《创新链和产业链融合下的产业政策》
附录 8

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1 理论模型推导与证明

1.1 封闭经济下社会最优资源配置与产出规模

1.1.1 社会最优劳动力资源配置

由中间产品市场出清条件有 $\sum_{j=1}^{n} M_{j,12} = M_{12} = Q_1$，将上游企业的生产函数代入到代表性下游企业 j 的生产函数，可得到下游企业产出表达式如下：

$$Q_{j,2} = n^{-\gamma_2} z_1^{\gamma_2} \phi_1^{\beta_2\theta+(1-\alpha_1)\gamma_2} z_2 \phi_2^{\beta_2} l_{p1}^{\alpha_1\gamma_2} l_{r1}^{\beta_2\theta+(1-\alpha_1)\gamma_2} l_{j,p2}^{\alpha_2} l_{j,r2}^{\beta_2}$$

社会最终产品产量最大化问题：

$$\max_{\{l_{p1}, l_{r1}, (l_{j,p2}), (l_{j,r2})\}} \left\{ \sum_{j=1}^{n} Q_{j,2}^{\frac{\sigma-1}{\sigma}} \right\}$$

$$s.t. \quad l_{p1} + l_{r1} + \sum_{j=1}^{n} (l_{j,p2} + l_{j,r2}) \leq L - nf$$

其一阶条件为：

$$(l_{j,p2}) \quad \frac{\sigma-1}{\sigma} Q_{j,2}^{\frac{-1}{\sigma}} n^{-\gamma_2} \psi_{12} \alpha_2 l_{p1}^{\alpha_1\gamma_2} l_{r1}^{\beta_2\theta+(1-\alpha_1)\gamma_2} l_{j,p2}^{\alpha_2-1} l_{j,r2}^{\beta_2} - \lambda = 0 \tag{1}$$

$$(l_{j,r2}) \quad \frac{\sigma-1}{\sigma} Q_{j,2}^{\frac{-1}{\sigma}} n^{-\gamma_2} \psi_{12} \beta_2 l_{p1}^{\alpha_1\gamma_2} l_{r1}^{\beta_2\theta+(1-\alpha_1)\gamma_2} l_{j,p2}^{\alpha_2} l_{j,r2}^{\beta_2-1} - \lambda = 0 \tag{2}$$

$$(l_{p1}) \quad \frac{\sigma-1}{\sigma} \alpha_1\gamma_2 n^{-\gamma_2} l_{p1}^{\alpha_1\gamma_2-1} l_{r1}^{\beta_2\theta+(1-\alpha_1)\gamma_2} \sum_{j=1}^{n} \left(Q_{j,2}^{\frac{-1}{\sigma}} \psi_{12} l_{j,p2}^{\alpha_2} l_{j,r2}^{\beta_2} \right) - \lambda = 0 \tag{3}$$

$$(l_{r1}) \quad \frac{\sigma-1}{\sigma} \left[\beta_2\theta + (1-\alpha_1)\gamma_2\right] n^{-\gamma_2} l_{p1}^{\alpha_1\gamma_2} l_{r1}^{\beta_2\theta+(1-\alpha_1)\gamma_2-1} \sum_{j=1}^{n} \left(Q_{j,2}^{\frac{-1}{\sigma}} \psi_{12} l_{j,p2}^{\alpha_2} l_{j,r2}^{\beta_2} \right) - \lambda = 0 \tag{4}$$

其中，$\psi_{12} = z_1^{\gamma_2} z_2 \phi_1^{(1-\alpha_1)\gamma_2+\beta_2\theta} \phi_2^{\beta_2}$。对于下游企业 j 有：

$$l_{j,p2} = \frac{\alpha_2}{\beta_2} l_{j,r2}$$

$$l_{p1} = \frac{\alpha_1\gamma_2}{\beta_2\theta + (1-\alpha_1)\gamma_2} l_{r1}$$

利用企业之间的对称性可知：

$$
\begin{aligned}
& n^{1-\gamma_2}\alpha_1\gamma_2 l_{p1}^{\alpha_1\gamma_2-1} l_{r1}^{\beta_2\theta+(1-\alpha_1)\gamma_2} Q_{j,2}^{\frac{-1}{\sigma}}\psi_{12}\left(\frac{l_{p2}}{n}\right)^{\alpha_2}\left(\frac{l_{r2}}{n}\right)^{\beta_2} \\
&= Q_{j,2}^{\frac{-1}{\sigma}} n^{-\gamma_2}\psi_{12}\alpha_2 l_{p1}^{\alpha_1\gamma_2} l_{r1}^{\beta_2\theta+(1-\alpha_1)\gamma_2}\left(\frac{l_{p2}}{n}\right)^{\alpha_2-1}\left(\frac{l_{r2}}{n}\right)^{\beta_2} \\
&\Rightarrow\ n\alpha_1\gamma_2\left(\frac{l_{p2}}{n}\right) = \alpha_2 l_{p1} \\
&\Rightarrow\ \frac{l_{p1}}{l_{p2}} = \frac{\alpha_1\gamma_2}{\alpha_2}
\end{aligned}
$$

从而劳动力资源配置方式可以表示为：

$$
\left(l_{p1}^*, l_{r1}^*, l_{j,p2}^*, l_{j,r2}^*\right) = \left(\alpha_1\gamma_2,\, (1-\alpha_1)\gamma_2+\beta_2\theta,\, \frac{\alpha_2}{n},\, \frac{\beta_2}{n}\right) \tag{5}
$$

以 $l_{p2}^* \equiv \sum_{j=1}^{n} l_{j,p2}^*$ 和 $l_{r2}^* \equiv \sum_{j=1}^{n} l_{j,r2}^*$ 分别表示下游企业生产活动和研发活动的总投入，由下游企业之间的对称性必有：

$$
(l_{p1}^*,\ l_{r1}^*,\ l_{p2}^*,\ l_{r2}^*) \propto (\alpha_1\gamma_2,\ (1-\alpha_1)\gamma_2+\beta_2\theta,\ \alpha_2,\ \beta_2)
$$

1.1.2 社会最终产品产出

(1) 社会最终产品的产出 利用劳动力比例方程 (5) 与劳动力市场出清方程可得社会最终产品的产出为：

$$
\begin{aligned}
Q_2 = \sum_{j=1}^{n} Q_{j,2} &= \psi_{12} n^{-\gamma_2} l_{p1}^{\alpha_1\gamma_2} l_{r1}^{\beta_2\theta+(1-\alpha_1)\gamma_2}\sum_{j=1}^{n} l_{j,p2}^{\alpha_2} l_{j,r2}^{\beta_2} \\
&= \psi_{12} n^{-\gamma_2} l_{p1}^{\alpha_1\gamma_2} l_{r1}^{\beta_2\theta+(1-\alpha_1)\gamma_2} n\left(\frac{l_{p2}}{n}\right)^{\alpha_2}\left(\frac{l_{r2}}{n}\right)^{\beta_2} \\
&= \psi_{12} l_{p1}^{\alpha_1\gamma_2} l_{r1}^{\beta_2\theta+(1-\alpha_1)\gamma_2} l_{p2}^{\alpha_2} l_{r2}^{\beta_2} \\
&= \psi_{12}\frac{(\alpha_1\gamma_2)^{\alpha_1\gamma_2}\left((1-\alpha_1)\gamma_2+\beta_2\theta\right)^{(1-\alpha_1)\gamma_2+\beta_2\theta}\alpha_2^{\alpha_2}\beta_2^{\beta_2}(L-nf)^{\alpha_1\gamma_2+(1-\alpha_1)\gamma_2+\beta_2\theta+\alpha_2+\beta_2}}{\left(\alpha_1\gamma_2+(1-\alpha_1)\gamma_2+\beta_2\theta+\alpha_2+\beta_2\right)^{\alpha_1\gamma_2+(1-\alpha_1)\gamma_2+\beta_2\theta+\alpha_2+\beta_2}} \\
&= \psi_{12}\frac{(\alpha_1\gamma_2)^{\alpha_1\gamma_2}\left((1-\alpha_1)\gamma_2+\beta_2\theta\right)^{(1-\alpha_1)\gamma_2+\beta_2\theta}\alpha_2^{\alpha_2}\beta_2^{\beta_2}(L-nf)^{1+\beta_2\theta}}{(1+\beta_2\theta)^{1+\beta_2\theta}} \\
&= \psi_2(L-nf)^{1+\beta_2\theta}
\end{aligned} \tag{6}
$$

