UTM UNIVERSITI TEKNOLOGI MALAYSIA RESEARCH UNIVERSITY	UTM DEGREE PROGRAMME UNIVERSITI TEKNOLOGI MALAYSIA

Discrete Structure

(SECI1013-09)

Assignment 1

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Question 1

Let the universe be the set $U = \{x \mid x \in \mathbb{Z}, 0 \leq x \leq 20\}$,

A, B, and C denote the subsets of U,

$A = \{p \mid p \in \text{prime numbers}, 0 \leq p \leq 20\}$,

$B = \{e \mid e \in \text{even numbers}, 10 \leq e \leq 20\}$,

$C = \{o \mid o \in \text{odd numbers}, 0 \leq o \leq 10\}$

$A = \{2, 3, 5, 7, 11, 13, 17, 19\}$

$B = \{10, 12, 14, 16, 18, 20\}$

$C = \{1, 3, 5, 7, 9\}$

a) $A \cap C \cup B$

$A \cap C = \{3, 5, 7\}$

$A \cap C \cup B = \{3, 5, 7, 10, 12, 14, 16, 18, 20\}$

b) $P(A \cap B \cup C)$

$A \cap B = \{\}$

$A \cap B \cup C = \{1, 3, 5, 7, 9\}$

$|P(A \cap B \cup C)| = 2^5$

$= 32$

$P(A \cap B \cup C) = \{\emptyset, \{1\}, \{3\}, \{5\}, \{7\}, \{9\}, \{1,3\}, \{1,5\}, \{1,7\}, \{1,9\}, \{3,5\}, \{3,7\}, \{3,9\},$
$\{5,7\}, \{5,9\}, \{7,9\}, \{1,3,5\}, \{1,3,7\}, \{1,3,9\}, \{1,5,7\}, \{1,5,9\}, \{1,7,9\},$
$\{3,5,7\}, \{3,5,9\}, \{3,7,9\}, \{5,7,9\}, \{1,3,5,7\}, \{1,3,5,9\}, \{1,5,7,9\},$
$\{1,3,7,9\}, \{3,5,7,9\}, \{1,3,5,7,9\}\}$

c) $A - C = \{2, 11, 13, 17, 19\}$

d) $|A| = 8$

$|B| = 6$

$|C| = 5$

Question 1

e) $|P(A \cap C)|$

$(A \cap C) = \{3, 5, 7\}$

$|P(A \cap C)| = 2^3$

$= 8$

f) $B \subset C'$ = True

$B = \{10, 12, 14, 16, 18, 20\}$

$C = \{1, 3, 5, 7, 9\}$

$C' = \{2, 4, 6, 8, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20\}$

$\therefore$ B is a proper subset of C' as every elements in B is in C' and C' has extra elements that are not in B.

g) $(A \cup B \cup C) \subseteq U$ = True

$(A \cup B \cup C) = \{1, 2, 3, 5, 7, 9, 10, 11, 12, 13, 14, 16, 17, 18, 19, 20\}$

$U = \{0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20\}$

$\therefore$ $(A \cup B \cup C)$ is a subset of U as every elements in $(A \cup B \cup C)$ is in U.

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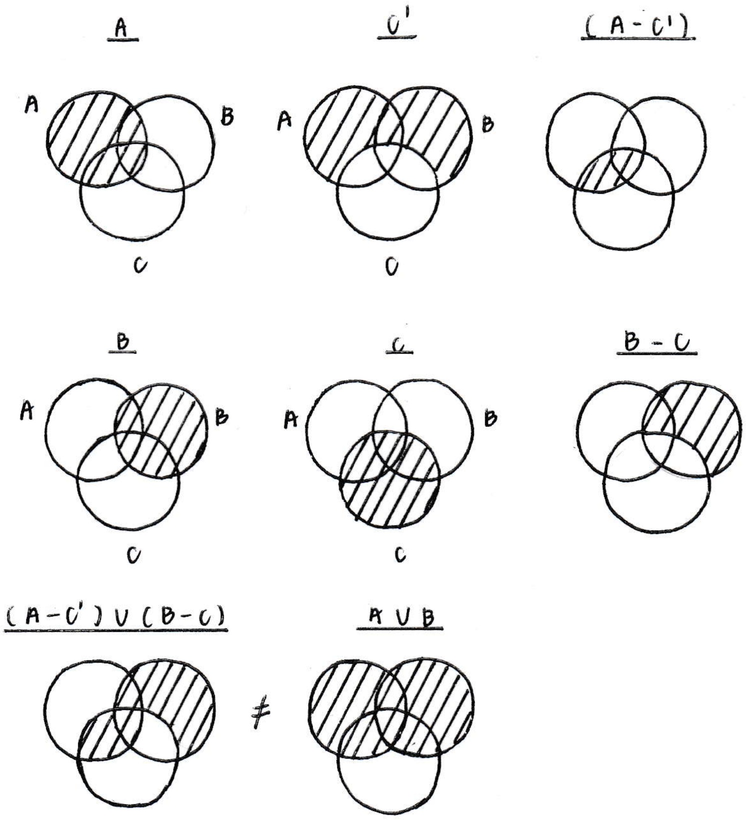
Question 2

