Semantics

1. Syntax

$$
\begin{array}{llll}
e & ::= & & \text{Expression} \\
  & | & n & \text{Integer Literal} \\
  & | & e_1 + e_2 & \text{Add} \\
  & | & x & \text{Variable} \\
  & | & \lambda x.\, e & \text{Abstraction} \\
  & | & e_1\, e_2 & \text{Application} \\
  & | & e_1\,,,\, e_2 & \text{Merge} \\
  & | & (e \Leftarrow A) & \text{Compile-time Annotation} \\
  & | & (e \Rightarrow A) & \text{Run-time Annotation} \\
a & ::= & & \text{Atomic Expression} \\
  & | & n & \\
  & | & \lambda x.\, e & \\
  & | & a_1\,,,\, a_2 & \\
v & ::= & & \text{Value} \\
  & | & (a \Rightarrow A) & \\
\end{array}
$$

2. Semantics

$\boxed{e_1 \rightsquigarrow e_2}$ Reduction

$$\frac{}{n \rightsquigarrow (n \Rightarrow \text{Int})}\ \text{R\_INT} \qquad \frac{e_1 \rightsquigarrow e_3}{e_1 + e_2 \rightsquigarrow e_3 + e_2}\ \text{R\_ADD1} \qquad \frac{e_1 \rightsquigarrow e_2}{v + e_1 \rightsquigarrow v + e_2}\ \text{R\_ADD2}$$

$$\frac{}{(m \Rightarrow \text{Int}) + (n \Rightarrow \text{Int}) \rightsquigarrow (m + n \Rightarrow \text{Int})}\ \text{R\_ADD3} \qquad \frac{}{(\lambda x.\, e_1 \Leftarrow A \rightarrow B) \rightsquigarrow (\lambda x.\, e_1 \Rightarrow A \rightarrow B)}\ \text{R\_ABS}$$

$$\frac{e_1 \rightsquigarrow e_3}{e_1\, e_2 \rightsquigarrow e_3\, e_2}\ \text{R\_APP1} \qquad \frac{e_2 \rightsquigarrow e_3}{(\lambda x.\, e_1 \Rightarrow A \rightarrow B)\, e_2 \rightsquigarrow (\lambda x.\, e_1 \Rightarrow A \rightarrow B)\, e_3}\ \text{R\_APP2}$$

$$\frac{}{(\lambda x.\, e_1 \Rightarrow A \rightarrow B)\,(a \Rightarrow C) \rightsquigarrow (\lambda x.\, e_1 \Rightarrow A \rightarrow B)\,((a \Rightarrow C) \Leftarrow A)}\ \text{R\_APP3}$$

$$\frac{}{(\lambda x.\, e_1 \Rightarrow A \rightarrow B)\,(a \Rightarrow A) \rightsquigarrow (e[x \mapsto (a \Rightarrow A)] \Leftarrow B)}\ \text{R\_APP4} \qquad \frac{e_1 \rightsquigarrow e_3}{e_1\,,,\, e_2 \rightsquigarrow e_3\,,,\, e_2}\ \text{R\_MERGE1}$$

$$\frac{e_1 \rightsquigarrow e_2}{v\,,,\, e_1 \rightsquigarrow v\,,,\, e_2}\ \text{R\_MERGE2} \qquad \frac{}{(a_1 \Rightarrow A)\,,,\, (a_2 \Rightarrow B) \rightsquigarrow (a_1\,,,\, a_2 \Rightarrow A\ \&\ B)}\ \text{R\_MERGE3}$$

$$\frac{e_1 \rightsquigarrow e_2}{(e_1 \Leftarrow A) \rightsquigarrow (e_2 \Leftarrow A)}\ \text{R\_ANN1}$$

$\boxed{e_1 \rightsquigarrow e_2}$ Sub/Ann Reduction $((v \Leftarrow A) \rightsquigarrow e)$

$$\frac{}{((a \Rightarrow \text{Int}) \Leftarrow \text{Int}) \rightsquigarrow (a \Rightarrow \text{Int})}\ \text{A\_INT}$$

$$\frac{}{((a \Rightarrow A \rightarrow B) \Leftarrow C \rightarrow D) \rightsquigarrow (\lambda y.\, (((a \Rightarrow A \rightarrow B)\,(y \Leftarrow A)) \Leftarrow D) \Rightarrow C \rightarrow D)}\ \text{A\_ABS}$$

$$\frac{A \leq C \qquad \text{ord } C}{((a_1\,,,\, a_2 \Rightarrow A\ \&\ B) \Leftarrow C) \rightsquigarrow ((a_1 \Rightarrow A) \Leftarrow C)}\ \text{A\_MERGE1}$$

$$\frac{B \leq C \qquad \text{ord } C}{((a_1\,,,\, a_2 \Rightarrow A\ \&\ B) \Leftarrow C) \rightsquigarrow ((a_2 \Rightarrow B) \Leftarrow C)}\ \text{A\_MERGE2}$$

$$\frac{}{((a_1\,,,\, a_2 \Rightarrow A\ \&\ B) \Leftarrow C\ \&\ D) \rightsquigarrow ((a_1\,,,\, a_2 \Rightarrow A\ \&\ B) \Leftarrow C)\,,,\, ((a_1\,,,\, a_2 \Rightarrow A\ \&\ B) \Leftarrow D)}\ \text{A\_MERGE3}$$