Credit Risk Modeling and Simulation

The primary objective of this report is to simulate bond losses over a one-year period and analyze the results by comparing two Monte Carlo-generated loss datasets with the true distribution. All three loss datasets incorporate both systematic and idiosyncratic risk, with values drawn from a normal distribution.

Loss Result

Here are the results for VaR and CVaR of the loss. Overall, Portfolio 1 exhibits significantly higher VaR and CVaR compared to Portfolio 2. This suggests that Portfolio 1 is more susceptible to extreme losses compared to Portfolio 2, likely due to the higher portfolio weights assigned to high-value bonds. The performance of Monte Carlo 1 and Monte Carlo 2 is quite similar, both closely approximating the actual values, though they slightly underestimate tail risk. In contrast, the Normal model severely underestimates tail risk. Notably, the actual 99.9% VaR is nearly twice the Normal approximation, and this underestimation extends to CVaR, which represents the average loss beyond the corresponding VaR threshold.

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Portfolio 1:

Out-of-sample: VaR 99.0% = $37282631.02, CVaR 99.0% = $45038705.42
In-sample MC1: VaR 99.0% = $37199183.26, CVaR 99.0% = $44573729.30
In-sample MC2: VaR 99.0% = $37185325.95, CVaR 99.0% = $44550141.17
In-sample No: VaR 99.0% = $26162490.95, CVaR 99.0% = $29053311.75
In-sample N1: VaR 99.0% = $26231617.31, CVaR 99.0% = $29123024.33
In-sample N2: VaR 99.0% = $26182559.60, CVaR 99.0% = $29069417.23

Out-of-sample: VaR 99.9% = $54519973.36, CVaR 99.9% = $62980086.24
In-sample MC1: VaR 99.9% = $53667406.01, CVaR 99.9% = $60367350.91
In-sample MC2: VaR 99.9% = $53614630.41, CVaR 99.9% = $60711359.62
In-sample No: VaR 99.9% = $32679080.84, CVaR 99.9% = $35040915.27
In-sample N1: VaR 99.9% = $32749528.68, CVaR 99.9% = $35111842.05
In-sample N2: VaR 99.9% = $32690215.57, CVaR 99.9% = $35048812.04


Portfolio 2:

Out-of-sample: VaR 99.0% = $27420869.74, CVaR 99.0% = $33729733.86
In-sample MC1: VaR 99.0% = $27436853.28, CVaR 99.0% = $33238940.71
In-sample MC2: VaR 99.0% = $27292814.67, CVaR 99.0% = $33422077.88
In-sample No: VaR 99.0% = $21087794.13, CVaR 99.0% = $23255162.77
In-sample N1: VaR 99.0% = $21052901.27, CVaR 99.0% = $23210506.97
In-sample N2: VaR 99.0% = $21089218.70, CVaR 99.0% = $23253053.43

Out-of-sample: VaR 99.9% = $41440595.67, CVaR 99.9% = $48613020.30
In-sample MC1: VaR 99.9% = $40547046.28, CVaR 99.9% = $45747478.94
In-sample MC2: VaR 99.9% = $40839008.02, CVaR 99.9% = $47508667.76
In-sample No: VaR 99.9% = $25973552.72, CVaR 99.9% = $27744318.26
In-sample N1: VaR 99.9% = $25916651.89, CVaR 99.9% = $27679441.00
In-sample N2: VaR 99.9% = $25967011.02, CVaR 99.9% = $27734889.31
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As shown in the figure below, Portfolio 1 exhibits greater variance compared to Portfolio 2. By averaging the mean and standard deviation over 100 trials, we observe that the sample variance remains consistent between Monte Carlo 1 and Monte Carlo 2. Since the results indicate a close alignment in mean and standard deviation between the actual loss sample and the Monte Carlo estimates, this suggests that a sample size of 5,000 may be sufficiently large to approximate the mean and variance of the loss distribution accurately.

