Chapter 1
THE SOLOW GROWTH MODEL

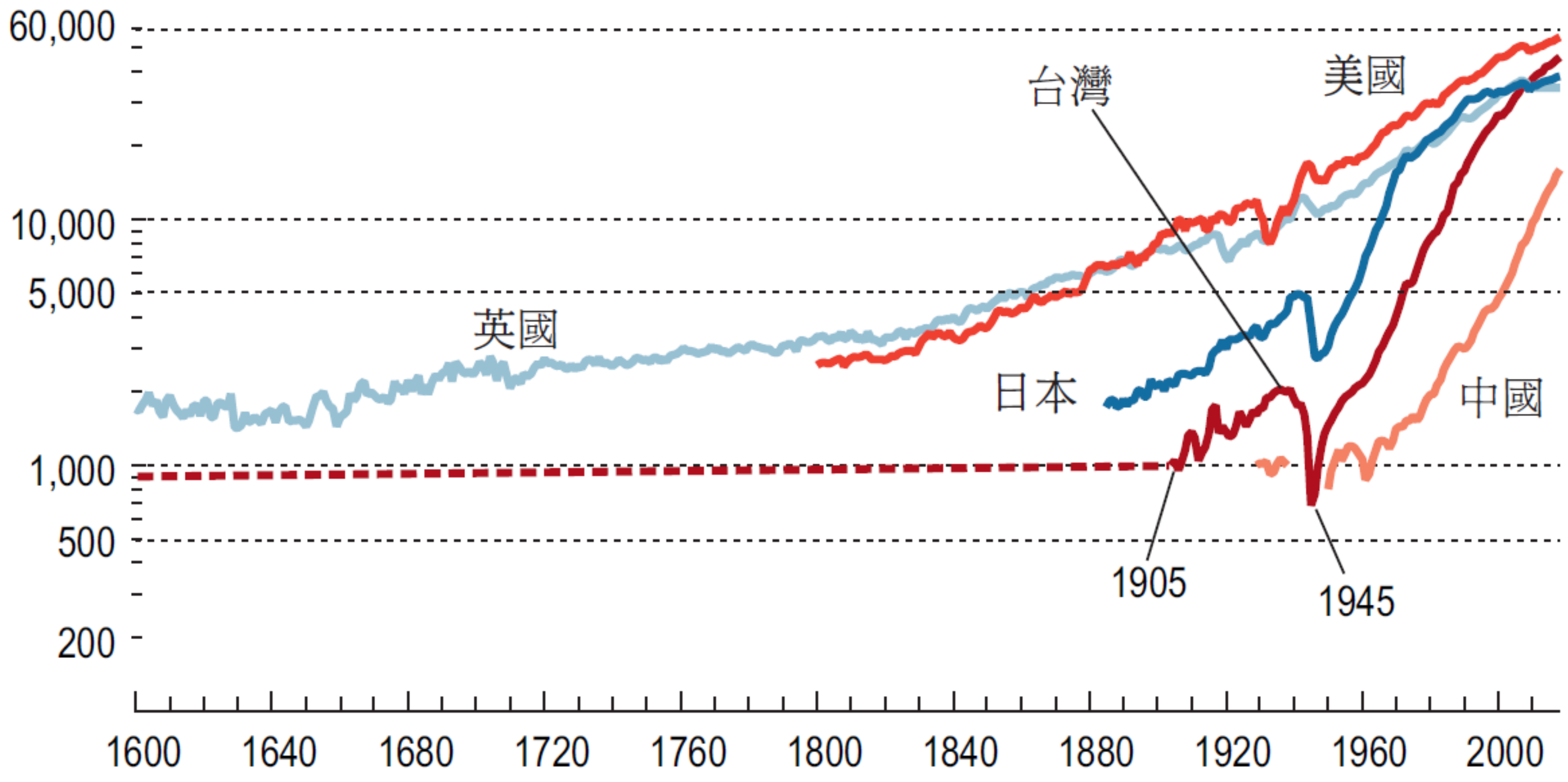
1.1 Some Basic Facts about Economic Growth

Introduction to Economic Growth

- Key Insight: Economic growth has drastically increased standards of living in industrialized countries over the past centuries.
- Magnitude of Growth: U.S. and Western European incomes today are 10 to 30 times larger than a century ago and 50 to 300 times larger than two centuries ago.
- Growth rates have been rising, with higher growth in the 20th century than in the 19th, and even lower growth before the Industrial Revolution.

Global Growth Patterns

- Rising Growth Rates: Over most of modern history, industrialized countries have experienced increasing growth rates.
- Growth Slowdown: A significant exception is the productivity growth slowdown from the early 1970s to mid-1990s in the U.S. and other industrialized countries, which has slightly rebounded since.



- 人均 GDP (PPP), 2011 US dollar
- 世界各國在工業革命 (大約 1800 年) 以前都是傳統農業經濟; 西歐國家在工業革命之後出現現代經濟成長 (modern economic growth)
- 台灣的現代經成長出現在日治初期

Cross-Country Income Differences

- Income Gaps: There are enormous differences in income levels between countries, with incomes in developed countries like the U.S. being around 20 times higher than in less developed countries such as Bangladesh and Kenya.
- Growth Miracles and Disasters: Miracles: Countries like Japan (post-WWII), East Asia's NICs, and China have experienced rapid growth, significantly improving their relative global income standings.
- Disasters: Examples include Argentina and many Sub-Saharan African countries like Chad and Mozambique, which have stagnated or even declined in relative terms.

The Implications of Growth Differences

- Impact on Human Welfare: The vast differences in living standards across countries translate into significant differences in health, education, life expectancy, and other welfare indicators.
- Long-Term vs. Short-Term: Long-term growth differences have a far greater impact on human welfare than short-term fluctuations like recessions.

The Productivity Growth Slowdown

- U.S. Case Study: The productivity growth slowdown in the U.S. since the early 1970s has reduced real income per person by approximately 25% compared to what it could have been.
- Global Examples: The Philippines would need 150 years to reach the current U.S. income level if its growth continues at 1.5%, while achieving higher growth rates (3% or 5%) would drastically shorten this period.

1.2 Assumptions

Introduction to the Solow Growth Model

Four Key Variables:

- Output $Y(t)$
- Capital $K(t)$
- Labor $L(t)$
- Knowledge or effectiveness of labor $A(t)$

Production Function: $Y(t)=F(K(t),A(t)L(t))$

Explanation: The production function shows how output changes over time depending on capital, labor, and knowledge.$A(t)$ represents labor-augmenting technological progress (Harrod-neutral technological change).

Constant Returns to Scale

- Assumption: The production function has constant returns to scale.

$$F(cK,cAL)=cF(K,AL)$$

For any constant $c \geq 0$.

