In [1]:
```
# Import libraries
import pandas as pd
import numpy as np
import math
import matplotlib.pyplot as plt
import matplotlib.dates as mdates
from scipy import sparse
import matplotlib.cm as cm
%matplotlib inline
```

In [2]:
```
# Cplex installation on Google Colab
try:
    import cplex
except:
    !pip install cplex
    import cplex

try:
  import sympy as sym
except:
  !pip install sympy
  import sympy as sym

try:
    import cyipopt as ipopt
except:
    if IN_COLAB:
        !apt install build-essential pkg-config python3-pip python3-de
        !pip install cyipopt
        import cyipopt as ipopt
    else:
        print("Please install cyipopt module with 'conda install -c cc
```

In [3]:
```
# Cyipopt installation on Google Colab
try:
    import cyipopt as ipopt
except:
    !apt install build-essential pkg-config python3-pip python3-dev cy
    !pip install cyipopt
    import cyipopt as ipopt

# from google.colab import drive
# drive.mount('/content/drive')


########## Data Source ##########
# Input file
input_file_prices = 'adjclose_2020_2021.csv' # path to close_2020_2021

# Read data into a dataframe
df = pd.read_csv(input_file_prices)
```

## Helper Function

In [4]:
```
#### Helper Function ######
# commission_rate=0.005
# num_stocks =30

# # Annual risk-free rate for years 2020-2021 is 1.5%
# r_rf = 0.015

#Portf_Retn (Minimum Expected Portfolio Return)
#rob_bnd (Risk Constraint or Maximum Allowed Variance)


# Annual risk-free rate for years 2020-2021 is 1.5%
r_rf = 0.015
# Annual risk-free rate for years 2008-2009 is 4.5%
r_rf2008_2009 = 0.045
# Annual risk-free rate for year 2022 is 3.75%
r_rf2022 = 0.0375
num_stocks = 30

# Weights of assets in the benchmark portfolio S&P30 for years 2020-20
w_b = np.array([0.14832533, 0.15556291, 0.01990254, 0.05079846, 0.0130
# Weights of assets in the benchmark portfolio S&P30 for years 2008-20
w_b2008_2009 = np.array([0.04515391, 0.09628167, 0.00962156, 0.0475155
# Weights of assets in the benchmark portfolio S&P30 for year 2022
w_b2022 = np.array([0.1994311, 0.18518391, 0.05464191, 0.06021769, 0.0


class config(object):
    def __init__ (self, commission_rate, num_stocks, r_rf, portf_retn,
        self.commission_rate = commission_rate
        self.num_stocks = num_stocks
        self.r_rf = r_rf
        self.portf_retn = portf_retn
        self.rob_bnd=rob_bnd
        self.w_b = w_b

config= config(0.005,  num_stocks, r_rf, 0.15, 0.1, w_b)
```

In [5]:
```



#recalculate the balance: before cash+before portfolio - after portfol
def calculate_balance(cash_init, x_optimal, x_init, cur_prices):
    commission_total = np.sum(np.abs(x_optimal-x_init) * cur_prices)*c
    cash_optimal = cash_init+ np.dot(x_init,cur_prices)- np.dot(x_opti
    return cash_optimal
```

```
#if cash_optimal < 0 after optimization then deducting each entry with
# def handle_insufficient_budget(x_init, x_optimal, cur_prices, cash_o
#     # deducted to those x_optimal[i] > 0, keep it to 0 if x_optimal[
#     # if -1.2 then sell more to -2
#     pos_num_shares = np.sum(x_optimal > 0)

#     # deducted_amount = cash_optimal / pos_num_shares
#     # np.where(x_optimal > 0, )
#     deducted_shares = np.floor([cash_optimal / pos_num_shares]*confi
#     #print("deducted shares:", deducted_shares)
#     x_optimal = np.where(x_optimal > 0, x_optimal+deducted_shares, 0
#     print()

#     return x_optimal, calculate_balance(cash_init, x_optimal,x_init,

def handle_insufficient_budget(x_init, x_optimal, cur_prices, cash_opt
    #print("insufficient_budget:",cash_optimal)
    deducted_amt = x_optimal*cur_prices/np.dot(x_optimal, cur_prices)
    x_optimal=np.floor( (x_optimal*cur_prices+deducted_amt)/cur_prices
    #print("New x_optimal", x_optimal)


    return x_optimal, calculate_balance(cash_init, x_optimal,x_init, c

def get_num_shares_from_weights(weights, total_asset, cur_prices):
    x_optimal = np.floor(weights * total_asset / cur_prices)
    return x_optimal


###################### Before Assignment requirement update: new funct

# # x_init is 100% at period 0
# # x_optimal and x_init are 200% afterwards
# def calculate_short(cash_init, x_optimal, x_init, cur_prices):
#     # sell stock and return by end of period
#      total_payable = debt.amount*(1+(debt.interest/12)*2)
#      if total_payable > 0:
#             cur_weight=((x_init*cur_prices)/ np.dot(x_init, cur_pric
#             deducted_shares=np.ceil(total_payable*cur_weight/cur_pri
#             # deducted_shares = np.ceil(np.where(x_init>0, (total_pa
#             total_gain = np.dot(deducted_shares, cur_prices)*(1-conf
#             # print("x_init:", x_init, " x-init - deducted shares:",
#             x_init -= deducted_shares
#             cash_init += (total_gain - debt.amount)
#             debt.renew_debt(0)


#      # short again and purchase
#      new_debt = np.dot(cur_prices, x_optimal/2)
#      debt.renew_debt(new_debt)
```

```
#      commission_total = np.sum(np.abs(x_optimal-x_init) * cur_prices
#      #print("comission total=", commission_total)

#      cash_optimal = cash_init + np.dot(x_init,cur_prices) + new_debt

#      #insufficient fund
#      if cash_optimal < 0:
#         #print("insufficient fund cash_optimal:", cash_optimal)
#         x_optimal, cash_optimal = handle_insufficient_budget(x_init,

#      #print("period x_opimal=", x_optimal," x_init:",x_init," cash_o

#      return x_optimal, cash_optimal





################################################################################
# Strategy: Equally risk contribution strat_equal_risk_contr

class erc(object):
    def __init__(self, mu: np.ndarray, Q: np.ndarray, num_stocks):
        self.mu = mu
        self.Q = Q
        self.n = num_stocks
        self.grad_f_num = self.gradient_formula()

    def gradient_formula(self):
        x_symbol = sym.symbols(f'x1:{(self.n)+1}')
        f = self.objective(x_symbol)
        grad = sym.Matrix([f]).jacobian(x_symbol)
        grad_f_num = sym.lambdify(x_symbol, grad, 'numpy')
        return grad_f_num


    def gradient(self, x):
        return self.grad_f_num(*x)

    def objective(self, x):
        # The callback for calculating the objective
        y = x * np.dot(self.Q, x) # y[i] represents the wi*Q*w
        fval = 0
        for i in range(self.n):
            for j in range(i,self.n):
                xij = y[i] - y[j] #wi*Q*w - wj*Q*w
                fval = fval + xij*xij
        fval = 2*fval # multiply by 2 because xij and xji are duplicat
```

```
        return fval


    def constraints(self, x):
        # The callback for calculating the constraints
        return [1.0] * self.n

    def jacobian(self, x):
        # The callback for calculating the Jacobian
        # print("Jacobian is called")
        return np.array([[1.0] * self.n])

    def intermediate(
            self,
            alg_mod,
            iter_count,
            obj_value,
            inf_pr,
            inf_du,
            mu,
            d_norm,
            regularization_size,
            alpha_du,
            alpha_pr,
            ls_trials
            ):

        # Example for the use of the intermediate callback.
        print("Objective value at iteration #%d is - %g" % (iter_count




##### Strategy Helper Function: Max Shape Ratio #######
def get_max_sharpe(mu, Q, r_rf):
    """
    Solve the maximum Sharpe ratio problem in CPLEX using the
    re-parameterized approach:
      min y^T Q y
      s.t. sum_i((mu_i - r_rf/252) * y_i) = 1
           sum_i(y_i) = kappa
           y_i >= 0, kappa >= small positive.

    The solution is y*, kappa*, and the portfolio weights are w_i = y_
    We also return the maximum Sharpe ratio = 1/sqrt(y^T Q y).

    Parameters
    ----------
    mu : list or np.ndarray
```

```
        Length-n array of expected (annual) returns.
    Q : list of lists or np.ndarray
        (n x n) covariance matrix of returns.
    r_rf : float
        Risk-free rate (annual). The code uses (r_rf/252) in the const
        if you want to interpret the Sharpe ratio on a daily basis.

    Returns
    -------
    w_ms : np.ndarray
        The optimal portfolio weights that maximize the Sharpe ratio.
    max_sharpe : float
        The maximum Sharpe ratio, i.e. 1 / sqrt(y^T Q y).
    """
    cpx = cplex.Cplex()
    cpx.objective.set_sense(cpx.objective.sense.minimize)

    # Number of assets
    n = len(mu)

    # 1) Linear objective coefficients => 0 for all variables (only a
    c = [0.0]*(n + 1)

    # 2) Variable bounds => y_i >= 0, kappa >= 1e-6
    lb = [0.0]*n + [1e-6]
    ub = [cplex.infinity]*(n+1)

    # ---------------------------------------------------------------
    # 3) Two constraints:
    #   (a) sum_i((mu_i - r_rf/252) * y_i) = 1
    #   (b) sum_i(y_i) - kappa = 0
    # ---------------------------------------------------------------
    Atilde = []
    for i in range(n):
        Atilde.append([
            [0, 1],                      # rows
            [(mu[i] - r_rf / 252.0), 1.0] # coefficients in those two
        ])

    # The (n+1)-th variable is kappa, so row 0 -> 0.0, row 1 -> -1.0
    Atilde.append([[0, 1], [0.0, -1.0]])

    my_sense = "EE"          # two equality constraints
    rhs_vals = [1.0, 0.0]    # right-hand sides

    cpx.linear_constraints.add(rhs=rhs_vals, senses=my_sense)

    var_names = [f"y_{i}" for i in range(n)] + ["kappa"]
    cpx.variables.add(obj=c, lb=lb, ub=ub, columns=Atilde, names=var_r
```

```
    # ---------------------------------------------------------------
    # 4) Quadratic objective: y^T Q y
    # ---------------------------------------------------------------
    Qmat = []
    for i in range(n):
        Qmat.append([list(range(n)), Q[i]])  # row for y_i
    # For kappa (the last var), we have zero row
    Qmat.append([[n], [0.0]])

    cpx.objective.set_quadratic(Qmat)

    # ---------------------------------------------------------------
    # 5) Solve the model
    # ---------------------------------------------------------------
    cpx.set_log_stream(None)
    cpx.set_error_stream(None)
    cpx.set_warning_stream(None)
    cpx.set_results_stream(None)
    alg = cpx.parameters.lpmethod.values
    cpx.parameters.qpmethod.set(alg.concurrent)
    cpx.set_results_stream(None)

    cpx.solve()

    # ---------------------------------------------------------------
    # 6) Retrieve the solution
    # ---------------------------------------------------------------
    numcols = cpx.variables.get_num()  # should be n+1
    x_opt = [cpx.solution.get_values(j) for j in range(numcols)]

    y_opt = x_opt[:n]
    kappa_opt = x_opt[n]

    # Portfolio weights: w_i = y_i / kappa
    w_ms = np.array([y_i / kappa_opt for y_i in y_opt])

    # Calculate y^T Q y
    #  => We'll do a straightforward summation
    yQy = 0.0
    for i in range(n):
        for j in range(n):
            yQy += y_opt[i] * Q[i][j] * y_opt[j]

    # Maximum Sharpe ratio = 1 / sqrt(y^T Q y)
    max_sharpe = 1.0 / math.sqrt(yQy)

    return w_ms, max_sharpe


### Helper function: Benchmark tracking #########
```

```
#############################################################################
# Define the Market Index Tracking function (no integer rounding)
#############################################################################

def strat_benchmark_tracking(mu, Q, w_benchmark, max_stocks):
    """
    Minimizes Tracking Error squared (TE^2) w.r.t. a benchmark w_bench
        TE^2(w) = (w - w_benchmark)^T * Q * (w - w_benchmark)
    subject to:
       sum(w_i) = 1,    w_i >= 0.

    Returns the continuous solution w_opt (no integer rounding).
    """
    n = len(w_benchmark)

    # Build the TE^2 objective = (w - w_b)^T Q (w - w_b)
    # Expand => w^T Q w - 2 w^T (Q w_b) + (w_b^T Q w_b) [constant wrt
    # So the w-dependent part is: w^T Q w - 2 (Q w_b)^T w
    # => linear term c_linear = -2 * (Q w_b),  quadratic term = Q
    Qw_b = Q @ w_benchmark
    c_linear = -2.0 * Qw_b  # shape (n, )

    # Set up CPLEX
    cpx = cplex.Cplex()
    cpx.objective.set_sense(cpx.objective.sense.minimize)  # we want t

   # Decision variables:
    # - w_0, ..., w_{n-1} (continuous weight variables, in [0,1])
    # - z_0, ..., z_{n-1} (binary selection variables, in {0,1})

    lb_w = [0.0] * n
    ub_w = [1.0] * n
    lb_z = [0] * n
    ub_z = [1] * n

    # Variable names
    w_names = [f"w_{i}" for i in range(n)]
    z_names = [f"z_{i}" for i in range(n)]

    #Add variables to CPLEX
    cpx.variables.add(obj=c_linear.tolist(),
                      lb=lb_w,
                      ub=ub_w,
                      names=w_names)

    cpx.variables.add(types=[cpx.variables.type.binary] * n,
                      lb=lb_z,
                      ub=ub_z,
                      names=z_names)
```

```
    # Sum of weights must equal 1: sum(w) = 1
    cpx.linear_constraints.add(lin_expr=[cplex.SparsePair(ind=w_names,
                               senses='E',
                               rhs=[1.0])

    # Cardinality constraint: sum(z) < max_stocks
    cpx.linear_constraints.add(lin_expr=[cplex.SparsePair(ind=z_names,
                               senses='L',
                               rhs=[max_stocks])

    # Linking constraint: w_i <= z_i for all i
    for i in range(n):
        cpx.linear_constraints.add(lin_expr=[[[w_names[i], z_names[i]]
                                   senses='L',
                                   rhs=[0.0])

    qmat = []
    var_names = [f"w_{i}" for i in range(n)] + [f"z_{i}" for i in rang

    for i in range(n):
        indices = list(range(n))  # only for w's
        values = [2 * Q[i, j] for j in range(n)]
        qmat.append([indices, values])
    # For binary variables, no quadratic terms.
    for i in range(n, 2*n):
        qmat.append([[], []])
    cpx.objective.set_quadratic(qmat)


    # Solve
    cpx.set_results_stream(None)
    cpx.set_log_stream(None)
    cpx.set_warning_stream(None)
    cpx.solve()

    # Extract optimal continuous weights
    w_opt = np.array(cpx.solution.get_values(w_names))
    return w_opt
```