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Portfolio 1:

Out-of-sample: Mean =  6316746.7493652  Std =  8530858.356532618
In-sample MC1 average of 100 trials: Mean =  6381848.654341359  Std =  8532588.300593285
In-sample MC2 average of 100 trials: Mean =  6364022.93721756  Std =  8519162.971568126

Portfolio 2:

Out-of-sample: Mean =  6208613.948005141  Std =  6395939.466982296
In-sample MC1 average of 100 trials: Mean =  6240744.568389106  Std =  6367128.867495955
In-sample MC2 average of 100 trials: Mean =  6234299.158832099  Std =  6385510.8296006145
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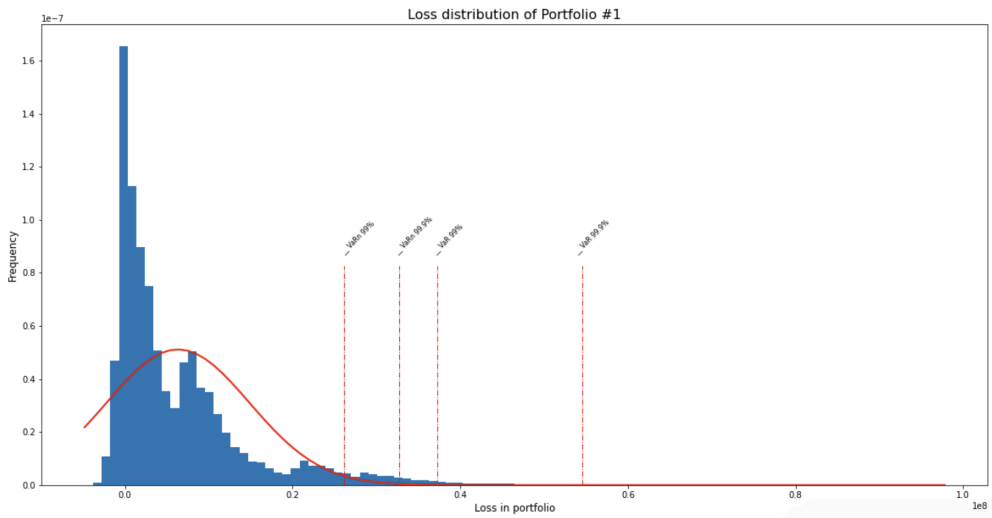
Loss Distribution

Portfolio 1

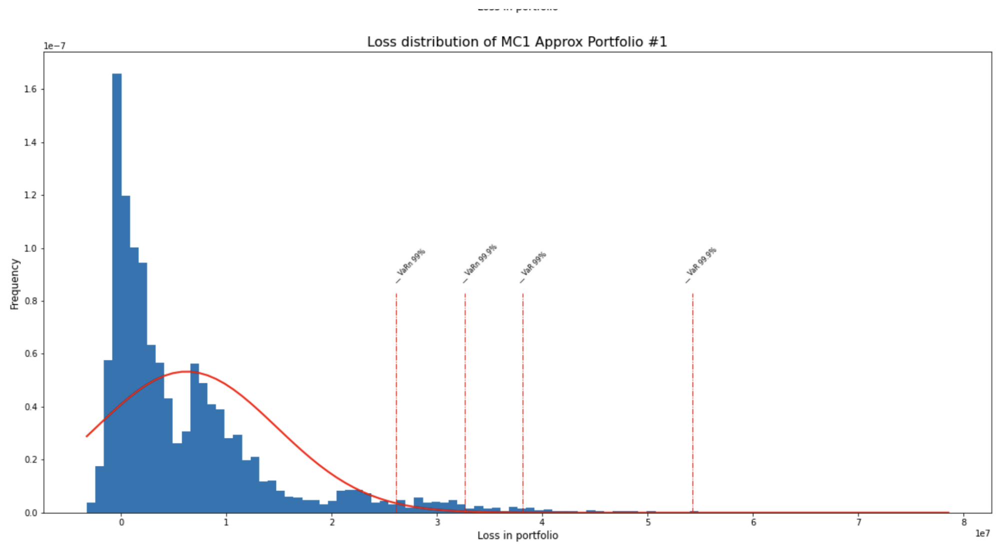
Plot the true loss distribution of Portfolio 1. The graph below reveals a fat right tail, indicating a higher probability of extreme losses. The red curve represents a normal distribution with the same mean and standard deviation as the out-of-sample data for comparison.

