Momentový odhad pro beta rozdělení

Beta rozdělení je dáno

$$rac{x^{lpha-1}(1-ar{x})^{eta-1}}{B(lpha,eta)}$$

where $B(\alpha,\beta)$ je beta funkce dána

$$B(lpha,eta) = rac{\Gamma(lpha)\Gamma(eta)}{\Gamma(lpha+eta)}$$

Chceme odhadnout parametry α a β , které jsou definované

$$ar{x}=rac{lpha}{lpha+eta} \ s^2=rac{lphaeta}{(lpha+eta)^2(lpha+eta+1)}$$

Ze dvou rovnic můžeme vyjádřit odhad parametrů

$$egin{aligned} ar{x} &= rac{lpha}{lpha + eta} \ ar{x}(lpha + eta) &= lpha = ar{x}lpha + ar{x}eta \ ar{x}eta &= lpha - ar{x}lpha \ eta &= rac{lpha(1 - ar{x})}{ar{x}} \end{aligned}$$

Dosazením do druhé rovnice

$$s^2 = rac{lpha \left[rac{lpha (1-ar{x})}{ar{x}}
ight]}{(lpha + \left[rac{lpha (1-ar{x})}{ar{x}}
ight])^2 (lpha + \left[rac{lpha (1-ar{x})}{ar{x}}
ight] + 1)} \ s^2 = rac{lpha^2 ar{x}^2 (1-ar{x})}{(lpha + lpha (1-ar{x}))^2 (lpha + (lpha (1-ar{x})) + 1)ar{x}} \ s^2 = rac{lpha^2 ar{x}^2 (1-ar{x})}{(lpha ar{x} + lpha (1-ar{x}))^2 (lpha ar{x} + (lpha (1-ar{x})) + ar{x})} \ s^2 = rac{lpha^2 ar{x}^2 (1-ar{x})}{lpha^2 (lpha + ar{x})} = rac{ar{x}^2 (1-ar{x})}{lpha + ar{x}}$$

Z toho vyjádříme α

$$lpha = rac{ar{x}^2(1-ar{x}^2) - s^2ar{x}}{s^2} = ar{x}(rac{ar{x}(1-ar{x})}{s^2} - 1)$$

Dosazením zpátky do β

$$\beta = (1 - \bar{x})(\frac{\bar{x}(1 - \bar{x})}{s^2} - 1)$$

Simulace odhadu

```
> x = rbeta(100, 0.5, 0.5)
> curve(dbeta(x, 0.5, 0.5))
```

```
dbeta(x, 0.5, 0.5)

1.0 1.5 2.0 2.5 3.0

0.0 0.2 0.4 0.6 0.8 1.0
```

```
> m = mean(x)
> v = var(x)
> alpha = m*((m*(1-m) /v) - 1)
> beta = (1-m)*((m*(1-m)/v) - 1)
```

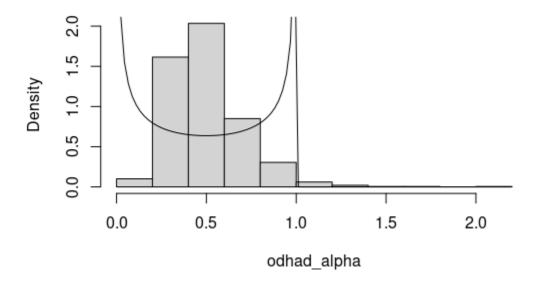
Hodnoty α a β jsou

```
> alpha
[1] 0.3953505
> beta
[1] 0.5416557
```

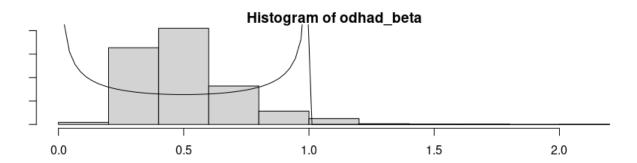
Simulace rozdělení odhadu

```
> simul.x=matrix(rbeta(1000*20, 0.5, 0.5), nrow = 20)
> odhad_alpha = apply(simul.x,2,function(x){mean(x)*((mean(x)*(1-mean(x)) /var(x)) - 1)})
> par(mfrow=c(2,1)
> hist(odhad_alpha, prob=TRUE)
> curve(dbeta(x, 0.5, 0.5), add=TRUE)
```

Histogram of odhad_alpha



```
> simul.x=matrix(rbeta(1000*20, 0.5, 0.5), nrow = 20)
> odhad_beta = apply(simul.x,2,function(x){(1-mean(x))*((mean(x)*(1-mean(x))/var(x)) -
    1)})
> par(mar=c(1,1,1,1))
> hist(odhad_beta, prob=TRUE)
> curve(dbeta(x, 0.5, 0.5), add=TRUE)
```



Konfidenční interval

Pro lpha

```
> margin = qt(0.975,df=(1000*20)-1)*var(odhad_alpha)/sqrt(1000*20)
> lowerinterval = mean(odhad_alpha) - margin
> upperinterval = mean(odhad_alpha) + margin
> lowerinterval
[1] 0.4999459
> upperinterval
[1] 0.4999651
```

Pro β

```
> margin = qt(0.975,df=n-1)*v/sqrt(n)
> lowerinterval = mean(odhad_beta) - margin
> upperinterval = mean(odhad_beta) + margin
> lowerinterval
[1] 0.5072402
> upperinterval
[1] 0.5107307
```

Simulace

tbc...