In [6]:

```
# Complete the following functions

def strat_buy_and_hold(x_init, cash_init, mu, Q, cur_prices):
   x_optimal = x_init
   cash_optimal = cash_init
   return x_optimal, cash_optimal

##Optimized solution
def strat_equally_weighted(x_init, cash_init, mu, Q, cur_prices):
```

```
    total_asset=np.dot(x_init, cur_prices)+cash_init
    optimized_weight=np.array([1/num_stocks] * num_stocks)

    x_optimal = get_num_shares_from_weights(optimized_weight, total_as
    cash_optimal = calculate_balance(cash_init,x_optimal,x_init, cur_p

    return x_optimal, cash_optimal


def strat_min_variance(x_init, cash_init, mu, Q, cur_prices):
    #create a cplex instance
    cpx = cplex.Cplex()
    cpx.set_problem_type(cplex.Cplex.problem_type.QP)  # Quadratic as
    cpx.objective.set_sense(cpx.objective.sense.minimize)  # Minimize

    #Add linear constraint and variables
    variables = [f"w_{i}" for i in range(num_stocks)]
    lb = [0] * num_stocks
    ub = [1] * num_stocks
    cpx.variables.add(names=variables, lb=lb, ub=ub)

    # linear_constraints: the sum of variables
    # ind: the variable names
    # val: the coefficient
    cpx.linear_constraints.add(
        lin_expr=[cplex.SparsePair(ind=variables, val=[1] * num_stocks
        senses=["E"],  # "E" means equality constraint
        rhs=[1]  # Sum of weights must equal 1
    )


    #Add quadratic part
    #CPLEX solver soling for 1/2 Q^TWQ
    qmat = []
    for i in range(num_stocks):
        qmat.append([[j for j in range(num_stocks)], [Q[i, j]  if i ==

    cpx.objective.set_quadratic(qmat)


    # Solve the problem with additional parameters
    cpx.set_log_stream(None)
    cpx.set_error_stream(None)
    cpx.set_warning_stream(None)
    cpx.set_results_stream(None)
    alg = cpx.parameters.lpmethod.values
    cpx.parameters.qpmethod.set(alg.concurrent)
    cpx.solve()

    # Get and print results
```

```
    optimized_weight = np.array(cpx.solution.get_values())
#     print("optimized_weight total:", np.sum(optimized_weight))

    total_assets=np.dot(x_init, cur_prices) + cash_init
#     print("total_assets:", total_assets)
    x_optimal=get_num_shares_from_weights(optimized_weight, total_asse

    cash_optimal = calculate_balance(cash_init,x_optimal,x_init, cur_p

    return x_optimal, cash_optimal


def strat_max_return(x_init, cash_init, mu, Q, cur_prices):
    cpx = cplex.Cplex()

    cpx.set_problem_type(cplex.Cplex.problem_type.LP)
    cpx.objective.set_sense(cpx.objective.sense.maximize)

    variables = [f"w_{i}" for i in range(num_stocks)]
    lb = [0] * num_stocks
    ub = [1] * num_stocks
    # all w_i > 0 and w_i <1
    # C stands for continuous
    cpx.variables.add(names=variables, lb=lb, ub=ub, types=['C']*num_s

    cpx.linear_constraints.add(
        lin_expr=[cplex.SparsePair(ind=variables, val=[1] * num_stocks
        senses=["E"],  # "E" means equality constraint
        rhs=[1]  # Sum of weights must equal 1
    )

    #Usage: cpx.objective.set_linear([('w_0', 0.05), ('w_1', 0.12), ('
    #cpx.objective.set_linear(mu)
    cpx.objective.set_linear(list(zip(variables, mu)))
    cpx.set_log_stream(None)
    cpx.set_error_stream(None)
    cpx.set_warning_stream(None)
    cpx.set_results_stream(None)

    alg = cpx.parameters.lpmethod.values
    cpx.parameters.qpmethod.set(alg.concurrent)
    cpx.solve()

    optimized_weight = np.array(cpx.solution.get_values())
#   print("optimized_weight total:", np.sum(optimized_weight))

    total_assets=np.dot(x_init, cur_prices) + cash_init
#   print("total_assets:", total_assets)
    x_optimal=get_num_shares_from_weights(optimized_weight, total_asse
#   print("x_optimal:", x_optimal)
```

```
    cash_optimal = calculate_balance(cash_init,x_optimal,x_init, cur_p


    return x_optimal, cash_optimal

def strat_max_Sharpe(x_init, cash_init, mu, Q, cur_prices):

   w_star, max_sr = get_max_sharpe(mu, Q, r_rf)

   total_assets = np.dot(x_init, cur_prices) + cash_init
   x_optimal=get_num_shares_from_weights(w_star, total_assets, cur_pri

   cash_optimal = calculate_balance(cash_init,x_optimal,x_init, cur_pr


   return x_optimal, cash_optimal



def strat_equal_risk_contr(x_init, cash_init, mu, Q, cur_prices):

   # Use "1/n portfolio" w0 as initial portfolio for starting IPOPT op

   lb = [0.0] * config.num_stocks  # lower bounds on variables
   ub = [1.0] * config.num_stocks  # upper bounds on variables
   cl = [1]        # lower bounds on constraints
   cu = [1]        # upper bounds on constraints

   #initial weight of each stock
   w0=x_init*cur_prices/np.dot(x_init,cur_prices)

   # Define IPOPT problem
   nlp = ipopt.Problem(n=len(w0), m=len(cl), problem_obj=erc(mu, Q, co

   # Set the IPOPT options
   nlp.add_option('jac_c_constant'.encode('utf-8'), 'yes'.encode('utf-
   nlp.add_option('hessian_approximation'.encode('utf-8'), 'limited-me
   nlp.add_option('mu_strategy'.encode('utf-8'), 'adaptive'.encode('ut
   nlp.add_option('tol'.encode('utf-8'), 1e-10)

   # Solve the problem
   w_erc, info = nlp.solve(w0)

   #print("\nOptimal solution: w = %s\n" % repr(w_erc))
   #print("Objective function value = %s\n" % repr(info['obj_val']))

   # Check constraint sum(w) = 1
#    print("sum(w) == 1:", np.allclose(np.sum(w_erc),1))
#    var_ERC = np.dot(w_erc, np.dot(Q, w_erc))
```

```
#    std_ERC = np.sqrt(var_ERC)
#    RC_ERC = (w_erc * np.dot(Q, w_erc)) / std_ERC

#    with np.printoptions(precision=6, suppress=True):
#     print("Risk contributions of assets in the ERC portfolio:\n{0}".

   #calibrate to integer
   x_optimal = get_num_shares_from_weights(w_erc, np.dot(x_init,cur_pr
   cash_optimal = calculate_balance(cash_init, x_optimal,x_init, cur_p

   return x_optimal, cash_optimal




##### Originally Proposed solution #######
# x_init are 200% position
def strat_lever_max_Sharpe(x_init, cash_init, mu, Q, cur_prices):
    w_star, max_sr = get_max_sharpe(mu, Q, config.r_rf)

    #borrow money and renew debt
    if debt.amount == 0:
        debt.renew_debt(np.dot(cur_prices, x_init))
        total_assets = np.dot(x_init, cur_prices)*2 + cash_init

#         x_optimal=get_num_shares_from_weights(w_star, total_assets,

#         commission_total = np.sum(np.abs(x_optimal-x_init) * cur_pri
#         cash_optimal = total_assets - np.dot(x_optimal,cur_prices) -
    else:
        #pay interest
        total_assets = np.dot(x_init, cur_prices) + cash_init - debt.a
#         x_optimal=get_num_shares_from_weights(w_star, total_assets,
#         commission_total = np.sum(np.abs(x_optimal-x_init) * cur_pri
#         cash_optimal = total_assets - np.dot(x_optimal,cur_prices) -
    x_optimal=get_num_shares_from_weights(w_star, total_assets, cur_pr

    commission_total = np.sum(np.abs(x_optimal-x_init) * cur_prices)*c
    cash_optimal = total_assets - np.dot(x_optimal,cur_prices) -commis

    if cash_optimal < 0:
        deducted_amt = x_optimal*cur_prices/np.dot(x_optimal, cur_pric
        x_optimal=np.floor( (x_optimal*cur_prices+deducted_amt)/cur_pr
        commission_total = np.sum(np.abs(x_optimal-x_init) * cur_price
        cash_optimal = total_assets - np.dot(x_optimal,cur_prices) -cc

#     #total assets excludes debt
#     #initially debt.amount =0
#     total_assets = np.dot(x_init, cur_prices) + cash_init - debt.amc
    # print("Before strategy total_assets:",total_assets," total debt:
```

```
    #x_optimal=get_num_shares_from_weights(w_star, total_assets, cur_p

    # borrow money to fufill 200% position under calculate_short()
    # calibrate x_optimal to ensure enough cash
    #x_optimal, cash_optimal = calculate_short(cash_init,x_optimal,x_i
    #cash_optimal = calculate_balance(cash_init, x_optimal,x_init, cur



    #print("x_optimal:",x_optimal," cash_optimal:", cash_optimal)

    return x_optimal, cash_optimal


def strat_robust_optim(x_init, cash_init, mu, Q, cur_prices):
    n = config.num_stocks
    rf = config.r_rf
    vol = np.sqrt(np.diag(Q))  # shape (n,)

    # Compute individual Sharpe ratio: (mu_i - rf)/vol_i
    # NOTE: some vol_i might be ~0 if Q was generated randomly.
    # We'll do a quick fix to avoid dividing by zero:
    small = 1e-8
    vol_safe = np.where(vol < small, small, vol)
    sharpe_indiv = (mu - rf) / vol_safe

    # Average Sharpe
    avg_sharpe = np.mean(sharpe_indiv)

    # Use half the average Sharpe ratio as the estimation-error bound
    eps_est_error = abs(avg_sharpe / 2)


    w0 = np.ones(n) / n
    ret_init = mu @ w0
    var_init = w0 @ Q @ w0

    # We'll set a "target return" 5% above that reference
    ret_minVar = ret_init * 1.05

    cpx = cplex.Cplex()
    cpx.objective.set_sense(cpx.objective.sense.minimize)  # minimize

    # We'll have 2 linear constraints:
    #   sum_i w_i = 1
    #   sum_i mu_i w_i >= ret_minVar
    # each variable  w_i in [0,1].
    c = [0.0]*n
    lb = [0.0]*n
    ub = [1.0]*n
```

```
    # Build "columns" for 2 constraints (row0 => sum(w)=1, row1 => sum
    cols = []
    for i in range(n):
        # row indices [0,1], coeffs [1.0, mu[i]]
        cols.append([[0, 1], [1.0, mu[i]]])

    cpx.linear_constraints.add(rhs=[1.0, ret_minVar],
                               senses="EG")
    # E => equality for row0, G => >= for row1

    var_names = [f"w_{i}" for i in range(n)]
    cpx.variables.add(obj=c, lb=lb, ub=ub, columns=cols, names=var_nam

    Qmat = []
    for i in range(n):
        Qmat.append([list(range(n)), list(Q[i, :])])

    cpx.objective.set_quadratic(Qmat)

    diagQ = np.diag(Q)
    Qcon = cplex.SparseTriple(ind1=var_names,
                              ind2=list(range(n)),
                              val=diagQ)
    cpx.quadratic_constraints.add(rhs=eps_est_error,
                                  quad_expr=Qcon,
                                  name="RobustConstraint")
    cpx.parameters.threads.set(4)  # single-thread for demonstration
    cpx.set_results_stream(None)
    cpx.set_warning_stream(None)
    cpx.solve()

    optimized_weight = np.array(cpx.solution.get_values())
    total_assets=np.dot(x_init, cur_prices) + cash_init
    x_optimal=get_num_shares_from_weights(optimized_weight, total_asse
    cash_optimal = calculate_balance(cash_init,x_optimal,x_init, cur_p

    return x_optimal, cash_optimal


# def strat_robust_optim(x_init, cash_init, mu, Q, cur_prices):
#     cpx = cplex.Cplex()
#     cpx.objective.set_sense(cpx.objective.sense.minimize)

#     c  = [0.0] * config.num_stocks
#     lb = [0.0] * config.num_stocks
#     ub = [1.0] * config.num_stocks

#     A = []
#     for k in range(config.num_stocks):
```

```
#         A.append([[0,1],[1.0,mu[k]]])

#     var_names = ["w_%s" % i for i in range(1,config.num_stocks+1)]

#     #config.Portf_Retn under the scale of yearly and mu is daily
#     cpx.linear_constraints.add(rhs=[1.0,config.portf_retn/252], sens
#     # Meaning:

#     # rhs=[1.0, Portf_Retn] defines right-hand sides of constraints.
#     # "E" ensures sum of weights = 1 (fully invested portfolio).
#     # "G" ensures portfolio return ≥ required return (Portf_Retn).

#     cpx.variables.add(obj=c, lb=lb, ub=ub, columns=A, names=var_name
#     Qmat = [[list(range(config.num_stocks)), list(2*Q[k,:])] for k i
#     cpx.objective.set_quadratic(Qmat)

#     # Required portfolio robustness
#     var_matr = np.diag(np.diag(Q))

#     Qcon = cplex.SparseTriple(ind1=var_names, ind2=range(config.num_
#     cpx.quadratic_constraints.add(rhs=config.rob_bnd, quad_expr=Qcon

#     cpx.parameters.threads.set(4)
#     cpx.parameters.timelimit.set(60)
#     cpx.parameters.barrier.qcpconvergetol.set(1e-12)

#     cpx.set_log_stream(None)
#     cpx.set_error_stream(None)
#     cpx.set_warning_stream(None)
#     cpx.set_results_stream(None)

#     cpx.solve()

#     optimized_weight = np.array(cpx.solution.get_values())
#     total_assets=np.dot(x_init, cur_prices) + cash_init
#     x_optimal=get_num_shares_from_weights(optimized_weight, total_as
#     cash_optimal = calculate_balance(cash_init,x_optimal,x_init, cur

#     return x_optimal, cash_optimal

def strat_tracking_index(x_init, cash_init, mu, Q, cur_prices):
    w_opt = strat_benchmark_tracking(mu, Q, config.w_b, 10)
    #print("w_opt strategy:", w_opt)

    total_assets=np.dot(x_init, cur_prices) + cash_init
    x_optimal=get_num_shares_from_weights(w_opt, total_assets, cur_pri
    #print("x_optimal:",x_optimal)
    cash_optimal = calculate_balance(cash_init,x_optimal,x_init, cur_p
    #print("cash_optimal:", cash_optimal)
```

```
    return x_optimal, cash_optimal
```