其中，$\psi_2 = \frac{\psi_{12}(\alpha_1\gamma_2)^{\alpha_1\gamma_2}((1-\alpha_1)\gamma_2+\beta_2\theta)^{(1-\alpha_1)\gamma_2+\beta_2\theta}\alpha_2^{\alpha_2}\beta_2^{\beta_2}}{(1+\beta_2\theta)^{1+\beta_2\theta}}$。

进一步地，社会计划者选择下游企业数目以最大化社会产出，由上式可知，由于减少进入固定成本消耗可以增加社会产出，因此社会最优均衡应该是使得下游企业数目尽可能少，即 $n^* = 1$ ，此时，该国的社会最优最终产品产出为 $Q_2^* = \psi_2(L-f)^{1+\beta_2\theta}$ 。

(2) 各部门的劳动力投入 上游生产性劳动力投入：

$$\begin{aligned} l_{p1}^{*} &= \frac{\alpha_1\gamma_2}{\alpha_1\gamma_2+(1-\alpha_1)\gamma_2+\beta_2\theta+\alpha_2+\beta_2}(L-n^{*}f) \\ &= \frac{\alpha_1\gamma_2}{1+\beta_2\theta}(L-f) \end{aligned}$$

同理,上游研发劳动力投入为 $l_{r1}^{*}=\frac{(1-\alpha_1)\gamma_2+\beta_2\theta}{1+\beta_2\theta}(L-f)$,下游生产性劳动力投入为 $l_{p2}^{*}=\frac{\alpha_2}{1+\beta_2\theta}(L-f)$，下游研发劳动力投入为 $l_{r2}^{*}=\frac{\beta_2}{1+\beta_2\theta}(L-f)$。

1.2 市场均衡与最优产业政策

1.2.1 上游成本最小化问题

$$\begin{aligned} \min_{\{l_{p1},l_{r1}\}} &\{(1+\tau_{p1})\,l_{p1}+(1+\tau_{r1})\,l_{r1}\} \\ &s.t.\quad Q_1\geq 1 \end{aligned}$$

拉格朗日函数为：

$$\mathcal{L}=(1+\tau_{p1})\,l_{p1}+(1+\tau_{r1})\,l_{r1}-\lambda\,(Q_1-1)$$

上游企业的成本最小化问题的一阶条件为：

$$1+\tau_{p1}-\lambda\alpha_1 z_1 l_{p1}^{\alpha_1-1}(\phi_1 l_{r1})^{1-\alpha_1}=0$$

$$1+\tau_{r1}-\lambda\,(1-\alpha_1)\,\phi_1 z_1 l_{p1}^{\alpha_1}(\phi_1 l_{r1})^{-\alpha_1}=0$$

同时，我们还知道 $\lambda=\frac{\partial\mathcal{L}}{\partial Q_1}=MC=p_1$，于是，我们有：

$$1+\tau_{p1}-\alpha_1 p_1 l_{p1}^{-1}Q_1=0$$

$$1+\tau_{r1}-(1-\alpha_1)\,p_1 l_{r1}^{-1}Q_1=0$$

即：

$$\frac{(1+\tau_{p1})\,\tilde{l}_{p1}}{p_1 Q_1}=\alpha_1 \tag{7}$$

$$\frac{(1+\tau_{r1})\,\tilde{l}_{r1}}{p_1 Q_1}=1-\alpha_1 \tag{8}$$

两式相除则有：

$$\frac{\tilde{l}_{p1}}{\tilde{l}_{r1}}=\frac{\alpha_1}{1-\alpha_1}\frac{1+\tau_{r1}}{1+\tau_{p1}} \tag{9}$$

1.2.2 下游企业利润最大化问题

我们假设下游企业购买关键核心技术中间产品时，存在产品市场摩擦，该市场摩擦导致下游企业支付的中间产品价格为：$(1+\chi_1)\,p_1$。则下游产业代表性企业的利润为：

$$\pi_{j,2}=p_{j,2}q_{j,2}-(1+\tau_{p2})\,l_{j,p2}-(1+\tau_{r2})\,l_{j,r2}-(1+\chi_1)\,p_1 M_{j,12}-f$$

由 $q_{j,2}=EP^{\sigma-1}p_{j,2}^{-\sigma}$ 可得 $p_{j,2}=(EP^{\sigma-1})^{\frac{1}{\sigma}}\,q_{j,2}^{-\frac{1}{\sigma}}$，将其代入上式则有：

$$\pi_{j,2}=\left(EP^{\sigma-1}\right)^{\frac{1}{\sigma}}\left[z_2 l_{j,p2}^{\alpha_2}\left(\phi_2 l_{j,r2}\,(\phi_1 l_{r_1})^{\theta}\right)^{\beta_2}M_{j,12}^{\gamma_2}\right]^{\frac{\sigma-1}{\sigma}}-(1+\tau_{p2})\,l_{j,p2}-(1+\tau_{r2})\,l_{j,r2}-(1+\chi_1)\,p_1 M_{j,12}-f$$

根据下游产业代表性企业的利润最大化问题关于 $l_{j,p2}$ 的一阶条件有生产性劳动成本-销售额支出之比为：

$$\begin{aligned}\alpha_2\frac{\sigma-1}{\sigma}l_{j,p2}^{-1}\left(EP^{\sigma-1}\right)^{\frac{1}{\sigma}}\left[z_2 l_{j,p2}^{\alpha_2}\left(\phi_2 l_{j,r2}\,(\phi_1 l_{r_1})^{\theta}\right)^{\beta_2}M_{j,12}^{\gamma_2}\right]^{\frac{\sigma-1}{\sigma}}-(1+\tau_{p2})=0\\ \Leftrightarrow\ \alpha_2\frac{\sigma-1}{\sigma}l_{j,p2}^{-1}p_{j,2}q_{j,2}=1+\tau_{p2}\\ \Leftrightarrow\ \frac{(1+\tau_{p2})\,l_{j,p2}}{p_{j,2}q_{j,2}}=\frac{\sigma-1}{\sigma}\alpha_2\end{aligned} \tag{10}$$

同理，由关于 $l_{j,2}$ 和 $M_{j,12}$ 的一阶条件可得到研发劳动成本-销售额和中间产品-销售额之比为：

$$\frac{(1+\tau_{r2})\,l_{j,r2}}{p_{j,2}q_{j,2}}=\frac{\sigma-1}{\sigma}\beta_2 \tag{11}$$

$$\frac{(1+\chi_1)\,p_1 M_{j,12}}{p_{j,2}q_{j,2}}=\frac{\sigma-1}{\sigma}\gamma_2 \tag{12}$$

将 (10) 式除以 (11) 式，可以得到下游企业生产性劳动投入与研发劳动投入的比例为：

$$\frac{\tilde{l}_{j,p2}}{\tilde{l}_{j,r2}}=\frac{(1+\tau_{r2})\alpha_2}{(1+\tau_{p2})\beta_2}$$

根据 (10)-(12) 则可得到下游产业代表性企业 j 的总成本为:

$$(1+\tau_{p2})\,l_{j,p2}+(1+\tau_{r2})\,l_{j,r2}+(1+\chi_1)\,p_1M_{j,12}=\frac{\sigma-1}{\sigma}p_{j,2}q_{j,2}$$

定义 $\rho_2\equiv\frac{\sigma-1}{\sigma}<1$ 那么，下游产业的总利润为:

$$(1-\rho_2)\sum_{j=1}^{n}p_{j,2}q_{j,2}$$

则中间产品市场出清条件为:

$$n\rho_2p_{j,2}q_{j,2}=n\,(1+\tau_{p2})\,\tilde{l}_{j,p2}+n\,(1+\tau_{r2})\,\tilde{l}_{j,r2}+(1+\chi_1)\,p_1Q_1 \tag{13}$$

1.2.3 政府最优的补贴和税收政策

将方程 (7)，(10)，(11) 代入 (13)，可以得到上游产业与下游产业的生产性劳动力配置之比为:

$$\frac{\tilde{l}_{p1}}{\tilde{l}_{j,p2}}=n\frac{\alpha_1\gamma_2}{\alpha_2}\frac{1+\tau_{p2}}{(1+\tau_{p1})\,(1+\chi_1)} \tag{14}$$

