Risk-based & Roust Portfolio Selection Strategies

The primary objective of this report is to analyze and compare the result of the following 5 different portfolio selection strategies: “Equal risk contributions”, “Leveraged max Sharpe Ratio”,“Robust mean-variance optimization” and “Benchmark tracking optimization”.

In particular, "Robust Mean-Variance Optimization" requires setting both the robustness bound and the minimum return bound. To determine the robustness bound, we first calculate the Sharpe ratio of each asset as follows:

$$\text{sharpe}_i = \frac{\mu_i - r_f}{\sigma_i}$$

Then, we take the mean of all individual Sharpe ratios and set the robustness bound to half of its absolute value.

$$\epsilon_{\text{rob}} = \left| \frac{\text{avg\_sharpe}}{2} \right|$$

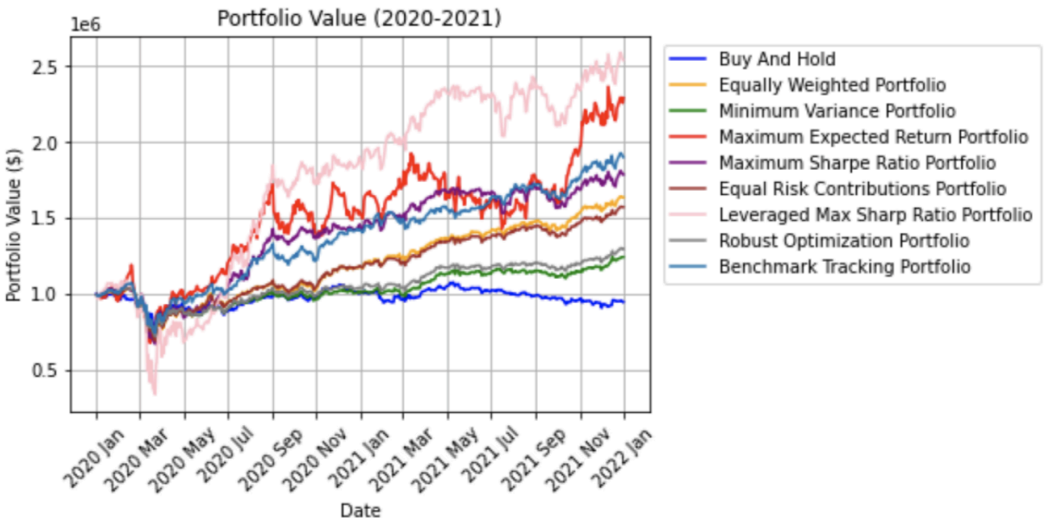
As for the minimum return bound, we select a reference portfolio,specifically, an equally weighted portfolio, and calculate its initial return. A reasonable expected return is then set at 5% above the reference portfolio's return.