In [7]:

```
:day_ind_end0-1,:] - 1



, 0, 0, 0, 0, 0, 0, 0, 0, 19385, 0])
```

```
, 0.02252249, 0.00227124, 0.01039192, 0.04658236, 0.03283954, 0.0096963
.00612178, 0.03599232, 0.00710205, 0.02258147, 0.03947036, 0.11549755,
958, 0.01735487, 0.00242781, 0.0111223 , 0.03079034, 0.02378042, 0.0107

e', 'strat_max_return', 'strat_max_Sharpe', 'strat_equal_risk_contr', '
rtfolio', 'Maximum Expected Return Portfolio', 'Maximum Sharpe Ratio Po

turn, strat_max_Sharpe, strat_equal_risk_contr, strat_lever_max_Sharpe,
```

```
s_array[:,1] == cur_month)) if val])
array[:,1] == cur_month+1)) if val])
], dates[day_ind_end]))















 curr_cash, mu, Q, cur_prices)







r_positions, x[strategy, period-1], cur_prices, cash[strategy, period-1
period-1])







 + cash[strategy, period-1]
size,1))



nd_start:day_ind_end+1]-debt.amount

t = ${3:.2f}'.format( strategy_names[strategy],
[0], cash[strategy, period-1]))
```

```
art:day_ind_end+1,:]* x[strategy, period-1] / np.reshape(np.dot(data_pr


t:day_ind_end,:] - 1
```