因此，根据 (9)、(10)、(11) 和 (14) 市场均衡条件下各部门劳动力的配置满足如下比例关系:

$$\left(\tilde{l}_{p1},\tilde{l}_{r1},\tilde{l}_{j,p2},\tilde{l}_{j,r2}\right)=\left(\frac{\alpha_1\gamma_2}{(1+\tau_{p1})\,(1+\chi_1)},\frac{(1-\alpha_1)\,\gamma_2}{(1+\tau_{r1})\,(1+\chi_1)},\frac{\alpha_2}{n\,(1+\tau_{p2})},\frac{\beta_2}{n\,(1+\tau_{r2})}\right) \tag{15}$$

根据 (5) 和 (15)，我们有:

$$\frac{l_{p1}^*}{l_{r2}^*}=\frac{\alpha_1\gamma_2}{\beta_2}$$

$$\frac{\tilde{l}_{p1}}{\tilde{l}_{r2}}=\frac{\alpha_1\gamma_2\,(1+\tau_{r2})}{(1+\tau_{p1})\,(1+\chi_1)\,\beta_2}$$

因此，便有 $\frac{1+\tau_{r2}^*}{1+\tau_{p1}^*}=1+\chi_1$，同理，可以得到其他政府政策的需要满足的条件。政府最优的补贴和税收政策总结如下:

$$\frac{1+\tau_{r2}^*}{1+\tau_{p1}^*}=1+\chi_1$$

$$\frac{1+\tau_{r2}^*}{1+\tau_{r1}^*}=(1+\chi_1)\left(1+\frac{\beta_2\theta}{(1-\alpha_1)\,\gamma_2}\right)$$

$$\frac{1+\tau_{p1}^*}{1+\tau_{r1}^*}=1+\frac{\beta_2\theta}{(1-\alpha_1)\,\gamma_2}$$

$$\tau_{p2}^*=\tau_{r2}^*$$

1.3 开放经济模型相关证明

1.3.1 依赖国外上游企业

(1) 下游企业的产出，边际成本与中间产品需求 在开放经济条件下，下游代表性企业的成本最小化问题为：

$$\min_{\left\{(l_{j,p2}),(l_{j,r2}),\left(M^x_{j,12}\right)\right\}} \left\{\left(1+\tau^x_{j,1}\right) p^x_1 M^x_{j,12} + \left(1+\tau_{j,p2}\right) l_{j,p2} + \left(1+\tau_{j,r2}\right) l_{j,r2}\right\}$$
$$s.t. \quad z_2 l^{\alpha_2}_{j,p2} \left(\phi_2 l_{j,r2}\right)^{\beta_2} \left(M^x_{j,12}\right)^{\gamma_2} \geq \bar{Q}_{j,2}$$

拉格朗日函数为：

$$\begin{aligned}\mathcal{L} = &\left(1+\tau^x_{j,1}\right) p^x_1 M^x_{j,12} + \left(1+\tau_{j,p2}\right) l_{j,p2} + \left(1+\tau_{j,r2}\right) l_{j,r2} \\ &+ \lambda_j \left(\bar{Q} - z_2 l^{\alpha_2}_{j,p2} \left(\phi_2 l_{j,r2}\right)^{\beta_2} \left(M^x_{j,12}\right)^{\gamma_2}\right)\end{aligned}$$

一阶条件为：

$$\frac{\partial \mathcal{L}}{\partial M^x_{j,12}} = \left(1+\tau^x_{j,1}\right) p^x_1 - \lambda_j \frac{\partial Q^x_{j,2}}{\partial M_{j,12}} = 0 \tag{16}$$

$$\frac{\partial \mathcal{L}}{\partial l_{j,p2}} = \left(1+\tau_{j,p2}\right) - \lambda_j \frac{\partial Q^x_{j,2}}{\partial l_{j,p2}} = 0 \tag{17}$$

$$\frac{\partial \mathcal{L}}{\partial l_{j,r2}} = \left(1+\tau_{j,r2}\right) - \lambda_j \frac{\partial Q^x_{j,2}}{\partial l_{j,r2}} = 0 \tag{18}$$

将上述方程两两相互对除可得：

$$l_{j,p2} = \frac{\left(1+\tau^x_{j,1}\right) p^x_1}{\left(1+\tau_{j,p2}\right)} \frac{\alpha_2}{\gamma_2} M^x_{j,12} \tag{19}$$

$$l_{j,r2} = \frac{\left(1+\tau^x_{j,1}\right) p^x_1}{\left(1+\tau_{j,r2}\right)} \frac{\beta_2}{\gamma_2} M^x_{j,12} \tag{20}$$

注意到 $\lambda_j = \frac{\partial \mathcal{L}}{\partial \bar{Q}_{j,2}}$ 即为生产边际成本，将上述两式带入到 (16) 可以得到：

$$\begin{aligned}c^x_{j,2} = \lambda_j &= \frac{\left(1+\tau^x_{j,1}\right) p^x_1}{\partial Q^x_{j,2} / \partial M_{j,12}} \\ &= \frac{\left(1+\tau^x_{j,1}\right) p^x_1}{\gamma_2 z_2 \left(\frac{\left(1+\tau^x_{j,1}\right) p^x_1}{\left(1+\tau_{j,p2}\right)} \frac{\alpha_2}{\gamma_2} M^x_{j,12}\right)^{\alpha_2} \left(\phi_2 \frac{\left(1+\tau^x_{j,1}\right) p^x_1}{\left(1+\tau_{j,r2}\right)} \frac{\beta_2}{\gamma_2} M^x_{j,12}\right)^{\beta_2} \left(M^x_{j,12}\right)^{\gamma_2 - 1}} \\ &= \frac{\left(1+\tau_{j,p2}\right)^{\alpha_2} \left(1+\tau_{j,r2}\right)^{\beta_2}}{z_2 \alpha^{\alpha_2}_2 \left(\beta_2 \phi_2\right)^{\beta_2} \gamma^{\gamma_2}_2} \left[\left(1+\tau^x_{j,1}\right) p^x_1\right]^{\gamma_2} &(21) \\ &\equiv \frac{1}{\gamma_2 B_2} \left[\left(1+\tau^x_{j,1}\right) p^x_1\right]^{\gamma_2} &(22)\end{aligned}$$

$$B_2 \equiv \frac{z_2 \alpha^{\alpha_2}_2 \left(\beta_2 \phi_2\right)^{\beta_2}}{\left(1+\tau_{j,p2}\right)^{\alpha_2} \left(1+\tau_{j,r2}\right)^{\beta_2} \gamma^{\alpha_2+\beta_2}_2} \tag{23}$$

此时企业 j 的产出可以表示为：

$$
\begin{aligned}
Q_{j,2}^{x} &= z_2\left(\frac{\left(1+\tau_{j,1}^{x}\right)p_{j,1}^{x}}{\left(1+\tau_{j,p2}\right)}\frac{\alpha_2}{\gamma_2}M_{j,12}^{x}\right)^{\alpha_2}\left(\phi_2\frac{\left(1+\tau_{j,1}^{x}\right)p_{j,1}^{x}}{\left(1+\tau_{j,r2}\right)}\frac{\beta_2}{\gamma_2}M_{j,12}^{x}\right)^{\beta_2}\left(M_{j,12}^{x}\right)^{\gamma_2} \\
&= \frac{\left[\left(1+\tau_{j,1}^{x}\right)p_{j,1}^{x}\right]^{\alpha_2+\beta_2} z_2\alpha_2^{\alpha_2}\left(\beta_2\phi_2\right)^{\beta_2}}{\left(1+\tau_{j,p2}\right)^{\alpha_2}\left(1+\tau_{j,r2}\right)^{\beta_2}\gamma_2^{\alpha_2+\beta_2}}M_{j,12}^{x} &(24)\\
&\equiv B_2\left[\left(1+\tau_{j,1}^{x}\right)p_{j,1}^{x}\right]^{\alpha_2+\beta_2}M_{j,12}^{x} &(25)
\end{aligned}
$$

