例题： 求右图带电球面球心处场强。

解1： 化为“点”的问题，取小面元

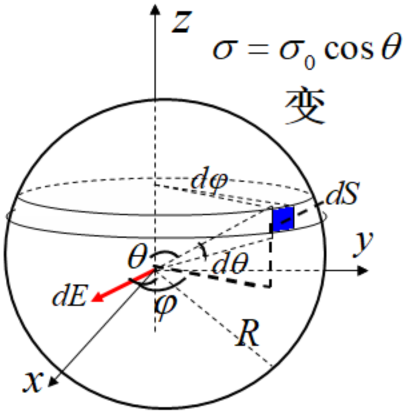
$$dS = R^2 \sin\theta d\theta d\varphi, \sigma = \sigma_0 \cos\theta$$

$$dq = \sigma dS = \sigma R^2 \sin\theta d\theta d\varphi = \sigma_0 \cos\theta R^2 \sin\theta d\theta d\varphi$$

$$dE = \frac{dq}{4\pi\varepsilon_0 R^2} = \frac{(\sigma_0 \cos\theta) R^2 \sin\theta d\theta d\varphi}{4\pi\varepsilon_0 R^2}$$

$$= \frac{\sigma_0 \cos\theta \sin\theta d\theta d\varphi}{4\pi\varepsilon_0}$$

由对称性分析 **E**方向向下



$$E = -\int dE \cos\theta = -\int_0^{2\pi} d\varphi \int_0^{\pi} \frac{\sigma_0 \cos^2\theta \sin\theta d\theta}{4\pi\varepsilon_0} = -\frac{\sigma_0}{3\varepsilon_0} \qquad \vec{E} = -\frac{\sigma_0}{3\varepsilon_0}\vec{k}$$

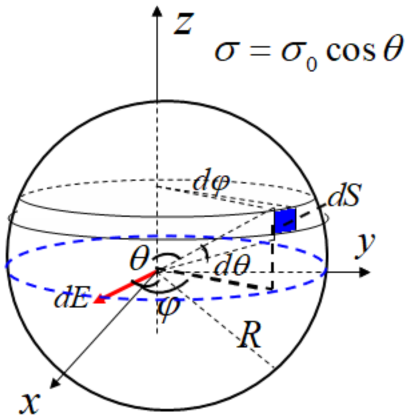
解2：

或化为“线”的问题，利用圆环结论： $E=\dfrac{qx}{4\pi\varepsilon_0(x^2+r^2)^{3/2}}$；

环带面积:$2\pi R\sin\theta\times Rd\theta$； $x^2+r^2=R^2$ ；$x=R\cos\theta$

$$dE=\frac{\sigma_0\cos\theta\times2\pi R\sin\theta\times Rd\theta\times R\cos\theta}{4\pi\varepsilon_0R^3}$$

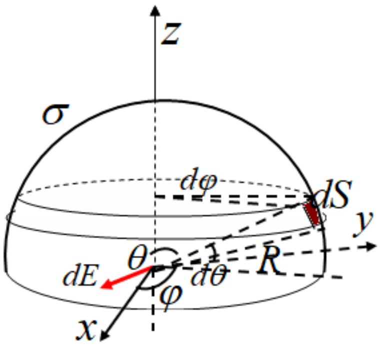
$$E=\int_0^\pi\frac{\sigma_0\cos\theta\times2\pi R\sin\theta\times Rd\theta\times R\cos\theta}{4\pi\varepsilon_0R^3}=\frac{\sigma_0}{3\varepsilon_0}$$



练习： 均匀带电半球面球心处的场强？

$$dq=\sigma dS \qquad dS=R^2\sin\theta d\theta d\varphi$$

$$dE=\frac{dq}{4\pi\varepsilon_0 r^2}$$



$$dE=\frac{\sigma R^2\sin\theta d\theta d\varphi}{4\pi\varepsilon_0 R^2},$$

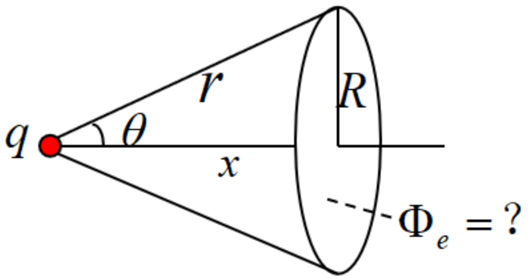
$$E=-\int dE\cos\theta=-\iint\frac{\sigma_0 R^2\sin\theta\cos\theta d\varphi}{4\pi\varepsilon_0 R^2}$$

$$=\int_0^{2\pi}d\varphi\int_0^{\frac{\pi}{2}}\cos\theta(\cos\theta)/4\pi\varepsilon_0$$

$$=-\frac{\sigma}{4\varepsilon_0}$$

$$\vec{E}=-\frac{\sigma}{4\varepsilon_0}\vec{k}$$

例题2：求圆面的电通量 $\Phi = \iint_S \vec{E} \cdot d\vec{S}$？

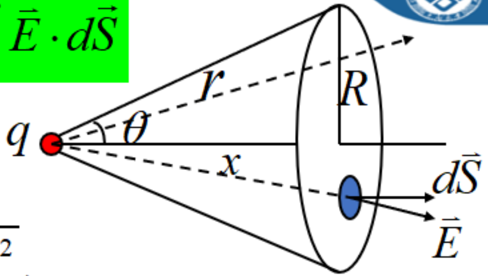


$$\Phi_e = \iint \vec{E}\cdot d\vec{S}$$

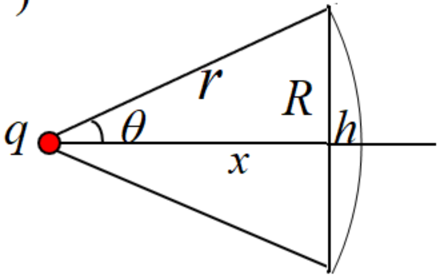
解:

球冠面积

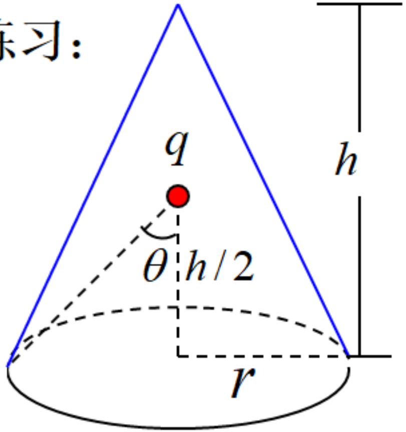
$$\Phi = \frac{q}{\varepsilon_0}\frac{2\pi rh}{4\pi r^2} \qquad h = r(1-\cos\theta)$$



$$\Phi_e = \frac{q}{2\varepsilon_0}(1-\cos\theta) = \frac{q}{2\varepsilon_0}\left(1-\frac{\sqrt{r^2-R^2}}{r}\right)$$



练习:



求侧面电通量

$$\Phi_{e侧} = \frac{q}{\varepsilon_0} - \Phi_{e底}$$

$$\Phi_{e底} = \frac{q}{2\varepsilon_0}(1-\cos\theta) = ?$$

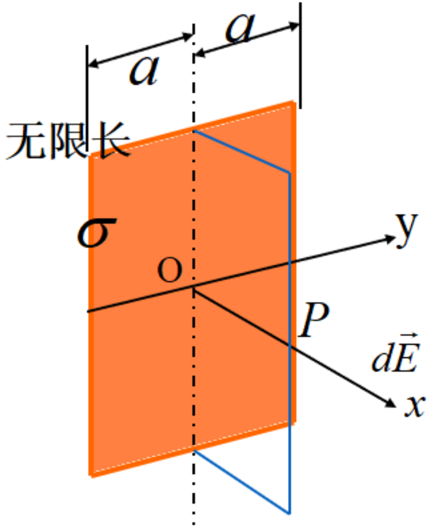
例题6：

求均匀无限长带电平板中垂面上场强。

? 想一想

能否用高斯定理直接求解？

无特殊对称性！

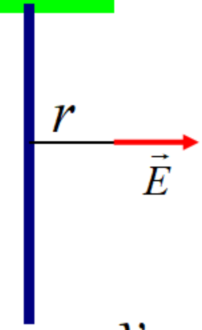


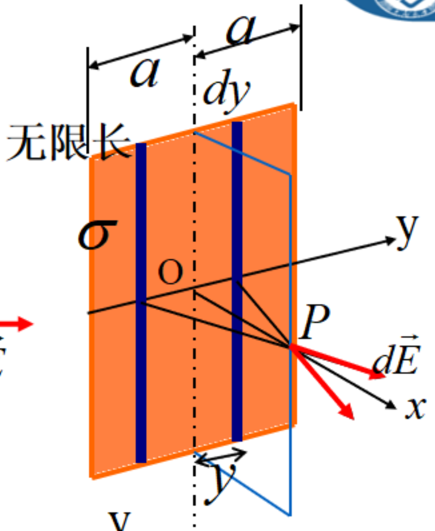
解： 视为无限多带电直线叠加

掌握基本方法！

高斯定理求带电直线场强结果：

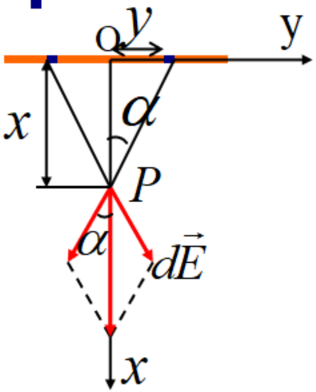
$$E = \frac{\lambda}{2\pi\varepsilon_0 r}$$