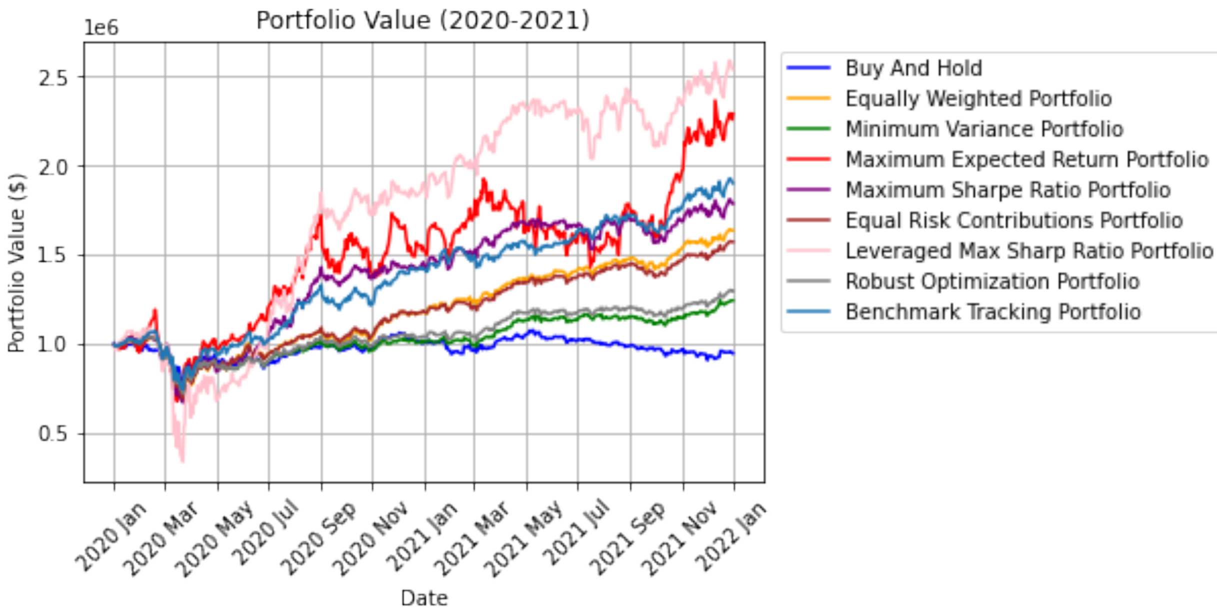
```
0.
  1287.     0.  6175.     0.     0.     0.     0.     0.  8116.    2
8.]  cash_opt: 330.682003853326
  Strategy "Mininum Variance Portfolio", value begin = $ 938635.26, v
alue end = $ 890187.07, cash account = $330.68
insufficient x_opt: [     0.      0.      0.      0.      0.      0.
0.      0.      0.
      0.      0.      0.      0.      0.      0.      0.      0.
0.
      0.      0.      0. 137587.      0.      0.      0.      0.
0.
      0.      0.      0.]  cash_opt: 51.47096007824803
  Strategy "Maximum Expected Return Portfolio", value begin = $ 94747
7.74, value end = $ 1001801.51, cash account = $51.47
insufficient x_opt: [    0.     0.     0.     0.     0.     0.     0.
0.     0.     0.
     0.     0.     0.     0.     0.     0.     0.     0.     0.
0.
 12114. 32944.     0.     0.     0.     0.     0.     0.     0.
0.]  cash_opt: 64.48482737544873
```

# Plot Results

In [8]:

```
# function to plot portfolio value
def plot_portfolio_value(portfolio_value, dates_array_plot,time_range)
  # Create the plot

  plt.plot(dates_array_plot, portfolio_value[0], label="Buy And Hold",
  plt.plot(dates_array_plot, portfolio_value[1], label="Equally Weight
  plt.plot(dates_array_plot, portfolio_value[2], label="Minimum Variar
  plt.plot(dates_array_plot, portfolio_value[3], label="Maximum Expect
  plt.plot(dates_array_plot, portfolio_value[4], label="Maximum Sharpe
  plt.plot(dates_array_plot, portfolio_value[5], label="Equal Risk Cor
  plt.plot(dates_array_plot, portfolio_value[6], label="Leveraged Max
  plt.plot(dates_array_plot, portfolio_value[7], label="Robust Optimiz
  plt.plot(dates_array_plot, portfolio_value[8], label="Benchmark Trac

  # Add labels and title
  plt.xlabel("Date")
  plt.ylabel("Portfolio Value ($)")
  title = "Portfolio Value "+time_range
  plt.title(title)

  plt.gca().xaxis.set_major_locator(mdates.MonthLocator(interval=2))
  plt.gca().xaxis.set_major_formatter(mdates.DateFormatter('%Y %b'))

  # Rotate labels for better readability
  plt.xticks(rotation=45)
  plt.grid(True)
  #plt.legend()
  plt.legend(loc="upper left", bbox_to_anchor=(1.01, 1))

  # Show the plot
  plt.show()
```

In [9]:
```
# Plot Line chart
# convert np.array[String] to np.array[datetime64]
dates_array_plot= pd.to_datetime(dates).to_numpy(dtype='datetime64[D]'
plot_portfolio_value(portf_value, dates_array_plot , "(2020-2021)")
```

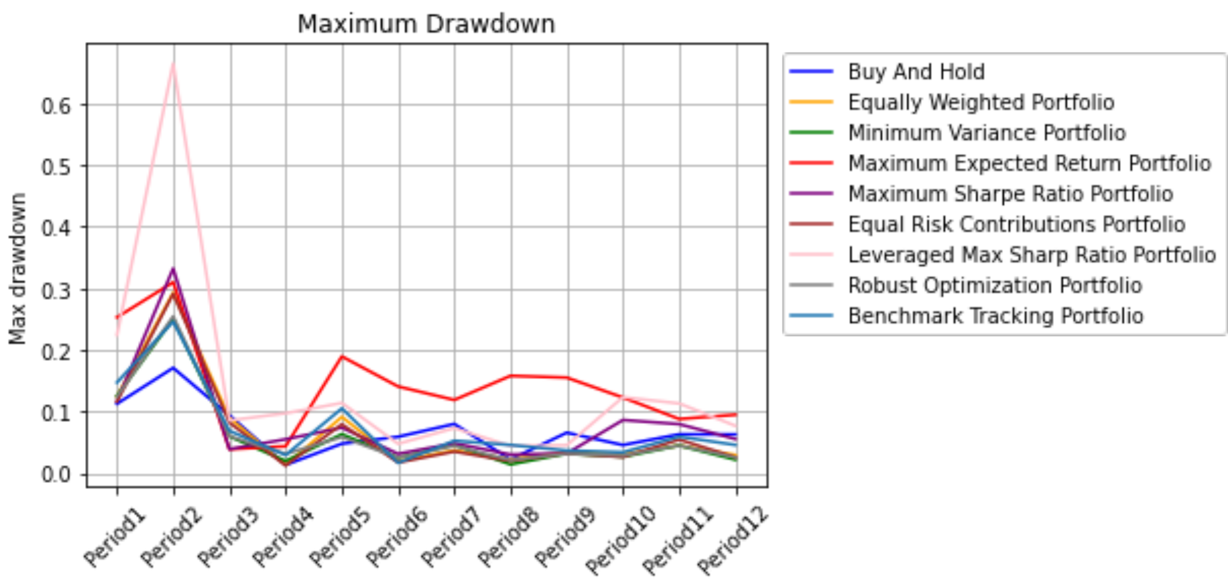



# Maximum Drawdown

In [10]:
```
et_max_drawdown_data(N_periods, dates_array):
tart_max_drawdown=[]
or i in range(len(strategy_functions)):
   max_drawdown=[]
   for period in range(1, N_periods+1):
      if dates_array[0, 0] == 20:
          cur_year  = 20 + math.floor(period/7)
      else:
          cur_year  = 2020 + math.floor(period/7)

      cur_month = 2*((period-1)%6) + 1

      day_ind_start = min([i for i, val in enumerate((dates_array[:,0]
      day_ind_end = max([i for i, val in enumerate((dates_array[:,0] ==


      #print(day_ind_end," ", day_ind_start)
      #print(np.maximum.accumulate(portf_value[i][day_ind_start-253: da
      #print('\nPeriod {0}: start date {1}, end date {2}'.format(perioc
      #print(day_ind_end," ", day_ind_start)


      peak = np.maximum.accumulate(portf_value[i][day_ind_start:day_inc
      drawdown = (peak - portf_value[i][day_ind_start:day_ind_end+1])/
      max_drawdown.append(np.max(drawdown))
   start_max_drawdown.append(max_drawdown)
eturn start_max_drawdown
rawdown_data=get_max_drawdown_data(N_periods, dates_array)
```

In [11]:
```
# function to plot portfolio value
def plot_max_drawdown(max_drawdown_data):
  # Create the plot
  x = np.arange(1, 13)  # 12 periods (1 to 12)
  #print(max_drawdown_data[0])
  plt.plot(x, max_drawdown_data[0], label="Buy And Hold", color="blue"
  plt.plot(x, max_drawdown_data[1],label="Equally Weighted Portfolio",
  plt.plot(x, max_drawdown_data[2],label="Minimum Variance Portfolio",
  plt.plot(x, max_drawdown_data[3],label="Maximum Expected Return Port
  plt.plot(x, max_drawdown_data[4],label="Maximum Sharpe Ratio Portfol
  plt.plot(x, max_drawdown_data[5],label="Equal Risk Contributions Por
  plt.plot(x, max_drawdown_data[6],label="Leveraged Max Sharp Ratio Po
  plt.plot(x, max_drawdown_data[7],label="Robust Optimization Portfoli
  plt.plot(x, max_drawdown_data[8],label="Benchmark Tracking Portfolic

  # Add labels and title
  #plt.xlabel("Date")
  plt.ylabel("Max drawdown")
  title = "Maximum Drawdown"
  plt.title(title)
```