由上式可知下游企业对中间产品的需求为：

$$
\begin{aligned}
M_{j,12}^{x} &= \frac{q_{j,2}^{d}+q_{j,2}^{f}}{B_2\left[\left(1+\tau_{j,1}^{x}\right)p_1^{x}\right]^{\alpha_2+\beta_2}} \\
&= \frac{EP^{\sigma-1}+E^{f}\left(P^{f}\right)^{\sigma-1}\left(1+\tau_{j,2}^{f}\right)^{-\sigma}}{B_2\left[\left(1+\tau_{j,1}^{x}\right)p_1^{x}\right]^{1-\gamma_2}\left(\frac{\mu_2}{\gamma_2B_2}\left[\left(1+\tau_{j,1}^{x}\right)p_1^{x}\right]^{\gamma_2}\right)^{\sigma}} \\
&= \frac{\left(EP^{\sigma-1}+E^{f}\left(P^{f}\right)^{\sigma-1}\left(1+\tau_{j,2}^{f}\right)^{-\sigma}\right)\left(\frac{\gamma_2}{\mu_2}\right)^{\sigma}}{B_2^{1-\sigma}\left[\left(1+\tau_{j,1}^{x}\right)p_1^{x}\right]^{\gamma_2(\sigma-1)+1}} \\
&= \Pi_1^{f}B_2^{\sigma-1}\left[\left(1+\tau_{j,1}^{x}\right)p_1^{x}\right]^{\gamma_2(1-\sigma)-1} &(26)
\end{aligned}
$$

其中，$\Pi_1^{f}\equiv\left(EP^{\sigma-1}+E^{f}\left(P^{f}\right)^{\sigma-1}\left(1+\tau_{j,2}^{f}\right)^{-\sigma}\right)\left(\frac{\gamma_2}{\mu_2}\right)^{\sigma}$。

(2) 下游企业的产品定价方程 在此情形下，下游企业可以同时通过国内市场销售和产品出口国外市场获取利润。下游企业所面临的出口从价关税 τ_2^{f}，假定国外消费者对下游企业 j 的需求函数为：

$$q_{j,2}^{f}=E^{f}P_f^{\sigma-1}\left[(1+\tau_{j,2}^{f})p_{j,2}^{f}\right]^{-\sigma}$$

其利润函数可以表示为：

$$\pi_{j,2}=(p_{j,2}^{d}-c_{j,2}^{x})q_{j,2}^{d}+(p_{j,2}^{f}-c_{j,2}^{x})q_{j,2}^{f}-f$$

利润函数关于价格求一阶条件：

$$
\begin{aligned}
\frac{\partial\pi_{j,2}}{\partial p_{j,2}^{d}} &= (p_{j,2}^{d}-c_{j,2}^{x})q_{j,2}^{d}+(p_{j,2}^{f}-c_{j,2}^{x})q_{j,2}^{f}-f=0 \\
\Rightarrow EP^{\sigma-1}\left(p_{j,2}^{d}\right)^{-\sigma}&+(p_{j,2}^{d}-c_{j,2}^{x})EP^{\sigma-1}(-\sigma)\left(p_{j,2}^{d}\right)^{-\sigma-1}=0 \\
&\Rightarrow \sigma(p_{j,2}^{d}-c_{j,2}^{x})\left(p_{j,2}^{d}\right)^{-1}=1 \\
&\Rightarrow p_{j,2}^{d}=\frac{\sigma}{\sigma-1}c_{j,2}^{x}
\end{aligned}
$$

同理可得 $p_{j,2}^{f}=\frac{\sigma}{\sigma-1}c_{j,2}^{x}$，于是，我们有：

$$p_{j,2}^{d}=p_{j,2}^{f}=\frac{\sigma}{\sigma-1}c_{j,2}^{x}\equiv\mu_2 c_{j,2}^{x}$$

其中，$\mu_2\equiv\frac{\sigma}{\sigma-1}$。

(3) 上游企业的利润函数和产品定价方程

$$\begin{aligned}\pi_1^f&=\sum_{j=1}^{n}(p_{j,1}^{x}-c_1^f)M_{j,12}^{x}\\&=\Pi_1^f\sum_{j=1}^{n}(p_{j,1}^{x}-c_1^f)\frac{(p_{j,1}^{x})^{\gamma_2(1-\sigma)-1}B_2^{\sigma-1}}{(1+\tau_1^x)^{\gamma_2(\sigma-1)+1}}\end{aligned}$$

利润函数关于价格求一阶条件有：

$$\begin{aligned}\frac{\partial\pi_1^f}{\partial p_{j,1}^{x}}=\Pi_1^f\left(\frac{(p_{j,1}^{x})^{\gamma_2(1-\sigma)-1}B_2^{\sigma-1}}{(1+\tau_1^x)^{\gamma_2(\sigma-1)+1}}+(\gamma_2(1\text{-}\sigma)\text{-}1)(p_{j,1}^{x}-c_1^f)\frac{(p_{j,1}^{x})^{\gamma_2(1-\sigma)-1}B_2^{\sigma-1}}{(1+\tau_1^x)^{\gamma_2(\sigma-1)+1}}(p_{j,1}^{x})^{-1}\right)&=0\\ \Rightarrow\ 1+(\gamma_2(1\text{-}\sigma)\text{-}1)(p_{j,1}^{x}-c_1^f)(p_{j,1}^{x})^{-1}&=0\\ \Rightarrow\ p_{j,1}^{x}=\frac{\gamma_2(1-\sigma)+1}{\gamma_2(1-\sigma)}c_1^f&\end{aligned}$$

令 $\mu_1^x\equiv\frac{\gamma_2(1-\sigma)+1}{\gamma_2(1-\sigma)}$，则有：

$$p_1^x\equiv p_{j,1}^{x}=\frac{\gamma_2(1-\sigma)+1}{\gamma_2(1-\sigma)}c_1^f\equiv\mu_1^x c_1^f$$

此时，国外上游企业的利润为：

$$\begin{aligned}
\pi_1^f &= \Pi_1^f \sum_{j=1}^{n} (p_{j,1}^x - c_1^f) \frac{(p_{j,1}^x)^{\gamma_2(1-\sigma)-1} B_2^{\sigma-1}}{(1+\tau_1^x)^{\gamma_2(\sigma-1)+1}} \\
&= \Pi_1^f n \left(1 - \frac{1}{\mu_1^x}\right) \mu_1^x c_1^f \frac{(\mu_1^x c_1^f)^{\gamma_2(1-\sigma)-1} B_2^{\sigma-1}}{(1+\tau_1^x)^{\gamma_2(\sigma-1)+1}} \\
&= \Pi_1^f n \left(1 - \frac{1}{\mu_1^x}\right) \frac{(\mu_1^x c_1^f)^{\gamma_2(1-\sigma)} B_2^{\sigma-1}}{(1+\tau_1^x)^{\gamma_2(\sigma-1)+1}} \\
&= \left(EP^{\sigma-1} + E^f \left(P^f\right)^{\sigma-1} \left(1+\tau_{j,2}^f\right)^{-\sigma}\right) \left(\frac{\gamma_2}{\mu_2}\right)^{\sigma} n \left(1 - \frac{1}{\mu_1^x}\right) \frac{(\mu_1^x c_1^f)^{\gamma_2(1-\sigma)} B_2^{\sigma-1}}{(1+\tau_1^x)^{\gamma_2(\sigma-1)+1}} \\
&= \left(1 - \frac{1}{\mu_1^x}\right) \left[\left(EP^{\sigma-1} + E^f \left(P^f\right)^{\sigma-1} \delta_{f2}^{\sigma}\right) \left(\frac{\gamma_2}{\mu_2}\right)^{\sigma} n (\mu_1^x c_1^f)^{\gamma_2(1-\sigma)} B_2^{\sigma-1} \left(\delta_{x1}\right)^{\gamma_2(\sigma-1)+1}\right] \\
&= \left(1 - \frac{1}{\mu_1^x}\right) [EP^{\sigma-1} \left(\frac{\gamma_2}{\mu_2}\right)^{\sigma} n (\mu_1^x c_1^f)^{\gamma_2(1-\sigma)} B_2^{\sigma-1} \left(\delta_{x1}\right)^{\gamma_2(\sigma-1)+1} \\
&\quad + E^f \left(P^f\right)^{\sigma-1} \delta_{f2}^{\sigma} \left(\frac{\gamma_2}{\mu_2}\right)^{\sigma} n (\mu_1^x c_1^f)^{\gamma_2(1-\sigma)} B_2^{\sigma-1} \left(\delta_{x1}\right)^{\gamma_2(\sigma-1)+1}] \\
&= \left(1 - \frac{1}{\mu_1^x}\right) [EP^{\sigma-1} \left(\frac{\gamma_2}{\mu_2}\right)^{\sigma} \sum_{j=1}^{n} (p_{j,1}^x)^{\gamma_2(1-\sigma)} B_2^{\sigma-1} \left(\delta_{x1}\right)^{\gamma_2(\sigma-1)+1} \\
&\quad + nE^f \left(P^f\right)^{\sigma-1} (\mu_1^x c_1^f)^{\gamma_2(1-\sigma)} B_2^{\sigma-1} \left(\delta_{x1}\right)^{\gamma_2(\sigma-1)+1} \left(\frac{\gamma_2 \delta_{f2}}{\mu_2}\right)^{\sigma}]
\end{aligned}$$