Let A, B and C denote the subsets of U.

a) $(A - C') \cup (B - C) = A \cup B$

$\text{LHS} = (A - C') \cup (B - C)$	
$= (A \cap (C')') \cup (B - C)$	Set Difference Laws
$= (A \cap C) \cup (B - C)$	Double Complement Laws
$= (A \cap C) \cup (B \cap C')$	Set Difference Laws
$= (A \cup B) \cap (C \cup C')$	Distributive Laws
$= (A \cup B) \cap U$	Complement Laws
$= A \cup B$	Identity Laws
$=$ RHS (proven)	



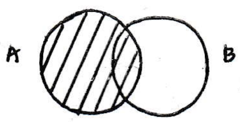
∴ Therefore, from the Venn Diagram, $(A - C') \cup (B - C) \neq A \cup B$.

Question 2

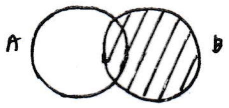
b) $(A \cap B) \cup (A - B) = A$

LHS	$= (A \cap B) \cup (A - B)$	
	$= (A \cap B) \cup (A \cap B')$	Set Difference Laws
	$= A \cap (B \cup B')$	Distributive Laws
	$= A \cap U$	Complement Laws
	$= A$	Identity Laws
	$=$ RHS (proven)	

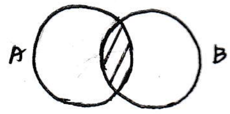
A



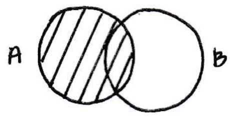
B



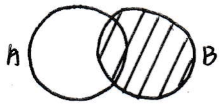
$A \cap B$



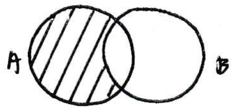
A



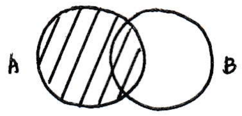
B



$A - B$

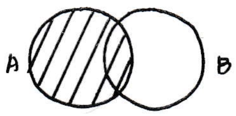


$(A \cap B) \cup (A - B)$



=

A



(proven)

Question 2

c) $(A \cap C) \cup (A \cup B')' = B \cup C$

$LHS = (A \cap C) \cup (A \cup B')'$

$= (A \cap C) \cup (A' \cap (B')')$ De Morgan's Laws

$= (A \cap C) \cup (A' \cap B)$ Double complement laws

$= (A \cup A') \cap (C \cup B)$ Distributive laws

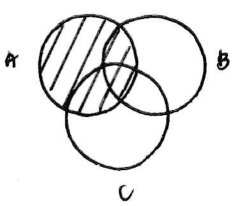
$= U \cap (C \cup B)$ Complement laws

$= C \cup B$ Identity laws

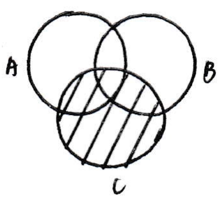
$= B \cup C$ Commutative laws

$= RHS$ (proven)

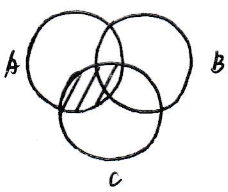
A



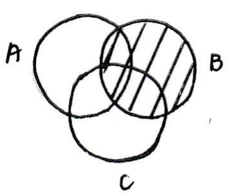
C



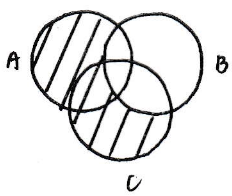
$(A \cap C)$



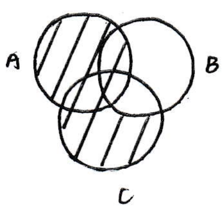
B



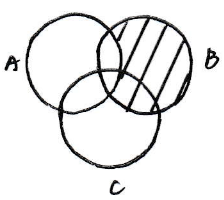
B'