Explanation: Doubling both capital and effective labor results in a doubling of output. Implies the economy has no further gains from specialization at large scales.

Intensive Form of the Production Function

$$F\left(\frac{K}{AL}, 1\right) = \frac{1}{AL} F(K, AL)$$

Capital per unit of effective labor:

$$k = \frac{\mathrm{K}}{\mathrm{AL}}$$

Output per unit of effective labor:

$$y = \frac{Y}{\mathrm{AL}}$$

Rewriting the Production Function:

$$F(K,AL)=AL \cdot f(k)$$

where:

$$y=f(k)$$

Explanation: This transformation simplifies the model by focusing on output and capital per unit of effective labor.

Intensive-Form Production Function and Properties

- Production Function in Intensive Form: y=$f(k)$
- Key Assumptions:$f(0)$=0, $f'(k)$>0, $f''(k)$<0

The marginal product of capital $f'(k)$ is positive but diminishes as capital increases ($f''(k)$<0).

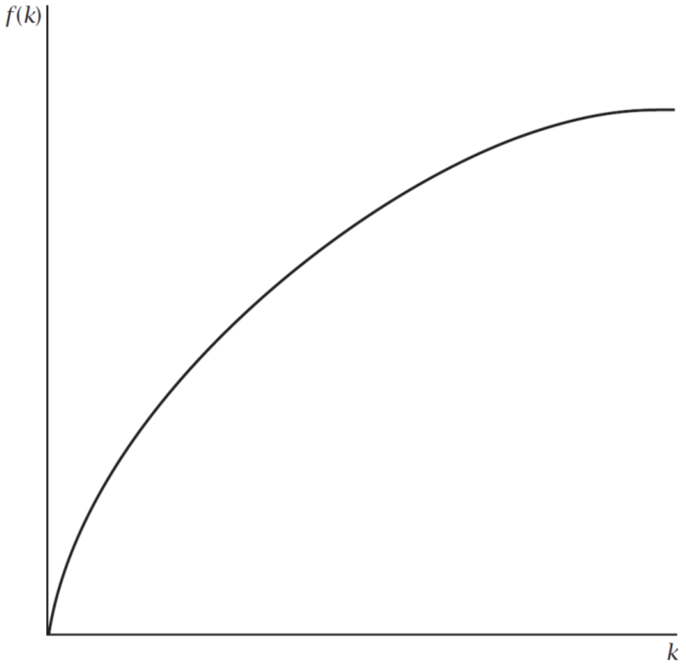


FIGURE 1.1 An example of a production function

Marginal Product of Capital and Inada Conditions

- The marginal product of capital is positive but decreases as capital per effective labor increases.
- Inada conditions

$$\lim_{k \to 0} f'(k) = \infty \quad , \qquad \lim_{k \to \infty} f'(k) = 0$$

These conditions ensure the economy does not diverge as capital approaches extreme values.

Cobb-Douglas Production Function

Cobb-Douglas Function:

$$F(K, AL) = K^{\alpha}(AL)^{1-\alpha}, \qquad 0 < \alpha < 1$$

Intensive Form:

$$f(k) = k^{\alpha}$$

Cobb-Douglas is commonly used in growth models because it reflects diminishing returns to capital while remaining analytically simple.

Marginal Product of Capital (MPK)

- Marginal Product of Capital:

$$f'(k) = \frac{d}{dk}(k^{\alpha}) = \alpha k^{\alpha-1}$$

- Diminishing Returns to Capital:

$$f''(k) = \frac{d^2}{dk^2}(k^{\alpha}) = \alpha(\alpha - 1)k^{\alpha-2}$$

- The marginal product of capital is positive but decreases as k increases, showing diminishing returns to capital accumulation.

Exogenous Growth of Labor

- Growth of Labor:

$$\dot{L}(t) = nL(t)$$

n: constant rate of labor growth.

- Growth of Knowledge:

$$\dot{A}(t) = gA(t)$$

g: constant rate of knowledge growth.

Both labor and knowledge grow at constant, exogenously determined rates, n and g, respectively.

Growth Rates and Logarithmic Growth

- General Growth Rate Formula:

$$\frac{dlnX(t)}{dt} = \frac{dlnX(t)}{dX(t)}\frac{dX(t)}{dt} = \frac{1}{X(t)}\dot{X}(t)$$

Example for Labor: $\dot{L}(t) = nL(t),\ lnL(t) = lnL(0) + nt$

Example for Knowledge: $\dot{A}(t) = gA(t),\ lnA(t) = lnA(0) + gt$

$$L(t) = L(0)e^{nt}$$
$$A(t) = A(0)e^{gt}$$

Growth rates of variables can be expressed as the rate of change of their natural logarithms, simplifying the analysis of proportional growth over time.

Capital Accumulation Equation

- Capital Accumulation:

$$\dot{K}(t) = sY(t) - \delta K(t)$$

  s: saving rate (fraction of output invested in capital)

  δ: depreciation rate of capital.

- This equation shows that capital stock grows through investment but depreciates over time.

Output in Terms of Capital and Labor

- Substitute Production Function into Capital Dynamics:

$$\dot{K}(t) = sF(K(t), A(t)L(t)) - \delta K(t)$$

- Investment drives capital growth, while depreciation diminishes it, leading to the dynamic accumulation process of capital in the economy.

Simplifications and the Purpose of Modeling

- The Solow Model is a highly simplified representation of economic growth.
- Many complexities, such as government and employment fluctuations, are intentionally omitted for clarity.
  - Single good economy
  - No government presence
  - Ignored employment fluctuations
  - Only three inputs in production (Capital, Labor, Knowledge)
  - Constant rates of savings, depreciation, population growth, and technological progress

1.3 The Dynamics of the Model

The Dynamics of the Solow Model

- To understand the evolution of the economy, we analyze the behavior of capital, labor, and knowledge.
- Core Focus: The evolution of capital per unit of effective labor k = K / A L.
- Use of the chain rule to derive the dynamics of k.

$$\begin{aligned}\dot{k}(t) &= \frac{\dot{K}(t)}{A(t)L(t)} - \frac{K(t)}{[A(t)L(t)]^2}[A(t)\dot{L}(t) + L(t)\dot{A}(t)] \\ &= \frac{\dot{K}(t)}{A(t)L(t)} - \frac{K(t)}{A(t)L(t)}\frac{\dot{L}(t)}{L(t)} - \frac{K(t)}{A(t)L(t)}\frac{\dot{A}(t)}{A(t)}.\end{aligned} \tag{1.16}$$

Simplification of Capital Dynamics

$$\dot{k}(t) = \frac{sY(t) - \delta K(t)}{A(t)L(t)} - k(t)n - k(t)g$$
$$= s\frac{Y(t)}{A(t)L(t)} - \delta k(t) - nk(t) - gk(t). \qquad (1.17)$$

- $s\frac{Y(t)}{A(t)L(t)}$:Output per unit of effective labor, fraction invested.
- $\delta k(t)$: Depreciation of capital per unit of effective labor.
- $nk(t)$: Growth of labor force.
- $gk(t)$: Growth of technology (effectiveness of labor).