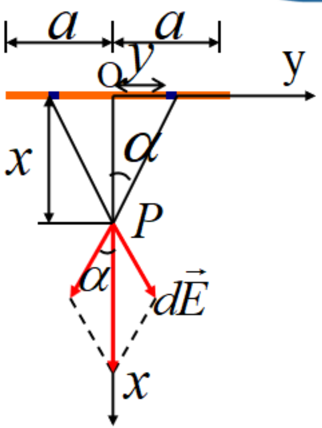
$$dE = \frac{\lambda}{2\pi\varepsilon_0 r} = \frac{\sigma dy \times 1}{2\pi\varepsilon_0\sqrt{x^2+y^2}}$$

$$dE_x = \frac{\sigma dy\cos\alpha}{2\pi\varepsilon_0\sqrt{x^2+y^2}}$$



$$E_y = 0$$

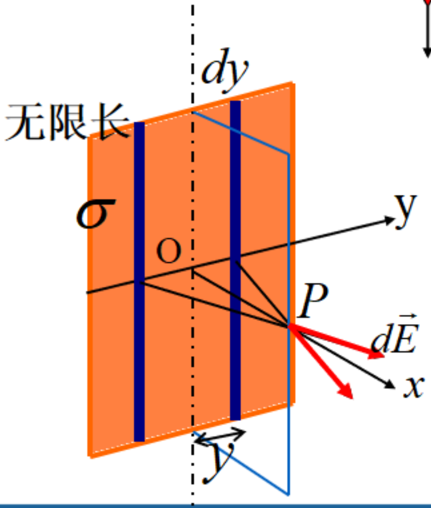
$$dE_x = \frac{\sigma dy \cos\alpha}{2\pi\varepsilon_0\sqrt{x^2 + y^2}}$$



$$E = \int dE_x = 2\int_0^a \frac{\sigma x dy}{2\pi\varepsilon_0(x^2 + y^2)} = \frac{\sigma}{\pi\varepsilon_0} tg^{-1}(\frac{a}{x})$$

$$a \to \infty \qquad E \to \frac{\sigma}{2\varepsilon_0}$$

无限大带电平面

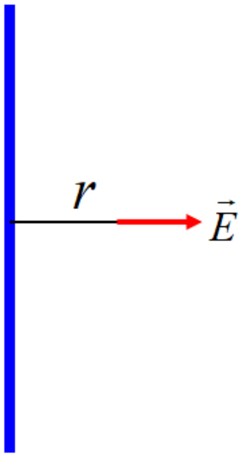


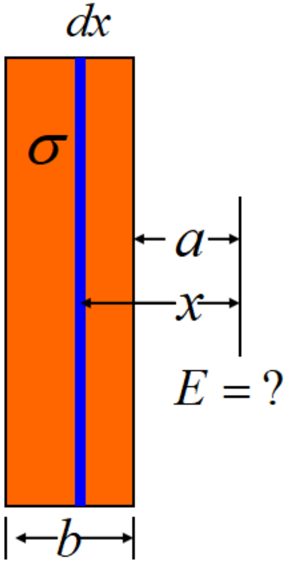
练习：宽为b且无限长带电平板，求距离为a处的场强

解： 视为无限长带电直线叠加

无限长带电直线叠加

$$E = \frac{\lambda}{2\pi\varepsilon_0 r}$$

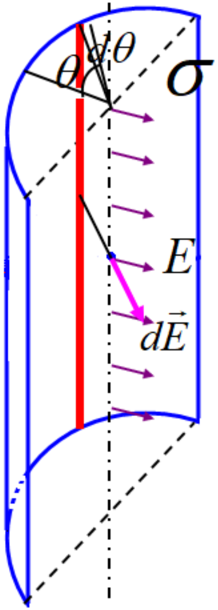


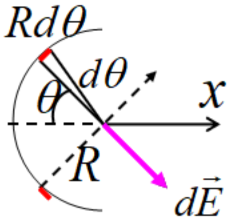


$$\lambda = \sigma dx \times 1 \qquad dE = \frac{\lambda}{2\pi\varepsilon_0 x}$$

$$E = \int_a^{a+b} \frac{\sigma dx}{2\pi\varepsilon_0 x} = \frac{\sigma}{2\pi\varepsilon_0} \ln \frac{a+b}{a}$$

练习： 无限长半圆柱面轴线上场强?



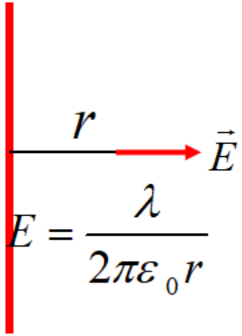


$$\lambda = \sigma R d\theta \times 1$$

$$dE = \frac{\lambda}{2\pi\varepsilon_0 R}$$

带电直线叠加

$$dE = \frac{\sigma R d\theta \times 1}{2\pi\varepsilon_0 R}$$



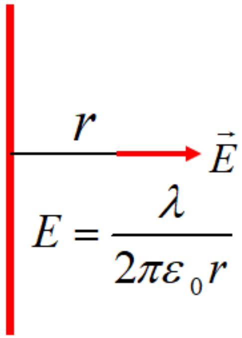
$$E = \frac{\lambda}{2\pi\varepsilon_0 r}$$

$$E = E_x = 2\int_0^{\frac{\pi}{2}} dE\cos\theta = \frac{\sigma}{\pi\varepsilon_0}$$ （对称性）

例题9：如图，无限长带电圆柱面，电荷面密度 $\sigma=\sigma_0\cos\varphi$

求轴线上场强。

解：　视无限多带电直线场强叠加

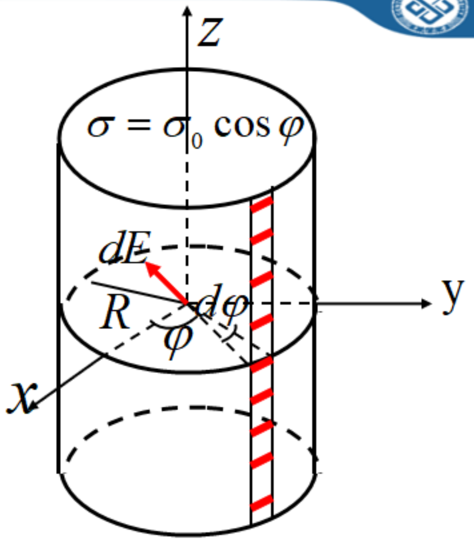


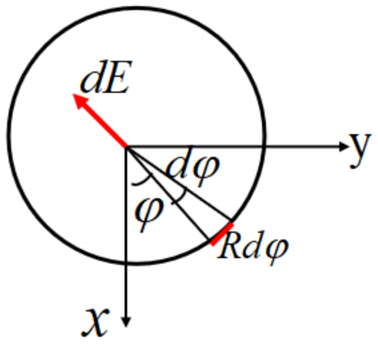
$$E=\frac{\lambda}{2\pi\varepsilon_0 r}$$

$$dE=\frac{\lambda}{2\pi\varepsilon_0 R}$$

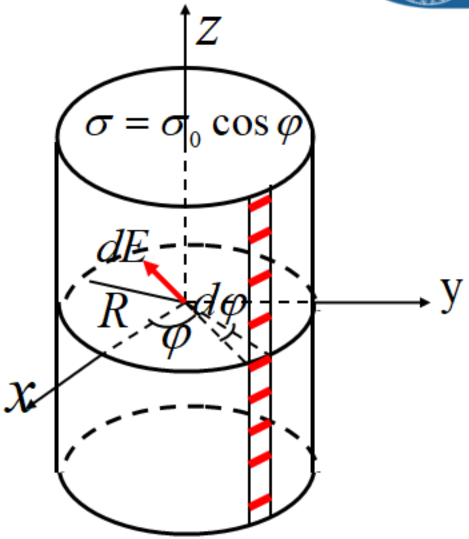
$$\lambda=\sigma R d\varphi\times 1$$

$$\sigma=\sigma_0\cos\varphi$$





$$\lambda = \sigma R d\varphi \times 1 \qquad dE = \frac{\lambda}{2\pi\varepsilon_0 R}$$

