

# Semantics

## 1. Syntax

$e ::=$	$n$ $e_1 + e_2$ $x$ $\lambda x. e$ $e_1 e_2$ $e_1 , , e_2$ $(e : A)$	Expression Integer Literal Add Variable Abstraction Application Merge Annotation
$v ::=$	$n$ $(\lambda x. e : A \rightarrow B)$ $v_1 , , v_2$	Value

## 2. Semantics

$v \in A$     Typed Values

$$\frac{}{n \in \text{Int}} \text{T\_INT} \qquad \frac{}{(\lambda x. e : A \rightarrow B) \in A \rightarrow B} \text{T\_ABS} \qquad \frac{v_1 \in A \quad v_2 \in B}{v_1 , , v_2 \in A \& B} \text{T\_MERGE}$$

$e_1 \rightsquigarrow e_2$     Reduction

$$\begin{array}{c}
\frac{e_1 \rightsquigarrow e_3}{e_1 + e_2 \rightsquigarrow e_3 + e_2} \text{R\_ADD1} \qquad \frac{v \in \text{Int} \quad e_1 \rightsquigarrow e_2}{v + e_1 \rightsquigarrow v + e_2} \text{R\_ADD2} \qquad \frac{r = m + n}{m + n \rightsquigarrow r} \text{R\_ADD3} \\
\\
\frac{e_1 \rightsquigarrow e_3}{e_1 e_2 \rightsquigarrow e_3 e_2} \text{R\_APP1} \qquad \frac{v \in A \rightarrow B \quad (e_1 : A) \rightsquigarrow e_2 \quad e_1 \neq e_2}{v e_1 \rightsquigarrow v e_2} \text{R\_APP2} \\
\\
\frac{v \in A}{(\lambda x. e : A \rightarrow B) v \rightsquigarrow (e[x \mapsto v] : B)} \text{R\_APP3} \qquad \frac{e_1 \rightsquigarrow e_3}{e_1 , , e_2 \rightsquigarrow e_3 , , e_2} \text{R\_MERGE1} \\
\\
\frac{e_1 \rightsquigarrow e_2}{v , , e_1 \rightsquigarrow v , , e_2} \text{R\_MERGE2} \qquad \frac{e_1 \rightsquigarrow e_2}{(e_1 : A) \rightsquigarrow (e_2 : A)} \text{R\_ANN1} \qquad \frac{e \neq \lambda x. e_2}{((e : A) : B) \rightsquigarrow (e : B)} \text{R\_ANN2} \\
\\
\frac{}{((\lambda x. e : A \rightarrow B) : C \rightarrow D) \rightsquigarrow (\lambda x. e : A \rightarrow D)} \text{R\_ANN2FUN} \qquad \frac{v \in A}{(v : A) \rightsquigarrow v} \text{R\_ANN3} \\
\\
\frac{\text{ord } A \quad v_1 , , v_2 \in B \& C \quad B \leq A}{(v_1 , , v_2 : A) \rightsquigarrow (v_1 : A)} \text{R\_ANN4} \qquad \frac{\text{ord } A \quad v_1 , , v_2 \in B \& C \quad C \leq A}{(v_1 , , v_2 : A) \rightsquigarrow (v_2 : A)} \text{R\_ANN5} \\
\\
\frac{v \in C \& D \quad C \& D \neq A \& B}{(v : A \& B) \rightsquigarrow (v : A) , , (v : B)} \text{R\_ANN6}
\end{array}$$