Core Equation of the Solow Model

Using the fact $\frac{Y(t)}{A(t)L(t)} = f(k(t))$

$$\dot{k}(t) = sf(k(t)) - (n + g + \delta)k(t)$$

- *sf(k)*: Actual investment per unit of effective labor.
- *(n + g + δ) k*: Break-even investment required to keep *k* constant.
- When actual investment exceeds break-even investment, capital per effective worker *k* rises.

Investment Dynamics

- Break-even investment grows linearly with k, reflecting labor growth, technology, and depreciation.
- sf(k) grows at a decreasing rate due to diminishing returns to capital.

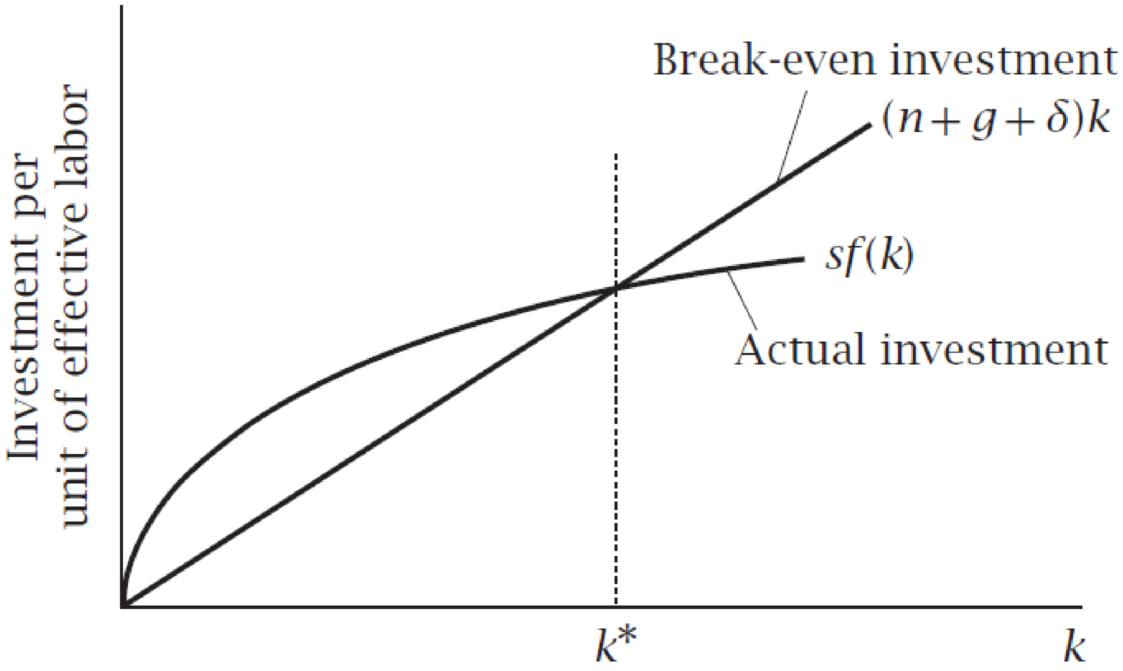


FIGURE 1.2 **Actual and break-even investment**

Phase Diagram of Capital Dynamics

- Phase Diagram of $\dot{k}$ as a function of k.
- When $k < k^*$, actual investment exceeds break-even investment, $\dot{k} > 0$.
- When $k > k^*$, break-even investment exceeds actual investment, $\dot{k} < 0$.
- $k = k^*$: The economy reaches a steady state where capital per effective worker remains constant.

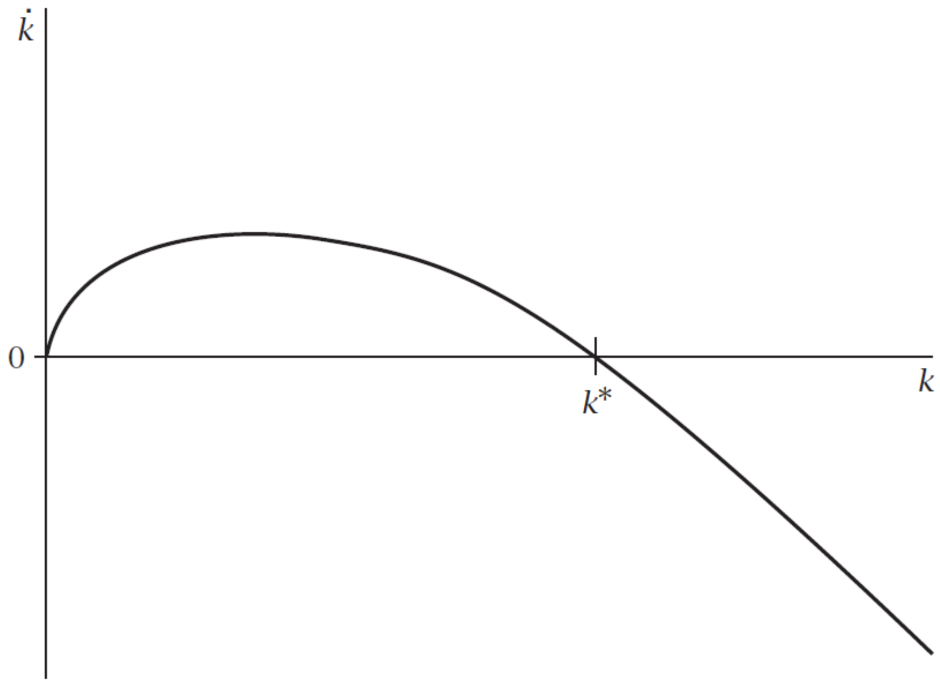


FIGURE 1.3 The phase diagram for *k* in the Solow model

The Balanced Growth Path

a balanced growth path—a situation where each variable of the model is growing at a constant rate

When $k = k^* \Rightarrow \dot{k} = 0$, $K = ALk^*$, $\frac{\dot{K}}{K} = n + g$

- At the balanced growth path, both capital and effective labor grow at rate $n + g$.
- Output grows at the same rate, ensuring capital-output ratio remains constant.
- Output per worker *Y/L* grows at rate *g*, <u>determined solely by technological progress.</u>

Convergence to the Balanced Growth Path

- Regardless of its starting point, the economy converges to the steady-state level of capital per worker k^*.
- At the steady state, all variables grow at constant rates.
- Long-run growth of output per worker $\frac{Y}{L}$ is solely determined by the rate of technological progress g.