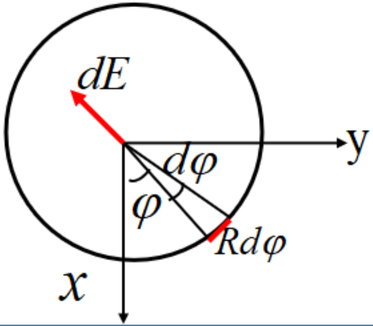


$$dE = \frac{\sigma R d\varphi \times 1}{2\pi\varepsilon_0 R} = \frac{\sigma_0 \cos\varphi d\varphi}{2\pi\varepsilon_0}$$

$$E_x = -\int dE\cos\varphi = -\frac{\sigma}{2\pi\varepsilon_0}\int_0^{2\pi}\cos^2\varphi d\varphi$$

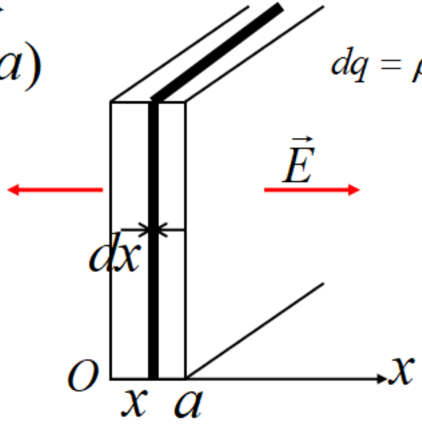
$$= -\frac{\sigma}{2\varepsilon_0}$$

$$E_y = -\int dE\sin\varphi = 0 \qquad \therefore \vec{E} = -\frac{\sigma}{2\varepsilon_0}\vec{i}$$



例题10：一带电平板，如图，电荷体密度为 $\rho = kx \quad (0 \le x \le a)$

求板外场强。



$dq = \rho S dx$;即 $\sigma = \dfrac{dq}{S} = \rho dx$

解：$dE = \dfrac{\sigma}{2\varepsilon_0} = \dfrac{\rho dx}{2\varepsilon_0} \qquad (S = 1)$

$$= \frac{kx}{2\varepsilon_0} dx$$

$\vec{E}$

$$E = \frac{\sigma}{2\varepsilon_0}$$

$$\therefore E = \int_0^a \frac{kx}{2\varepsilon_0} dx = \frac{ka^2}{4\varepsilon_0}$$

视为无限多带电平面产生的场强的叠加。

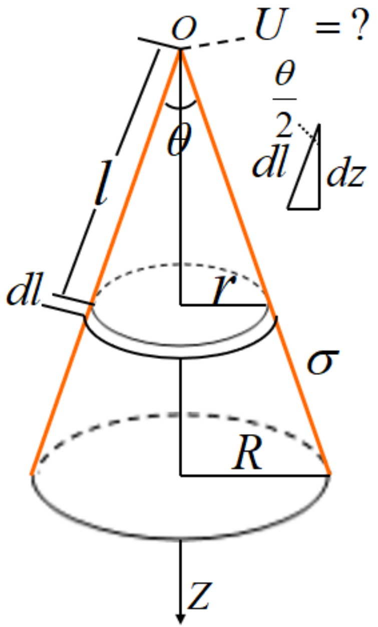
例题11： 如图，均匀带电圆锥面（σ ）。求尖锥处电势。

解：视无限多圆环构成 $dq = \sigma 2\pi r dl$

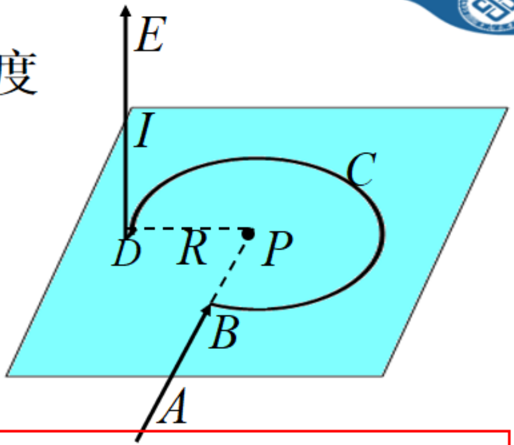
$$U = \int \frac{dq}{4\pi\varepsilon_0 l} = \int \frac{\sigma 2\pi r dl}{4\pi\varepsilon_0 l} = \int \frac{\sigma r dl}{2\varepsilon_0 l}$$

$$\because \frac{r}{l} = \sin\frac{\theta}{2}$$

$$\therefore U = \int \frac{\sigma r dl}{2\varepsilon_0 l} = \int_0^L \frac{\sigma \sin\frac{\theta}{2} dl}{2\varepsilon_0} = \frac{\sigma \sin\frac{\theta}{2}}{2\varepsilon_0} L = \frac{\sigma R}{2\varepsilon_0}$$



例题12：如图，求P点的磁感应强度

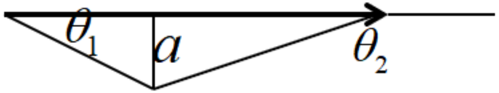


解：
$$\vec{B}=\vec{B}_{AB}+\vec{B}_{BCD}+\vec{B}_{DE}$$

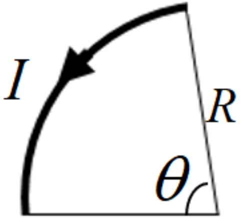
磁场叠加原理

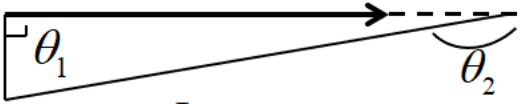
$$\vec{B}_{AB}=0 \qquad \vec{B}_{BCD}\perp\vec{B}_{DE}$$

直线 $B=\dfrac{\mu_0 I}{4\pi a}(\cos\theta_1-\cos\theta_2)$



$$B=\frac{\mu_0 I}{2R}\frac{\theta}{2\pi}$$





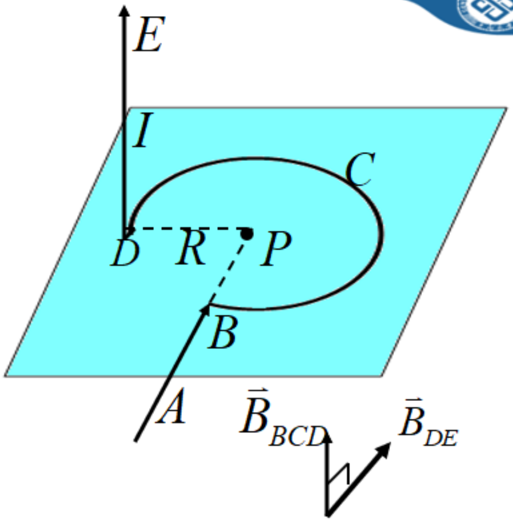
$$B=\frac{\mu_0 I}{4\pi a}$$

$$\vec{B} = \vec{B}_{AB} + \vec{B}_{BCD} + \vec{B}_{DE}$$

$$\vec{B}_{AB} = 0 \quad \vec{B}_{BCD} \perp \vec{B}_{DE}$$

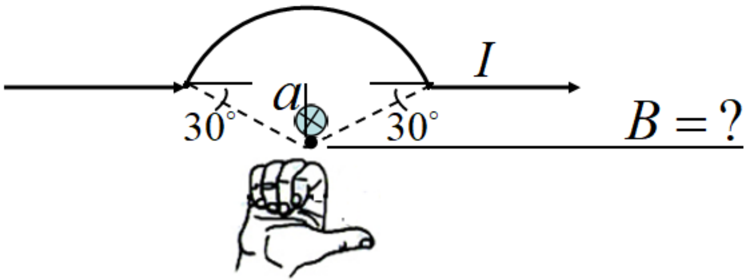
$$B = \frac{\mu_0 I}{4\pi a} \longrightarrow B_{DE} = \frac{\mu_0 I}{4\pi R}$$

$$\mathrm{B} = \frac{\mu_0 I}{2R}\frac{\theta}{2\pi} \longrightarrow \mathrm{B}_{\mathrm{BCD}} = \frac{\mu_0 I}{2R}\frac{3}{4}$$



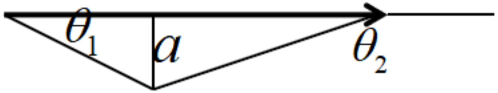
$$B_P = \sqrt{\left(\frac{\mu_0 I}{4\pi R}\right) + \left(\frac{\mu_0 I}{2R}\frac{3}{4}\right)^2} = \frac{\mu_0 I}{4\pi R}\sqrt{\frac{1}{\pi^2} + \frac{9}{16}}$$

思考：

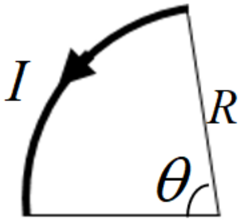


$$B=\frac{\mu_0 I}{4\pi a}(\cos 0^\circ-\cos 30^\circ)+\frac{\mu_0 I}{4\pi a}(\cos 150^\circ-\cos\pi)+\frac{\mu_0 I}{2R}\times\frac{1}{3}$$