由 (22) 可知 $\left(\frac{p_1^x}{\delta_{x1}}\right)^{\gamma_2} = \gamma_2 B_2 c_{j,2}^x$，我们有 $p_1^x = \left(\gamma_2 B_2 c_{j,2}^x\right)^{\frac{1}{\gamma_2}} \delta_{x1} = \left(\gamma_2 B_2 \frac{p_{j,2}^d}{\mu_2}\right)^{\frac{1}{\gamma_2}} \delta_{x1}$，代入上式可得：

$$\begin{aligned}
\pi_1^f &= \left(1 - \frac{1}{\mu_1^x}\right) [EP^{\sigma-1} \left(\frac{\gamma_2}{\mu_2}\right)^{\sigma} \sum_{j=1}^{n} (\gamma_2 B_2 \frac{p_{j,2}}{\mu_2})^{\frac{\gamma_2(1-\sigma)}{\gamma_2}} B_2^{\sigma-1} \delta_{x1} \\
&\quad + nE^f \left(P^f\right)^{\sigma-1} (\mu_1^x c_1^f)^{\gamma_2(1-\sigma)} B_2^{\sigma-1} \left(\delta_{x1}\right)^{\gamma_2(\sigma-1)+1} \left(\frac{\gamma_2 \delta_{f2}}{\mu_2}\right)^{\sigma}] \\
&= \left(1 - \frac{1}{\mu_1^x}\right) \left[EP^{\sigma-1} \frac{\gamma_2}{\mu_2} \sum_{j=1}^{n} p_{j,2}^{1-\sigma} \delta_{x1} + nE^f \left(P^f\right)^{\sigma-1} (\mu_1^x c_1^f)^{\gamma_2(1-\sigma)} B_2^{\sigma-1} \left(\delta_{x1}\right)^{\gamma_2(\sigma-1)+1} \left(\frac{\gamma_2 \delta_{f2}}{\mu_2}\right)^{\sigma}\right] \\
&= \left(1 - \frac{1}{\mu_1^x}\right) \left[\frac{\gamma_2 E}{\mu_2} \delta_{x1} + nE^f \left(P^f\right)^{\sigma-1} (\mu_1^x c_1^f)^{\gamma_2(1-\sigma)} B_2^{\sigma-1} \left(\delta_{x1}\right)^{\gamma_2(\sigma-1)+1} \left(\frac{\gamma_2 \delta_{f2}}{\mu_2}\right)^{\sigma}\right]
\end{aligned} \tag{27}$$

(4) 国内下游产业代表性企业的净利润和企业数目 由成本加成率为常数可知，国内下游企业利润为：

$$\pi_{j,2} = \frac{1}{n\sigma} \left(E + E^f \delta_{f2}\right) - f$$

由于国内下游产业内的企业可以自由进入，所以均衡状态下 $\pi_{j,2} = 0$，因此，均衡状态下国内下游产业之中的企业数目为：

$$\hat{n}=\frac{E+E^f\delta_{f2}}{\sigma f}$$

(5) 消费者福利

$$U_f=\frac{E}{P}=\frac{E}{\left(\sum_{j=1}^{n}p_{j,2}^{1-\sigma}\right)^{\frac{1}{1-\sigma}}}=\frac{E}{\hat{n}^{\frac{1}{1-\sigma}}p_{j,2}}=\frac{E}{\left(\frac{E+E^f\delta_{f2}}{\sigma f}\right)^{\frac{1}{1-\sigma}}\frac{\mu_2\left(\mu_1^x c_1^f\right)^{\gamma_2}}{\gamma_2 B_2\delta_{x1}^{\gamma_2}}}$$
$$=\frac{\gamma_2 E B_2\delta_{x1}^{\gamma_2}}{\mu_2\left(\mu_1^x c_1^f\right)^{\gamma_2}}\left(\frac{E+E^f\delta_{f2}}{\sigma f}\right)^{\frac{1}{\sigma-1}}$$

1.3.2 国内上游企业与国外上游企业竞争性提供关键技术产品

(1) 中间产品的需求 假定国内下游行业生产所需的关键核心技术中间产品，可以由国外上游企业与国内上游企业竞争性地提供。此时，我们使用 p_1^d 表示国内上游企业提供的关键核心技术中间产品的销售价格，仍用 p_1^x 表示国外上游企业关键核心技术中间产品的销售价格。国外上游企业与国内企业之间进行 Betrand 竞争（即价格竞争），从而国内下游企业的关键核心技术中间产品价格为：

$$p_{12}^M=\min\left\{(1+\chi_1)p_1^d,\quad (1+\tau_1^x)p_1^x\right\}$$

在该情形下，国内上游企业的研发投入会对国内下游企业造成技术溢出效应，所以国内下游企业的生产函数为：

$$Q_{j,2}=z_2\phi_1^{\beta_2\theta}\phi_2^{\beta_2}l_{r1}^{\beta_2\theta}l_{j,p2}^{\alpha_2}l_{j,r2}^{\beta_2}M_{j,12}^{\gamma_2}$$

与 (26) 的推导过程相似，我们可以得到在国内上游企业与国外上游企业竞争性提供关键技术产品的情境下，若 $(1+\tau_1^x)\,p_1^x\le(1+\chi_1)\,p_1^d$，即国内下游企业全部从国外上游企业购买关键核心技术中间产品，此时，

$$M_{j,12}^x=\Pi_1^f B_2^{\sigma-1}\left(\phi_1 l_{r1}\right)^{\beta_2\theta(\sigma-1)}\left[\left(1+\tau_{j,1}^x\right)p_1^x\right]^{\gamma_2(1-\sigma)-1} \tag{28}$$

为了保证社会最终产品生产函数关于上游企业研发劳动投入满足规模报酬递减,我们限制 $\beta_2\theta(\sigma-1)<1$。由于国内上游企业的研发活动对国内下游企业存在正向溢出效应，国内下游企业对关键核心技术中间产品的需求则受到国内上游企业研发规模的影响。

类比 (22)，此时有国内下游企业的成本为：

$$c_{j,2}^x=\frac{\left[\left(1+\tau_{j,1}^x\right)p_1^x\right]^{\gamma_2}}{\gamma_2 B_2\left(\phi_1 l_{r1}\right)^{\beta_2\theta(\sigma-1)}}$$

(2) 国内上游企业的边际成本 上游企业的成本最小化问题为：

$$\min_{\{l_{p1},l_{r1}\}}\left\{\left(1+\tau_{p1}\right)l_{p1}+\left(1+\tau_{r1}\right)l_{r1}\right\}$$
$$s.t.\,Q\geq 1$$

由一阶条件可知：

$$1+\tau_{p1}=\lambda_1\frac{\partial Q_1}{\partial l_{p1}}=\lambda_1 z_1\alpha_1 l_{p1}^{\alpha_1-1}\left(\phi_1 l_{r1}\right)^{1-\alpha_1} \tag{29}$$

$$1+\tau_{r1}=\lambda_1\frac{\partial Q_1}{\partial l_{r1}}=\lambda_1 z_1\left(1-\alpha_1\right)l_{p1}^{\alpha_1}\phi_1^{1-\alpha_1}l_{r1}^{-\alpha_1} \tag{30}$$

可知：

$$\frac{l_{p1}}{l_{r1}}=\frac{\alpha_1}{1-\alpha_1}\frac{1+\tau_{r1}}{1+\tau_{p1}} \tag{31}$$

带入一阶条件，可知边际生产成本为:

$$\begin{aligned}c_1^d=\lambda_1&=\frac{1+\tau_{p1}}{z_1\alpha_1\phi_1^{1-\alpha_1}}\left(l_{p1}/l_{r1}\right)^{1-\alpha_1}\\&=\frac{\left(1+\tau_{p1}\right)^{\alpha_1}\left(1+\tau_{r1}\right)^{1-\alpha_1}}{z_1\phi_1^{1-\alpha_1}\alpha_1^{\alpha_1}\left(1-\alpha_1\right)^{1-\alpha_1}}\end{aligned} \tag{32}$$