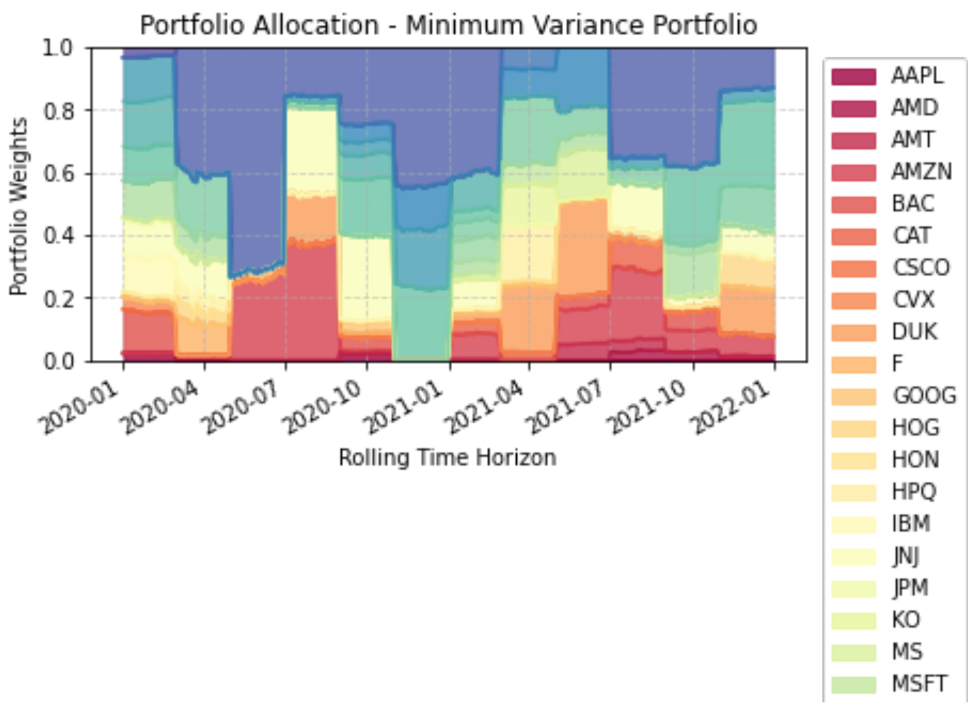
```
    x_labels = [f'Period{i}' for i in range(1, 13)]


    # Rotate labels for better readability
    plt.xticks(ticks=x, labels=x_labels, rotation=45)
    plt.grid(True)
    #plt.legend()
    plt.legend(loc="upper left", bbox_to_anchor=(1.01, 1))

    # Show the plot
    plt.show()

plot_max_drawdown(max_drawdown_data)
```




## Dynamic Weight

In [12]:
```
# Function to plot portfolio allocation
def plot_pandas_portfolio(df, strategy_name):
    #df.plot.area(figsize=(12, 6), alpha=0.8, cmap="Spectral")
    df.plot.area(figsize=(6, 3), alpha=0.8, cmap=cm.get_cmap("Spectral
    plt.title(f"Portfolio Allocation - {strategy_name}")
    plt.xlabel("Rolling Time Horizon")
    plt.ylabel("Portfolio Weights")
    plt.ylim(0, 1)  # Portfolio weights must sum to 1
    plt.grid(True, linestyle="--", alpha=0.6)
    plt.legend(loc="upper left", bbox_to_anchor=(1.01, 1))
    plt.show()
```

In [13]:
```
#time_horizon = np.arange(time_periods)
dates_array_plot= pd.to_datetime(dates).to_numpy(dtype='datetime64[D]'
asset_labels = df.columns[1:]


# Convert data into a Pandas DataFrame
df_3 = pd.DataFrame(portfolio_allocation[2], columns=asset_labels, inc
df_4 = pd.DataFrame(portfolio_allocation[3], columns=asset_labels, inc
df_5 = pd.DataFrame(portfolio_allocation[4], columns=asset_labels, inc
df_8 = pd.DataFrame(portfolio_allocation[7], columns=asset_labels, inc
# df_5 = d.DataFrame(strategy_5, columns=asset_labels, index=time_hori


# Plot for
plot_pandas_portfolio(df_3, "Minimum Variance Portfolio")

# # Plot for Strategy 4
plot_pandas_portfolio(df_4, "Maximum Expected Return Portfolio")

# # Plot for Strategy 5
plot_pandas_portfolio(df_5, "Maximum Sharpe Ratio Portfolio")

# # Plot for Strategy 8
plot_pandas_portfolio(df_8, "Robust Optimization Portfolio")
```



# trading strategies for years 2008 and 2009:

In [14]:
```
########## Data Source ##########
# Input file
input_file_prices = 'adjclose_2008_2009.csv' # path to close_2020_2021

# Read data into a dataframe
df = pd.read_csv(input_file_prices)

# Annual risk-free rate for years 2008-2009 is 4.5%
r_rf2008_2009 = 0.045

num_stocks = 30

# Weights of assets in the benchmark portfolio S&P30 for years 2008-20
w_b2008_2009 = np.array([0.04515391, 0.09628167, 0.00962156, 0.0475155

class config(object):
    def __init__ (self, commission_rate, num_stocks, r_rf, portf_retn,
        self.commission_rate = commission_rate
        self.num_stocks = num_stocks
        self.r_rf = r_rf
        self.portf_retn = portf_retn
        self.rob_bnd=rob_bnd
        self.w_b = w_b


config= config(0.005,  num_stocks, r_rf2008_2009, 0.015, 0.005, w_b200
```

In [15]:
```
class debt(object):
    def __init__(self, interest):
        self.amount=0.0
        self.interest = interest

    def renew_debt(self, amount):
        self.amount = amount

debt = debt(config.r_rf)

dates_array = np.array(list(df['Date'].apply(convert_date_to_array)))
data_prices = df.iloc[:, 1:].to_numpy()
dates = np.array(df['Date'])
# Find the number of trading days in Nov-Dec 2019 and
# compute expected return and covariance matrix for period 1
day_ind_start0 = 0
```

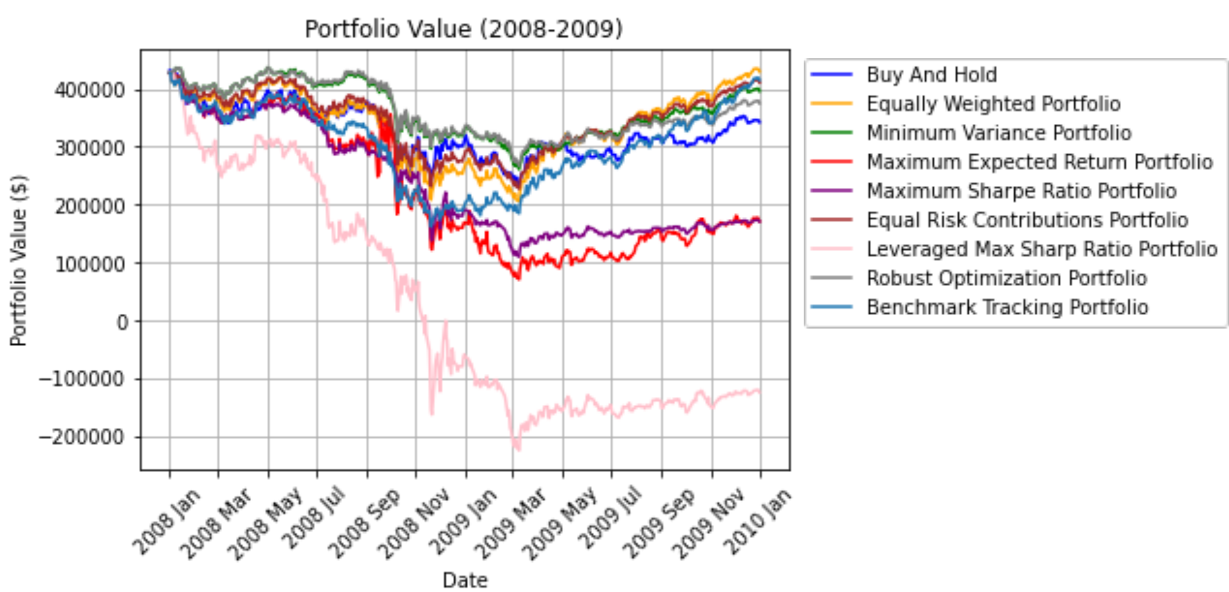
```
day_ind_end0 = len(np.where(dates_array[:,0]==2007)[0])   # for 2008-2

cur_returns0 = data_prices[day_ind_start0+1:day_ind_end0,:] / data_pri
mu = np.mean(cur_returns0, axis = 0)
Q = np.cov(cur_returns0.T)

# Remove datapoints for year 2019
data_prices = data_prices[day_ind_end0:,:]
dates_array = dates_array[day_ind_end0:,:]
dates = dates[day_ind_end0:]

# Initial positions in the portfolio
init_positions = np.array([0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 3447, 0, 0

# Initial value of the portfolio
init_value = np.dot(data_prices[0,:], init_positions)
print('\nInitial portfolio value = $ {}\n'.format(round(init_value, 2)

# Initial portfolio weights
w_init = (data_prices[0,:] * init_positions) / init_value

# Number of periods, assets, trading days
N_periods = 6*len(np.unique(dates_array[:,0])) # 6 periods per year
N = len(df.columns)-1
N_days = len(dates)

# Number of strategies
strategy_functions = ['strat_buy_and_hold', 'strat_equally_weighted',
strategy_names     = ['Buy and Hold', 'Equally Weighted Portfolio', 'M
N_strat = 9  # comment this in your code
#N_strat = len(strategy_functions)  # uncomment this in your code
fh_array = [strat_buy_and_hold, strat_equally_weighted, strat_min_vari


portfolio_allocation = np.zeros((N_strat, N_days, num_stocks),  dtype=
portf_value = [0] * N_strat
x = np.zeros((N_strat, N_periods),  dtype=np.ndarray)
cash = np.zeros((N_strat, N_periods),  dtype=np.ndarray)
for period in range(1, N_periods+1):
   # Compute current year and month, first and last day of the period

   # Depending on what data/csv (i.e time period) uncomment code
#    if dates_array[0, 0] == 20:
#        cur_year  = 20 + math.floor(period/7)
#    else:
#        cur_year  = 2020 + math.floor(period/7)

   # example for 2008-2009 data
   if dates_array[0, 0] == 8:
      cur_year  = 8 + math.floor(period/7)
```

```
    else:
       cur_year  = 2008 + math.floor(period/7)

    cur_month = 2*((period-1)%6) + 1
    day_ind_start = min([i for i, val in enumerate((dates_array[:,0] ==
    day_ind_end = max([i for i, val in enumerate((dates_array[:,0] == c
    print('\nPeriod {0}: start date {1}, end date {2}'.format(period, c

    # Prices for the current day
    cur_prices = data_prices[day_ind_start,:]

    # Execute portfolio selection strategies
    for strategy  in range(N_strat):

       # Get current portfolio positions
     if period == 1:
         curr_positions = init_positions
         curr_cash = 0
         portf_value[strategy] = np.zeros((N_days, 1))
     else:
         curr_positions = x[strategy, period-2]
         curr_cash = cash[strategy, period-2]

     # Compute strategy
     x[strategy, period-1], cash[strategy, period-1] = fh_array[strateg

     # Verify that strategy is feasible (you have enough budget to re-b
     # Check that cash account is >= 0
     # Check that we can buy new portfolio subject to transaction costs

     ###################### Insert your code here ####################
     if cash[strategy, period-1] < 0 :
         x[strategy, period-1], cash[strategy, period-1] = handle_insuf

     # throw exception is cash less than 0
     if cash[strategy, period-1] < 0:
         raise ValueError("Negative cash balance is not allowed")


     # Compute portfolio value
     p_values = np.dot(data_prices[day_ind_start:day_ind_end+1,:], x[st
     portf_value[strategy][day_ind_start:day_ind_end+1] = np.reshape(p_

     # deducted the debt and interest
     if strategy_names[strategy] == "Leveraged Max Sharpe Ratio Portfol
         portf_value[strategy][day_ind_start:day_ind_end+1] = portf_val

     print('  Strategy "{0}", value begin = $ {1:.2f}, value end = $ {2
             portf_value[strategy][day_ind_start][0], portf_value[stra
```