相等，同向

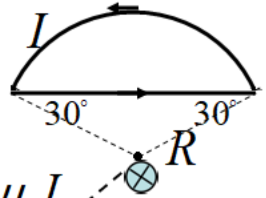


$$B=\frac{\mu_0 I}{4\pi a}(\cos\theta_1-\cos\theta_2)$$

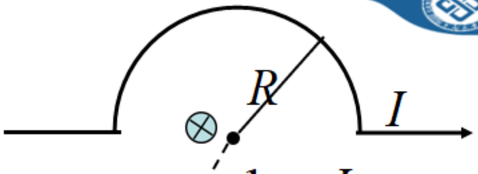


$$B=\frac{\mu_0 I}{2R}\frac{\theta}{2\pi}$$

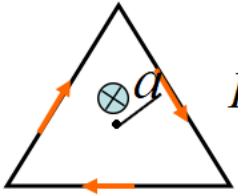
$$B=\frac{\mu_0 Il}{4\pi R^2}$$



$$B=\frac{\mu_0 I}{4\pi R\sin 30^\circ}\cdot 2\cos 30^\circ-\frac{1}{3}\frac{\mu_0 I}{2R}$$

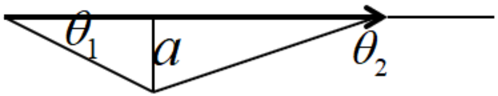


$$B'=\frac{1}{2}\frac{\mu_0 I}{2R}$$

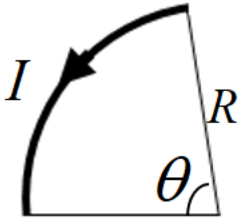


$$B=3\frac{\mu_0 I}{4\pi a}\cdot 2\cos 30^\circ$$

3倍

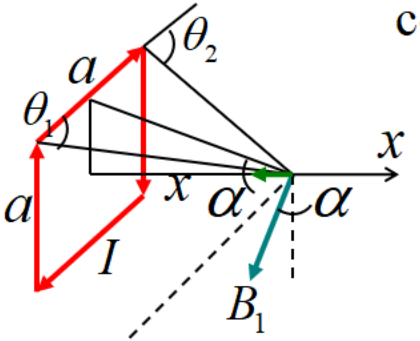


$$B=\frac{\mu_0 I}{4\pi a}(\cos\theta_1-\cos\theta_2)$$



$$B=\frac{\mu_0 I}{2R}\frac{\theta}{2\pi}$$

$$B=\frac{\mu_0 Il}{4\pi R^2}$$



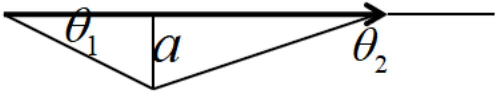
$$\cos\theta_1 = -\cos\theta_2$$

$$= \frac{a/2}{\sqrt{x^2 + (a/2)^2 + (a/2)^2}}$$

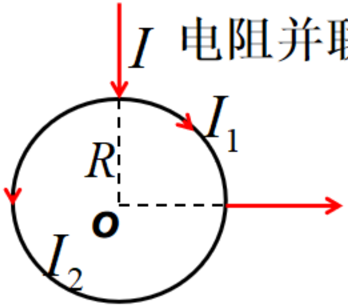
$$\sin\alpha = \frac{a/2}{\sqrt{x^2 + (a/2)^2}}$$

$$B = 4\frac{\mu_0 I(\cos\theta_1 - \cos\theta_2)}{4\pi\sqrt{x^2 + (a/2)^2}}\sin\alpha$$

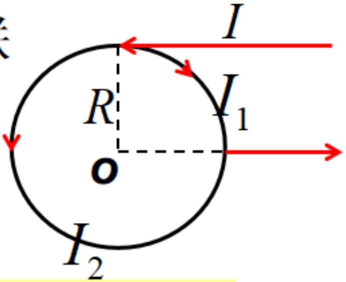
$$B = \frac{\mu_0 I}{4\pi a}(\cos\theta_1 - \cos\theta_2)$$



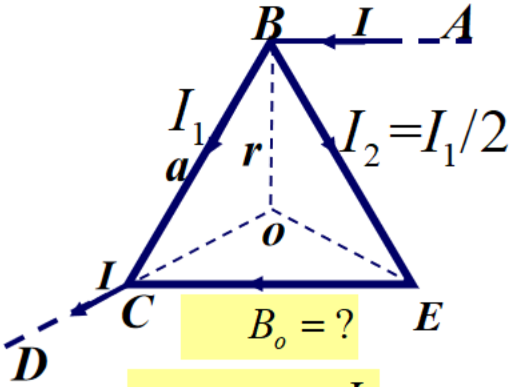
电阻并联



$B_o = 0$



$$B_o = \frac{\mu_0 I}{4\pi R}$$

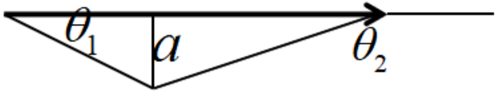


$B_o = ?$

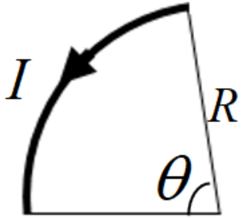
$$B = \frac{\mu_0 I}{4\pi r}$$

$$I_1 r_1 = I_2 r_2 \quad \text{电阻 } r \propto l$$

$$I_1 l_1 = I_2 l_2$$

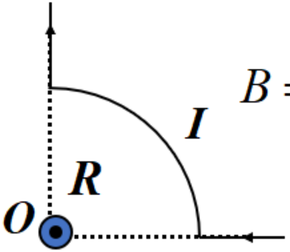


$$B = \frac{\mu_0 I}{4\pi a}(\cos\theta_1 - \cos\theta_2)$$



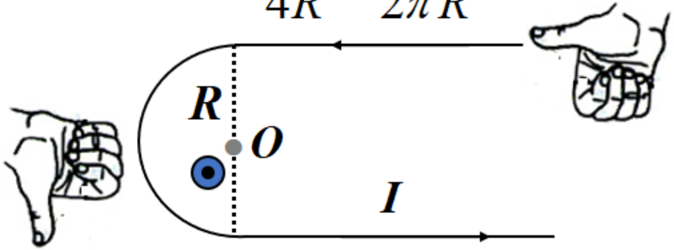
$$B = \frac{\mu_0 I}{2R}\frac{\theta}{2\pi}$$

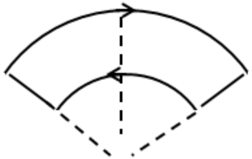
$$B = \frac{\mu_0 I l}{4\pi R^2}$$



$$B=\frac{\mu_0 I}{8R}$$

$$B=\frac{\mu_0 I}{4R}+\frac{\mu_0 I}{2\pi R}$$





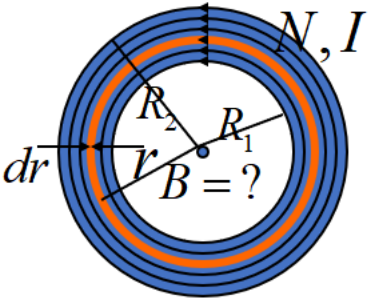
$$B=\frac{\mu_0 I}{2}\left(\frac{1}{R_1}-\frac{1}{R_2}\right)\frac{\theta}{2\pi}$$

$$B=\frac{\mu_0 I}{4\pi a}(\cos\theta_1-\cos\theta_2)$$

$$B=\frac{\mu_0 I}{2R}\frac{\theta}{2\pi}$$

$$B=\frac{\mu_0 I}{4\pi a}$$

例题9： N匝电流组成的平面螺绕环，求中心的磁感应强度



解：视为无限多圆环电流磁场的叠加

根据圆环电流中心磁场表达式 $B=\dfrac{\mu_0 I}{2R}$

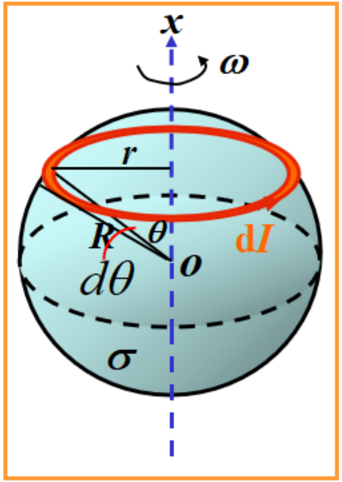
$$I \to dI, R \to r, B \to dB$$

$$dI=\frac{NI}{R_2-R_1}dr \qquad dB=\frac{\mu_0 dI}{2r} \qquad dB=\frac{\mu_0 NI}{2r(R_2-R_1)}dr$$

$$B=\int_{R_1}^{R_2}\frac{\mu_0 NI}{2r(R_2-R_1)}dr=\frac{\mu_0 NI}{2(R_2-R_1)}\ln\frac{R_2}{R_1}$$

