Exercise 2

Beta rozdělení

Odhad parametru α pro zafixované β

β=0.5

Momentova metoda

$E[X] = \frac{\alpha}{\alpha+\beta} = \overline{Xn}$

$\overline{Xn} = \frac{\alpha}{\alpha+0,5}$

$\overline{Xn}.(\alpha+0,5) = \alpha$

$\overline{Xn}.\alpha + 0,5.\overline{Xn}$

$\alpha - \overline{Xn}.\alpha = 0,5.\overline{Xn}$

$\alpha.(1-\overline{Xn}) = 0,5.\overline{Xn}$

$\alpha = \frac{0,5.\overline{Xn}}{1-\overline{Xn}}$

Odhad α na zaklade jedne simulace

```
# Vybereme 1000 čísel z Beta rozdělení s parametry alpha=1 a beta=0.5
x = rbeta(1000, 1,0.5)

# spocitame jejich prumer
x_mean= mean(x)

# spocitame odhad alpha na zaklade vzorce vyse
odhad_alfa = (0.5)* x_mean / (1-x_mean)
odhad_alfa
```

```
## [1] 0.9801403
```

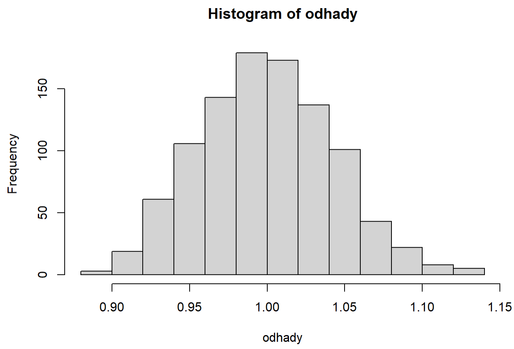
Porovnani pro 1000 simulaci

```
# vyvorime vektor pro uchovani odhadu
odhady = c(rep(0,1000))

# pro 1000 opakovani
for (i in 1:1000) {

  # vybereme 1000 cisel z beta rozdeleni s parametry alpha=1 a beta=0.5
  x = rbeta(1000, 1,0.5)

  # spocitame odhad
  x_mean= mean(x)
  odhad_alfa = (0.5)* x_mean / (1-x_mean)
  odhady[i] = odhad_alfa
}

# prumerna hodnota odhadu alpha na zaklade 1000 simulaci
mean(odhady)
```

```
## [1] 0.9994612
```

Histogram odhadu α pri 1000 opakovani

```
hist(odhady)
```



Metoda maximalni verohodnosti

Pro Beta distribuci je metoda maximalni verohodnosti komplikovana, viz wiki Ulohu jsem tedy resila numericky pomoci funkce mle ze stats4 knihovny. a potrebne vzorce jsem cerpala z wiki.

$ln(L(\alpha,\beta|X)) = \sum_{i=1}^{N} ln(L_i(\alpha,\beta|X_i))$

$\frac{\partial ln(L(\alpha,\beta|X))}{\partial\alpha} = \sum_{i=1}^{N} ln(X_i) - N.\frac{\partial ln(B(\alpha,\beta))}{\partial\alpha} = 0$

$\frac{\partial B(\alpha,\beta)}{\partial\alpha} = -\psi(\alpha+\beta) + \psi(\alpha) + 0$

ψ je digamma funkce wiki

Odhad α

```
# reseni numericky pomoci mle funkce z stat4 knihovny

library(stats4)

beta_nll <- function(alpha, x=x){
  x_log_sum = sum(log(x))
  derivation = -digamma(alpha+0.5) + digamma(alpha) + 0
  N = length(x)
  result = -(x_log_sum -N*(derivation))
  return(result)
}


fit0 <- mle(minuslogl = beta_nll, start = list(alpha = 0.5))
```

```
odhady2 = c(rep(0,1000))
for (i in 1:1000){
  name = paste('x', as.character(i), sep='')
  odhad2 = coef(fit0)[name][1]
  odhady2[i] = as.double(odhad2)
}
```

```
hist(odhady2)
```