Mean-Variance Portfolio Selection Strategies

This primary objective of this report is to analyze and compare the result of the following 5 different portfolio selection strategies: Buy and Hold', 'Equally Weighted Portfolio', 'Minimum Variance Portfolio', 'Maximum Expected Return Portfolio', 'Maximum Sharpe Ratio Portfolio'.

Mathematical Formulation

The mathematical formulation is the underlying implementation logic of each strategy

Minimum Variance Portfolio
This quadratic objective function aims to minimize variance, where the weight vector is the variable to be optimized, and **Q** represents the covariance matrix derived from historical stock price data. The constraints ensure that each weight w_i is non-negative and that the total portfolio weight sums to 1.

$$\begin{aligned} \min_{w} \quad & w^T Q w \\ \text{s.t.} \quad & \textstyle\sum_i w_i = 1 \\ & w \geq 0 \end{aligned}$$

Maximum Expected Return Portfolio
This linear-quadratic function aims to maximize expected return, where the weight vector is the variable to be optimized, and **μ** represents the vector of daily expected returns for each stock. The constraints ensure that each stock's weight is greater than zero and that the total portfolio weight sums exactly to one.

$$\begin{aligned} \max_{w} \quad & \mu^T w \\ \text{s.t.} \quad & \textstyle\sum_i w_i = 1 \\ & w \geq 0 \end{aligned}$$

Maximum Sharpe Ratio Portfolio
The sharpe ratio formula and its variables are defined below, where both the daily expected return and daily risk free rate are calculated based on 252 trading days per year.

$$\max_{w \in \mathcal{F}} \quad \frac{\mu^T w - r_f}{\sqrt{w^T Q w}} \sqrt{252},$$

μ is the vector of daily expected returns,
r_f is the daily risk-free rate,
Q is the covariance matrix,
$\mathcal{F}$ is the feasible set (e.g., $\{w : \mathbf{1}^T w = 1,\ w \geq 0\}$).

However, the quadratic objective function inputted into the CPLEX solver is derived by transforming the original Sharpe ratio formula into its reciprocal form. This reformulation ensures a convex optimization problem, making it more suitable for quadratic programming. Additionally, the constraints are designed to ensure that the sum of all portfolio weights equals one.

$$\min_{y \in \mathbb{R}^n,\ \kappa > 0} \quad y^T Q y$$

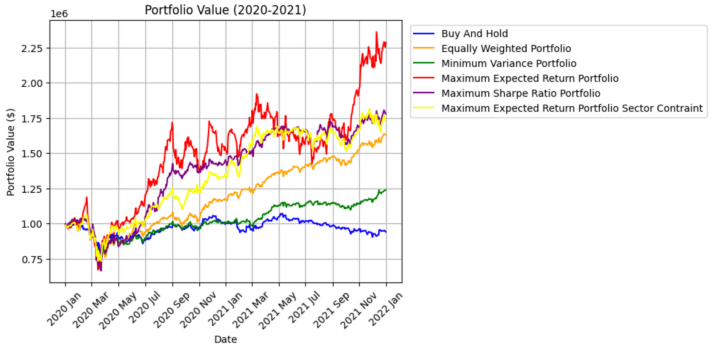
$$\text{s.t.} \quad \sum_{i=1}^{n} (\mu_i - r_f)\, y_i = 1,$$

$$\sum_{i=1}^{n} y_i = \kappa,$$

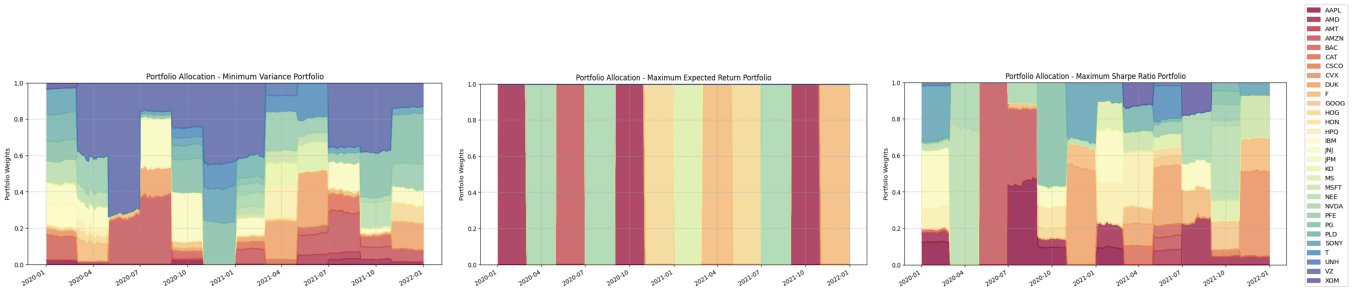
$$y \geq 0.$$

Result Analysis

The initial total asset is $1 million, and various portfolio strategies were applied over 2020 to 2021 (12 periods), with a rebalancing strategy implemented every two months.
The dynamic portfolio value fluctuates significantly across different strategies. Notably, the Maximum Expected Return Portfolio achieved the highest total asset, surpassing $2.25 million by the end of 2021. In contrast, the Buy and Hold strategy remained relatively stable, fluctuating around $1 million throughout the period.