例题10： 均匀带电球面(R , σ)，绕直径以 ω 匀速旋转

求球心处 $\vec{B}_0$



解： 旋转带电球面 $\xrightarrow{\text{等效}}$ 许多环形电流

取半径 r 的环带

$$\mathrm{d}q=\sigma\mathrm{d}S=\sigma\cdot 2\pi rR\,\mathrm{d}\theta$$

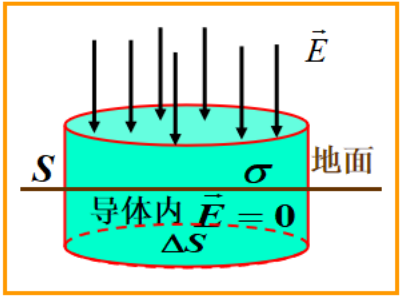
$$dI=\frac{dq}{T},T=\frac{2\pi}{\omega}$$

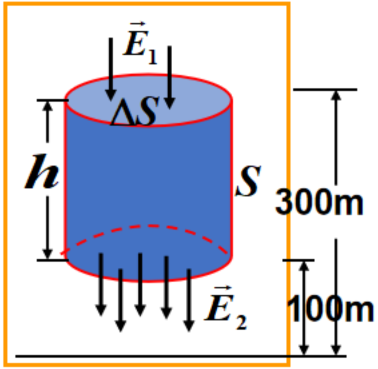
等效圆电流:

$$\mathrm{d}I=\frac{\omega\,\mathrm{d}q}{2\pi}=\sigma R^2\omega\sin\theta\mathrm{d}\theta$$

$$B=\frac{\mu_0}{2}\frac{Ir^2}{(x^2+r^2)^{3/2}}=\frac{\mu_0 Ir^2}{2\mathrm{R}^3}=\frac{\mu_0 I}{2R}\sin^2\theta$$

例题5：

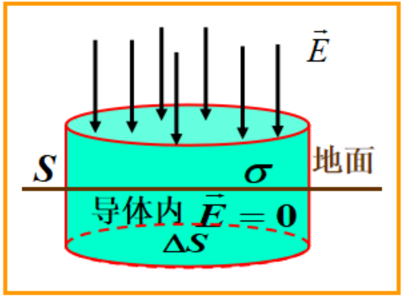


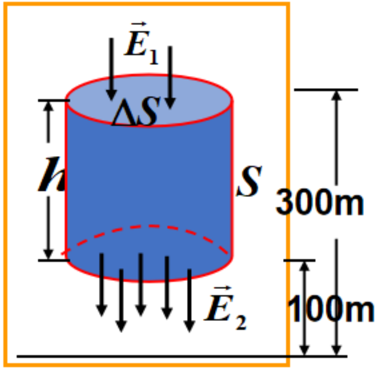


（1）晴天大气平均电场强度约120V/m，方向向下。求地球表面上的过剩电荷密度，以每平方厘米的额外电子数表示。

已知电场分布求电荷

例题5：

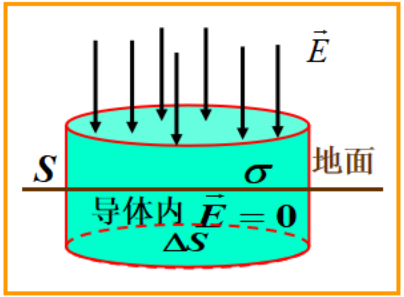




（2）地球表面上方电场强度方向向下，大小随高度变化。在地面上方100m高处场强为150N/C，300m高处场强为100N/C.求两高度之间的平均电荷体密度。

已知电场分布求电荷

解：（1） $\oiint \vec{E}\cdot d\vec{S} = \dfrac{\sum q}{\varepsilon_0}$



$$-E\Delta S = \frac{\sigma \Delta S}{\varepsilon_0}, \boxed{E = -\frac{\sigma}{\varepsilon_0}}$$

$$\sigma = -E\varepsilon_0 = -120\times 8.85\times 10^{-12} = -1.062\times 10^{-9}\, C/m^2$$

地球表面电子数密度=

$$\frac{1.062\times 10^{-9}}{1.6\times 10^{-19}\times 10^{4}} = 6.64\times 10^{5}\text{个}/cm^2$$

高斯面上电通量已知！

（2）地球表面上方电场强度方向向下，大小随高度变化。在地面上方100m高处场强为150N/C，300m高处场强为100N/C.求两高度之间的平均电荷体密度。

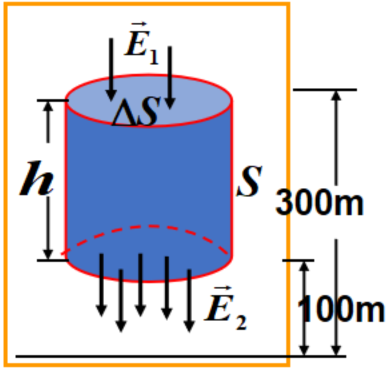
$$\oiint \vec{E}\cdot d\vec{S}=\frac{\sum q}{\varepsilon_0}$$

$$E_2\Delta S-E_1\Delta S=\rho h\Delta S/\varepsilon_0,$$

$$h=200m$$

$$\rho=(E_2-E_1)\varepsilon_0/h=50\varepsilon_0/200$$

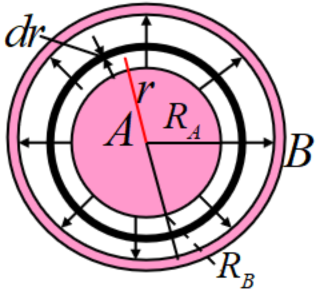
$$=2.21\times10^{-12}C/m^3=1.38\times10^{7}个电子/m^3$$



高斯面上电通量已知！

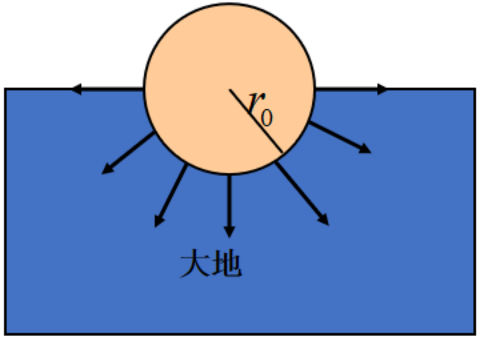
$$R=\int\rho\frac{dl}{S}$$

例：求球形电容器漏电电阻



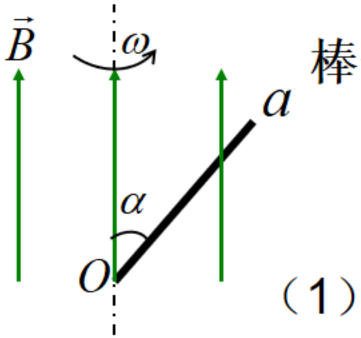
$$R=\int_{R_1}^{R_2}\frac{\rho dr}{4\pi r^2}=\frac{\rho}{4\pi}\left(\frac{1}{R_1}-\frac{1}{R_2}\right)$$

球形电极一半入地，大地电阻

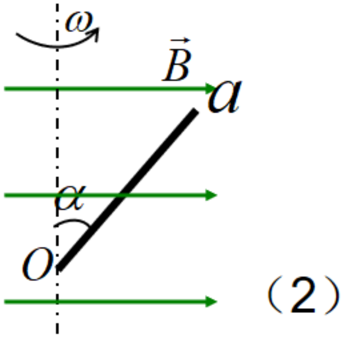


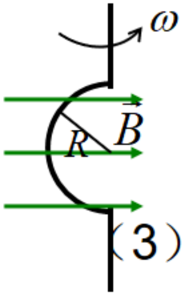
$$R_{地}=\int_{r_0}^{\infty}\rho\frac{dr}{2\pi r^2}=\frac{\rho}{2\pi r_0}$$

例题：　求图中位置导体的电动势

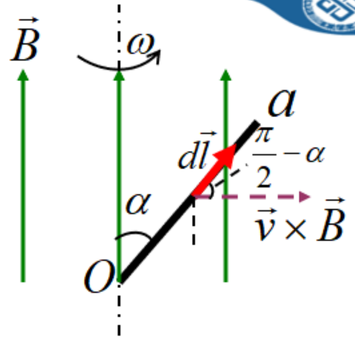


棒长L



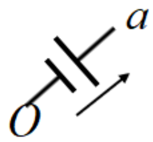


解：（1） $\boxed{\varepsilon_{oa} = \int_o^a \vec{v} \times \vec{B} \cdot d\vec{l}}$

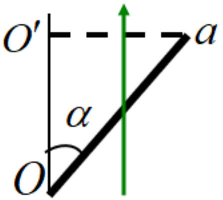


$$\varepsilon_{oa} = \int vBdl\cos(\frac{\pi}{2}-\alpha) = \int_0^L vB\sin\alpha dl$$

$$= \int_0^L l\omega B\sin^2\alpha dl \qquad (v = l\sin\alpha\omega)$$



$$= \frac{1}{2}L^2\omega B\sin^2\alpha \qquad (\frac{1}{2}\omega B(L\sin\alpha)^2)$$



