Semantics

1. Syntax

$$
\begin{array}{llll}
e & ::= & & \text{Expression} \\
& \mid & n & \text{Integer Literal} \\
& \mid & e_1 + e_2 & \text{Add} \\
& \mid & x & \text{Variable} \\
& \mid & \lambda x.\ e & \text{Abstraction} \\
& \mid & e_1\ e_2 & \text{Application} \\
& \mid & e_1\ ,,\ e_2 & \text{Merge} \\
& \mid & (e : A) & \text{Annotation} \\
v & ::= & & \text{Value} \\
& \mid & n & \\
& \mid & (\lambda x.\ e : A \to B) & \\
& \mid & v_1\ ,,\ v_2 &
\end{array}
$$

2. Semantics

$\boxed{v \in A}$ Typed Values

$$
\frac{}{n \in \text{Int}}\ \text{T\_INT}
\qquad
\frac{}{(\lambda x.\ e : A \to B) \in A \to B}\ \text{T\_ABS}
\qquad
\frac{v_1 \in A \quad v_2 \in B \quad A * B}{v_1\ ,,\ v_2 \in A\ \&\ B}\ \text{T\_MERGE}
$$

$\boxed{e_1 \rightsquigarrow e_2}$ Reduction

$$
\frac{e_1 \rightsquigarrow e_3}{e_1 + e_2 \rightsquigarrow e_3 + e_2}\ \text{R\_ADD1}
\qquad
\frac{v \in \text{Int} \quad e_1 \rightsquigarrow e_2}{v + e_1 \rightsquigarrow v + e_2}\ \text{R\_ADD2}
\qquad
\frac{r = m + n}{m + n \rightsquigarrow r}\ \text{R\_ADD3}
$$

$$
\frac{e_1 \rightsquigarrow e_3}{e_1\ e_2 \rightsquigarrow e_3\ e_2}\ \text{R\_APP1}
\qquad
\frac{v \in A \to B \quad e_1 \notin A \quad (e_1 : A) \rightsquigarrow e_2}{v\ e_1 \rightsquigarrow v\ e_2}\ \text{R\_APP2}
$$

$$
\frac{v \in A}{(\lambda x.\ e : A \to B)\, v \rightsquigarrow (e[x \mapsto v] : B)}\ \text{R\_APP3}
\qquad
\frac{e_1 \rightsquigarrow e_3}{e_1\ ,,\ e_2 \rightsquigarrow e_3\ ,,\ e_2}\ \text{R\_MERGE1}
$$

$$
\frac{e_1 \rightsquigarrow e_2}{v\ ,,\ e_1 \rightsquigarrow v\ ,,\ e_2}\ \text{R\_MERGE2}
\qquad
\frac{e_1 \rightsquigarrow e_2}{(e_1 : A) \rightsquigarrow (e_2 : A)}\ \text{R\_ANN1}
\qquad
\frac{v \in A \quad A \neq C \to D}{((v : A) : B) \rightsquigarrow (v : B)}\ \text{R\_ANN2}
$$

$$
\frac{}{((\lambda x.\ e : A \to B) : C \to D) \rightsquigarrow (\lambda y.\ (((\lambda x.\ e : A \to B)\,(y : A)) : D) : C \to D)}\ \text{R\_ANN2ABS}
$$

$$
\frac{}{((\lambda x.\ e : A \to B) : C \to D) \rightsquigarrow (\lambda x.\ e : A \to D)}\ \text{R\_ANN2ABSALT}
\qquad
\frac{v \in A \quad A \neq C \to D}{(v : A) \rightsquigarrow v}\ \text{R\_ANN3}
$$

$$
\frac{\text{ord}\ A \quad v_1\ ,,\ v_2 \in B\ \&\ C \quad B \leq A}{(v_1\ ,,\ v_2 : A) \rightsquigarrow (v_1 : A)}\ \text{R\_ANN4}
\qquad
\frac{\text{ord}\ A \quad v_1\ ,,\ v_2 \in B\ \&\ C \quad C \leq A}{(v_1\ ,,\ v_2 : A) \rightsquigarrow (v_2 : A)}\ \text{R\_ANN5}
$$

$$
\frac{v \in C\ \&\ D \quad C\ \&\ D \neq A\ \&\ B}{(v : A\ \&\ B) \rightsquigarrow (v : A)\ ,,\ (v : B)}\ \text{R\_ANN6}
$$