Asset Pricing

This report aims to analyze and compare the performance of option pricing estimation using the Black-Scholes model and Geometric Brownian Motion. The Black-Scholes formula provides a one-step price estimation after one year, whereas Geometric Brownian Motion enables a Monte Carlo simulation to model dynamic price fluctuations over a one-year period.

Result analysis

The Geometric Brownian Motion simulates option prices using two methods: single-step and multi-step. In both approaches, a sample size of 1,000,000 is used. The multi-step Monte Carlo process divides the one-year period into 10 time intervals, simulating price movements at each step until the end of the period.

For the European option, the average call option price is 8.0, slightly higher than the average put option price of 7.9. Since the market and asset setup align with the Black-Scholes formula assumptions, we can consider the Black-Scholes model as providing a reliable benchmark for option pricing. The price estimation from the multi-step Monte Carlo method is significantly closer to this true value compared to the one-step Monte Carlo estimation.

For the Barrier option, the average call option price is considerably higher than the put option price due to the option expiring worthless if the underlying asset fails to exceed the barrier price of 110. Notably, the one-step put option is priced at approximately 0.0, as the stock price cannot simultaneously exceed 110 and remain below the strike price of 105 in a single step. In contrast, the multi-step barrier put option has a nonzero value, as a limited number of simulated paths surpass 110 at some point before reverting below 105 by the end of the period. Consequently, the barrier put option is priced significantly lower than the European put option.

However, the Barrier call option is only slightly lower in price compared to the European call option. This is because a substantial number of price paths exceed both the 110 barrier and the 105 strike price by the end of the period, resulting in minimal impact on the overall valuation.

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Black-Scholes price of an European call option is 8.021352235143176
Black-Scholes price of an European put option is 7.9004418077181455
One-step MC price of an European call option is 8.033377607552046
One-step MC price of an European put option is 7.898817657222702
Multi-step MC price of an European call option is 8.020499120477375
Multi-step MC price of an European put option is 7.903030234256964
One-step MC price of an Barrier call option is 7.793726511954623
One-step MC price of an Barrier put option is 0.0
Multi-step MC price of an Barrier call option is 7.949122867523895
Multi-step MC price of an Barrier put option is 1.200745316600754
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Under the below scenario of 10% rally or decline of volatility, one could find a positive correlated pattern between option price and volatility. A much volatile stock leads to a much expensive option price. As a much higher volatility could reflect a much greater amount of movement at each step. However, in terms of call option, for those who move below strike price will end up to zero anyways, but those paths move above strike price by the end then become much higher than the regular volatility and Vice Versa. Similar to the put option, both sides' movements end up with a much greater amount than before, and so for those paths moves below strike price become much lower stock price with higher profitability.