Key Insights of Solow Model Dynamics

- The Solow model separates the growth of output per worker into components driven by capital accumulation and technological progress.
- Capital accumulation leads to short-term growth but converges to a steady-state path.
- Long-term growth is driven by technological improvements (g), as it determines the growth rate of output per worker.

1.4 The Impact of a Change in the Saving Rate

The Solow Model: Saving Rate and Investment

- The Solow model focuses on how savings affect investment and thus the capital stock.
- Policy changes, such as government spending on consumption vs. investment, taxes, or borrowing, can impact the fraction of output invested.
- Investigating the effects of a permanent change in the saving rate helps to understand the growth dynamics in the model.

The Effect of a Higher Saving Rate on Capital

- An increase in the saving rate shifts the actual investment curve upwards, causing capital per worker (k) to rise over time.
- Initially, capital remains at its old steady-state level, but investment now exceeds depreciation, leading to capital accumulation.
- The economy converges towards a new, higher steady-state level of k*, where capital per worker remains constant.

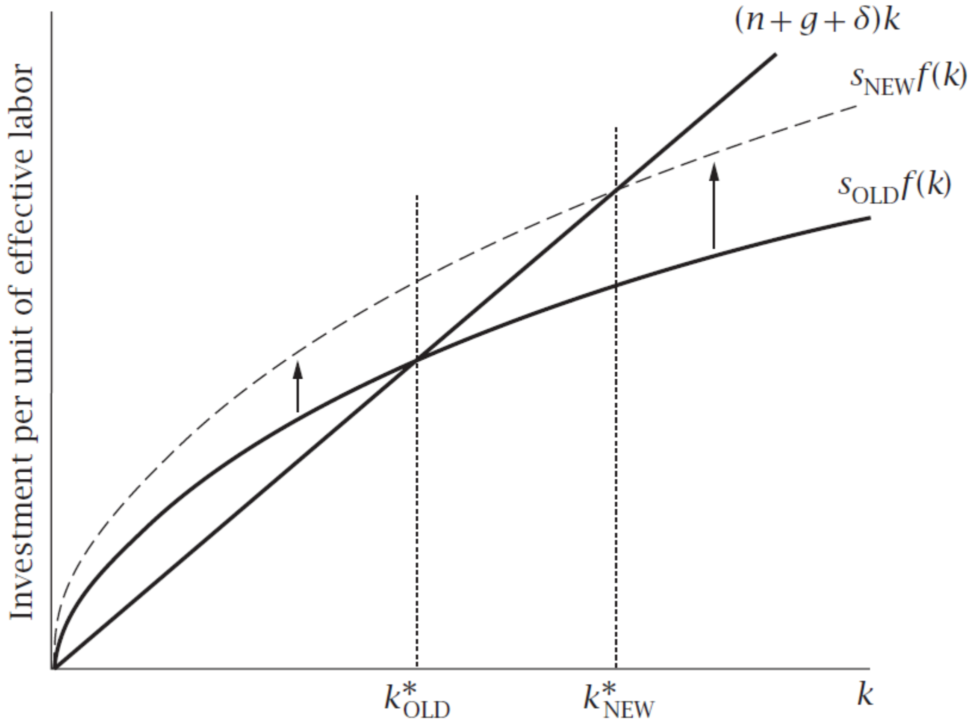
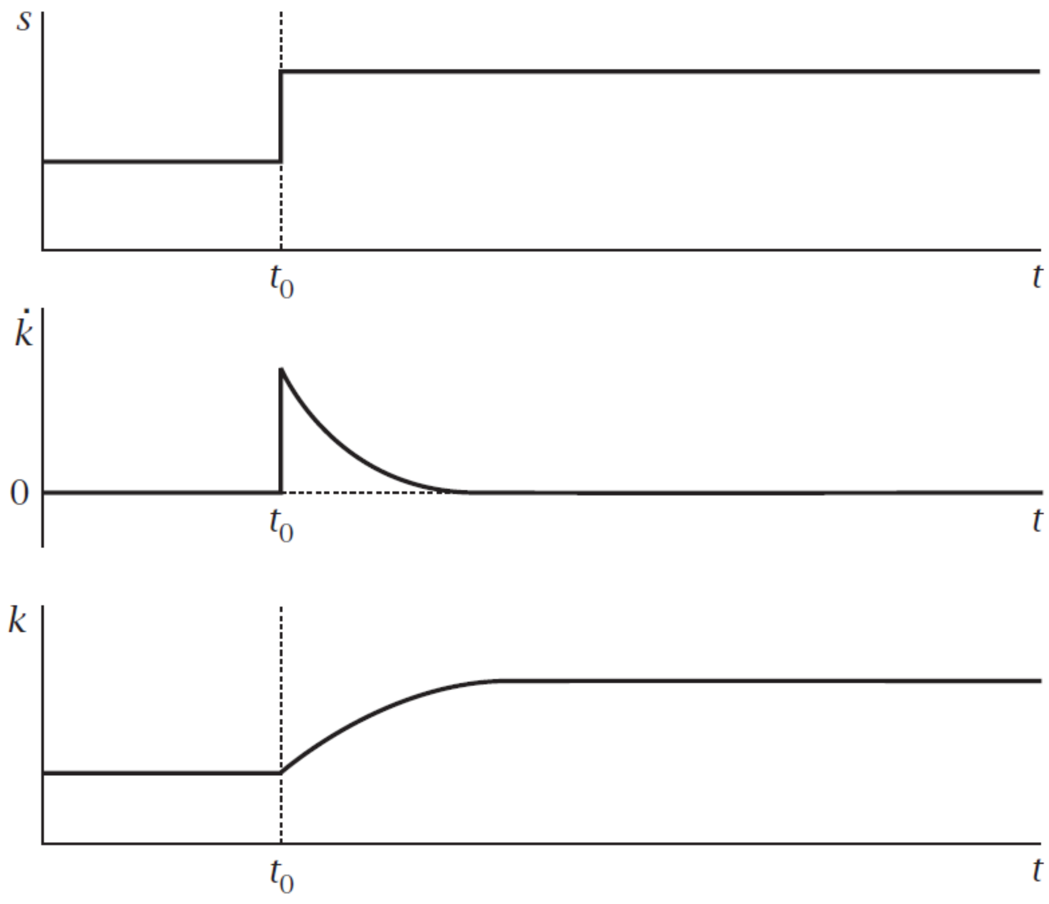