(3) 对国外上游企业产生实质性的竞争效应的条件

$$\begin{aligned}\tilde{c}_1^d&<\left(1+\tau_1^x\right)\mu_1^x c_1^f\\ \Leftrightarrow\ \frac{\left(1+\chi_1\right)\left(1+\tau_{p1}\right)^{\alpha_1}\left(1+\tau_{r1}\right)^{1-\alpha_1}}{z_1\phi_1^{1-\alpha_1}\alpha_1^{\alpha_1}\left(1-\alpha_1\right)^{1-\alpha_1}}&<\left(1+\tau_1^x\right)\mu_1^x c_1^f\\ \Leftrightarrow\ \frac{\left(1+\tau_{p1}\right)^{\alpha_1}\left(1+\tau_{r1}\right)^{1-\alpha_1}}{1+\tau_1^x}<\frac{\alpha_1^{\alpha_1}\left(1-\alpha_1\right)^{1-\alpha_1}z_1\phi_1^{1-\alpha_1}\mu_1^x c_1^f}{\left(1+\chi_1\right)}&\end{aligned}$$

此时，国外上游企业利润方程：

$$\begin{aligned}\pi_1^{f*}&=\sum_{j=1}^{n}\left((p_{j,1}^{x*}-c_1^f)M_{j,12}^x\right)\\&=n\Pi_1^f B_2^{\sigma-1}(\delta_{x1}\tilde{c}_1^d-c_1^f)\left(\phi_1 l_{r1}\right)^{\beta_2\theta(\sigma-1)}\left(\tilde{c}_1^d\right)^{\gamma_2(1-\sigma)-1}\end{aligned}$$

且满足 $\pi_1^{f*}<\pi_1^f$, 其中 π_1^f 为方程 (27) 刻画的垄断利润。

(4) 国内上游企业为关键核心技术产品的供应商时的分析 上游企业成本最小化问题：

$$\min_{\{l_{p1},l_{r1}\}} \{(1+\tau_{p1})\,l_{p1}+(1+\tau_{r1})\,l_{r1}\}$$
$$s.t. Q \geq 1$$

与国内上游企业成本最小化的问题相似，拉格朗日乘子也代表边际成本，且表达式与 (32) 相同。那么关于研发劳动投入 l_{r2} 的一阶条件为：

$$(1+\tau_{r1})-\frac{(1+\tau_{p1})^{\alpha_1}(1+\tau_{r1})^{1-\alpha_1}}{z_1\phi_1^{1-\alpha_1}\alpha_1^{\alpha_1}(1-\alpha_1)^{1-\alpha_1}}\frac{1-\alpha}{l_{r1}}Q_1=0$$

于是我们有：

$$l_{r1}^*=\frac{(1+\tau_{p1})^{\alpha_1}(1+\tau_{r1})^{-\alpha_1}}{z_1\phi_1^{1-\alpha_1}\alpha_1^{\alpha_1}(1-\alpha_1)^{-\alpha_1}}Q_1 \tag{33}$$

与全部从国外上游企业购买关键核心技术中间产品的情景类似，当国内下游企业全部从国外上游企业购买关键核心技术中间产品的时候，下游代表性企业 j 生产所需要的中间产品为：

$$M_{j,12}^d=\Pi_1^f B_2^{\sigma-1}(\phi_1 l_{r1})^{\beta_2\theta(\sigma-1)}\left[(1+\chi_1)\,p_1^d\right]^{\gamma_2(1-\sigma)-1}$$

又由于在市场出清的情况下，

$$Q_1=M_{12}=nM_{j,12}^d=n\Pi_1^f B_2^{\sigma-1}(\phi_1 l_{r1})^{\beta_2\theta(\sigma-1)}\left[(1+\chi_1)\,p_1^d\right]^{\gamma_2(1-\sigma)-1}$$

将上式带入 (33) 可得：

$$\begin{aligned}
l_{r1}^* &= \frac{n(1-\alpha_1)^{\alpha_1}\Pi_1^f B_2^{\sigma-1}\delta_{r1}^{\alpha_1}\left[(1+\chi_1)\,p_1^{d*}\right]^{\gamma_2(1-\sigma)-1}}{z_1\phi_1^{1-\alpha_1-\beta_2\theta(\sigma-1)}\alpha_1^{\alpha_1}\delta_{p1}^{\alpha_1}} \\
&= \frac{\left(E+E^f\delta_{f2}\right)(1-\alpha_1)^{\alpha_1}\Pi_1^f B_2^{\sigma-1}\delta_{r1}^{\alpha_1}\left[(1+\chi_1)\,p_1^{d*}\right]^{\gamma_2(1-\sigma)-1}}{z_1\phi_1^{1-\alpha_1-\beta_2\theta(\sigma-1)}\alpha_1^{\alpha_1}\delta_{p1}^{\alpha_1}\sigma f}
\end{aligned} \tag{34}$$

此时，消费者福利为：

$$\begin{aligned}
U_d=\frac{E}{P} &= \frac{E}{\left(\sum_{j=1}^n p_{j,2}^{1-\sigma}\right)^{\frac{1}{1-\sigma}}}=\frac{E}{\hat{n}^{\frac{1}{1-\sigma}}p_{j,2}}=\frac{E}{\left(\frac{E+E^f\delta_{f2}}{\sigma f}\right)^{\frac{1}{1-\sigma}}\frac{\mu_2\left((1+\chi_1)p_1^{d*}\right)^{\gamma_2}}{\gamma_2 B_2\left(\phi_1 l_{r1}^*\right)^{\beta_2\theta(\sigma-1)}}} \\
&= \frac{\gamma_2 E B_2(\phi_1 l_{r1}^*)^{\beta_2\theta(\sigma-1)}}{\mu_2\left((1+\chi_1)\,p_1^{d*}\right)^{\gamma_2}}\left(\frac{E+E^f\delta_{f2}}{\sigma f}\right)^{\frac{1}{\sigma-1}}
\end{aligned}$$

将国内上游企业研发活动的劳动力投入方程 (34) 代入上式可得：

$$U_d=U_0B_2^{\frac{1}{1-\beta_2\theta(\sigma-1)}}\left(E+E^f\delta_{f2}\right)^{\frac{1}{(\sigma-1)[1-\beta_2\theta(\sigma-1)]}}\left(\Pi_1^f\right)^{\frac{\beta_2\theta}{1-\beta_2\theta(\sigma-1)}}\left(\frac{\delta_{r1}}{\delta_{p1}}\right)^{\frac{\alpha_1\beta_2\theta}{1-\beta_2\theta(\sigma-1)}}\left((1+\chi_1)p_1^{d*}\right)^{-\frac{\gamma_2+\beta_2\theta}{1-\beta_2\theta(\sigma-1)}}$$

$$\text{其中，}U_0\equiv\frac{\gamma_2E}{\mu_2z_1^{\frac{\beta_2\theta}{1-\beta_2\theta(\sigma-1)}}(\sigma f)^{\frac{1}{(\sigma-1)[1-\beta_2\theta(\sigma-1)]}}}\left(\frac{1-\alpha_1}{\alpha_1}\phi\right)^{\frac{\alpha_1\beta_2\theta}{1-\beta_2\theta(\sigma-1)}}$$

(5) 不同进口关税水平下的分析 首先，当进口关税处于中等水平时，即

$$\begin{aligned}
&\tilde{c}_1^d<\left(1+\tau_1^x\right)c_1^f<\mu_1^x\tilde{c}_1^d\\
\Leftrightarrow&\frac{\left(1+\chi_1\right)\left(1+\tau_{p1}\right)^{\alpha_1}\left(1+\tau_{r1}\right)^{1-\alpha_1}}{\alpha_1^{\alpha_1}\left(1-\alpha_1\right)^{1-\alpha_1}z_1\phi_1^{1-\alpha_1}}<\left(1+\tau_1^x\right)c_1^f<\mu_1^x\frac{\left(1+\chi_1\right)\left(1+\tau_{p1}\right)^{\alpha_1}\left(1+\tau_{r1}\right)^{1-\alpha_1}}{\alpha_1^{\alpha_1}\left(1-\alpha_1\right)^{1-\alpha_1}z_1\phi_1^{1-\alpha_1}}\\
\Leftrightarrow&\frac{1}{\alpha_1^{\alpha_1}\left(1-\alpha_1\right)^{1-\alpha_1}z_1\phi_1^{1-\alpha_1}c_1^f}<\frac{1+\tau_1^x}{\left(1+\chi_1\right)\left(1+\tau_{p1}\right)^{\alpha_1}\left(1+\tau_{r1}\right)^{1-\alpha_1}}<\mu_1^x\frac{1}{\alpha_1^{\alpha_1}\left(1-\alpha_1\right)^{1-\alpha_1}z_1\phi_1^{1-\alpha_1}c_1^f}\\
\Leftrightarrow&\frac{1}{\alpha_1^{\alpha_1}\left(1-\alpha_1\right)^{1-\alpha_1}z_1\phi_1^{1-\alpha_1}c_1^f}<\frac{\delta_{p1}^{\alpha_1}\delta_{r1}^{1-\alpha_1}}{\left(1+\chi_1\right)\delta_{x1}}<\frac{\mu_1^x}{\alpha_1^{\alpha_1}\left(1-\alpha_1\right)^{1-\alpha_1}z_1\phi_1^{1-\alpha_1}c_1^f}
\end{aligned}$$

