 and 1 would probably be chosen. Such a filter's Bode plot magnitude and phase characteristics are close to straight lines, and this simple asymptotic presentation of filter behaviour is sometimes used as convenient shorthand. By using equations (1) and (2), G, Q and ω_n are calculated as:

$$\omega_n = \frac{1}{\sqrt{R_2R_3C_5C_6}} \tag{3a}$$

$$\frac{1}{Q} = \sqrt{\frac{C_6}{C_5}}\left(\frac{\sqrt{R_2R_3}}{R_1}+\sqrt{\frac{R_3}{R_2}}+\sqrt{\frac{R_2}{R_3}}\right) \tag{3b}$$

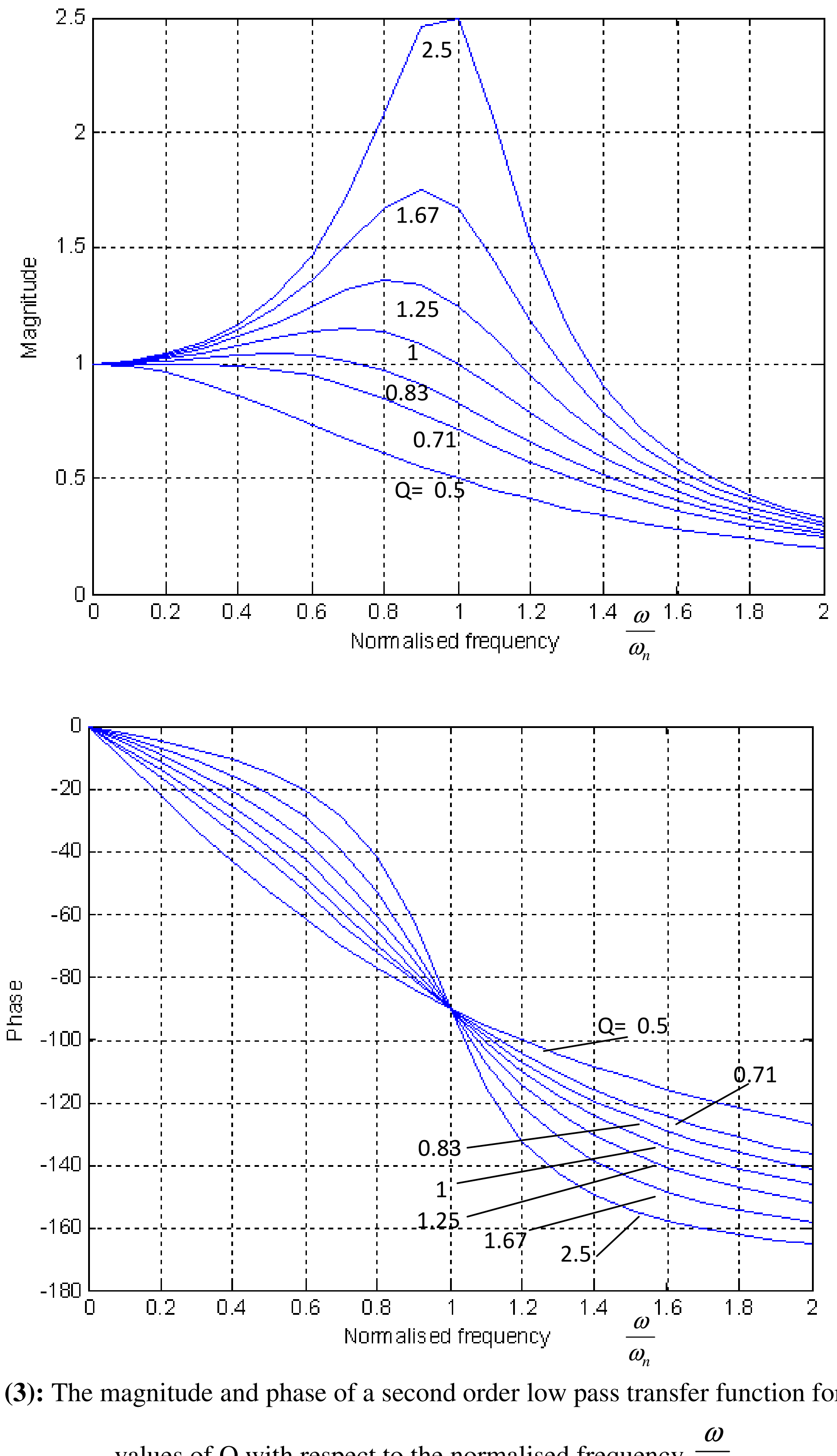

Figure (3): The magnitude and phase of a second order low pass transfer function for various values of Q with respect to the normalised frequency $\frac{\omega}{\omega_n}$

$$G = \frac{R_2}{R_1} \tag{3c}$$

Action 2: Verify equations (3a), (3b) and (3c), by comparing equations (1) and (2).