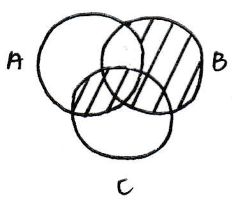
$(A \cup B')$



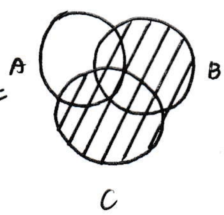
$(A \cup B')'$



$(A \cap C) \cup (A \cup B')'$ $B \cup C$



$\neq$



∴ Therefore, from the Venn Diagram, $(A \cap C) \cup (A \cup B')' \neq B \cup C$.

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Question 3 :

let $A_i = \{ (\frac{1}{x}, x) \mid x = i \}$,

$B_i = \{ (\frac{x}{x}, \frac{x^2}{x^2} \mid x = i \}$

a) $\bigcup_{i=1}^{3} A_i$

$A_1 = \{ (\frac{1}{1}, 1) \}$ $\quad A_2 = \{ (\frac{1}{2}, 2) \}$ $\quad A_3 = \{ (\frac{1}{3}, 3) \}$

$= \{ (1,1) \}$

$= \bigcup_{i=1}^{3} A_i \{ (\frac{1}{i}, i) \}$

$= A_1 \cup A_2 \cup A_3$

$= \{ (1,1), (\frac{1}{2}, 2), (\frac{1}{3}, 3) \}$

$= \{ (\frac{1}{3}, 3), (\frac{1}{2}, 2), (1,1) \}$

b) $\bigcap_{i=1}^{3} B_i$

$B_1 = \{ (\frac{1}{1}, \frac{1^2}{1^2}) \}$ $\quad B_2 = \{ (\frac{2}{2}, \frac{2^2}{2^2}) \}$ $\quad B_3 = \{ (\frac{3}{3}, \frac{3^2}{3^2}) \}$

$= \{ (1,1) \}$ $\quad = \{ (1,1) \}$ $\quad = \{ (1,1) \}$

$= \bigcap_{i=1}^{3} B_i \{ (\frac{i}{i}, \frac{i^2}{i^2}) \}$

$= B_1 \cap B_2 \cap B_3$

$= \{ (1,1) \}$

c) $A_2 \times B_2$

$= \{ (\frac{1}{2}, 2) \} \times \{ (1,1) \}$

$= \{ ((\frac{1}{2}, 2), (1,1)) \}$

$A_2 \times A_3$

$= \{ (\frac{1}{2}, 2) \} \times \{ (\frac{1}{3}, 3) \}$

$= \{ ((\frac{1}{2}, 2), (\frac{1}{3}, 3)) \}$

d) $|A_1 \times A_2 \times A_3| = 1 \cdot 1 \cdot 1$

$= 1$

$A_1 = \{ (1,1) \}$

$A_2 = \{ (\frac{1}{2}, 2) \}$

$A_3 = \{ (\frac{1}{3}, 3) \}$

$|B_1 \times A_2| = 1 \cdot 1$

$= 1$

$B_1 = \{ (1,1) \}$

$A_2 = \{ (\frac{1}{2}, 2) \}$

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4a. $Q = (p \wedge r) \vee (q \vee \neg r)$, $R = (p \vee q) \vee \neg r$

p	q	r	$\neg r$	$p \wedge r$	$q \vee \neg r$	$p \vee q$	$(p \wedge r) \vee (q \vee \neg r)$	$(p \vee q) \vee \neg r$
T	T	T	F	T	T	T	T	T
T	T	F	T	F	T	T	T	T
T	F	T	F	T	F	T	T	T
T	F	F	T	F	T	T	T	T
F	T	T	F	F	T	T	T	T
F	T	F	T	F	T	T	T	T
F	F	T	F	F	F	F	F	F
F	F	F	T	F	T	F	T	T

$\therefore Q \equiv R$. $(p \wedge r) \vee (q \vee \neg r)$ and $(p \vee q) \vee \neg r$ are logically equivalent. Both results columns gives T, T, T, T, T, T, F, T.