国内上游企业利润方程为：

$$\begin{aligned}
\pi_1^d&=\sum_{j=1}^{n}(p_{j,1}^d-c_1^d)M_{j,12}^d\\
&=n(p_{j,1}^d-c_1^d)\Pi_1^fB_2^{\sigma-1}\left(\phi_1l_{r1}\right)^{\beta_2\theta(\sigma-1)}\left[\left(1+\chi_1\right)p_1^d\right]^{\gamma_2(1-\sigma)-1}\\
&=n\Pi_1^fB_2^{\sigma-1}\left(\phi_1l_{r1}\right)^{\beta_2\theta(\sigma-1)}\left(\left(1+\tau_1^x\right)c_1^f-c_1^d\right)\left[\left(1+\chi_1\right)\left(1+\tau_1^x\right)c_1^f\right]^{\gamma_2(1-\sigma)-1}
\end{aligned}$$

此时消费者福利为：

$$\begin{aligned}
U_d&=U_0B_2^{\frac{1}{1-\beta_2\theta(\sigma-1)}}\left(E+E^f\delta_{f2}\right)^{\frac{1}{(\sigma-1)[1-\beta_2\theta(\sigma-1)]}}\left(\Pi_1^f\right)^{\frac{\beta_2\theta}{1-\beta_2\theta(\sigma-1)}}\left(\frac{\delta_{r1}}{\delta_{p1}}\right)^{\frac{\alpha_1\beta_2\theta}{1-\beta_2\theta(\sigma-1)}}\left((1+\chi_1)p_1^{d*}\right)^{-\frac{\gamma_2+\beta_2\theta}{1-\beta_2\theta(\sigma-1)}}\\
&=U_0B_2^{\frac{1}{1-\beta_2\theta(\sigma-1)}}\left(E+E^f\delta_{f2}\right)^{\frac{1}{(\sigma-1)[1-\beta_2\theta(\sigma-1)]}}\\
&\times\left(\left(EP^{\sigma-1}+E^f\left(P^f\right)^{\sigma-1}\delta_{f2}^{\sigma}\right)\left(\frac{\gamma_2}{\mu_2}\right)^{\sigma}\right)^{\frac{\beta_2\theta}{1-\beta_2\theta(\sigma-1)}}\left(\frac{\delta_{r1}}{\delta_{p1}}\right)^{\frac{\alpha_1\beta_2\theta}{1-\beta_2\theta(\sigma-1)}}\left(\frac{(1+\chi_1)c_1^f}{\delta_{x1}}\right)^{-\frac{\gamma_2+\beta_2\theta}{1-\beta_2\theta(\sigma-1)}}
\end{aligned}$$

然后，当进口关税较高时，即

$$\frac{\delta_{p1}^{\alpha_1}\delta_{r1}^{1-\alpha_1}}{\left(1+\chi_1\right)\delta_{x1}}>\frac{\mu_1^x}{\alpha_1^{\alpha_1}\left(1-\alpha_1\right)^{1-\alpha_1}z_1\phi_1^{1-\alpha_1}c_1^f}$$

国内上游产业的其利润方程为：

$$
\begin{aligned}
\pi_1^d &= \Pi_1^f \sum_{j=1}^{n} \left\{ (p_{j,1}^d - c_1^d) B_2^{\sigma-1} \left(\phi_1 l_{r1}\right)^{\beta_2\theta(\sigma-1)} \left[\left(1+\chi_1\right) p_1^d \right]^{\gamma_2(1-\sigma)-1} \right\} \\
&= \Pi_1^f n (1 - \frac{1}{\mu_1^x}) \frac{\left(\mu_1^x c_1^d\right)^{\gamma_2(1-\sigma)} B_2^{\sigma-1} \left(\phi_1 l_{r1}\right)^{\beta_2\theta(\sigma-1)}}{\left(1+\chi_1\right)^{\gamma_2(\sigma-1)+1}} \\
&= (1 - \frac{1}{\mu_1^x}) \left(EP^{\sigma-1} + E^f \left(P^f\right)^{\sigma-1} \left(1+\tau_{j,2}^f\right)^{-\sigma} \right) \left(\frac{\gamma_2}{\mu_2}\right)^{\sigma} n \frac{\left(\mu_1^x c_1^d\right)^{\gamma_2(1-\sigma)} B_2^{\sigma-1} \left(\phi_1 l_{r1}\right)^{\beta_2\theta(\sigma-1)}}{\left(1+\chi_1\right)^{\gamma_2(\sigma-1)+1}} \\
&= (1 - \frac{1}{\mu_1^x}) [EP^{\sigma-1} \left(\frac{\gamma_2}{\mu_2}\right)^{\sigma} \sum_{j=1}^{n} \left(\frac{\left(\mu_1^x c_1^d\right)^{\gamma_2(1-\sigma)}}{\left(1+\chi_1\right)^{\gamma_2(\sigma-1)+1}} \right) B_2^{\sigma-1} \left(\phi_1 l_{r1}\right)^{\beta_2\theta(\sigma-1)} \\
&\quad + nE^f \left(P^f\right)^{\sigma-1} \frac{\left(B_2 \left(\phi_1 l_{r1}\right)^{\beta_2\theta}\right)^{\sigma-1} \left(\mu_1^x c_1^d\right)^{\gamma_2(1-\sigma)}}{\left(1+\chi_1\right)^{\gamma_2(\sigma-1)+1}} \left(\frac{\gamma_2 \delta_{f2}}{\mu_2}\right)^{\sigma}] \\
&= (1 - \frac{1}{\mu_1^x}) [EP^{\sigma-1} \left(\frac{\gamma_2}{\mu_2}\right)^{\sigma} \left(\phi_1 l_{r1}\right)^{\beta_2\theta(\sigma-1)} \left(1+\chi_1\right)^{-1} \sum_{j=1}^{n} \left(\left(1+\chi_1\right) \mu_1^x c_1^d \right)^{\gamma_2(1-\sigma)} B_2^{\sigma-1} \\
&\quad + nE^f \left(P^f\right)^{\sigma-1} \frac{\left(B_2 \left(\phi_1 l_{r1}\right)^{\beta_2\theta}\right)^{\sigma-1} \left(\mu_1^x c_1^d\right)^{\gamma_2(1-\sigma)}}{\left(1+\chi_1\right)^{\gamma_2(\sigma-1)+1}} \left(\frac{\gamma_2 \delta_{f2}}{\mu_2}\right)^{\sigma}] \\
&= (1 - \frac{1}{\mu_1^x}) [EP^{\sigma-1} \left(\frac{\gamma_2}{\mu_2}\right)^{\sigma} \left(\phi_1 l_{r1}\right)^{\beta_2\theta(\sigma-1)} \left(1+\chi_1\right)^{-1} \sum_{j=1}^{n} \left(\gamma_2 B_2 \left(\phi_1 l_{r1}\right)^{\beta_2\theta} \frac{p_{j,2}^d}{\mu_2} \right)^{(1-\sigma)} B_2^{\sigma-1} \\
&\quad + nE^f \left(P^f\right)^{\sigma-1} \frac{\left(B_2 \left(\phi_1 l_{r1}\right)^{\beta_2\theta}\right)^{\sigma-1} \left(\mu_1^x c_1^d\right)^{\gamma_2(1-\sigma)}}{\left(1+\chi_1\right)^{\gamma_2(\sigma-1)+1}} \left(\frac{\gamma_2 \delta_{f2}}{\mu_2}\right)^{\sigma}] \\
&= (1 - \frac{1}{\mu_1^x}) [\frac{\gamma_2 E}{\mu_2 \left(1+\chi_1\right)} + nE^f \left(P^f\right)^{\sigma-1} \frac{\left(B_2 \left(\phi_1 l_{r1}\right)^{\beta_2\theta}\right)^{\sigma-1} \left(\mu_1^x c_1^d\right)^{\gamma_2(1-\sigma)}}{\left(1+\chi_1\right)^{\gamma_2(\sigma-1)+1}} \left(\frac{\gamma_2 \delta_{f2}}{\mu_2}\right)^{\sigma}]
\end{aligned}
$$

