

# Semantics

## 1. Syntax

$e$	$::=$	Expression
	$n$	Integer Literal
	$e_1 + e_2$	Add
	$x$	Variable
	$\lambda x. e$	Abstraction
	$e_1 e_2$	Application
	$e_1 , , e_2$	Merge
	$(e : A)$	Annotation
$v$	$::=$	Value
	$(n : A)$	
	$(\lambda x. e : A \rightarrow B)$	
	$v_1 , , v_2$	

## 2. Semantics

$v \in A$     Typed Values

$$\frac{}{(n : \text{Int}) \in \text{Int}} \text{T\_INT} \quad \frac{}{(\lambda x. e : A \rightarrow B) \in A \rightarrow B} \text{T\_ABS} \quad \frac{v_1 \in A \quad v_2 \in B}{v_1 , , v_2 \in A \& B} \text{T\_MERGE}$$

$e_1 \rightsquigarrow e_2$     Reduction

$$\begin{array}{c} \frac{}{n \rightsquigarrow (n : \text{Int})} \text{R\_INT} \quad \frac{e_1 \rightsquigarrow e_3}{e_1 + e_2 \rightsquigarrow e_3 + e_2} \text{R\_ADD1} \quad \frac{v \in \text{Int} \quad e_1 \rightsquigarrow e_2}{v + e_1 \rightsquigarrow v + e_2} \text{R\_ADD2} \\ \\ \frac{r = m + n}{(m : \text{Int}) + (n : \text{Int}) \rightsquigarrow (r : \text{Int})} \text{R\_ADD3} \quad \frac{e_1 \rightsquigarrow e_3}{e_1 e_2 \rightsquigarrow e_3 e_2} \text{R\_APP1} \quad \frac{v \in A \rightarrow B \quad e_1 \rightsquigarrow e_2}{v e_1 \rightsquigarrow v e_2} \text{R\_APP2} \\ \\ \frac{v_1 \in A \rightarrow B \quad v_2 \in C \quad A \neq C}{v_1 v_2 \rightsquigarrow v_1 (v_2 : C)} \text{R\_APP3} \quad \frac{v \in A}{(\lambda x. e : A \rightarrow B) v \rightsquigarrow (e[x \mapsto v] : B)} \text{R\_APP4} \\ \\ \frac{e_1 \rightsquigarrow e_3}{e_1 , , e_2 \rightsquigarrow e_3 , , e_2} \text{R\_MERGE1} \quad \frac{e_1 \rightsquigarrow e_2}{v , , e_1 \rightsquigarrow v , , e_2} \text{R\_MERGE2} \quad \frac{(e_1 : A) \notin A \quad e_1 \rightsquigarrow e_2}{(e_1 : A) \rightsquigarrow (e_2 : A)} \text{R\_ANN1} \\ \\ \frac{v \in \text{Int}}{(v : \text{Int}) \rightsquigarrow v} \text{R\_ANN2} \\ \\ \frac{}{((\lambda x. e : A \rightarrow B) : C \rightarrow D) \rightsquigarrow (\lambda y. (((\lambda x. e : A \rightarrow B) (y : A)) : D) : C \rightarrow D)} \text{R\_ANN3} \\ \\ \frac{}{((\lambda x. e : A \rightarrow B) : C \rightarrow D) \rightsquigarrow (\lambda x. e : A \rightarrow D)} \text{R\_ANN3ALT} \\ \\ \frac{\text{ord } A \quad v_1 , , v_2 \in B \& C \quad B \leq A}{(v_1 , , v_2 : A) \rightsquigarrow (v_1 : A)} \text{R\_ANN4} \quad \frac{\text{ord } A \quad v_1 , , v_2 \in B \& C \quad C \leq A}{(v_1 , , v_2 : A) \rightsquigarrow (v_2 : A)} \text{R\_ANN5} \\ \\ \frac{v \in C \& D}{(v : A \& B) \rightsquigarrow (v : A) , , (v : B)} \text{R\_ANN6} \end{array}$$