$$\epsilon_{\text{ret}} = 1.05 \times r_{init}$$

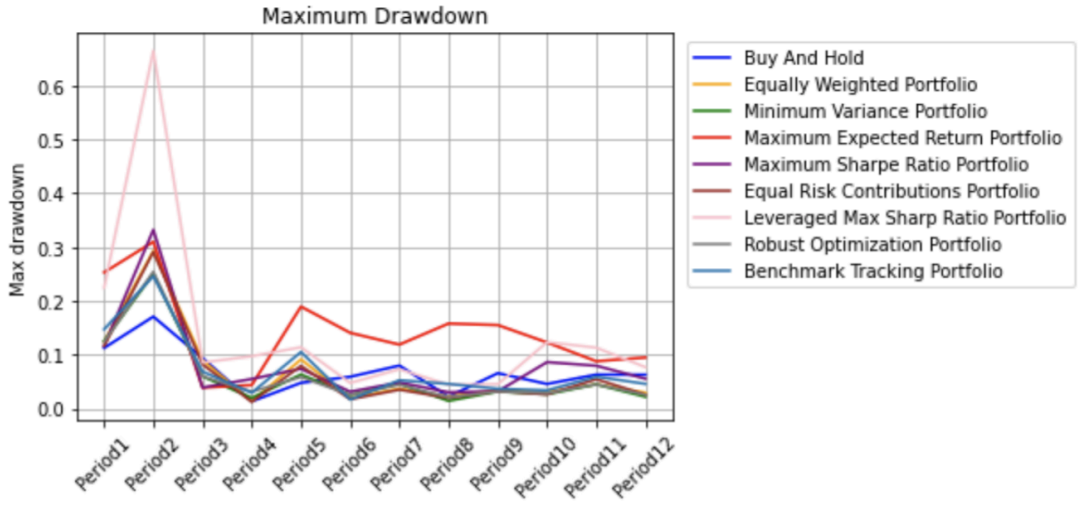
2020-2021 Result Analysis

The initial total asset is around $1 million, and various portfolio strategies were applied from 2020 to 2021 (12 periods), with a rebalancing strategy implemented every two months.
The dynamic portfolio value fluctuates significantly across different strategies. Notably, the Leveraged Max Sharpe Ratio Portfolio achieved the highest total asset, surpassing $2.5 million by the end of 2021.
The benchmark tracking portfolio and equally weighted strategy curves exhibited nearly identical performance curves, suggesting that most stocks contributed similar levels of risk. While the benchmark tracking strategy showed an overall upward trend, its performance remained slightly below that of the maximum expected return strategy.

Robust optimization strategies consistently outperformed the minimum variance portfolio, as it built upon and optimized its framework.

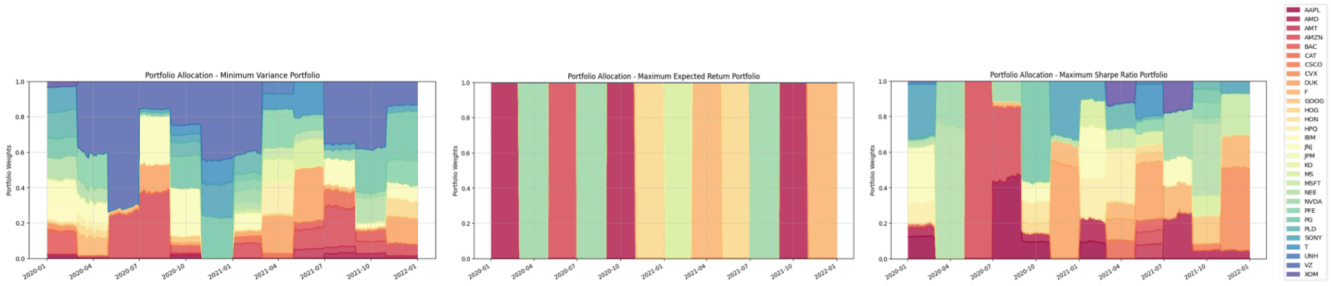


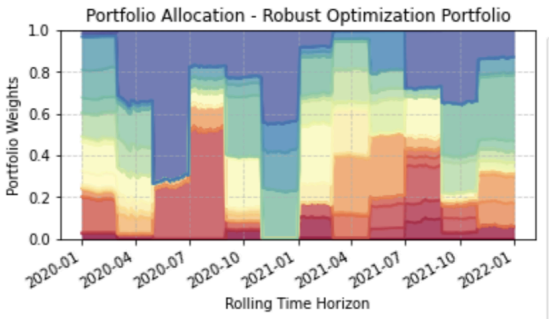
The maximum drawdown curve tracks the largest decline in portfolio value within each period. While all curves follow a similar overall trend, the Leveraged Max Sharpe Ratio strategy experiences a significantly higher drawdown during the bearish phase at the beginning of 2020 compared to other strategies. Conversely, during the bullish period, the Maximum Expected Return strategy exhibits the highest maximum drawdown, likely due to the large fluctuations in high-return stocks.



Robust Portfolio Optimization is a variation of the Minimum Variance Portfolio, incorporating additional constraints on variance and return bounds. As a result, their portfolio allocation plots nearly overlap throughout the entire period, with the same stock selections but slight differences in weight distribution.

