Semantics

1. Syntax

e	$::=$		Expression
	$\mid$	n	Integer Literal
	$\mid$	$e_1 + e_2$	Add
	$\mid$	x	Variable
	$\mid$	$\lambda x.\ e$	Abstraction
	$\mid$	$e_1\ e_2$	Application
	$\mid$	$e_1\ ,,\ e_2$	Merge
	$\mid$	$(e \Leftarrow A)$	Compile-time Annotation
	$\mid$	$(e \Rightarrow A)$	Run-time Annotation
a	$::=$		Atomic Expression
	$\mid$	n	
	$\mid$	$\lambda x.\ e$	
v	$::=$		Value
	$\mid$	$(a \Rightarrow A)$	
	$\mid$	$(v_1\ ,,\ v_2)$	

2. Semantics

$\boxed{v\ \in\ A}$ Typed Values

$$\frac{}{(n \Rightarrow \text{Int})\ \in\ \text{Int}}\ \text{TV\_INT} \qquad \frac{}{(\lambda x.\ e \Rightarrow A \to B)\ \in\ A \to B}\ \text{TV\_ABS}$$

$$\frac{v_1\ \in\ A \qquad v_2\ \in\ B \qquad A * B}{(v_1\ ,,\ v_2)\ \in\ A \,\&\, B}\ \text{TV\_MERGE}$$

$\boxed{e_1 \rightsquigarrow e_2}$ Reduction

$$\frac{}{n \rightsquigarrow (n \Rightarrow \text{Int})}\ \text{R\_INT} \qquad \frac{e_1 \rightsquigarrow e_3}{e_1 + e_2 \rightsquigarrow e_3 + e_2}\ \text{R\_ADD1} \qquad \frac{e_1 \rightsquigarrow e_2}{v + e_1 \rightsquigarrow v + e_2}\ \text{R\_ADD2}$$

$$\frac{}{(m \Rightarrow \text{Int}) + (n \Rightarrow \text{Int}) \rightsquigarrow (m + n \Rightarrow \text{Int})}\ \text{R\_ADD3} \qquad \frac{}{(\lambda x.\ e_1 \Leftarrow A \to B) \rightsquigarrow (\lambda x.\ e_1 \Rightarrow A \to B)}\ \text{R\_ABS}$$

$$\frac{e_1 \rightsquigarrow e_3}{e_1\ e_2 \rightsquigarrow e_3\ e_2}\ \text{R\_APP1} \qquad \frac{e_1 \rightsquigarrow e_2}{v\ e_1 \rightsquigarrow v\ e_2}\ \text{R\_APP2} \qquad \frac{v_1\ \in\ A \to B \qquad v_2\ \notin\ A}{v_1\ v_2 \rightsquigarrow v_1\ (v_2 \Leftarrow A)}\ \text{R\_APP3}$$

$$\frac{v\ \in\ A}{(\lambda x.\ e_1 \Rightarrow A \to B)\ v \rightsquigarrow (e[x \mapsto v] \Leftarrow B)}\ \text{R\_APP4} \qquad \frac{e_1 \rightsquigarrow e_3}{e_1\ ,,\ e_2 \rightsquigarrow e_3\ ,,\ e_2}\ \text{R\_MERGE1}$$

$$\frac{e_1 \rightsquigarrow e_2}{v\ ,,\ e_1 \rightsquigarrow v\ ,,\ e_2}\ \text{R\_MERGE2} \qquad \frac{e_1 \rightsquigarrow e_2}{(e_1 \Leftarrow A) \rightsquigarrow (e_2 \Leftarrow A)}\ \text{R\_ANN}$$

$\boxed{e_1 \rightsquigarrow e_2}$ Sub/Ann Reduction ($(v \Leftarrow A) \rightsquigarrow e$)

$$\frac{}{((n \Rightarrow \mathrm{Int}) \Leftarrow \mathrm{Int}) \rightsquigarrow (n \Rightarrow \mathrm{Int})}\ \text{A\_INT}$$

$$\frac{}{((\lambda x.\ e \Rightarrow A \to B) \Leftarrow C \to D) \rightsquigarrow (\lambda y.\ (((\lambda x.\ e \Rightarrow A \to B)\,(y \Leftarrow A)) \Leftarrow D) \Rightarrow C \to D)}\ \text{A\_ABS}$$

$$\frac{}{((\lambda x.\ e \Rightarrow A \to B) \Leftarrow C \to D) \rightsquigarrow (\lambda x.\ e \Rightarrow A \to D)}\ \text{A\_ABSALT}$$

$$\frac{v_1\ \in\ A \qquad A \le B \qquad \mathrm{ord}\, B}{((v_1\ ,,\ v_2) \Leftarrow B) \rightsquigarrow (v_1 \Leftarrow B)}\ \text{A\_MERGE1} \qquad \frac{v_2\ \in\ A \qquad A \le B \qquad \mathrm{ord}\, B}{((v_1\ ,,\ v_2) \Leftarrow B) \rightsquigarrow (v_2 \Leftarrow B)}\ \text{A\_MERGE2}$$

$$\frac{}{(v \Leftarrow A \,\&\, B) \rightsquigarrow (v \Leftarrow A)\ ,,\ (v \Leftarrow B)}\ \text{A\_MERGE3}$$