The Minimum Variance Portfolio strategy consistently holds a broader selection of stocks compared to other strategies, with stock choices remaining relatively stable while only the weights adjust over time. Notably, XOM maintains a high allocation for much of the period, as its low price volatility aligns well with the variance minimization objective.In contrast, the Maximum Expected Return Portfolio tends to concentrate entirely on a single stock—the one with the highest expected return at each rebalancing point. Meanwhile, the Maximum Sharpe Ratio Portfolio exhibits greater variability in stock selection over time, adjusting more dynamically compared to the other two strategies.



In terms of risk measures, the Minimum Variance Portfolio strategy exhibits the lowest variance, making it the most stable among all strategies. In contrast, the Maximum Expected Return Portfolio experiences the highest variance and largest maximum drawdown, reflecting its aggressive return-seeking nature. Both the Maximum Expected Return Portfolio and Maximum Sharpe Ratio Portfolio achieve higher Sharpe ratios (around 1) compared to the Minimum Variance Portfolio, indicating better risk-adjusted returns.

```
Variance over 2020-2021
   Stategy - strat_buy_and_hold: 2632184618.58
   Stategy - strat_equally_weighted: 54444957267.77
   Stategy - strat_min_variance: 11160870476.50
   Stategy - strat_max_return: 128331795595.33
   Stategy - strat_max_Sharpe: 92754872835.84
   Stategy - strat_max_return_sector_constraints: 87276705614.73
Maximum Drawdown (MDD) over 2020-2021
   Stategy - strat_buy_and_hold: 0.22
   Stategy - strat_equally_weighted: 0.34
   Stategy - strat_min_variance: 0.31
   Stategy - strat_max_return: 0.43
   Stategy - strat_max_Sharpe: 0.36
   Stategy - strat_max_return_sector_constraints: 0.31
Sharpe Ratio over 2020-2021
   Stategy - strat_buy_and_hold: -0.11
   Stategy - strat_equally_weighted: 1.02
   Stategy - strat_min_variance: 0.57
   Stategy - strat_max_return: 1.09
   Stategy - strat_max_Sharpe: 1.04
   Stategy - strat_max_return_sector_constraints: 1.01
```

Overall, while the Maximum Expected Return Portfolio delivers the highest returns, it comes with significant volatility, making it suitable for risk-tolerant investors. On the other hand, the Maximum Sharpe Ratio Portfolio provides a balanced approach, optimizing both returns and risk, making it a stronger choice for investors seeking a well-rounded strategy.

Possible Improvements

We believe different variation on the current strategies may lead to better result, such as selecting the “1/n” portfolio at the beginning and hold it till the end of period 12 resulted in a final total asset value of $1,660,288.84, outperforming the standard Equally Weighted Strategy. However, this approach still yielded lower returns compared to the Maximum Expected Return Portfolio and Maximum Sharpe Ratio Portfolio.

Another improvement involves introducing sector diversification constraints, limiting each sector’s allocation to ≤ 25%. By applying this constraint to the Maximum Expected Return Portfolio, we obtained a smoother portfolio value curve over time. However, the final asset value of $1,399,437.35 was lower than the unconstrained model. The Portfolio Value Over Time graph indicates that while this constrained model maintained an upward trend, it underperformed the standard Maximum Expected Return Portfolio during bullish markets. During the second half of 2024, a market pullback caused the constrained portfolio to achieve an almost identical final return to the standard Maximum Expected Return Portfolio, but with significantly lower risk and fluctuation.

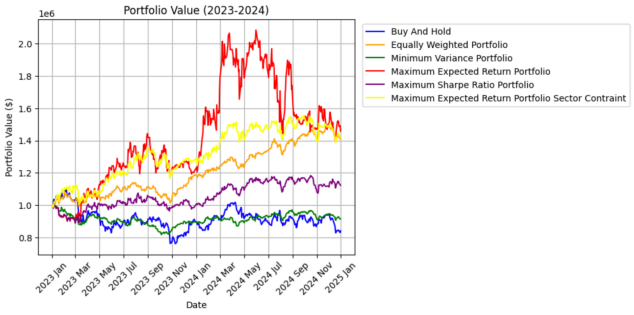
The mean-variance model is highly sensitive to estimation risk, often leading to unstable optimal solutions due to misestimation of risk and return. Additionally, the mean-variance framework assumes a single-period model, whereas long-term investors may require multi-period approaches. Several improvements could be enforced, such as: Resampling and risk parity strategies to achieve better diversification, Robust optimization to mitigate sensitivity to input estimates and Multi-period optimization, accounting for bullish, bearish, and steady market conditions, to better align with long-term investment goals.