□

One of the main advantages of Rausch filters over other second order filters is that it enjoys the low sensitivities of circuit parameters such as G, Q and ω_n with respect to the variations of elements (resistors and capacitors). A design procedure is therefore inferred by employing equations (3a) – (3c) as follows:

Design Procedure: *Given: G, Q* and ω_n

Choose: C_5 (*a convenient value*)

Calculate:

$$m \le \frac{1}{4Q^2(1+G)} \quad \text{and} \quad C_6 = mC_5 \tag{4a}$$

$$R_2 = \frac{1}{2\omega_n C_5 mQ}\left[1 \pm \sqrt{1-4mQ^2(1+G)}\right] \tag{4b}$$

$$R_1 = \frac{R_2}{G} \tag{4c}$$

$$R_3 = \frac{1}{\omega_n^2 C_5^2 R_2 m} \tag{4d}$$

Example 1: It is desired to realise a second-order low-pass Butterworth (Q=0.7071) filter function in which $f_n = \omega_n / 2\pi = 100Hz$, and G=1.

Solution: Using the design procedure of (4), we choose $C_5 = 0.1\ \mu\text{F}$. Since from (4a), m must be less than 0.25, we select *m=0.1* so that $C_6 = C_5/10 = 0.01\ \mu\text{F}$. We then find $R_1 = R_2 = 199.7\ \text{k}\Omega$ and $R_3 = 12.68\ \text{k}\Omega$

4. Higher order filters

A general low pass filter of order n has the transfer function

$$H(s) = \frac{1}{s^n + b_{n-1}s^{n-1} + b_{n-2}s^{n-2} + \ldots + b_1 s + b_0}$$

The denominator of this function is just an nth order polynomial in s with real coefficients and so can always be factorised to give n roots (or poles), so that H(s) becomes

$$H(s) = \frac{a}{(s+p_1)(s+p_2)...(s+p_{n-1})(s+p_n)}$$

Because the b_n coefficients are all positive and real, the poles will be either real and negative, or pairs of complex conjugates. Therefore we can group together all the complex conjugate pair terms. Pairing together any real terms gives H(s) of an even-order filter in the form

$$H(s) = a\frac{1}{(s+p_1)(s+p_2)(s+p_3)(s+p_4)...}$$

If we now multiply out the paired terms we get a product of realisable second order sections. This means that we can realise any filter as a series of Rausch circuits having the appropriate individual values of ω_n and Q. To realise an even order Butterworth filter, successive stages of second order filters with identical natural un-damped frequencies ω_n but different values of Q are cascaded. Table 1 shows the Q values necessary for Butterworth filters of various even orders.

Q values for n even

2	4	6	8	10
0.707	0.541	0.518	0.510	0.506
	1.307	0.707	0.601	0.561
		1.932	0.900	0.707
			2.563	1.101
				3.196

Table 1-Q required for each section of even-order Butterworth filters

Action 3: Design a forth order low pass Butterworth filter with the following specifications:

i) Pass-band small-signal voltage gain : 100 ± 5 or 40dB $\pm$ 0.4dB
ii) Input Impedance: ≥ 10 kΩ
iii) Pass band: *DC* to 4.5 kHz
iv) Voltage gain at 10 kHz: *between* 34 to 36 dB

Realise this filter with two 741 op-amps to provide two Rausch filters. Two second order Rausch filters should be cascaded. The first Rausch filter should have Q=0.541 and the second one should have Q=1.307 (see Table 1). Both Rausch filters have $\omega_n = 6.28 \times 10^4 \ \frac{rad}{s}$. Use design procedure (4) to design each Rausch filter.□

Action 4: Simulate your design in PSpice.□

Action 5: Build and test the filter you have designed in action 3. The output signal should be allowed to oscillate between +10 and -10 volts without any saturation. Plot the amplitude and phase frequency response with respect to the frequency by measuring the gain and the phase in the filter output in different frequencies. If the frequency response is different from what you expect, explain why? Compare the frequency response of your filter with the simulation results using PSpice. □

<u>Action 6:</u> Add two sine wave signals with frequencies 1kHz and 100kHz using a passive or active adder to construct a composite signal. Apply this composite signal to the input of the filter you have built in action 4. What do you observe at the output of the filter? What do you observe at the output of the first filter? Print the plots of the inputs and outputs.□

<u>Action 7:</u> Apply a square wave signal with 1kHz to the filter you have built in action 4. Plot the output. Increase the input frequency of the square wave signal gradually to 10kHz and plot the output signal after each increase. Does the general shape of the output signal change as the input frequency increases? Why?

<u>Action 8:</u> Add two square wave signals with frequencies 1kHz and 100kHz using the adder you designed in action 6 to construct a composite signal. Apply this composite signal to the input of the filter you built in action 4. Print the plots of inputs and outputs.

5. References

1. L.P. Huelsman, P.E. Allen, "*Introduction to the Theory and Design of Active Filters*", McGraw-Hill Book Company, 1980.
2. D.E. Johnson, "*Introduction to Filter Theory*", Prentice-Hall, Electrical Engineering Series, 1976.
3. A.S. Sedra, K.C. Smith, "*Microelectronic Circuits*", Oxford University Press, 5th edition , 2003.

SM3 June 2010