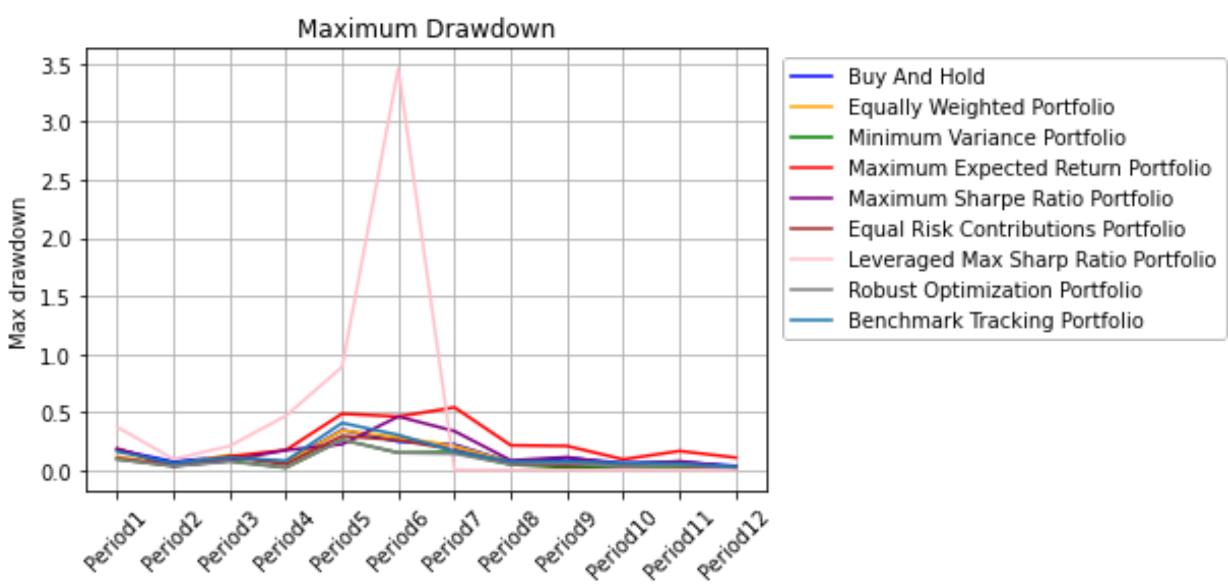
```
     portfolio_allocation[strategy][day_ind_start:day_ind_end+1] = data


    # Compute expected returns and covariances for the next period
    cur_returns = data_prices[day_ind_start+1:day_ind_end+1,:] / data_p
    mu = np.mean(cur_returns, axis = 0)
    Q = np.cov(cur_returns.T)
```

```
Objective value at iteration #12 is - 2.95674e-11
Objective value at iteration #13 is - 1.46511e-11
Objective value at iteration #14 is - 9.20081e-12
Objective value at iteration #15 is - 4.2531e-12
Objective value at iteration #16 is - 3.32572e-12
Objective value at iteration #17 is - 1.45067e-12
Objective value at iteration #18 is - 8.21512e-13
  Strategy "Equal Risk Contributions Portfolio", value begin = $ 4170
57.03, value end = $ 374639.71, cash account = $201.93
  Strategy "Leveraged Max Sharpe Ratio Portfolio", value begin = $ 30
3612.79, value end = $ 248131.62, cash account = $57.33
  Strategy "Robust Optimization Portfolio", value begin = $ 434648.3
7, value end = $ 409427.97, cash account = $188.74
  Strategy "Benchmark Tracking Portfolio", value begin = $ 387001.33,
value end = $ 350841.50, cash account = $104.02

Period 4: start date 07/01/2008, end date 08/29/2008
  Strategy "Buy and Hold", value begin = $ 361827.08, value end = $ 3
69082.82, cash account = $0.00
  Strategy "Equally Weighted Portfolio", value begin = $ 364572.28, v
```

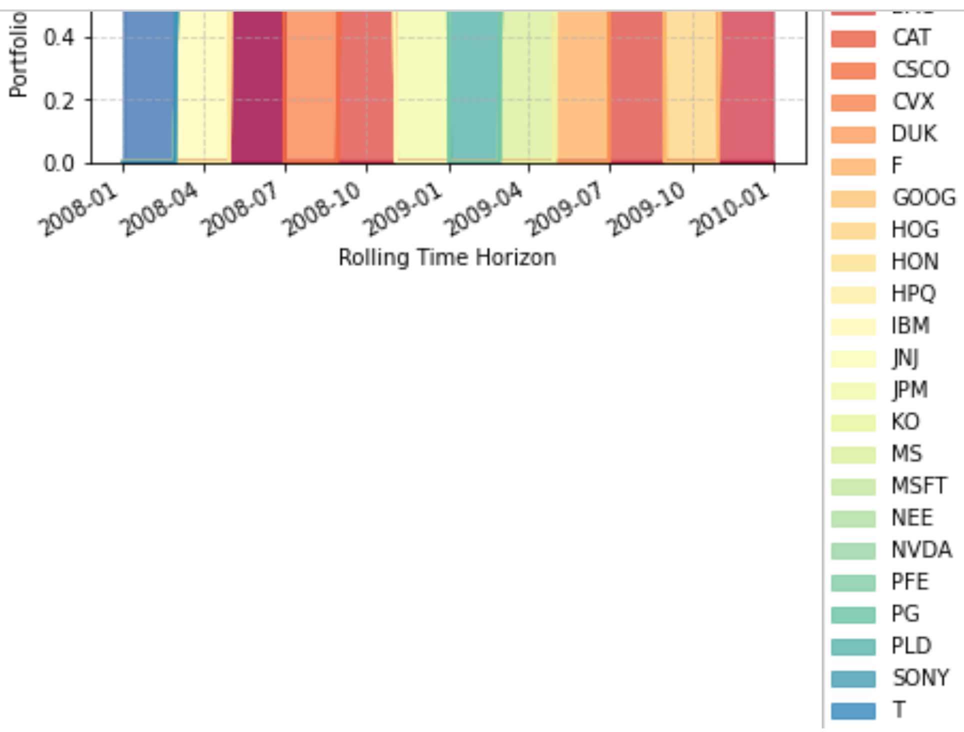
In [16]:
```
# Plot Line chart
# convert np.array[String] to np.array[datetime64]
dates_array_plot= pd.to_datetime(dates).to_numpy(dtype='datetime64[D]'
plot_portfolio_value(portf_value, dates_array_plot , "(2008-2009)")
```



In [17]:
```
def get_max_drawdown_data_2008(N_periods, dates_array):
    start_max_drawdown=[]
    for i in range(len(strategy_functions)):
        max_drawdown=[]
        for period in range(1, N_periods+1):
           if dates_array[0, 0] == 8:
              cur_year  = 8 + math.floor(period/7)
           else:
              cur_year  = 2008 + math.floor(period/7)

           cur_month = 2*((period-1)%6) + 1

           day_ind_start = min([i for i, val in enumerate((dates_array
           day_ind_end = max([i for i, val in enumerate((dates_array[:


           #print(day_ind_end," ", day_ind_start)
           #print(np.maximum.accumulate(portf_value[i][day_ind_start-2
           #print('\nPeriod {0}: start date {1}, end date {2}'.format(
           #print(day_ind_end," ", day_ind_start)


           peak = np.maximum.accumulate(portf_value[i][day_ind_start:d
           drawdown = (peak - portf_value[i][day_ind_start:day_ind_enc
           max_drawdown.append(np.max(drawdown))
        start_max_drawdown.append(max_drawdown)
    return start_max_drawdown

# Plot max drawdown
max_drawdown_data=get_max_drawdown_data_2008(N_periods, dates_array)
plot_max_drawdown(max_drawdown_data)
```