FIGURE 1.4 The effects of an increase in the saving rate on investment

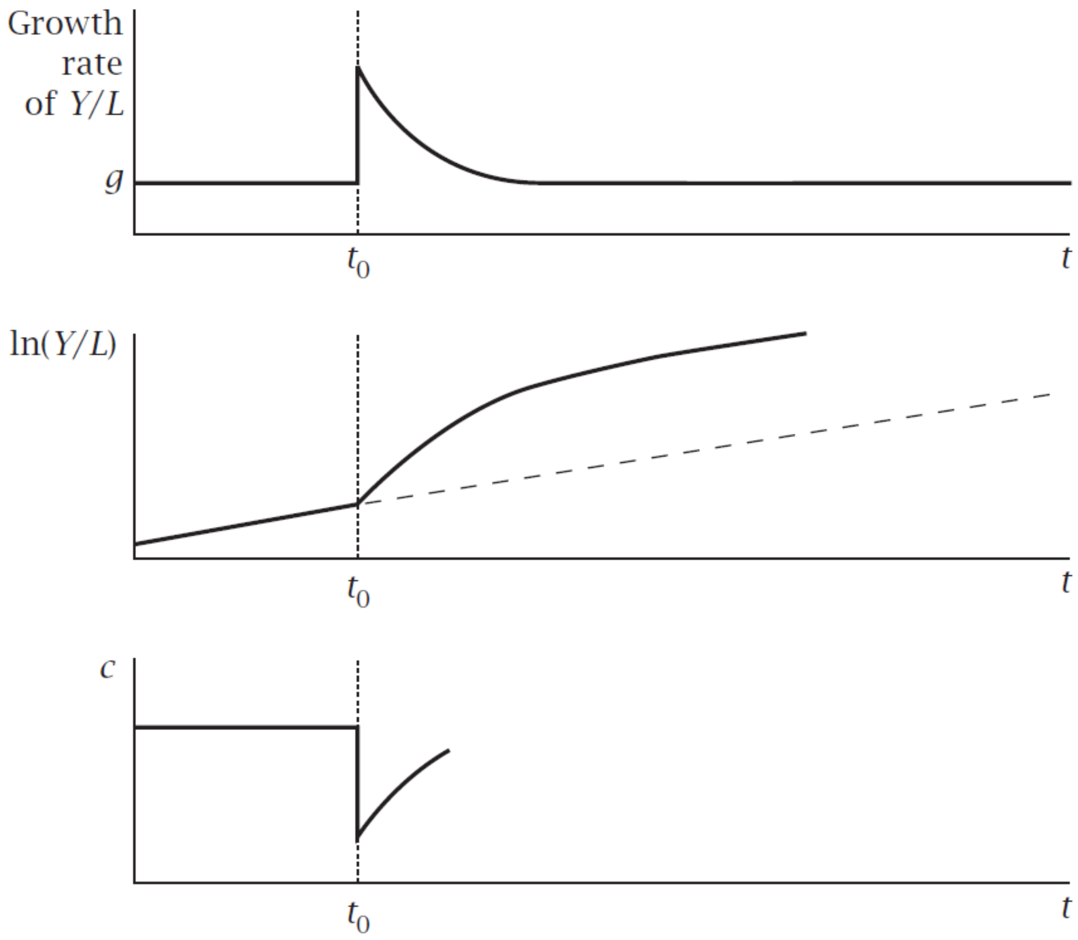
Temporary Growth Acceleration

- During the transition period, capital per worker increases, resulting in a temporary rise in output growth beyond the rate of technological progress.
- Once capital reaches the new steady-state (k^*), growth returns to the long-run rate driven by technological progress (g).



Output per Worker and Growth Rates

- Initially, the growth rate of output per worker exceeds the steady-state growth due to increasing capital.
- Over time, output per worker rises above its previous trajectory, but the long-term growth rate remains unchanged, driven solely by technology.



Permanent vs. Temporary Effects

- A change in the saving rate results in a "level effect" (a higher level of capital and output) but not a "growth effect".
- The growth rate of output per worker on the balanced growth path is unaffected by saving rate changes—only technological progress influences long-term growth.

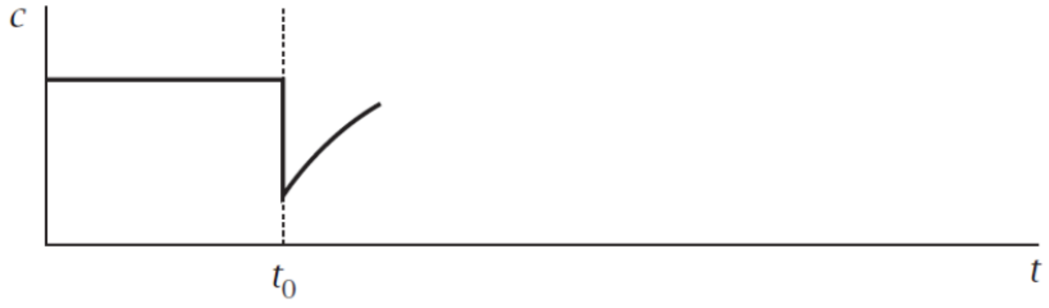
The Impact on Consumption

- Households' welfare depends on consumption rather than output.
- Investment is an input into future production, making consumption a key variable of interest.
- Consumption per unit of effective labor equals output per unit of effective labor ($f(k)$), multiplied by the fraction of that output consumed (1 - s).

$$c = (1 - s)f(k)$$

Initial Drop in Consumption

- At the time of the increase in the saving rate (t_0), consumption per unit of effective labor initially drops.
- This happens because *k* does not immediately rise, and a higher fraction of output is being devoted to investment.
- Consumption then rises gradually as *k* rises over time.