Retest 2023-2024 data

Considering the economic context of 2020, the pandemic significantly impacted the stock market, causing a substantial decline in Q1 and Q2 across all strategies. However, stock prices began to rise sharply after that, continuing until the peak at the end of 2021. During the bearish period, all strategies showed similar poor performance, but in the subsequent bullish phase, the ‘Maximum Expected Return Portfolio’ strategy outperformed others, benefiting from its higher sensitivity to market conditions. In contrast, the ‘Buy and Hold’ and ‘Minimum Variance Portfolio’ strategies were more stable and less sensitive to market fluctuations.

When retesting all strategies from 2023 to 2024, we found that the ‘Maximum Expected Return Portfolio’ still had the highest final portfolio value. However, in the second half of 2024, the market experienced a pullback, resulting in nearly a 25% loss, mainly due to a high concentration in a single stock and insufficient risk diversification. The tech sector, which typically offers the highest returns, also carries the highest risk compared to other sectors. Meanwhile, the ‘Minimum Variance Portfolio’ strategy focused solely on minimizing risk, leading to a portfolio loss after 12 periods due to neglecting potential returns.

Ultimately, by diversifying the portfolio into other sectors—similar to the approaches used by the ‘Maximum Sharpe Ratio’ and ‘Equally Weighted Portfolio’ models—risk could be spread across a broader base, with only a small sacrifice in returns. Balancing both risk and return is crucial for achieving optimal performance.



Chatgpt generated Summary

Below is the report generated by the AI tool (Chatgpt), it archives a high clarity and accuracy of the overall contents:

Executive Summary: Mean-Variance Portfolio Selection Strategies

Overview

This report evaluates five portfolio selection strategies—**Buy and Hold**, **Equally Weighted Portfolio**, **Minimum Variance Portfolio**, **Maximum Expected Return Portfolio**, and **Maximum Sharpe Ratio Portfolio**—using a mean-variance optimization framework. The analysis spans from 2020 to 2021 with portfolio rebalancing every two months. The primary objective is to assess risk-return trade-offs and identify the most effective strategy for various investor profiles.

Key Findings

1. Performance Comparison:
    - The **Maximum Expected Return Portfolio** achieved the highest final asset value (> $2.25M), but at the cost of **high volatility and risk**.
    - The **Buy and Hold strategy** remained stable around $1M, showing limited growth but low risk.
    - The **Minimum Variance Portfolio** exhibited **low variance** and **higher diversification**, with ExxonMobil (XOM) maintaining a **strong presence** due to its historically stable stock prices.
    - The **Maximum Sharpe Ratio Portfolio** balanced return and risk, achieving a Sharpe ratio of ~1, outperforming Minimum Variance but with **higher stock turnover**.
2. Risk Analysis:
    - The **Minimum Variance Portfolio** had the **lowest variance** and drawdown risk.

    - The **Maximum Expected Return Portfolio** had the **highest volatility** and was prone to market pullbacks.
    - The **Maximum Sharpe Ratio Portfolio** provided a balance between risk and return, making it suitable for moderate-risk investors.

Recommendations & Improvements

- **Sector Diversification Constraint:** By limiting sector exposure to **≤25%**, the Maximum Expected Return Portfolio became more stable but had a lower final asset value (~$1.39M). This suggests that **diversification reduces risk but can cap upside potential**.
- Alternative Strategies:
    - A **1/n strategy** (holding an equal-weighted portfolio without rebalancing) resulted in a final asset value of ~$1.66M, outperforming standard equally weighted portfolios.
    - **Multi-period modeling** and **robust optimization** can improve performance by accounting for **market cycles and estimation errors**.

Retesting on 2023-2024 Data

- The **Maximum Expected Return Portfolio** remained the top performer but suffered **~25% loss in the second half of 2024** due to its **concentration in a single stock**.
- The **Minimum Variance Portfolio** focused on risk reduction but ultimately underperformed after 12 periods.
- **Blended strategies** (e.g., Maximum Sharpe Ratio or Equally Weighted) achieved **better risk-adjusted returns** by **spreading risk across multiple sectors**.

Conclusion

The **Maximum Expected Return Portfolio** offers the highest potential gain but is suited for **high-risk investors**. The **Maximum Sharpe Ratio Portfolio** presents a **balanced approach**, making it a strong candidate for investors seeking a **better risk-return trade-off**. Future improvements could include **multi-period optimization** and **risk-adjusted sector constraints** to enhance portfolio stability during volatile market conditions.