闭合回路 $\varepsilon = 0$

$$\varepsilon_{oa} = \varepsilon_{o'a}$$

（2） $\boxed{\varepsilon_{oa}=\int_o^a \vec{v}\times\vec{B}\cdot\vec{dl}}$ $\quad v=l\sin\alpha\cdot\omega$

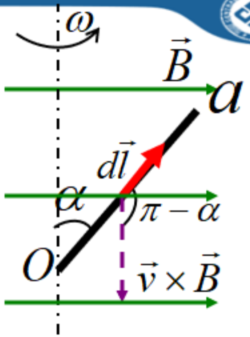
$$\varepsilon_{oa}=\int_0^a vBdl\cos(\pi-\alpha)$$

$$=-\int_0^L l\sin\alpha\omega Bdl\cos\alpha$$

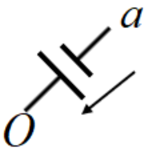
$$=-\frac{\sin 2\alpha}{4}\omega BL^2$$

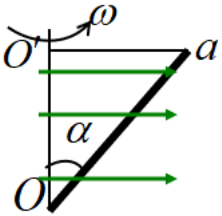
或 $\quad \Phi=BS\sin\omega t$

$$\boxed{\varepsilon=-\frac{d\Phi}{dt}}=-BS\omega\cos\omega t$$



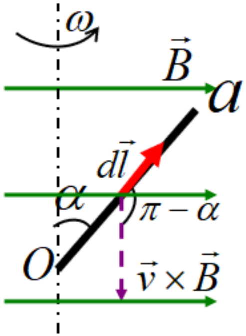
（2）

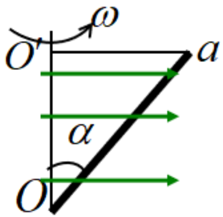




$$\boxed{\varepsilon = -\frac{d\Phi}{dt}} = -BS\omega\cos\omega t$$

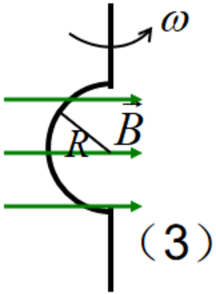
$$t = 0 \quad \varepsilon = BS\omega$$





$$S = \frac{1}{2}L\cos\alpha \cdot L\sin\alpha = \frac{1}{4}L^2\sin 2\alpha$$

$$\therefore \varepsilon = -\frac{\sin 2\alpha}{4}\omega B L^2$$



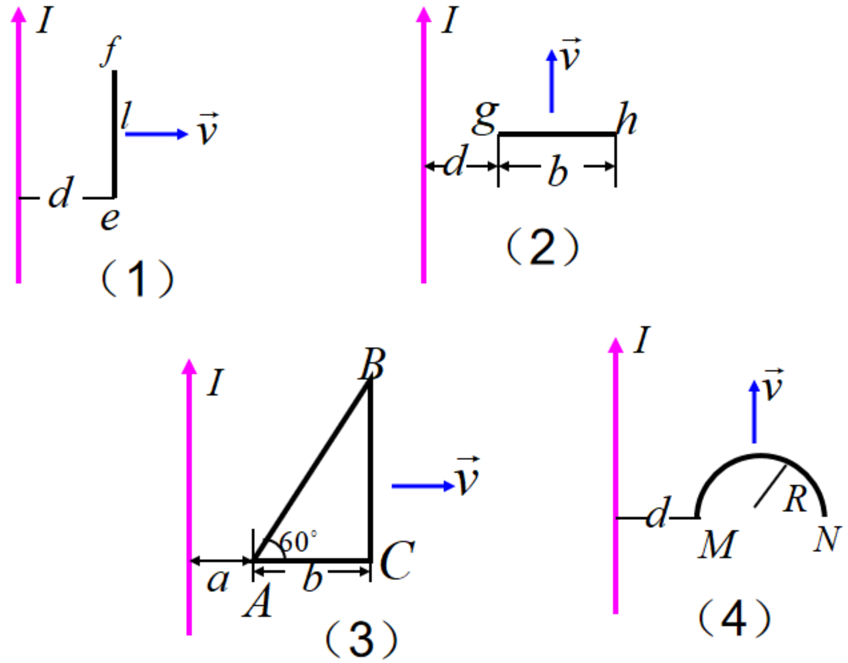
（3） $\Phi = BS\sin\omega t$ $\quad S = \frac{1}{2}\pi R^2$

$$\boxed{\varepsilon = -\frac{d\Phi}{dt}} = -BS\omega\cos\omega t$$

$$t = 0 \quad \varepsilon = BS\omega = \frac{1}{2}\pi R^2 B\omega$$

用$\varepsilon = \int \vec{v}\times\vec{B}\cdot d\vec{l}$计算复杂

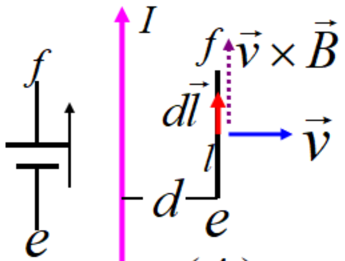
例题4： 求下列运动导体的感应电动势



简单非均匀磁场中的动生电动势

解： （1） $B=\dfrac{\mu_0 I}{2\pi d}$

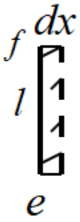
$$\boxed{\varepsilon_{ef}=\int\vec{v}\times\vec{B}\cdot\vec{dl}}=\int vBdl=vBl=\frac{\mu_0 I}{2\pi d}vl$$



（1）

另 $d\Phi=Bdxl$ $B=\dfrac{\mu_0 I}{2\pi d}$

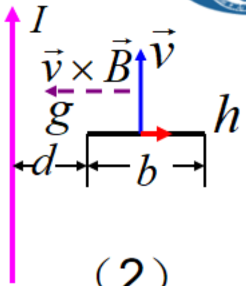
$$\frac{d\Phi}{dt}=B\frac{dx}{dt}l=Bvl$$



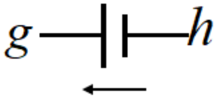
（2） $\boxed{\varepsilon = \int \vec{v} \times \vec{B} \cdot \vec{dl}}$

$$\varepsilon_{gh} = \int_d^{d+b} vBdl\cos\pi = \int_d^{d+b} -v\frac{\mu_0 I}{2\pi x}dx$$

$$= -\frac{\mu_0 I v}{2\pi}\ln\frac{d+b}{d} < 0$$



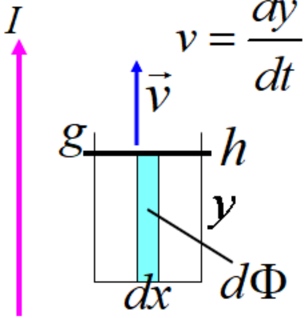
（2）



另 $\Phi = \int_d^{d+b} \frac{\mu_0 I}{2\pi x} y dx = \frac{\mu_0 I}{2\pi} y \ln\frac{d+b}{d}$

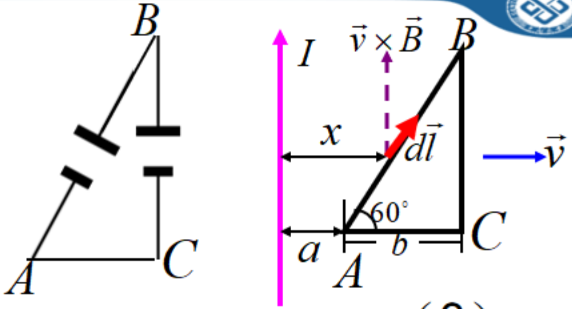
$$\boxed{\varepsilon_{gh} = -\frac{d\Phi}{dt}} = \frac{d\Phi}{dy}\frac{dy}{dt} = -v\frac{d\Phi}{dy}$$

$$= -\frac{\mu_0 I v}{2\pi}\ln\frac{d+b}{d} < 0$$



（3）

$$\varepsilon = \int \vec{v} \times \vec{B} \cdot \vec{dl}$$

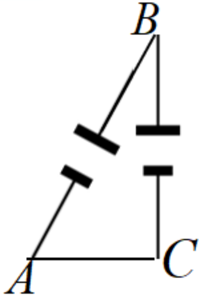


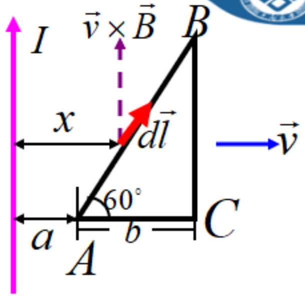
$$\varepsilon_{AB} = \int_A^B vBdl\cos 30^\circ$$

$$= \int_a^{a+b} \frac{\mu_0 I}{2\pi x} v \frac{dx}{\cos 60^\circ} \cos 30^\circ = \frac{\mu_0 \sqrt{3}}{2\pi} Iv \ln\frac{a+b}{a}$$

$$\varepsilon_{BC} = \int \vec{v} \times \vec{B} \cdot \vec{dl} = \int Bvdl = Blv$$

$$\varepsilon_{BC} = \frac{\mu_0 I}{2\pi(a+b)} (b\sqrt{3}) \cdot v$$





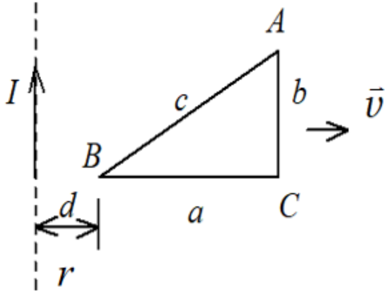
$$\varepsilon_{AB}=\frac{\mu_0\sqrt{3}}{2\pi}Iv\ln\frac{a+b}{a}$$

$$\varepsilon_{BC}=\frac{\mu_0 I}{2\pi(a+b)}(b\sqrt{3})\cdot v$$

$$\varepsilon=\varepsilon_{AB}-\varepsilon_{BC}=\frac{\mu_0\sqrt{3}Iv}{2\pi}(\ln\frac{a+b}{a}-\frac{b}{a+b})$$