Most of the sample data is highly concentrated around zero loss, with a gradual decline in frequency as losses increase. However, the normal distribution overestimates the probability of losses around 2×1072×107 while significantly underestimating the frequency of extreme losses beyond 3×1073×107.

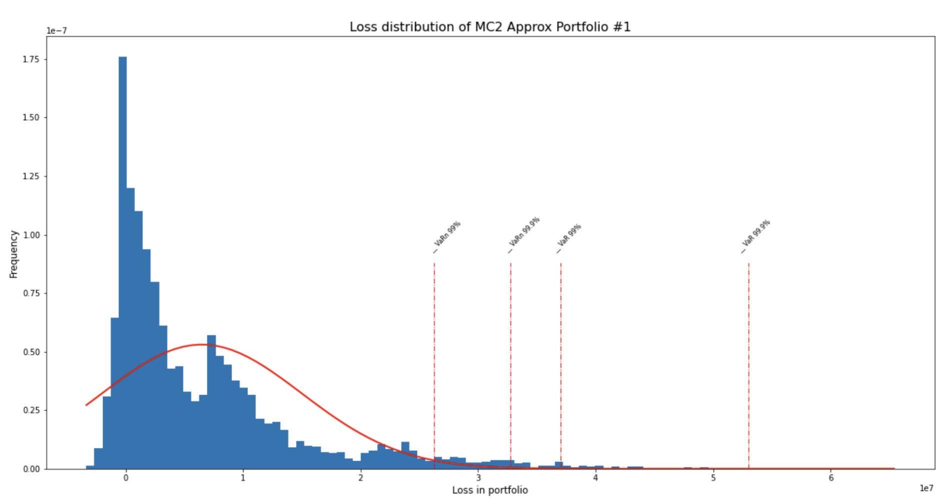
By examining the difference between VaRnVaRn and VaRVaR, we can see that the marginal portfolio loss at the 99.9% VaR level is even more underestimated than at the 99% level. As shown in the graph, VaRnVaRn at 99.9% is lower than the actual 99% VaR, further highlighting the inability of the normal approximation to capture tail risk effectively.



The Monte Carlo distribution shown is from one of the 100 trials. Compared to the true distribution, Monte Carlo 1 exhibits a higher concentration of samples around zero loss, with fewer data points distributed on the right tail, indicating a potential underestimation of extreme losses.Despite these differences, the overall shape of the Monte Carlo distribution aligns closely with the true distribution, as reflected in the almost overlapping bar plots.

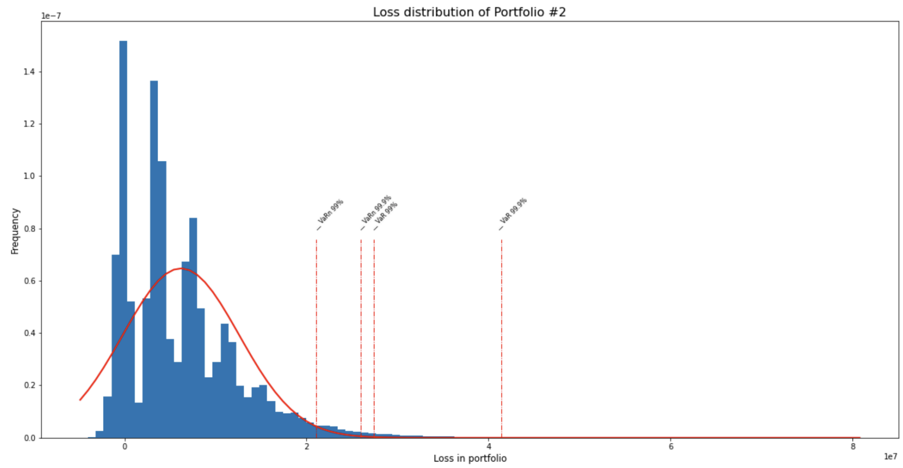


Below is the approximation of Monte Carlo 2. Compared to both the true distribution and Monte Carlo 1, Monte Carlo 2 exhibits an even higher concentration of samples around zero loss, suggesting a potential bias toward underestimating larger losses.