In [18]:
```
#time_horizon = np.arange(time_periods)
dates_array_plot= pd.to_datetime(dates).to_numpy(dtype='datetime64[D]'
asset_labels = df.columns[1:]


# Convert data into a Pandas DataFrame
df_3 = pd.DataFrame(portfolio_allocation[2], columns=asset_labels, inc
df_4 = pd.DataFrame(portfolio_allocation[3], columns=asset_labels, inc
df_5 = pd.DataFrame(portfolio_allocation[4], columns=asset_labels, inc
df_8 = pd.DataFrame(portfolio_allocation[7], columns=asset_labels, inc
# df_5 = d.DataFrame(strategy_5, columns=asset_labels, index=time_hori


# Plot for
plot_pandas_portfolio(df_3, "Minimum Variance Portfolio")

# # Plot for Strategy 4
plot_pandas_portfolio(df_4, "Maximum Expected Return Portfolio")

# # Plot for Strategy 5
plot_pandas_portfolio(df_5, "Maximum Sharpe Ratio Portfolio")

# # Plot for Strategy 8
plot_pandas_portfolio(df_8, "Robust Optimization Portfolio")
```




# trading strategies for year 2022

In [19]:
```
########## Data Source ##########
# Input file
input_file_prices = 'adjclose_2022.csv' # path to close_2020_2021

# Read data into a dataframe
df = pd.read_csv(input_file_prices)


r_rf2022 = 0.0375
num_stocks = 30

# Weights of assets in the benchmark portfolio S&P30 for year 2022
w_b2022 = np.array([0.1994311, 0.18518391, 0.05464191, 0.06021769, 0.0


class config(object):
    def __init__ (self, commission_rate, num_stocks, r_rf, portf_retn,
        self.commission_rate = commission_rate
        self.num_stocks = num_stocks
        self.r_rf = r_rf
        self.portf_retn = portf_retn
        self.rob_bnd=rob_bnd
        self.w_b = w_b

config= config(0.005,  num_stocks, r_rf2022, 0.15, 0.005, w_b2022)
```

In [20]:
```
class debt(object):
    def __init__(self, interest):
        self.amount=0.0
        self.interest = interest

    def renew_debt(self, amount):
        self.amount = amount

debt = debt(config.r_rf)


dates_array = np.array(list(df['Date'].apply(convert_date_to_array)))
data_prices = df.iloc[:, 1:].to_numpy()
dates = np.array(df['Date'])
# Find the number of trading days in Nov-Dec 2019 and
# compute expected return and covariance matrix for period 1
day_ind_start0 = 0
day_ind_end0 = len(np.where(dates_array[:,0]==2021)[0])   # for 2022 c

cur_returns0 = data_prices[day_ind_start0+1:day_ind_end0,:] / data_pri
mu = np.mean(cur_returns0, axis = 0)
Q = np.cov(cur_returns0.T)
```

```
# Remove datapoints for year 2019
data_prices = data_prices[day_ind_end0:,:]
dates_array = dates_array[day_ind_end0:,:]
dates = dates[day_ind_end0:]

# Initial positions in the portfolio
init_positions = np.array([0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 3447, 0, 0

# Initial value of the portfolio
init_value = np.dot(data_prices[0,:], init_positions)
print('\nInitial portfolio value = $ {}\n'.format(round(init_value, 2)

# Initial portfolio weights
w_init = (data_prices[0,:] * init_positions) / init_value

# Number of periods, assets, trading days
N_periods = 6*len(np.unique(dates_array[:,0])) # 6 periods per year
N = len(df.columns)-1
N_days = len(dates)

# Number of strategies
strategy_functions = ['strat_buy_and_hold', 'strat_equally_weighted',
strategy_names     = ['Buy and Hold', 'Equally Weighted Portfolio', 'M
N_strat = 9  # comment this in your code
#N_strat = len(strategy_functions)  # uncomment this in your code
fh_array = [strat_buy_and_hold, strat_equally_weighted, strat_min_vari


portfolio_allocation = np.zeros((N_strat, N_days, num_stocks),  dtype=
portf_value = [0] * N_strat
x = np.zeros((N_strat, N_periods),  dtype=np.ndarray)
cash = np.zeros((N_strat, N_periods),  dtype=np.ndarray)
for period in range(1, N_periods+1):
   cur_year = 2022
   # Compute current year and month, first and last day of the period

   # Depending on what data/csv (i.e time period) uncomment code
#    if dates_array[0, 0] == 20:
#        cur_year  = 20 + math.floor(period/7)
#    else:
#        cur_year  = 2020 + math.floor(period/7)

   # example for 2008-2009 data
#    if dates_array[0, 0] == 8:
#       cur_year  = 8 + math.floor(period/7)
#    else:
#       cur_year  = 2008 + math.floor(period/7)

   cur_month = 2*((period-1)%6) + 1
   day_ind_start = min([i for i, val in enumerate((dates_array[:,0] ==
```

```
    day_ind_end = max([i for i, val in enumerate((dates_array[:,0] == c
    print('\nPeriod {0}: start date {1}, end date {2}'.format(period, c

    # Prices for the current day
    cur_prices = data_prices[day_ind_start,:]

    # Execute portfolio selection strategies
    for strategy  in range(N_strat):

        # Get current portfolio positions
      if period == 1:
          curr_positions = init_positions
          curr_cash = 0
          portf_value[strategy] = np.zeros((N_days, 1))
      else:
          curr_positions = x[strategy, period-2]
          curr_cash = cash[strategy, period-2]

      # Compute strategy
      x[strategy, period-1], cash[strategy, period-1] = fh_array[strateg

      # Verify that strategy is feasible (you have enough budget to re-b
      # Check that cash account is >= 0
      # Check that we can buy new portfolio subject to transaction costs

      ###################### Insert your code here #####################
      if cash[strategy, period-1] < 0 :
          x[strategy, period-1], cash[strategy, period-1] = handle_insuf

      # throw exception is cash less than 0
      if cash[strategy, period-1] < 0:
          raise ValueError("Negative cash balance is not allowed")


      # Compute portfolio value
      p_values = np.dot(data_prices[day_ind_start:day_ind_end+1,:], x[st
      portf_value[strategy][day_ind_start:day_ind_end+1] = np.reshape(p_

      # deducted the debt and interest
      if strategy_names[strategy] == "Leveraged Max Sharpe Ratio Portfol
          portf_value[strategy][day_ind_start:day_ind_end+1] = portf_val

      print('  Strategy "{0}", value begin = $ {1:.2f}, value end = $ {2
              portf_value[strategy][day_ind_start][0], portf_value[stra

      portfolio_allocation[strategy][day_ind_start:day_ind_end+1] = data


    # Compute expected returns and covariances for the next period
    cur_returns = data_prices[day_ind_start+1:day_ind_end+1,:] / data_p
```

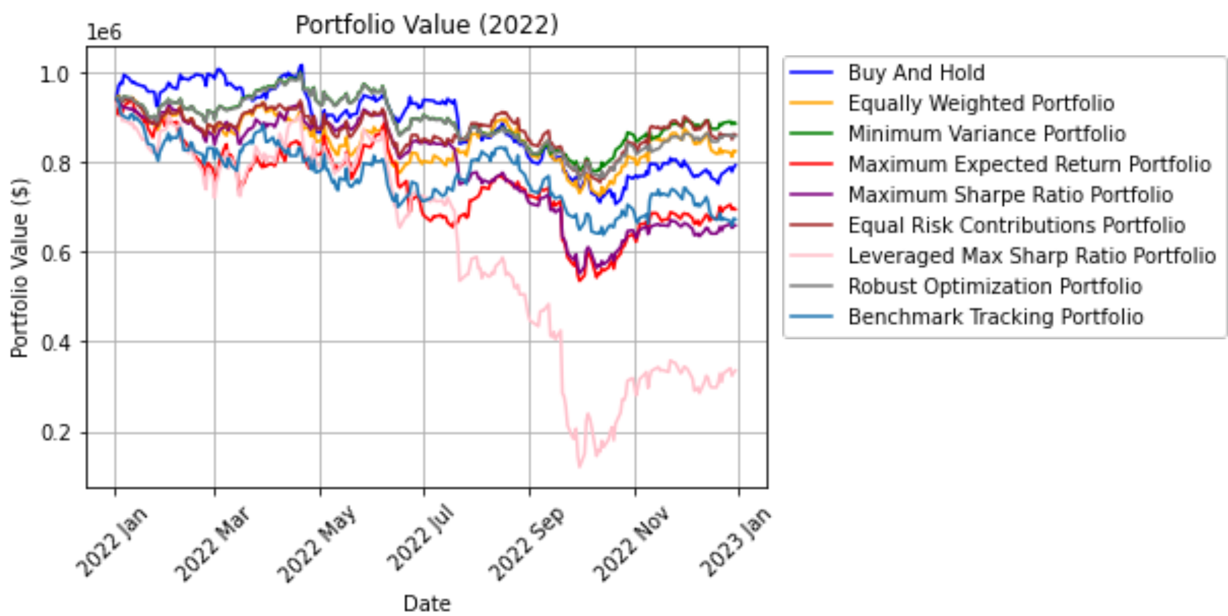
```
    mu = np.mean(cur_returns, axis = 0)
    Q = np.cov(cur_returns.T)
```

```
Initial portfolio value = $ 950205.84


Period 1: start date 01/03/2022, end date 02/28/2022
  Strategy "Buy and Hold", value begin = $ 950205.84, value end = $ 9
92093.09, cash account = $0.00
  Strategy "Equally Weighted Portfolio", value begin = $ 941384.79, v
alue end = $ 879062.82, cash account = $1788.78
  Strategy "Mininum Variance Portfolio", value begin = $ 944111.34, v
alue end = $ 910355.96, cash account = $766.42
  Strategy "Maximum Expected Return Portfolio", value begin = $ 94075
1.46, value end = $ 785443.70, cash account = $82.23
  Strategy "Maximum Sharpe Ratio Portfolio", value begin = $ 940752.3
6, value end = $ 860358.54, cash account = $261.70
Objective value at iteration #0 is - 5.10405e-07
Objective value at iteration #1 is - 5.10397e-07
Objective value at iteration #2 is - 2.14131e-07
Objective value at iteration #3 is - 2.14097e-07
```