由楞次定律可以判断，电动势方向为 $ABCA$

另解：无限长直导线，通以常定电流I．有一与之共面的直角三角形线圈ABC．已知AC边长为b，且与长直导线平行，BC边长为a．若线圈以垂直于导线方向的速度 $\vec{v}$ 向右平移，当B点与长直导线的距离为d 时，求线圈ABC内的感应电动势的大小和感应电动势的方向。



解：建立坐标系，长直导线为y轴，BC边为x轴，原点在长直导线上，则斜边的方程为

$$y=(bx/a)-br/a$$

式中r是t时刻B点与长直导线的距离．三角形中磁通量

$$\Phi=\frac{\mu_0 I}{2\pi}\int_r^{a+r}\frac{y}{x}\mathrm{d}x=\frac{\mu_0 I}{2\pi}\int_r^{a+r}(\frac{b}{a}-\frac{br}{ax})\mathrm{d}x=\frac{\mu_0 I}{2\pi}(b-\frac{br}{a}\ln\frac{a+r}{r})$$

$$\varepsilon=-\frac{\mathrm{d}\Phi}{\mathrm{d}t}=\frac{\mu_0 Ib}{2\pi a}(\ln\frac{a+r}{r}-\frac{a}{a+r})\frac{\mathrm{d}r}{\mathrm{d}t}$$

当$r=d$时，
$$\varepsilon=\frac{\mu_0 Ib}{2\pi a}(\ln\frac{a+d}{d}-\frac{a}{a+d})v$$

方向：$ACBA$(即顺时针)

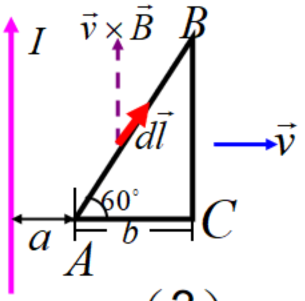
或 $\Phi=\Phi(z)$ $\quad \varepsilon=\frac{d\Phi}{dz}\frac{dz}{dt}=v\frac{d\Phi}{dz}$

$$B=\frac{\mu_0 I}{2\pi(z+x)}$$

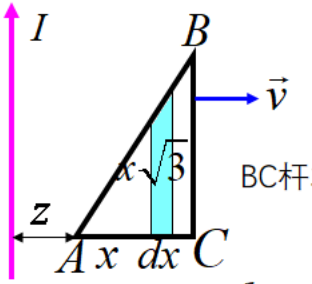
$$\Phi=\int BdS=\int_0^b\frac{\mu_0 I\sqrt{3}xdx}{2\pi(z+x)}$$

$$\int\frac{x}{z+x}dx=\int\frac{x+z-z}{z+x}dx=\int(1-\frac{z}{z+x})dx$$

$$\Phi=\frac{\mu_0 I\sqrt{3}}{2\pi}\int_0^b\frac{xdx}{(z+x)}=\frac{\mu_0 I\sqrt{3}}{2\pi}[b-z\ln\frac{b+z}{z}]$$



（3）

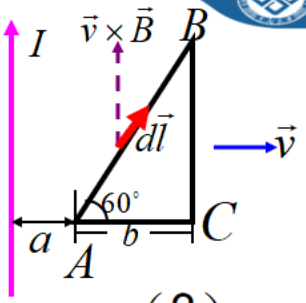


BC杆水平运动?

$$v=\frac{dz}{dt}$$

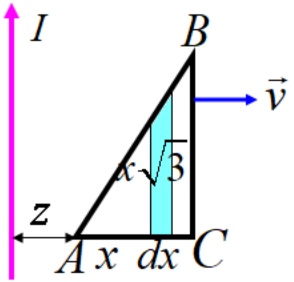
$$\Phi = \frac{\mu_0 I\sqrt{3}}{2\pi}[b - z\ln\frac{b+z}{z}]$$

$$\Phi = \Phi(z)$$



（3）

$$\boxed{\varepsilon = -\frac{d\Phi}{dt}} = -\frac{d\Phi}{dz}\frac{dz}{dt} = \frac{\mu_0\sqrt{3}Iv}{2\pi}(\ln\frac{z+b}{z} - \frac{b}{z+b})$$

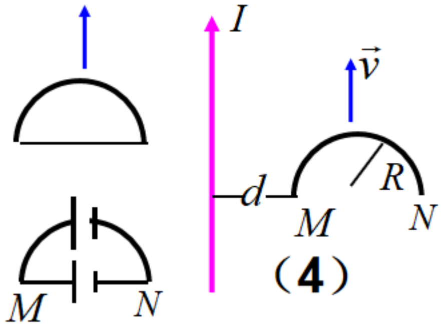


取$z = a$

$$\therefore \varepsilon = \frac{\mu_0\sqrt{3}Iv}{2\pi}(\ln\frac{a+b}{a} - \frac{b}{a+b})$$

$$v = \frac{dz}{dt}$$

（4） $\varepsilon_{\overset{\frown}{MN}} = \varepsilon_{MN} = \dfrac{\mu_0 I}{2\pi} v \ln \dfrac{d+2R}{d}$



二、感生电动势的计算

方法一： $\varepsilon = \int \vec{E}_{感} \cdot d\vec{l}$ （个别）

方法二：法拉第定律

$$\varepsilon = -\frac{d\Phi}{dt}$$ 闭合导体回路

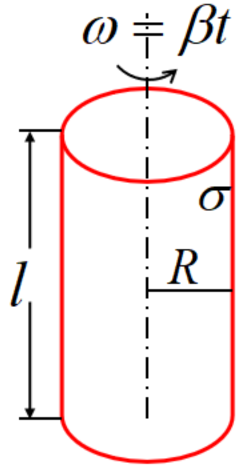
对非闭合导体，须假设回路且假设部分无电动势。

例题2：如图，半径为 R 长为 l 的均匀带电的长圆柱面（电荷面密度σ）以角速度$\omega=\beta t$绕轴线加速转动，求

（1）圆柱面内磁感应强度

（2）柱面内接近内壁处的电场强度和坡印亭矢量

（3）证明进入圆筒的能流等于圆筒内磁能的增加率。



解：（1） 圆柱面内磁感应强度

类似于长螺线管的磁场

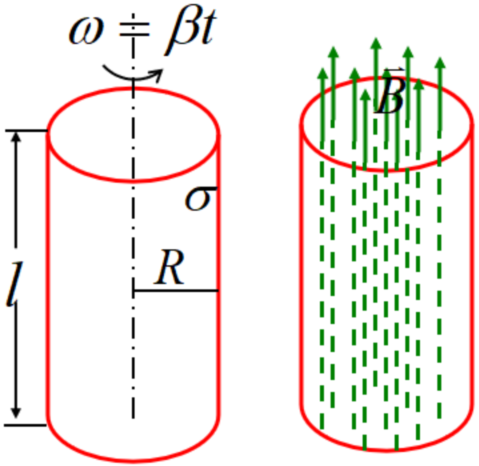
$$B = \mu_0 nI = \mu_0 i$$

$$i = \frac{\Delta I}{\Delta l} = nI$$ ——理解为单位宽的电流

此处， $i = \dfrac{dq}{dt}, \dfrac{q}{t}$ $\qquad q = 2\pi R \times 1 \times \sigma, t = T$

$$i = \frac{2\pi R \times 1 \times \sigma}{T} = \omega\sigma R, B = \mu_0 i$$

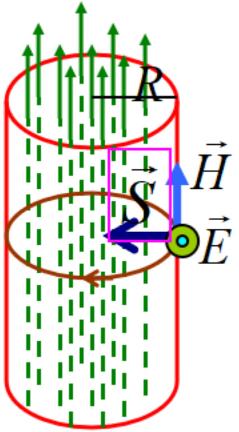
$\therefore B = \mu_0 \beta\sigma R t$ 不断增大