Long-Run Impact on Consumption

On the Balanced Growth Path

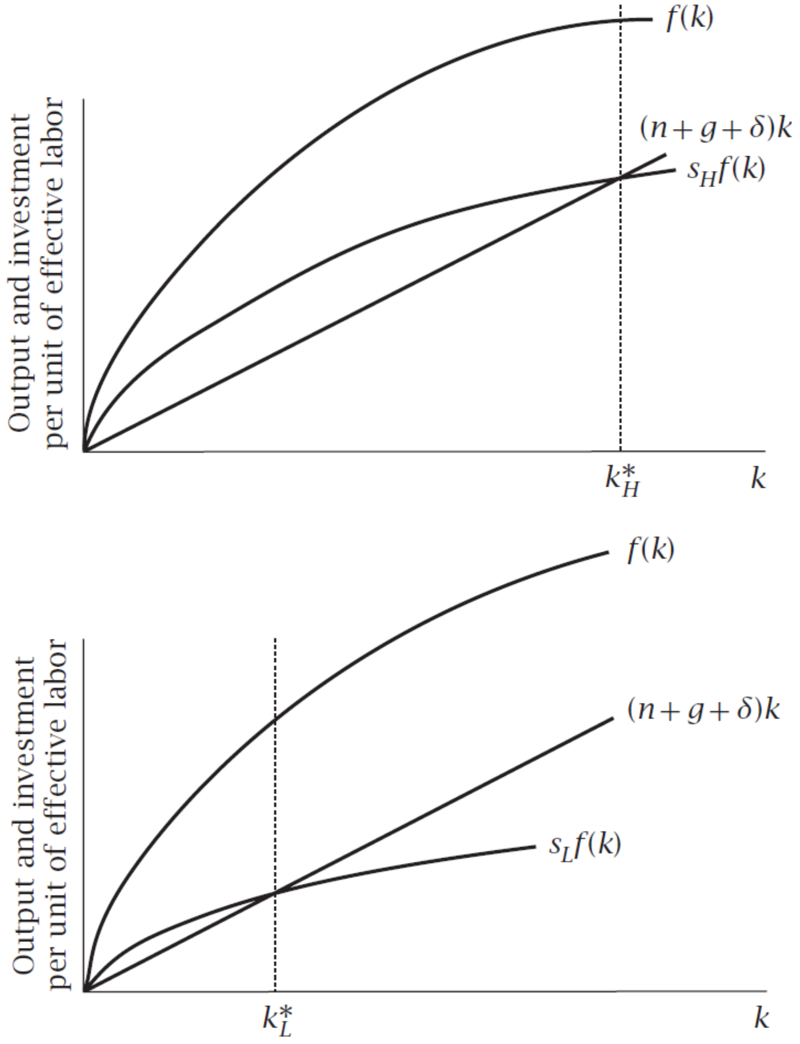
$$c^* = f(k^*) - (n + g + \delta)k^*$$

$$\frac{\partial c^*}{\partial s} = [f'(k^*(s, n, g, \delta)) - (n + g + \delta)] \frac{\partial k^*(s, n, g, \delta)}{\partial s}$$

- In the long run, whether consumption exceeds its previous level depends on whether $f'(k^*)$ is greater or less than $(n + g + \delta)$.

Long-Run Impact on Consumption

- If $f'(k^*)$ is less than $(n + g + \delta)$, consumption must fall in the long run to maintain the higher capital stock.
- If $f'(k^*)$ exceeds $(n + g + \delta)$, consumption rises because the additional output is enough to maintain k and allow more consumption.

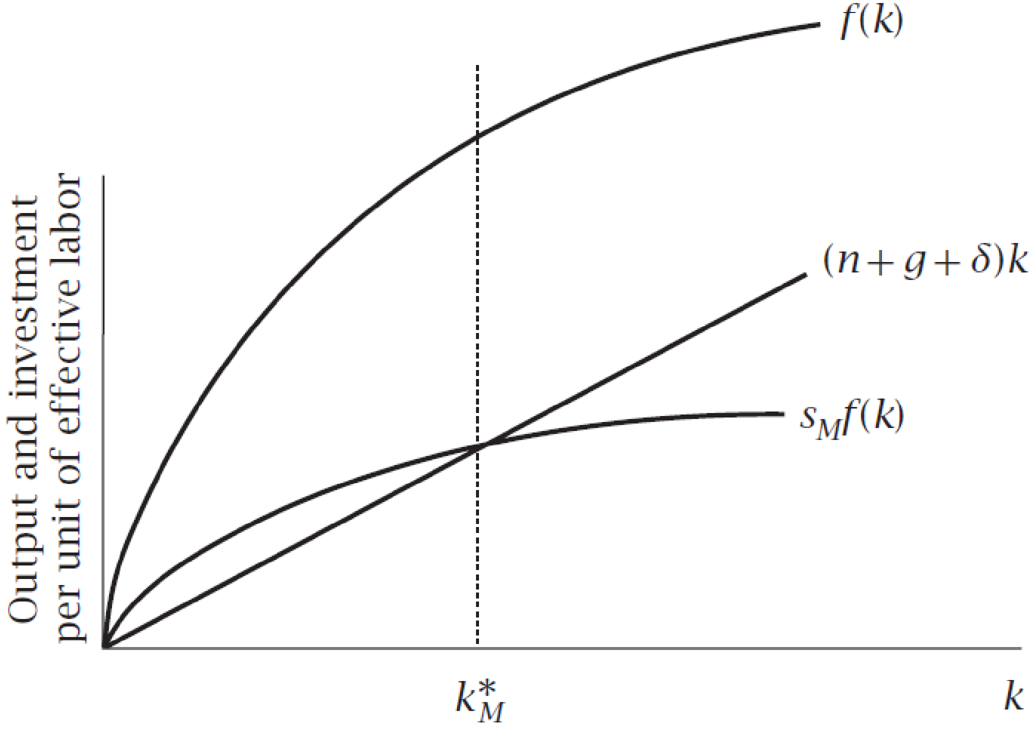


Golden-Rule Level of Capital

- The golden-rule level of capital is the level where

  $f'(k^*) = (n + g + \delta).$

- At this level, the economy maximizes consumption per unit of effective labor.
- Any deviation from this level results in lower consumption either due to insufficient or excessive capital accumulation.



Summary: Impact on Consumption

- Initially, consumption declines due to the higher saving rate.
- Over time, consumption may rise or fall depending on the marginal product of capital and the balanced growth path.
- The golden-rule capital stock ensures maximum consumption over time.

1.5 Quantitative Implications

Quantitative Implications: Overview

- Quantitative predictions of models are key for understanding long-term impacts.
- For example, a permanent increase in saving affects output growth differently based on the assumptions of the model.
- Here we focus on how saving impacts output in the long run, specifically examining elasticity and capital-labor dynamics.

The Long-Run Effect of Saving on Output

- The long-run effect of saving on output is expressed as:

$$\frac{\partial y^*}{\partial s} = f'(k^*)\,\frac{\partial k^*(s,n,g,\delta)}{\partial s}$$

where $y^* = f(k^*)$ represents output per unit of effective labor, and k^* represents capital per worker on the balanced growth path.

Deriving $\frac{\partial k^*}{\partial s}$

- We use the equation $sf(k^*(s,n,g,\delta)) = (n+g+\delta)k^*(s,n,g,\delta)$ to express investment and capital accumulation.

$$sf'(k^*)\frac{\partial k^*}{\partial s} + f(k^*) = (n+g+\delta)\frac{\partial k^*}{\partial s}$$

$$\frac{\partial k^*}{\partial s} = \frac{f(k^*)}{(n+g+\delta) - sf'(k^*)}$$

- Substituting this into the original equation for $\partial y^*/\partial s$ leads to the final expression for output.