b. $Q = (p \wedge r) \vee \neg(p \wedge \neg q)$, $R = (p \wedge r) \rightarrow (q \vee r)$

p	q	r	$\neg q$	$(p \wedge r)$	$\neg(p \wedge \neg q)$	$q \vee r$	$(p \wedge r) \vee \neg(p \wedge \neg q)$	$(p \wedge r) \rightarrow (q \vee r)$
T	T	T	F	T	T	T	T	T
T	T	F	F	F	T	T	T	T
T	F	T	T	T	F	T	T	T
T	F	F	T	F	F	F	F	T
F	T	T	F	F	T	T	T	T
F	T	F	F	F	T	T	T	T
F	F	T	T	F	T	T	T	T
F	F	F	T	F	T	F	T	T

$\therefore Q \not\equiv R$. $(p \wedge r) \vee \neg(p \wedge \neg q)$ and $(p \wedge r) \rightarrow (q \vee r)$ are not logically equivalent. First column gives T, T, T, F, T, T, T, T. Second column gives T, T, T, T, T, T, T, T.

c. $Q = (p \vee \neg q) \rightarrow (p \vee r)$, $R = (p \vee r) \leftrightarrow (q \rightarrow p)$

p	q	r	$\neg q$	$p \vee \neg q$	$p \vee r$	$q \rightarrow p$	$(p \vee \neg q) \rightarrow (p \vee r)$	$(p \vee r) \leftrightarrow (q \rightarrow p)$
T	T	T	F	T	T	T	T	T
T	T	F	F	T	T	T	T	T
T	F	T	T	T	T	T	T	T
T	F	F	T	T	T	T	T	T
F	T	T	F	F	T	F	T	F
F	T	F	F	F	F	F	T	T
F	F	T	T	T	T	T	T	T
F	F	F	T	T	F	T	F	F

$\therefore Q \not\equiv R$. $(p \vee \neg q) \rightarrow (p \vee r)$ and $(p \vee r) \leftrightarrow (q \rightarrow p)$ are not logically equivalent. First column gives T, T, T, T, T, T, T, F. Second column gives T, T, T, T, F, T, T, F.

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Question 5 :

a. Direct Proof

Assume $x-y \leq 0$ is true.

$x-y \leq 0$

$x \leq y$

let $x = 1, 0, -1, \ldots$

$y = 2, 3, 4, \ldots$

let $x = 1$, $y = 2$

$x \leq y$

$1 \leq 2$

let $x = 0$, $y = 3$

$x \leq y$

$0 \leq 3$

let $x = -1$, $y = 4$

$x \leq y$

$-1 \leq 4$

$\ldots$

$\therefore$ Since there are too many real numbers, let it be infinite.

$\therefore$ It is not suitable as it doesn't prove definitely.

b. Indirect proof

$x-y \leq 0 \rightarrow (x \leq 1 \vee y \leq 1) \equiv \neg(x \leq 1 \vee y \leq 1) \rightarrow \neg(x-y \leq 0)$

Show $x-y \leq 0 \rightarrow (x \leq 1 \vee y \leq 1)$ is true by showing the implication.

$\neg(x \leq 1 \vee y \leq 1) \rightarrow \neg(x-y \leq 0)$ is true.

If $(x \leq 1 \vee y \leq 1)'$, then $(x-y \leq 0)'$.

$(x \leq 1 \vee y \leq 1)' = (x \leq 1)' \wedge (y \leq 1)'$

$= (x > 1) \wedge (y > 1)$

$= (x > 1)$ and $(y > 1)$

$x > 1$

$y > 1$

$x-y > 1-1$

$x-y > 0 = \neg(x-y \leq 0)$

$\therefore$ Therefore, the statement is correct.

c. Proof by contradiction

We begin by letting x and y be arbitrary real numbers.

We then suppose the conclusion is false, that is,

$$\neg(x \leq 1 \vee y \leq 1)$$

De Morgan's Law:

$$\neg(x \leq 1 \vee y \leq 1) \equiv \neg(x \leq 1) \wedge \neg(y \leq 1)$$
$$\equiv (x > 1) \wedge (y > 1)$$

We may add these inequalities to obtain:

$$x - y > 1 - 1$$
$$x - y > 0$$

At this point, we have derived contradiction $p \wedge \neg p$, where

$$p: x - y \leq 0$$

$\therefore$ Thus, we can conclude that the statement is true.

Conclusion: Proof by contradiction is the most suitable to solve this equation. It is powerful as it can be used to prove any statement, in several fields of mathematics. As we all know, proof by contradiction in logic and mathematics is a proof that determines the truth of a statement by assuming the proposition is false, then working to show its falsity until the result of that assumption is a contradiction.