Compared to the Maximum Expected Return Portfolio and the Maximum Sharpe Ratio Portfolio, Robust Portfolio Optimization exhibits a more diverse selection of stocks.





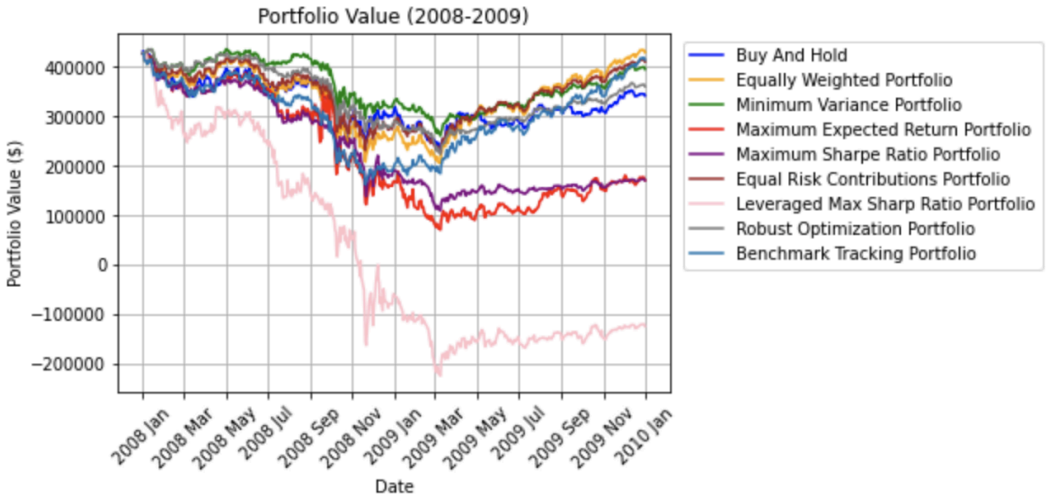
After comparing the performance of these strategies from 2020 to 2021, I personally prefer the Maximum Sharpe Ratio Benchmark Tracking Portfolio strategies. The reason is their stable upward trend and lower volatility compared to the Leveraged Max Sharpe Ratio and Maximum Expected Return strategies. Although the latter two generate higher returns during bullish markets, they also come with greater fluctuations. Meanwhile, the remaining strategies performed slightly below the average market return for 2020 and 2021.

2008-2009 Result Analysis

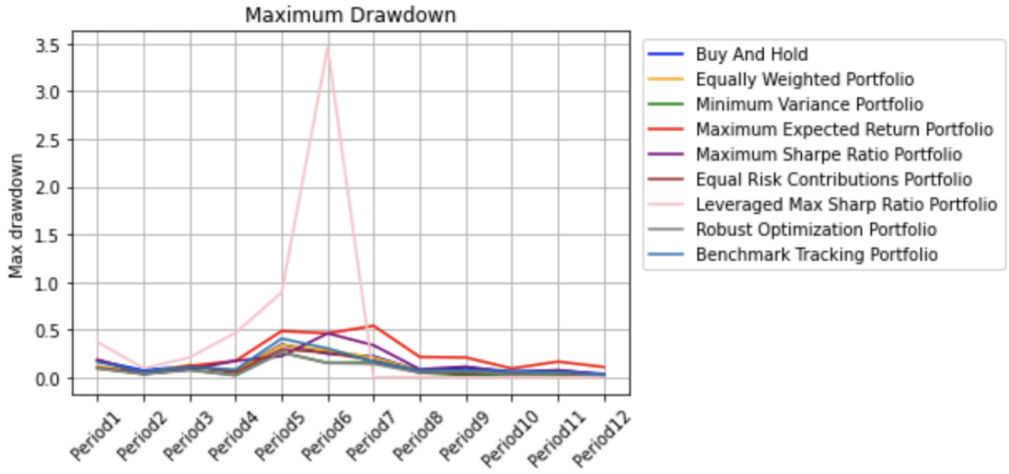
When applying all nine strategies during the financial crisis period, we observe a stark contrast in performance. While 2020–2021 was a bullish period with an overall upward trend, here, all portfolios started with an initial value of approximately $40,000. Over time, most strategies—including the Benchmark Tracking Portfolio, Equal Risk Contribution Portfolio, and Robust Optimization Portfolio—experienced declines in value by early 2009 but gradually recovered to near their initial levels by the end of the year.