Elasticity of Output with Respect to Saving

$$\frac{\partial y^*}{\partial s} = \frac{f'(k^*)f(k^*)}{(n+g+\delta) - sf'(k^*)}$$

- The expression can be transformed into an elasticity form:

$$\begin{aligned}\frac{s}{y^*}\frac{\partial y^*}{\partial s} &= \frac{s}{f(k^*)}\frac{f'(k^*)f(k^*)}{(n+g+\delta) - sf'(k^*)} \\ &= \frac{(n+g+\delta)k^*f'(k^*)}{f(k^*)[(n+g+\delta) - (n+g+\delta)k^*f'(k^*)/f(k^*)]} \\ &= \frac{k^*f'(k^*)/f(k^*)}{1 - [k^*f'(k^*)/f(k^*)]}.\end{aligned}$$

Elasticity of Output with Respect to Capital

- Elasticity of output with respect to capital at $k = k^*$

$$k^* \frac{f'(k^*)}{f(k^*)} = \alpha_k \, (k^*)$$

The elasticity of the balanced-growth-path level of output with respect to the saving rate is

$$\frac{s}{y^*} \frac{\partial y^*}{\partial s} = \frac{\alpha_K(k^*)}{1 - \alpha_K(k^*)}$$

Capital's Share of Output $\alpha_k\ (k^*)$

- Assumptions:
  - Competitive markets.
  - No externalities.
  - Capital earns its marginal product.
- **Income earned by capital** (per unit of effective labor):

$$k^* f'(k^*)$$

- **Capital's share of total income**:

$$k^* \ \frac{f'(k^*)}{f(k^*)} = \alpha_k\ (k^*)$$

Real-World Implications

- In most economies, capital earns its marginal product, and $\alpha_k\ (k^*)$ can be estimated using the share of income going to capital.
- For example, if $\alpha_k\ (k^*)$ is about one-third, the long-run elasticity with respect to the saving rate is about one-half.
- Thus, a 10% increase in the saving rate results in approximately a 5% rise in output per worker over the long term.

The Speed of Convergence

- In practice, we are interested in how rapidly changes (like saving rate) impact the equilibrium.
- We focus on how rapidly *k* approaches *k**.
- Key equation of the model:

$$\dot{k}(t) = sf(k(t)) - (n + g + \delta)k(t)$$

Taylor Approximation for Speed of Convergence

- Using a first-order Taylor-series approximation of $\dot{k}(k)$ around $k = k^*$, we get:

$$\dot{k} \simeq \left[\left. \frac{\partial \dot{k}(k)}{\partial k} \right|_{k=k^*} \right] (k - k^*)$$

- This approximation allows us to see that $\dot{k}$ is the product of the difference (k - k^*) and the derivative of $\dot{k}$ with respect to k.

Stability and Convergence

- At ***k = k*,*** the system is stable, and *k* converges to *k** over time.
- The growth rate of *k(t) - k** is approximately proportional to its distance from *k**.

Let $\lambda = \left.\frac{\partial \dot{k}(k)}{\partial k}\right|_{k=k^*}$

$$\dot{k}(t) \simeq -\lambda[k(t) - k^*]$$

- This leads to the expression: $k(t) \simeq k^* + e^{-\lambda t}[k(0) - k^*]$

Finding λ

- To find λ, we differentiate expression $\dot{k}(t) = sf(k(t)) - (n+g+\delta)k(t)$ for $\dot{k}$ with respect to k and evaluate at $k = k^*$

$$\begin{aligned}\lambda \equiv -\left.\frac{\partial \dot{k}(k)}{\partial k}\right|_{k=k^*} &= -[sf'(k^*) - (n+g+\delta)] \\ &= (n+g+\delta) - sf'(k^*) \\ &= (n+g+\delta) - \frac{(n+g+\delta)k^* f'(k^*)}{f(k^*)} \\ &= [1-\alpha_K(k^*)](n+g+\delta).\end{aligned} \tag{1.31}$$

Convergence to Balanced-Growth-Path

- k converges to its balanced-growth-path value at rate

  $[1 - \alpha_k\,(k^*)]\,(n + g + \delta)$

- Additionally, y approaches y^* at the same rate that k approaches k^*.
- Thus,

$$y(t) - y^* \simeq e^{-\lambda t}[y(0) - y^*]$$

Calibrating Equation (1.31)

- We can calibrate (1.31) to see how quickly economies approach their balanced growth paths.
- Typically, $n + g + \delta \approx 6\%$ per year, arising from:

  1-2% population growth

  1-2% output per worker

  3-4% depreciation
- With capital's share ≈ one-third, $(1 - \alpha_k)(n + g + \delta) \approx 4\%$.
- Thus, k and y move 4% of the remaining distance toward k* and y* each year.

Example of Saving Rate Change

- Example: A 10% increase in the saving rate

  - After 1 year: output is 0.04(5%) = 0.2% above the previous path.

  - After 17 years: output is 0.5(5%) = 2.5% above the previous path.

- The overall impact of a substantial change in the saving rate is modest and occurs slowly.

1.6 The Solow Model and the Central Questions of Growth Theory

An overview of capital and labor in economic growth theory

Capital vs. Labor Effectiveness

- Two primary sources of variation in output per worker:

1. Capital per worker (K/L)
2. Effectiveness of labor (A)

Only growth in labor effectiveness leads to permanent growth in output per worker.

Capital Accumulation Limitations

- The Solow model concludes that differences in capital cannot account for large cross-country income differences.
- Key Insight: Differences in the effectiveness of labor explain the vast income differences across time and space.

Evidence of Capital Differences

- No evidence supports such extreme differences in capital stocks.
- Capital-output ratios are roughly constant over time.
- Capital stock per worker is around 10 times higher in industrialized countries than 100 years ago, not 100-1000 times larger.

Marginal Product of Capital

- Elasticity of output with respect to capital: α.
- Example: A tenfold difference in capital leads to a hundredfold difference in marginal product.
- Yet, there's no significant evidence of such variations in the rates of return on capital across countries.

Effectiveness of Labor

- The Solow model's second source of variation is the effectiveness of labor.
- Effectiveness of labor could represent:
  - Abstract knowledge
  - Education, skills, cultural attitudes, etc.

Potential Future Directions

- Future considerations include:

1. Knowledge accumulation and how it explains differences between countries.
2. Reconsidering the role of capital if it encompasses more than just physical capital (e.g., positive externalities).

Growth Accounting

- Growth accounting breaks down growth into contributions from various factors (capital, labor, etc.) and other forces.
- Pioneered by Abramovitz (1956) and Solow (1957).

