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1. (a): Since every iterated dominance equilibrium s^* adopts the dominant strategy.

$\forall i, U_i(s^*) \geq U_i(s_i', s_{-i}^*)$

The definition of nash equilibrium is

$\forall i, u_i(s^*) \geq U_i(s_i', s_{-i}^*)$. So, s^* satisfy the condition of being Nash equilibrium.

So, every iterated dominance equilibrium S^* is a Nash equilibrium.

(b): For example.

	A	B
a	0,0	1,1
b	1,1	0,0

No dominance strategy. So, we cannot find a dominance equilibrium. However, (b, A) and (a, B) are in Nash Equilibrium.

2. (a): $2X < 2$ and $X^2 < 3$

and $X < 1$

So, $-\sqrt{3} < X < 1$

So, if $X = 0$, the game has no pure Nash equilibrium.

(b): $x^2 > 3$, $x < 4$

$\sqrt{3} < x < 4$ or $x < -\sqrt{3}$

So, if $x = 2$, (c, C) is pure Nash equilibrium.

3.

	0	2	4	6	8	10	......	1000
1	(0,0)	(1,0)	(1,0)	(1,0)	(1,0)	(1,0)		(1,0)
3	(0,0)	(0,2)	(3,0)	(3,0)	(3,0)	(3,0)		(3,0)
5	(0,0)	(0,2)	(0,4)	(5,0)	(5,0)	(5,0)		(5,0)
7	(0,0)	(0,2)	(0,4)	(0,6)	(7,0)	(7,0)		(7,0)
9	(0,0)	(0,2)	(0,4)	(0,6)	(0,8)	(9,0)		(9,0)
11	(0,0)	(0,2)	(0,4)	(0,6)	(0,8)	(0,10)		(11,0)
⋮								
999	(0,0)	(0,2)	(0,4)	(0,6)	(0,8)	(0,10)	...	(999,0)

So, strategy (0,0) when player 1 announce 1, and player 2 announce 2 is Nash equilibrium.

4. (a) $V(S3) = R(S3) = 3$

$V(S4) = R(S4) = 4$

$V(S5) = R(S5) = 5$

$V(S6) = R(S6) = 6$

$V(S1) = \frac{1}{6}\left(-6 + V(S1) + V(S2) + V(S3) + V(S4) + V(S5) + V(S6)\right)$

$V(S2) = \frac{1}{6}\left(-6 + V(S1) + V(S2) + V(S3) + V(S4) + V(S5) + V(S6)\right)$

So, $V(S1) = V(S2) = 3$

So,

State	s_1	s_2	s_3	s_4	s_5	s_6
$\pi(s)$	Roll	Roll	Stop	Stop	Stop	Stop
$V^\pi(s)$	3	3	3	4	5	6

(b): If $\pi(S2)$ is stop. $V(S2) = 2$. So, $\pi'(S_2)$ is roll.

If $\pi(S3)$ is roll,

$V(S3) = \frac{1}{6}(-6+3V(3)+15) \Rightarrow V(S3) = 3.$

So stop and roll is the same.

If $\pi(S4)$ is roll,

$V(S4) = \frac{1}{6}(-6+3V(S4)+14) \Rightarrow V(S4) = \frac{8}{3} < 4$

$\pi'(S4)$ is stop.

If $\pi(S5)$ is roll,

$V(S5) = \frac{1}{6}(-6+3V(S5)+13) \Rightarrow V(S5) = \frac{7}{3} < 5$

$\pi'(S5)$ is roll

So,

State	s_1	s_2	s_3	s_4	s_5	s_6
$\pi(s)$	Roll	Roll	Stop	Stop	Stop	Stop
$\pi'(s)$	Roll	Roll	Roll/Stop	Stop	Stop	Stop

(c): Yes.

Because the improved policy ($\pi'(s)$) is the same as $\pi(s)$ in part (a). It has already converged. So, $\pi(s)$ from part (a) is optimal.

(d) B