The Leveraged Max Sharpe Ratio Portfolio, which had outperformed all other strategies in 2020–2021, performed the worst during 2008–2009, even reaching negative portfolio value—likely due to the loss of both the original capital and borrowed funds. Similarly, the Maximum Sharpe Ratio and Maximum Expected Return portfolios ended the period with less than 50% of their initial value.

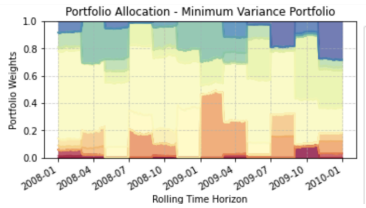
Unlike the 2020–2021 period, the Minimum Variance Portfolio slightly outperformed the Robust Optimization Portfolio during this downturn.

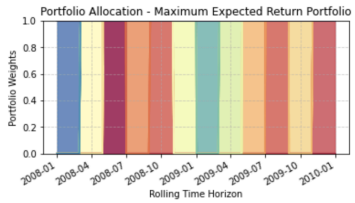


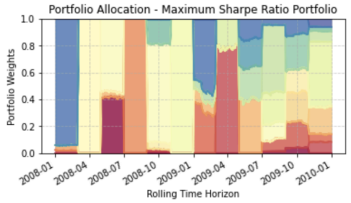
Similar to the maximum drawdown plot for 2020–2021, most strategies experienced their largest drawdowns between Period 5 and Period 7, with values remaining below 0.5. However, the Leveraged Max Sharpe Ratio Portfolio once again exhibited a significantly larger drawdown—approximately 6 to 7 times greater than all other strategies. This substantial decline in portfolio value reveals the high-risk nature of leverage techniques.

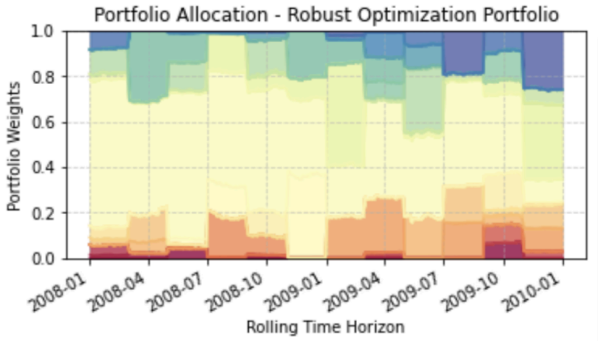


Examining the portfolio allocation of the Robust Optimization Portfolio, we observe that stock selection was far more limited during 2008–2009 compared to 2020–2021, even a less diverse stock selection than the Maximum Sharpe Ratio Portfolio, likely due to the high variance in stock prices during the financial crisis. While the stock choices across these strategies remain largely similar, the allocation weights differ significantly.