（2）柱面内接近内壁处的电场强度和坡印亭矢量

$B=\mu_0\beta\sigma Rt$ 不断增大

$$\boxed{S=EH=EB/\mu_0}$$



$$\oint\vec{E}\cdot d\vec{l}=-\frac{d\Phi_m}{dt},\Phi_m=BS'=\pi R^2B$$

$$\oint\vec{E}\cdot d\vec{l}=2\pi RE=-\pi R^2\frac{dB}{at}=-\pi R^2\mu_0\beta\sigma R$$

$\therefore|\mathrm{E}|=\dfrac{1}{2}\mu_0\sigma R^2\beta$ 或直接利用结论E$=\dfrac{r}{2}\dfrac{\partial B}{\partial t}$进行计算

$\therefore S=\dfrac{1}{2}\mu_0\sigma^2R^3\beta^2t$ $\vec{S}=\vec{E}\times\vec{H}$,方向如图

方向指向轴线

(3)进入圆筒的能流等于圆筒内磁能的增加率。

$$S=\frac{1}{2}\mu_0\sigma^2R^3\beta^2t$$

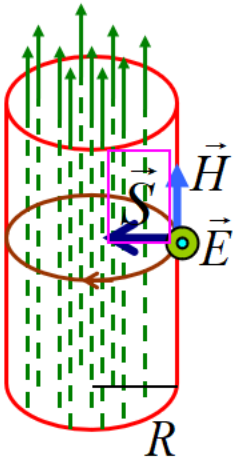
单位时间从侧面进入柱体的电磁能量：

$$P=SA=2\pi RlS=\pi\mu_0R^4l\beta^2\sigma^2t$$

$$W_m=\frac{1}{2}\frac{B^2}{\mu_0}\cdot V=\frac{B^2}{2\mu_0}\pi R^2l$$

$A=2\pi Rl$ —面积

$B=\mu_0\beta\sigma Rt$



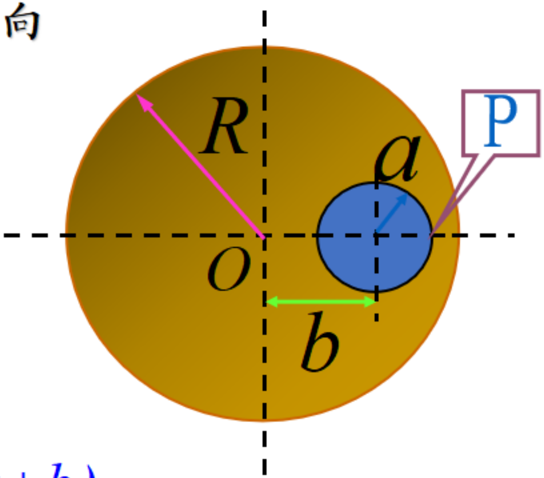
单位时间柱体内电磁能量增加量：

$$\frac{dW_m}{dt}=\frac{\pi R^2B}{\mu_0}l\frac{dB}{dt}=\pi\mu_0R^4l\beta^2\sigma^2t\quad\therefore P=\frac{dW_m}{dt}$$

讨论.半径为R的无限长直导体，内部有一与导体轴平行、半径为a的圆柱形孔洞，两轴相距为b。设导体横截面上均匀通有电流I，求(1)P点处的磁感应强度。(2)圆柱形孔洞内的磁场.

解：(1) P点：设导体中电流密度方向垂直于纸面向外，电流密度大小为

$$j=\frac{I}{\pi(R^2-a^2)}$$



挖补法：设想在空洞里同时存在密度为 $\vec{j}$ 和 $-\vec{j}$ 的电流

a. 对半径为 R 的无限长载流导体

$$B_1=\frac{\mu_0}{2\pi(a+b)}\cdot\pi(a+b)^2 j=\frac{\mu_0(a+b)}{2}j$$

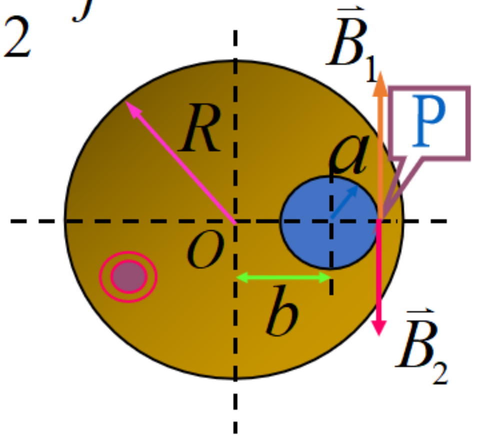
b. 对半径为a的无限长载流圆柱体

$$B_2 = \frac{\mu_0}{2\pi a}\cdot \pi a^2 j = \frac{\mu_0 a}{2} j$$ 方向如图

$$\therefore B_P = B_1 - B_2 = \frac{\mu_0 (a+b)}{2} j - \frac{\mu_0 a}{2} j$$

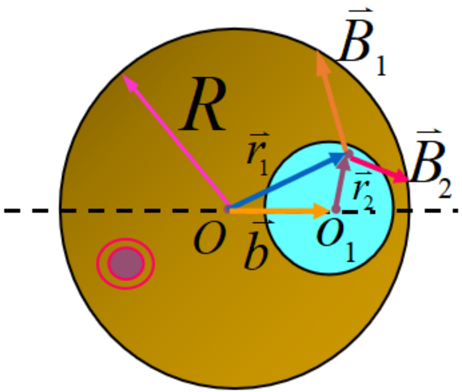
$$= \frac{\mu_0 b}{2} j = \frac{\mu_0 b I}{2\pi (R^2 - a^2)}$$

方向竖直向上



(2) 对空腔内的点，方法一：

$$B2\pi r = \mu_0 \pi r^2 \times j; B = \frac{\mu_0 j r}{2}, (r < R)$$



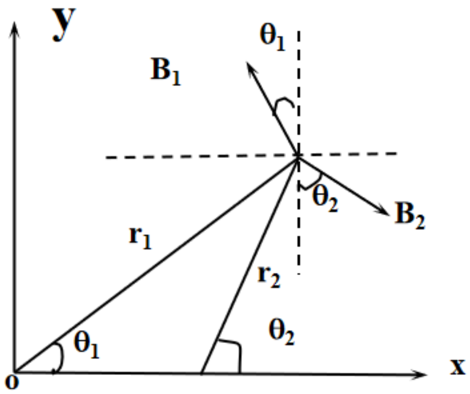
将其改写为矢量式 $\vec{B} = \dfrac{\mu_0}{2} \vec{j} \times \vec{r}$

$$\vec{B}_1 = \frac{\mu_0}{2} \vec{j} \times \vec{r}_1 \qquad \vec{B}_2 = \frac{\mu_0}{2} (-\vec{j}) \times \vec{r}_2$$

$$\vec{B} = \vec{B}_1 + \vec{B}_2 = \frac{\mu_0}{2} \vec{j} \times (\vec{r}_1 - \vec{r}_2) = \frac{\mu_0}{2} \vec{j} \times \vec{b}$$

结论: 空腔内的磁场为均匀磁场

方法二：分解投影法



$$B_1 = \frac{1}{2}\mu_0 j r_1$$

$$B_2 = -\frac{1}{2}\mu_0 j r_2$$

如图，将$\vec{B}_1$，$\vec{B}_2$在坐标轴投影得:

$$B_x = \frac{1}{2}\mu_0 j(r_1 \sin\theta_1 - r_2 \sin\theta_2)$$

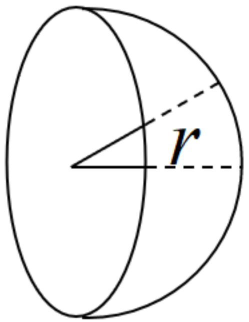
$$\because \quad r_1 \sin\theta_1 = r_2 \sin\theta \qquad \therefore \quad B_x = 0$$

$$B_y = \frac{1}{2}\mu_0 j(r_1 \cos\theta_1 - r_2 \cos\theta_2) = \frac{1}{2}\mu_0 jb$$

例题：广播电台平均辐射功率**10kW**,能流分布在以电台为中心的半球面上，求

（**1**）距电台**10km**处坡印亭矢量

（**2**）**10km**处的磁场强度和电场强度最大值



解：（1） 已知$\overline{P}$=10kW，r=10km

$$\overline{P}=\overline{S}2\pi r^2 \qquad \overline{S}=\frac{\overline{P}}{2\pi r^2}=1.58\times10^{-5}W\cdot m^{-2}\cdot s^{-1}$$

(2) $$\overline{S}=\frac{1}{2}E_0H_0=\frac{1}{2}\sqrt{\frac{\varepsilon_0}{\mu_0}}E_0^2=\frac{1}{2}\varepsilon_0 cE_0^2,\therefore E_0=\sqrt{\frac{2S}{\varepsilon_0 c}}=0.1V\cdot m^{-1}$$

$$H_0=\sqrt{\frac{\varepsilon_0}{\mu_0}}E_0=2.9\times10^{-4}A\cdot m^{-1}$$