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Increase volatility by 10%:
One-step MC price of an Barrier call option is 8.605381857748053
One-step MC price of an Barrier put option is 0.0
Multi-step MC price of an Barrier call option is 8.747058920749303
Multi-step MC price of an Barrier put option is 1.504789646231103
Decrease volatility by 10%:
One-step MC price of an Barrier call option is 6.997131424015212
One-step MC price of an Barrier put option is 0.0
Multi-step MC price of an Barrier call option is 7.137963829804451
Multi-step MC price of an Barrier put option is 0.9103999398036619
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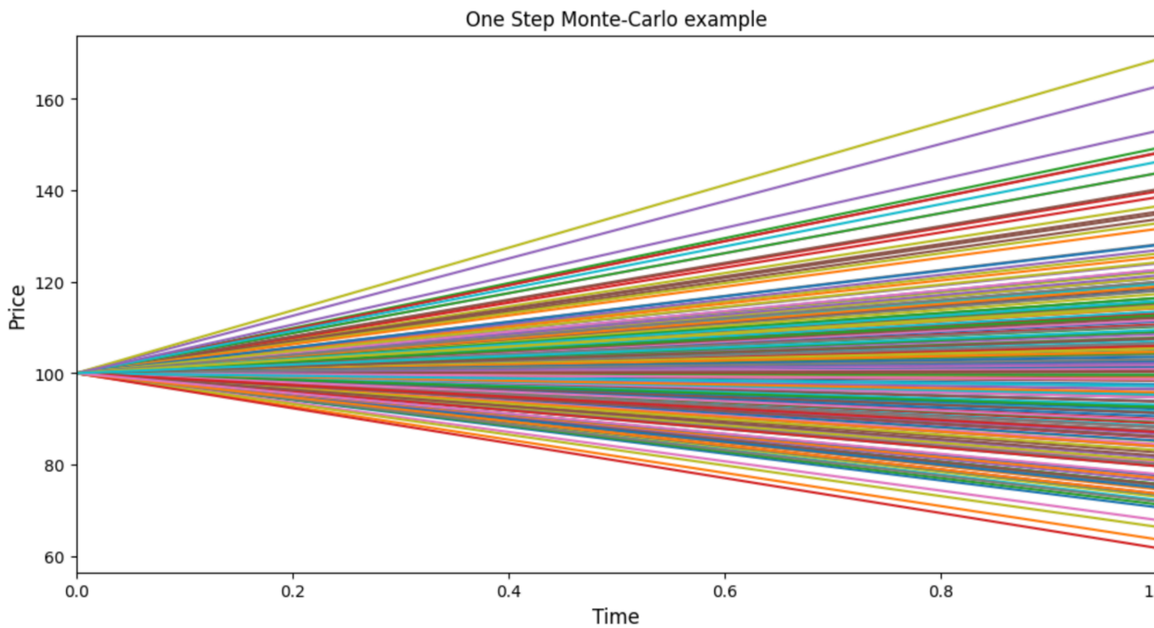
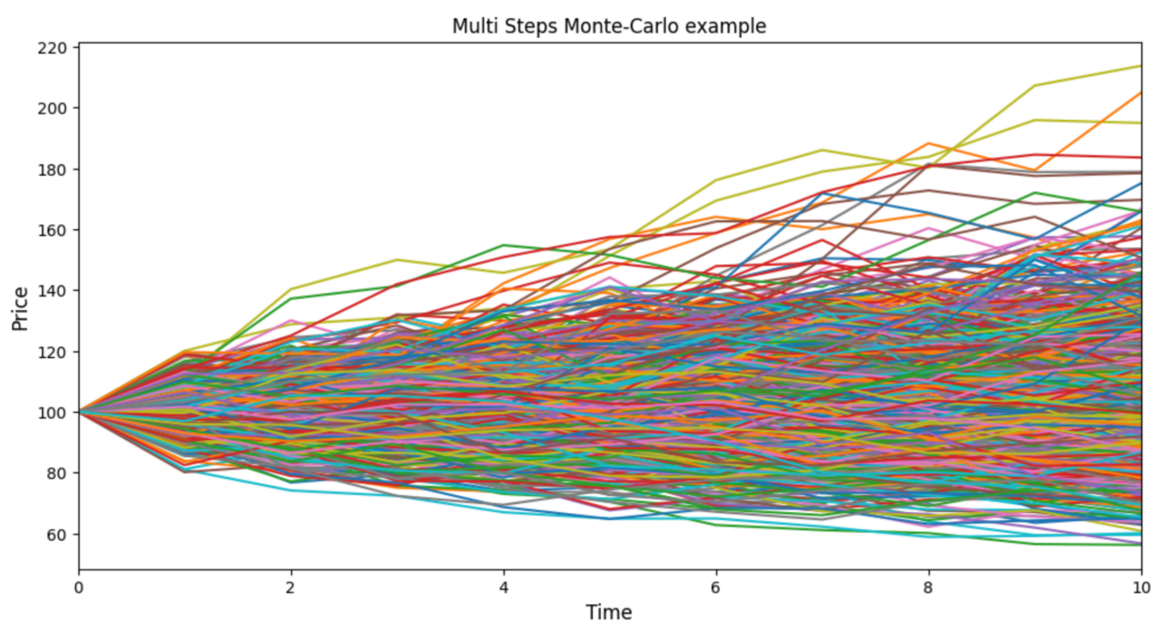
Plot

Here is the plot of the one-step simulation, displaying the first 200 selected paths. The majority of these sample paths conclude within the price range of 80 to 120, reflecting the natural variability of the simulated stock movements under the Geometric Brownian Motion model.



For the multi-step simulation, the first 1000 paths are displayed in the graph below. Similar to the one-step simulation, most paths are concentrated within the 80 to 120 range. Notably, the fluctuations are more pronounced for paths above the initial price compared to those below. This is because the same multiplicative factor applied to a higher price results in a larger absolute change in subsequent steps, leading to greater variability in upward-moving paths.



Strategies Improvement

To improve accuracy, we can establish a predefined absolute error threshold of 0.005, ensuring that the difference between the true option price and the Monte Carlo estimation remains within this limit. The process begins with a small number of time steps N and a limited number of simulation paths M. We then iteratively refine the model by increasing N to divide the fixed time horizon into smaller intervals—potentially down to each second—and increasing M to incorporate more simulated paths. This approach helps to smooth out fluctuations in results, reduce variance, and enhance the overall precision of the estimation until the target error threshold is achieved.