

Semantics

1. Syntax

e	$::=$	Expression
	n	Integer Literal
	$e_1 + e_2$	Add
	x	Variable
	$\lambda x. e$	Abstraction
	$e_1 e_2$	Application
	$e_1 , , e_2$	Merge
	$(e \Leftarrow A)$	Compile-time Annotation
	$(e \Rightarrow A)$	Run-time Annotation
a	$::=$	Atomic Expression
	n	
	$\lambda x. e$	
	$a_1 , , a_2$	
v	$::=$	Value
	$(a \Rightarrow A)$	

2. Semantics

$e_1 \rightsquigarrow e_2$	Reduction
$\frac{}{n \rightsquigarrow (n \Rightarrow \text{Int})}$	R_INT
$\frac{e_1 \rightsquigarrow e_3}{e_1 + e_2 \rightsquigarrow e_3 + e_2}$	R_ADD1
$\frac{e_1 \rightsquigarrow e_2}{v + e_1 \rightsquigarrow v + e_2}$	R_ADD2
$\frac{}{(m \Rightarrow \text{Int}) + (n \Rightarrow \text{Int}) \rightsquigarrow (m + n \Rightarrow \text{Int})}$	R_ADD3
$\frac{}{(\lambda x. e_1 \Leftarrow A \rightarrow B) \rightsquigarrow (\lambda x. e_1 \Rightarrow A \rightarrow B)}$	R_ABS
$\frac{e_1 \rightsquigarrow e_3}{e_1 e_2 \rightsquigarrow e_3 e_2}$	R_APP1
$\frac{e_1 \rightsquigarrow e_2}{v e_1 \rightsquigarrow v e_2}$	R_APP2
$\frac{}{(\lambda x. e_1 \Rightarrow A \rightarrow B) (a \Rightarrow C) \rightsquigarrow (\lambda x. e_1 \Rightarrow A \rightarrow B) ((a \Rightarrow C) \Leftarrow A)}$	R_APP3
$\frac{}{(\lambda x. e_1 \Rightarrow A \rightarrow B) (a \Rightarrow A) \rightsquigarrow (e[x \mapsto (a \Rightarrow A)] \Leftarrow B)}$	R_APP4
$\frac{e_1 \rightsquigarrow e_3}{e_1 , , e_2 \rightsquigarrow e_3 , , e_2}$	R_MERGE1
$\frac{e_1 \rightsquigarrow e_2}{v , , e_1 \rightsquigarrow v , , e_2}$	R_MERGE2
$\frac{}{(a_1 \Rightarrow A) , , (a_2 \Rightarrow B) \rightsquigarrow (a_1 , , a_2 \Rightarrow A \& B)}$	R_MERGE3
$\frac{e_1 \rightsquigarrow e_2}{(e_1 \Leftarrow A) \rightsquigarrow (e_2 \Leftarrow A)}$	R_ANN

$e_1 \rightsquigarrow e_2$	Sub/Ann Reduction $((v \Leftarrow A) \rightsquigarrow e)$
$\frac{}{((n \Rightarrow \text{Int}) \Leftarrow \text{Int}) \rightsquigarrow (n \Rightarrow \text{Int})}$	A_INT
$\frac{}{((\lambda x. e \Rightarrow A \rightarrow B) \Leftarrow C \rightarrow D) \rightsquigarrow (\lambda y. (((\lambda x. e \Rightarrow A \rightarrow B) (y \Leftarrow A)) \Leftarrow D) \Rightarrow C \rightarrow D)}$	A_ABS
$\frac{A \leq C \quad \text{ord } C}{((a_1 , , a_2 \Rightarrow A \& B) \Leftarrow C) \rightsquigarrow ((a_1 \Rightarrow A) \Leftarrow C)}$	A_MERGE1
$\frac{B \leq C \quad \text{ord } C}{((a_1 , , a_2 \Rightarrow A \& B) \Leftarrow C) \rightsquigarrow ((a_2 \Rightarrow B) \Leftarrow C)}$	A_MERGE2
$\frac{}{((a \Rightarrow A \& B) \Leftarrow C \& D) \rightsquigarrow ((a \Rightarrow A \& B) \Leftarrow C) , , ((a \Rightarrow A \& B) \Leftarrow D)}$	A_MERGE3