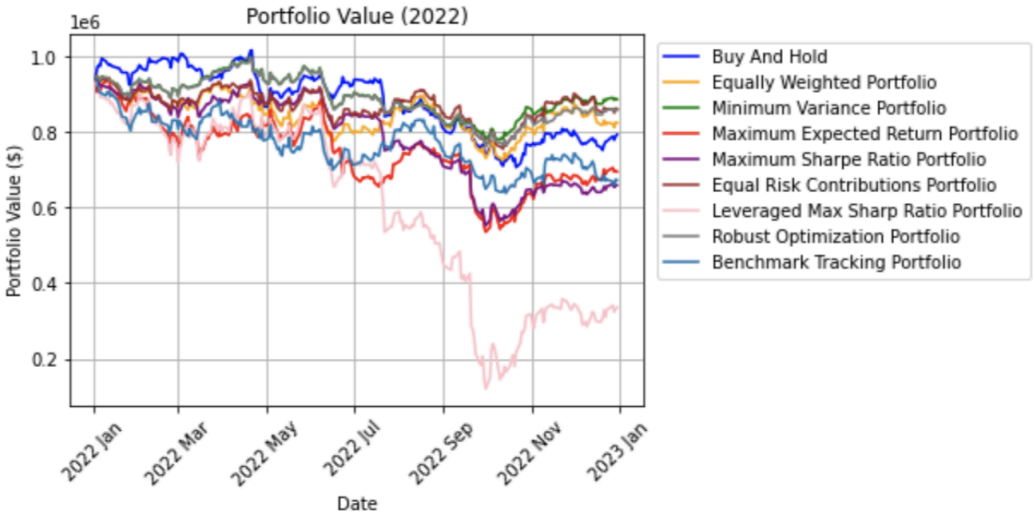
During the 2008–2009 financial crisis, I would prefer to choose a Minimum Variance Portfolio as my strategy. This approach minimizes substantial losses, experiences lower maximum drawdowns, and has less fluctuation, all while being able to recover from the crisis.

2022 Result Analysis

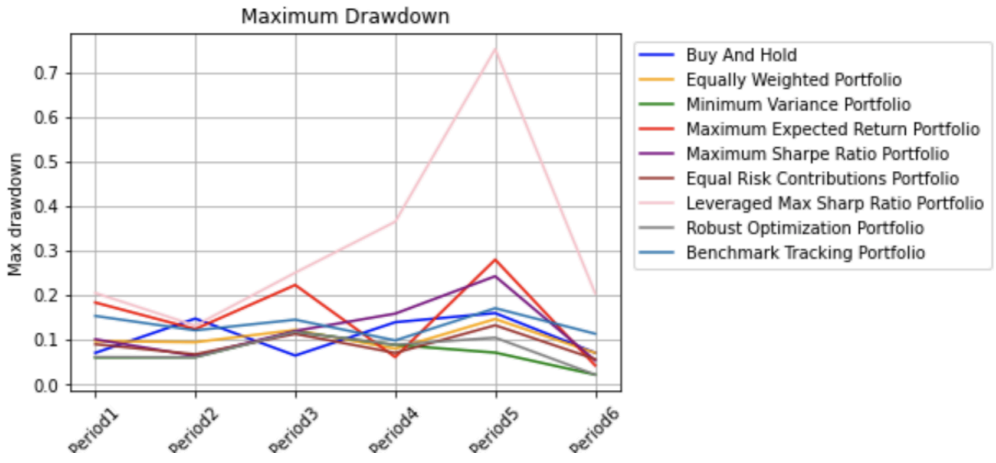
During the bearish period in 2022, all strategies followed a declining trend and ended below their initial values. Unlike the financial crisis, where all portfolios experienced a sharp drop over a short period, the curves here oscillated downward over time. As before, the Leveraged Sharpe Ratio portfolio had the worst performance, losing more than 60% by the end of the period.

The Robust Optimization Portfolio consistently overlapped with the Minimum Variance Portfolio at the beginning but slightly underperformed by the end. The Minimum Variance Portfolio consistently outperformed other strategies during the bearish period.

Both the Equal Risk Contribution and Benchmark Tracking Portfolios also performed better than the Leveraged Max Sharpe Ratio strategy.



Although the stock market in 2022 again faced bearish conditions, the overall maximum drawdown was less severe than during the 2008–2009 financial crisis. The Leveraged Max Sharpe Ratio portfolio once again experienced the highest drawdown, with the Maximum Expected Return portfolio ranking second in terms of drawdown. The Minimum Variance Portfolio experienced the least drop in portfolio value on average, suffering the smallest overall loss.



Compared to the 2008–2009 financial crisis triggered by Lehman Brothers, which had a more dramatic impact on the market, the 2022 recession was also a period of economic downturn but with less severe consequences. Despite the differences, one commonality between both periods is that the portfolio value did not continue to decrease indefinitely; instead, it began to recover and bounce back after a certain point.