In [21]:
```
config.r_rf
```

Out[21]:
```
0.0375
```

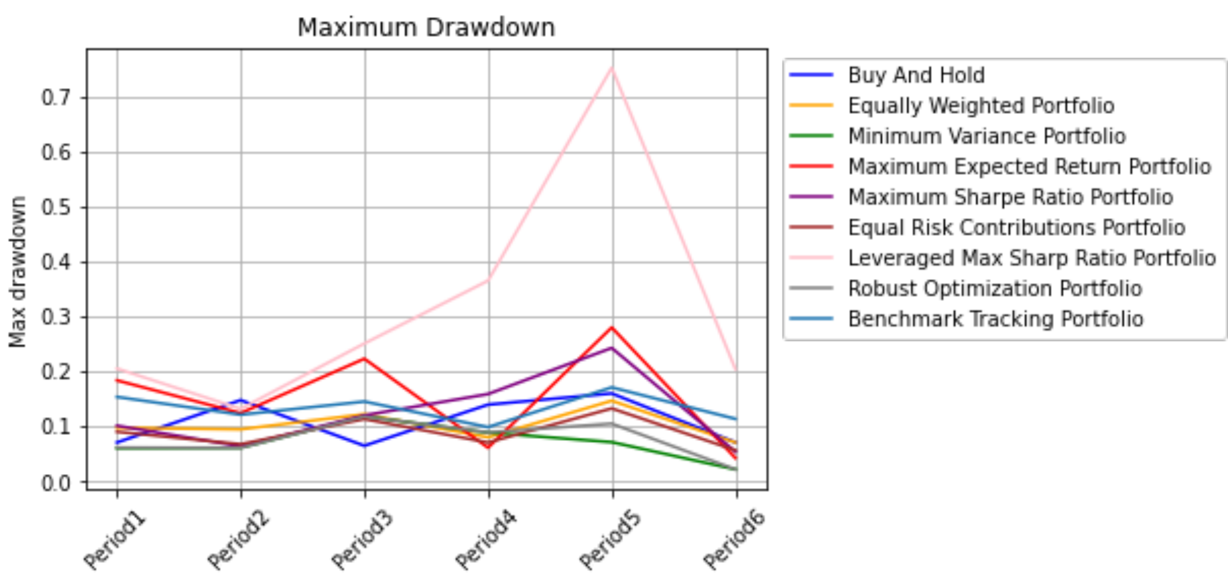
In [22]:
```
# Plot Line chart
# convert np.array[String] to np.array[datetime64]
dates_array_plot= pd.to_datetime(dates).to_numpy(dtype='datetime64[D]'
plot_portfolio_value(portf_value, dates_array_plot , "(2022)")
```



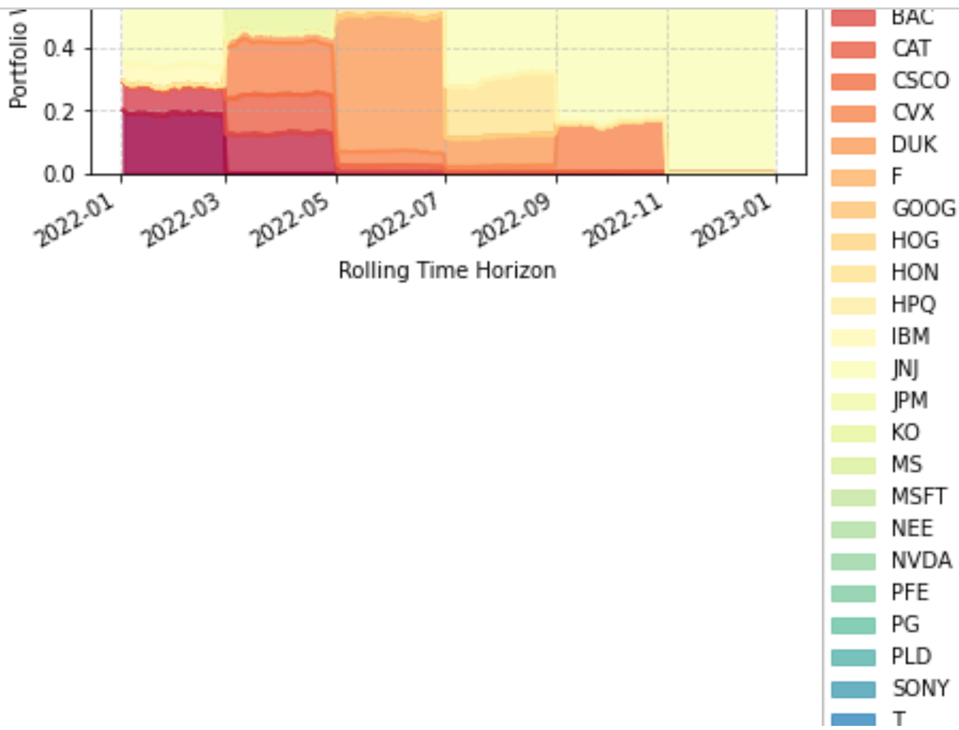
In [23]:
```
# Plot max drawdown

def get_max_drawdown_data_2022(N_periods, dates_array):
    start_max_drawdown=[]
    for i in range(len(strategy_functions)):
        max_drawdown=[]
        for period in range(1, N_periods+1):
           cur_year = 2022

           cur_month = 2*((period-1)%6) + 1

           day_ind_start = min([i for i, val in enumerate((dates_array
           day_ind_end = max([i for i, val in enumerate((dates_array[:


           #print(day_ind_end," ", day_ind_start)
           #print(np.maximum.accumulate(portf_value[i][day_ind_start-2
           #print('\nPeriod {0}: start date {1}, end date {2}'.format(
           #print(day_ind_end," ", day_ind_start)


           peak = np.maximum.accumulate(portf_value[i][day_ind_start:c
           drawdown = (peak - portf_value[i][day_ind_start:day_ind_enc
           max_drawdown.append(np.max(drawdown))
        start_max_drawdown.append(max_drawdown)
    return start_max_drawdown

# function to plot portfolio value
def plot_max_drawdown(max_drawdown_data):
  # Create the plot
  x = np.arange(1, 7)  # 12 periods (1 to 12)
  #print(max_drawdown_data[0])
  plt.plot(x, max_drawdown_data[0], label="Buy And Hold", color="blue"
  plt.plot(x, max_drawdown_data[1],label="Equally Weighted Portfolio",
  plt.plot(x, max_drawdown_data[2],label="Minimum Variance Portfolio",
  plt.plot(x, max_drawdown_data[3],label="Maximum Expected Return Port
  plt.plot(x, max_drawdown_data[4],label="Maximum Sharpe Ratio Portfol
  plt.plot(x, max_drawdown_data[5],label="Equal Risk Contributions Por
  plt.plot(x, max_drawdown_data[6],label="Leveraged Max Sharp Ratio Po
  plt.plot(x, max_drawdown_data[7],label="Robust Optimization Portfoli
  plt.plot(x, max_drawdown_data[8],label="Benchmark Tracking Portfolio

  # Add labels and title
  #plt.xlabel("Date")
  plt.ylabel("Max drawdown")
  title = "Maximum Drawdown"
  plt.title(title)
  x_labels = [f'Period{i}' for i in range(1, 7)]
```

```
    # Rotate labels for better readability
    plt.xticks(ticks=x, labels=x_labels, rotation=45)
    plt.grid(True)
    #plt.legend()
    plt.legend(loc="upper left", bbox_to_anchor=(1.01, 1))

    # Show the plot
    plt.show()

max_drawdown_data=get_max_drawdown_data_2022(N_periods, dates_array)
print(max_drawdown_data)
plot_max_drawdown(max_drawdown_data)
```

```
[[0.07023202410195113, 0.14714007641663682, 0.06425654743875601, 0.13
898561858185285, 0.15937478007682224, 0.07042413679045122], [0.096964
92419609738, 0.09416494921394769, 0.12162811953890988, 0.079736117591
73424, 0.14608386756413008, 0.07022876869980707], [0.0600385933639826
4, 0.06071182384985307, 0.11755005736101108, 0.08810507019662567, 0.0
7082153696804412, 0.021394684864875318], [0.1830948192429032, 0.12359
088230789121, 0.22245241082662007, 0.06073816579175777, 0.27926104290
9582, 0.04201258211394035], [0.10068684109583255, 0.0632731119363393,
0.11949877162851695, 0.15822225496205328, 0.24205726953771553, 0.0533
3511801317898], [0.08971576886175821, 0.06708226295782996, 0.11281982
645410894, 0.07018547681748907, 0.1321313236968682, 0.056169511152335
74], [0.20452158625778816, 0.1313968232172084, 0.2500922794484809, 0.
3642808929387523, 0.7511343257231117, 0.20374749107608536], [0.061010
811875237735, 0.060447650039015706, 0.11739075406951731, 0.0885084992
0510544, 0.10443055558619278, 0.021399719844857067], [0.1530407748969
1606, 0.12074014511839994, 0.14462532519269827, 0.09812310300506398,
0.1703522848510444, 0.11312723537257943]]
```



In [24]:
```
#time_horizon = np.arange(time_periods)
dates_array_plot= pd.to_datetime(dates).to_numpy(dtype='datetime64[D]'
asset_labels = df.columns[1:]


# Convert data into a Pandas DataFrame
df_3 = pd.DataFrame(portfolio_allocation[2], columns=asset_labels, inc
df_4 = pd.DataFrame(portfolio_allocation[3], columns=asset_labels, inc
df_5 = pd.DataFrame(portfolio_allocation[4], columns=asset_labels, inc
df_8 = pd.DataFrame(portfolio_allocation[7], columns=asset_labels, inc
# df_5 = d.DataFrame(strategy_5, columns=asset_labels, index=time_hori


# Plot for
plot_pandas_portfolio(df_3, "Minimum Variance Portfolio")

# # Plot for Strategy 4
plot_pandas_portfolio(df_4, "Maximum Expected Return Portfolio")

# # Plot for Strategy 5
plot_pandas_portfolio(df_5, "Maximum Sharpe Ratio Portfolio")

# # Plot for Strategy 8
plot_pandas_portfolio(df_8, "Robust Optimization Portfolio")
```




In [ ]: