Momentový odhad pro beta rozdělení

Beta rozdělení je dáno

$$\frac{x^{\alpha-1}(1-\bar{x})^{\beta-1}}{B(\alpha,\beta)}$$

where $B(\alpha,\beta)$ je beta funkce dána

$$B(\alpha,\beta)=\frac{\Gamma(\alpha)\Gamma(\beta)}{\Gamma(\alpha+\beta)}$$

Chceme odhadnout parametry α a β, které jsou definované

$$\bar{x}=\frac{\alpha}{\alpha+\beta}$$

$$s^2=\frac{\alpha\beta}{(\alpha+\beta)^2(\alpha+\beta+1)}$$

Ze dvou rovnic můžeme vyjádřit odhad parametrů

$$\bar{x}=\frac{\alpha}{\alpha+\beta}$$

$$\bar{x}(\alpha+\beta)=\alpha=\bar{x}\alpha+\bar{x}\beta$$

$$\bar{x}\beta=\alpha-\bar{x}\alpha$$

$$\beta=\frac{\alpha(1-\bar{x})}{\bar{x}}$$

Dosazením do druhé rovnice

$$s^2 = \frac{\alpha\left[\frac{\alpha(1-\bar{x})}{\bar{x}}\right]}{(\alpha + \left[\frac{\alpha(1-\bar{x})}{\bar{x}}\right])^2(\alpha + \left[\frac{\alpha(1-\bar{x})}{\bar{x}}\right] + 1)}$$

$$s^2 = \frac{\alpha^2\bar{x}^2(1-\bar{x})}{(\alpha + \alpha(1-\bar{x}))^2(\alpha + (\alpha(1-\bar{x})) + 1)\bar{x}}$$

$$s^2 = \frac{\alpha^2\bar{x}^2(1-\bar{x})}{(\alpha\bar{x} + \alpha(1-\bar{x}))^2(\alpha\bar{x} + (\alpha(1-\bar{x})) + \bar{x})}$$

$$s^2 = \frac{\alpha^2\bar{x}^2(1-\bar{x})}{\alpha^2(\alpha+\bar{x})} = \frac{\bar{x}^2(1-\bar{x})}{\alpha+\bar{x}}$$

Z toho vyjádříme α

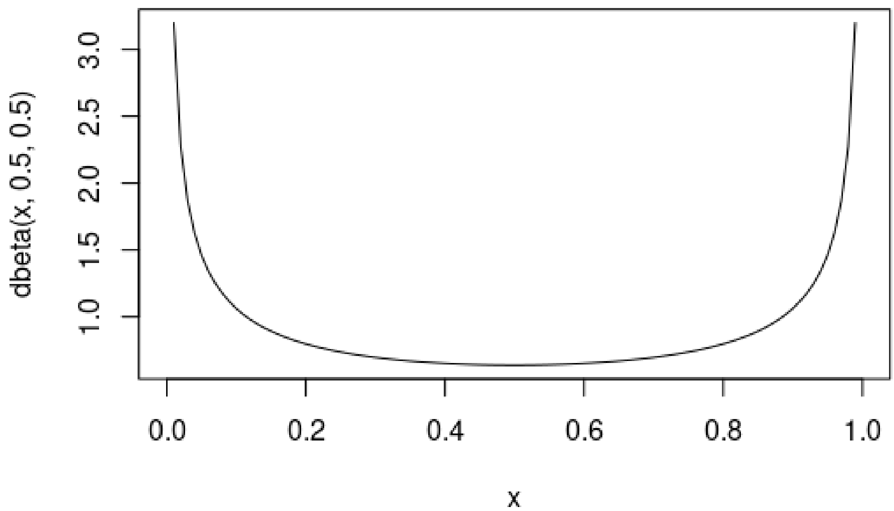
$$\alpha = \frac{\bar{x}^2(1-\bar{x}^2) - s^2\bar{x}}{s^2} = \bar{x}(\frac{\bar{x}(1-\bar{x})}{s^2} - 1)$$

Dosazením zpátky do β

$$\beta = (1-\bar{x})(\frac{\bar{x}(1-\bar{x})}{s^2} - 1)$$

Simulace odhadu

```
> x = rbeta(100, 0.5, 0.5)
> curve(dbeta(x, 0.5, 0.5))
```



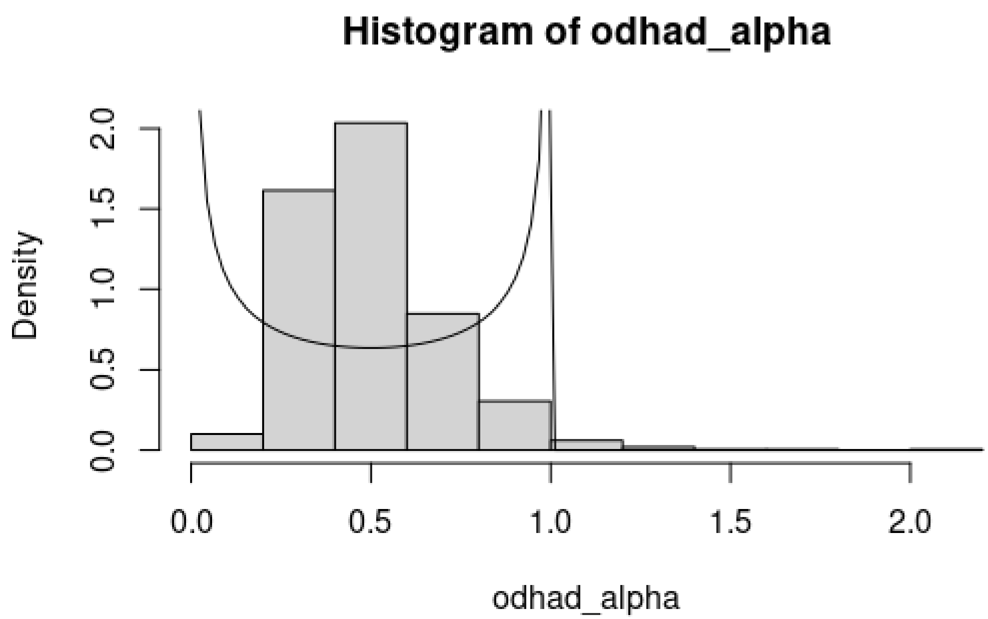

```
> m = mean(x)
> v = var(x)
> alpha = m*((m*(1-m) /v) - 1)
> beta = (1-m)*((m*(1-m)/v) - 1)
```

Hodnoty α a β jsou

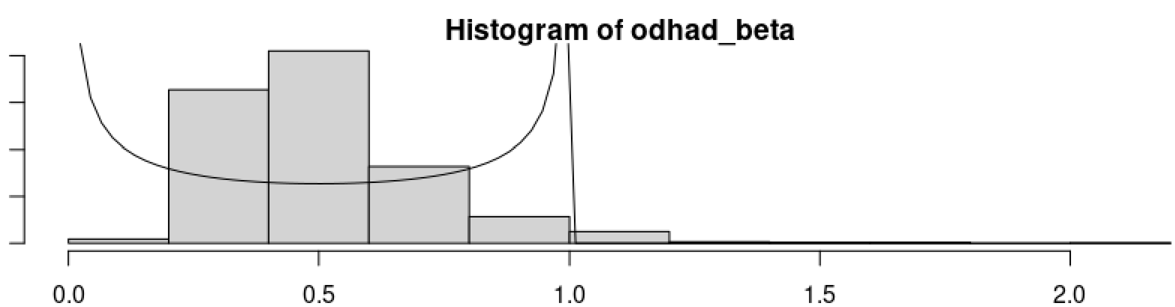
```
> alpha
[1] 0.3953505
> beta
[1] 0.5416557
```

Simulace rozdělení odhadu

```
> simul.x=matrix(rbeta(1000*20, 0.5, 0.5), nrow = 20)
> odhad_alpha = apply(simul.x,2,function(x){mean(x)*((mean(x)*(1-mean(x)) /var(x)) - 1)})
> par(mfrow=c(2,1)
> hist(odhad_alpha, prob=TRUE)
> curve(dbeta(x, 0.5, 0.5), add=TRUE)
```




```
> simul.x=matrix(rbeta(1000*20, 0.5, 0.5), nrow = 20)
> odhad_beta = apply(simul.x,2,function(x){(1-mean(x))*((mean(x)*(1-mean(x))/var(x)) -
 1)})
> par(mar=c(1,1,1,1))
> hist(odhad_beta, prob=TRUE)
> curve(dbeta(x, 0.5, 0.5), add=TRUE)
```



Konfidenční interval

Pro α

```
> margin = qt(0.975,df=(1000*20)-1)*var(odhad_alpha)/sqrt(1000*20)
> lowerinterval = mean(odhad_alpha) - margin
> upperinterval = mean(odhad_alpha) + margin
> lowerinterval
[1] 0.4999459
> upperinterval
[1] 0.4999651
```

Pro β

```
> margin = qt(0.975,df=n-1)*v/sqrt(n)
> lowerinterval = mean(odhad_beta) - margin
> upperinterval = mean(odhad_beta) + margin
> lowerinterval
[1] 0.5072402
> upperinterval
[1] 0.5107307
```

Simulace

tbc...