此时消费者福利为：

$$
\begin{aligned}
U_d &= U_0 B_2^{\frac{1}{1-\beta_2\theta(\sigma-1)}} \left(E+E^f\delta_{f2}\right)^{\frac{1}{(\sigma-1)[1-\beta_2\theta(\sigma-1)]}} \left(\Pi_1^f\right)^{\frac{\beta_2\theta}{1-\beta_2\theta(\sigma-1)}} \left(\frac{\delta_{r1}}{\delta_{p1}}\right)^{\frac{\alpha_1\beta_2\theta}{1-\beta_2\theta(\sigma-1)}} \left((1+\chi_1)p_1^{d*}\right)^{-\frac{\gamma_2+\beta_2\theta}{1-\beta_2\theta(\sigma-1)}} \\
&= U_0 B_2^{\frac{1}{1-\beta_2\theta(\sigma-1)}} \left(E+E^f\delta_{f2}\right)^{\frac{1}{(\sigma-1)[1-\beta_2\theta(\sigma-1)]}} \\
&\times \left(\left(EP^{\sigma-1}+E^f\left(P^f\right)^{\sigma-1}\delta_{f2}^{\sigma}\right)\left(\frac{\gamma_2}{\mu_2}\right)^{\sigma}\right)^{\frac{\beta_2\theta}{1-\beta_2\theta(\sigma-1)}} \left(\frac{\delta_{r1}}{\delta_{p1}}\right)^{\frac{\alpha_1\beta_2\theta}{1-\beta_2\theta(\sigma-1)}} \left((1+\chi_1)\mu_1^x c_1^d\right)^{-\frac{\gamma_2+\beta_2\theta}{1-\beta_2\theta(\sigma-1)}} \\
&= U_0 B_2^{\frac{1}{1-\beta_2\theta(\sigma-1)}} \left(E+E^f\delta_{f2}\right)^{\frac{1}{(\sigma-1)[1-\beta_2\theta(\sigma-1)]}} \left(\left(EP^{\sigma-1}+E^f\left(P^f\right)^{\sigma-1}\delta_{f2}^{\sigma}\right)\left(\frac{\gamma_2}{\mu_2}\right)^{\sigma}\right)^{\frac{\beta_2\theta}{1-\beta_2\theta(\sigma-1)}} \\
&\times \left(\frac{\delta_{r1}}{\delta_{p1}}\right)^{\frac{\alpha_1\beta_2\theta}{1-\beta_2\theta(\sigma-1)}} \left((1+\chi_1)\mu_1^x \frac{\left(1+\tau_{p1}\right)^{\alpha_1}\left(1+\tau_{r1}\right)^{1-\alpha_1}}{z_1\phi_1^{1-\alpha_1}\alpha_1^{\alpha_1}\left(1-\alpha_1\right)^{1-\alpha_1}}\right)^{-\frac{\gamma_2+\beta_2\theta}{1-\beta_2\theta(\sigma-1)}} \\
&= U_0 B_2^{\frac{1}{1-\beta_2\theta(\sigma-1)}} \left(E+E^f\delta_{f2}\right)^{\frac{1}{(\sigma-1)[1-\beta_2\theta(\sigma-1)]}} \left(\left(EP^{\sigma-1}+E^f\left(P^f\right)^{\sigma-1}\delta_{f2}^{\sigma}\right)\left(\frac{\gamma_2}{\mu_2}\right)^{\sigma}\right)^{\frac{\beta_2\theta}{1-\beta_2\theta(\sigma-1)}} \\
&\times \delta_{r1}^{\frac{\beta_2\theta+(1-\alpha_1)\gamma_2}{1-\beta_2\theta(\sigma-1)}} \delta_{p1}^{\frac{\alpha_1\gamma_2}{1-\beta_2\theta(\sigma-1)}} \left(\frac{(1+\chi_1)\mu_1^x}{z_1\phi_1^{1-\alpha_1}\alpha_1^{\alpha_1}\left(1-\alpha_1\right)^{1-\alpha_1}}\right)^{-\frac{\gamma_2+\beta_2\theta}{1-\beta_2\theta(\sigma-1)}}
\end{aligned}
$$

1.3.3 国外上游产业产品（关键核心技术中间产品）存在断供风险

(1) 消费者的预期福利 从国内上游企业购买关键核心技术中间产品时，消费者福利为：

$$
U_d = \frac{\gamma_2 E B_2\left(\phi_1 l_{r1}^*\right)^{\beta_2\theta}}{\mu_2\left(\left(1+\chi_1\right)p_1^{d*}\right)^{\gamma_2}}\left(\frac{E+E^f\delta_{f2}}{\sigma f}\right)^{\frac{1}{\sigma-1}}
$$

当存在国外技术断供风险时，消费者的预期福利：

$$
\begin{aligned}
\bar{U}_f = \frac{E}{P} = \frac{E}{n_f^{\frac{1}{1-\sigma}}p_{j,2}} &= \frac{E}{(1-q)^{\frac{1}{1-\sigma}}p_{j,2}}\left(\frac{E+E^f\delta_{f2}}{\sigma f}\right)^{\frac{1}{\sigma-1}} \\
&= \frac{\gamma_2\left(1-q\right)^{\frac{1}{\sigma-1}}EB_2\delta_{x1}^{\gamma_2}}{\mu_2\left(\mu_1^x c_1^f\right)^{\gamma_2}}\left(\frac{E+E^f\delta_{f2}}{\sigma f}\right)^{\frac{1}{\sigma-1}}
\end{aligned}
$$

此时，

$$
\frac{U_d}{\bar{U}_f} = \frac{\frac{\gamma_2 E B_2\left(\phi_1 l_{r1}^*\right)^{\beta_2\theta}}{\mu_2\left(\left(1+\chi_1\right)p_1^{d*}\right)^{\gamma_2}}\left(\frac{E+E^f\delta_{f2}}{\sigma f}\right)^{\frac{1}{\sigma-1}}}{\frac{\gamma_2(1-q)^{\frac{1}{\sigma-1}}EB_2\delta_{x1}^{\gamma_2}}{\mu_2\left(\mu_1^x c_1^f\right)^{\gamma_2}}\left(\frac{E+E^f\delta_{f2}}{\sigma f}\right)^{\frac{1}{\sigma-1}}} = \frac{\left(\phi_1 l_{r1}^*\right)^{\beta_2\theta}\left(\mu_1^x c_1^f\right)^{\gamma_2}}{\left(\left(1+\chi_1\right)p_1^{d*}\right)^{\gamma_2}\left(1-q\right)^{\frac{1}{\sigma-1}}\delta_{x1}^{\gamma_2}}
$$

令 $\frac{U_d}{\bar{U}_f} > 1$，则有：

$$
\begin{aligned}
&\frac{\left(\phi_1 l_{r1}^*\right)^{\beta_2\theta}\left(\mu_1^x c_1^f\right)^{\gamma_2}}{\left(\left(1+\chi_1\right)p_1^{d^*}\right)^{\gamma_2}\left(1-q\right)^{\frac{1}{\sigma-1}}\delta_{x1}^{\gamma_2}}>1\\
\Leftrightarrow&\left(1-q\right)^{\frac{1}{\sigma-1}}<\left(\phi_1 l_{r1}^*\right)^{\beta_2\theta}\left(\frac{\mu_1^x c_1^f}{\delta_{x1}\left(1+\chi_1\right)p_1^{d^*}}\right)^{\gamma_2}\\
\Leftrightarrow& q>1-\left(\left(\phi_1 l_{r1}^*\right)^{\beta_2\theta}\left(\frac{\mu_1^x c_1^f}{\delta_{x1}\left(1+\chi_1\right)p_1^{d^*}}\right)^{\gamma_2}\right)^{\sigma-1}
\end{aligned}
$$