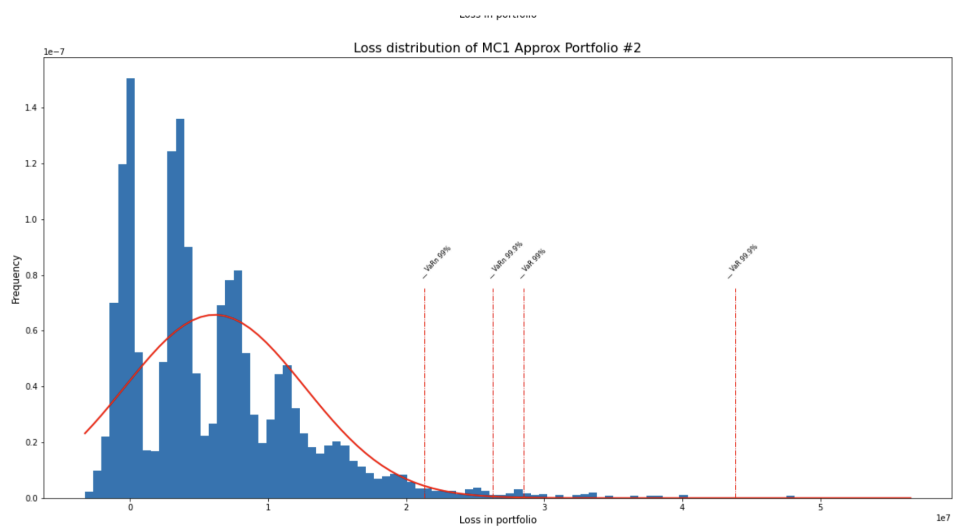


Portfolio 2

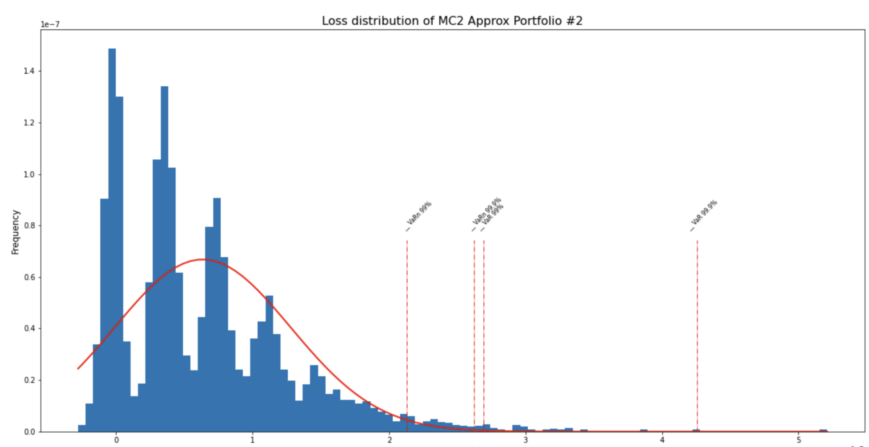
However, the true distribution of portfolio 2 exhibits multiple peaks while still maintaining a fat-tailed shape. Notably, the highest concentration remains around zero loss, but several local maxima are observed between 0 and 2e7. The height of each peak gradually decreases as the loss increases. Similar to portfolio 1, the distribution around zero is significantly higher than that of the normal distribution. However, unlike portfolio 1, the gap between VaRn and VaR is much smaller, potentially indicating that portfolio 2's tail behavior is more aligned with a normal distribution compared to portfolio 1.



Here is the Monte Carlo 1 distribution, where the data is even more concentrated around zero loss compared to the true distribution. This indicates that Monte Carlo 1 underestimates the probability of extreme losses, leading to fewer observations in the tail region.



For Monte Carlo 2, it also shows a similar pattern to the true distribution with a fat tail shape



Sampling Error

The sampling error is calculated based on the Z-score, standard deviation, and sample size. Portfolio 1 exhibits a significantly larger sampling error compared to Portfolio 2. This is likely due to Portfolio 1's heavier investment in higher-priced bonds, which may contribute more to the sampling error. As a result, Portfolio 1 experiences greater variability in its sample compared to Portfolio 2. Additionally, Monte Carlo simulations tend to show larger sampling errors, indicating a greater deviation relative to the sample size. This suggests that Monte Carlo's approximation may not be as precise as expected, especially when dealing with portfolios heavily influenced by high-price bonds.

	Out-of-Sample	MC1	MC2
Portfolio 1	52873.836307	236506.935148	236134.810851
Portfolio 2	39641.715086	176484.564952	176994.077583

Model Error

To evaluate the model error, the absolute error of VaR and VaRn is calculated, as shown in the table below. Since the mean and standard deviation are consistent between the Normal and Non-Normal models, the analysis focuses on how well each model captures the tail risk.

For Portfolio 1, it is observed that as we approach the tail of the loss distribution, the absolute error increases. Monte Carlo 2 tends to have slightly larger errors than Monte Carlo 1, but overall, both have smaller errors compared to the true model. This reinforces the conclusion that smaller sample sizes are less likely to capture extreme loss events, potentially leading to an underestimation of risk. This finding highlights the importance of using sufficiently large sample sizes in Monte Carlo simulations to accurately model extreme tail risks.

	VaR 99%	VaR 99.9%	CVaR 99%	CVaR 99.9%
True	1.112014e+07	2.184089e+07	1.598539e+07	2.793917e+07
MonteCarlo1	1.096757e+07	2.091788e+07	1.545070e+07	2.525551e+07
MonteCarlo2	1.100277e+07	2.092441e+07	1.548072e+07	2.566255e+07

For Portfolio 2, the overall model error is smaller compared to Portfolio 1, which may imply that the loss distribution of Portfolio 2 is much closer to a Normal distribution. This suggests that Portfolio 2 exhibits less extreme tail risk and a more symmetric distribution.

	VaR 99%	VaR 99.9%	CVaR 99%	CVaR 99.9%
True	6.333076e+06	1.546704e+07	1.047457e+07	2.086870e+07
MonteCarlo1	6.383952e+06	1.463039e+07	1.002843e+07	1.806804e+07
MonteCarlo2	6.203596e+06	1.487200e+07	1.016902e+07	1.977378e+07

Discussion

The impact of sampling error in the in-sample data could result in the underestimation of tail risk. Since a smaller sample size is insufficient to capture the low probability of extreme losses, a larger sample size in Monte Carlo simulations would provide a more accurate approximation of VaR and CVaR.

However, the primary contributor to the underestimation of loss is likely model error. The smaller tail of the Normal distribution fails to account for extreme loss events, significantly underestimating the risk. Regulatory bodies typically require investment companies to reserve sufficient capital, such as three times the VaR loss, to protect against extreme losses. If the actual loss exceeds the financial reserves, it could lead to a financial crisis.

To minimize sampling error, one can increase the sample size in Monte Carlo simulations and apply variance reduction techniques. Additionally, it is beneficial to incorporate a wider range of Monte Carlo scenarios, including multiple systematic and idiosyncratic risks. To reduce model error, selecting the appropriate distribution model for the Monte Carlo simulation is essential. Rather than relying solely on the Normal distribution, models like the T-distribution, which captures fat tails, can better reflect the risk of extreme events. Furthermore, conducting stress tests to simulate extreme economic conditions helps identify fat tails and extreme losses, improving the model’s ability to predict rare, high-impact events.