Growth Accounting Equation

- Production function: Y(t) = F(K(t), A(t)L(t))
- Decomposition of growth:

$$\dot{Y}(t) = \frac{\partial Y(t)}{\partial K(t)} \dot{K}(t) + \frac{\partial Y(t)}{\partial L(t)} \dot{L}(t) + \frac{\partial Y(t)}{\partial A(t)} \dot{A}(t)$$

$$\frac{\dot{Y}(t)}{Y(t)} = \frac{K(t)}{Y(t)} \frac{\partial Y(t)}{\partial K(t)} \frac{\dot{K}(t)}{K(t)} + \frac{L(t)}{Y(t)} \frac{\partial Y(t)}{\partial L(t)} \frac{\dot{L}(t)}{L(t)} + \frac{A(t)}{Y(t)} \frac{\partial Y(t)}{\partial A(t)} \frac{\dot{A}(t)}{A(t)}$$

$$\equiv \alpha_K(t)\frac{\dot{K}(t)}{K(t)} + \alpha_L(t)\frac{\dot{L}(t)}{L(t)} + R(t).$$

α_k : Elasticity of output w.r.t. capital

α_L : Elasticity of output w.r.t. labor

R(t) : Solow residual

Growth Rate of Output per Worker

- Subtracting growth in labor from both sides:

$$\frac{\dot{Y}(t)}{Y(t)} - \frac{\dot{L}(t)}{L(t)} = \alpha_K(t)\left[\frac{\dot{K}(t)}{K(t)} - \frac{\dot{L}(t)}{L(t)}\right] + R(t)$$

- Decomposes growth into:
  - Contribution of capital growth per worker
  - Solow residual

Solow Residual Interpretation

- Often interpreted as the contribution of technological progress.
- Captures all sources of growth not attributed to capital accumulation.
- Can be adjusted for different types of capital, labor quality, or market imperfections.

Limitations of Growth Accounting

- Focuses on immediate determinants like factor accumulation but doesn’t explain underlying sources.
- Incorrectly attributes much growth to the residual in balanced growth economies.

Applications in Economic Studies

- Rapid growth of newly industrialized countries (NICs) in East Asia.
- Studies on productivity slowdowns and rebounds (e.g., U.S. productivity growth).

Historical Examples

- NICs' success attributed to:
  - Rising investment
  - Improved labor quality
  - Increased labor force participation
- U.S. productivity rebound (mid-1990s):
  - IT played a key role in productivity growth rebound.

What is Convergence?

- Definition: The concept that poorer countries tend to grow faster than richer ones.

Countries with lower initial income per capita will 'catch up' over time due to higher growth rates.

Solow Growth Model and Convergence

- First prediction: Countries with lower initial output per worker grow faster, leading to convergence.
- Second prediction: Return on capital is lower in richer countries, leading to capital flows to poorer nations.
- Third prediction: Knowledge diffusion lags cause temporary differences in growth rates but will diminish as technology spreads.

Baumol's Empirical Evidence

- Baumol studied 16 industrialized countries (1870–1979).
- Key finding: Strong evidence of convergence among these countrie

$$\ln\left[\left(\frac{Y}{N}\right)_{i,1979}\right] - \ln\left[\left(\frac{Y}{N}\right)_{i,1870}\right] = a + b\ln\left[\left(\frac{Y}{N}\right)_{i,1870}\right] + \varepsilon_i$$

- Negative b indicates convergence. Result: b = -0.995 suggesting near-perfect convergence.

$$\ln\left[\left(\frac{Y}{N}\right)_{i,1979}\right] - \ln\left[\left(\frac{Y}{N}\right)_{i,1870}\right] = 8.457 - \underset{(0.094)}{0.995}\ \ln\left[\left(\frac{Y}{N}\right)_{i,1870}\right], \tag{1.37}$$

$$R^2 = 0.87, \qquad \text{s.e.e.} = 0.15,$$

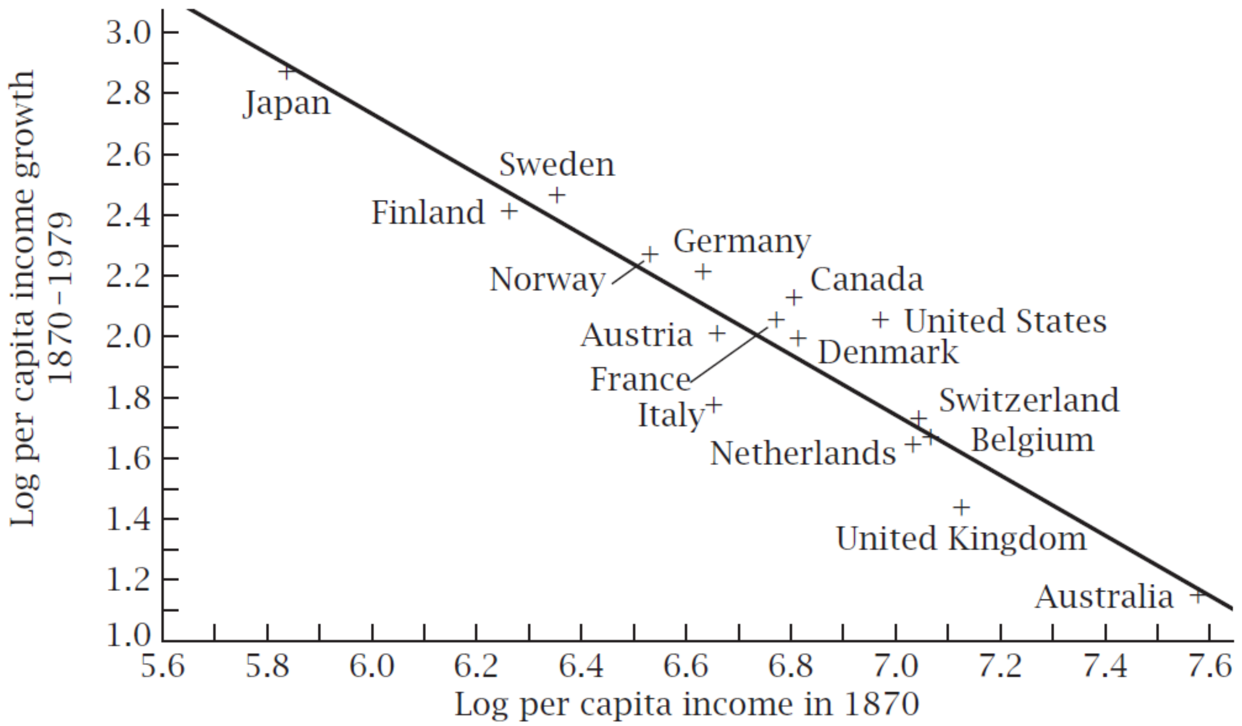


FIGURE 1.7 Initial income and subsequent growth in Baumol's sample (from DeLong, 1988; used with permission)

Critique: DeLong (1988)

- Sample Selection Bias: Baumol’s sample only includes countries that experienced rapid growth, thus biasing results.
- Measurement Error: Income estimates for 1870 are imprecise, affecting growth rate estimates.
- DeLong's recalculated b = -0.566 suggests weaker convergence.

Correcting Biases and Findings

- Revised Sample: Including additional countries weakens evidence of convergence.
- Revised Model: Addresses measurement error with a modified equation.
- DeLong's conclusion: Convergence is much weaker than Baumol suggested.

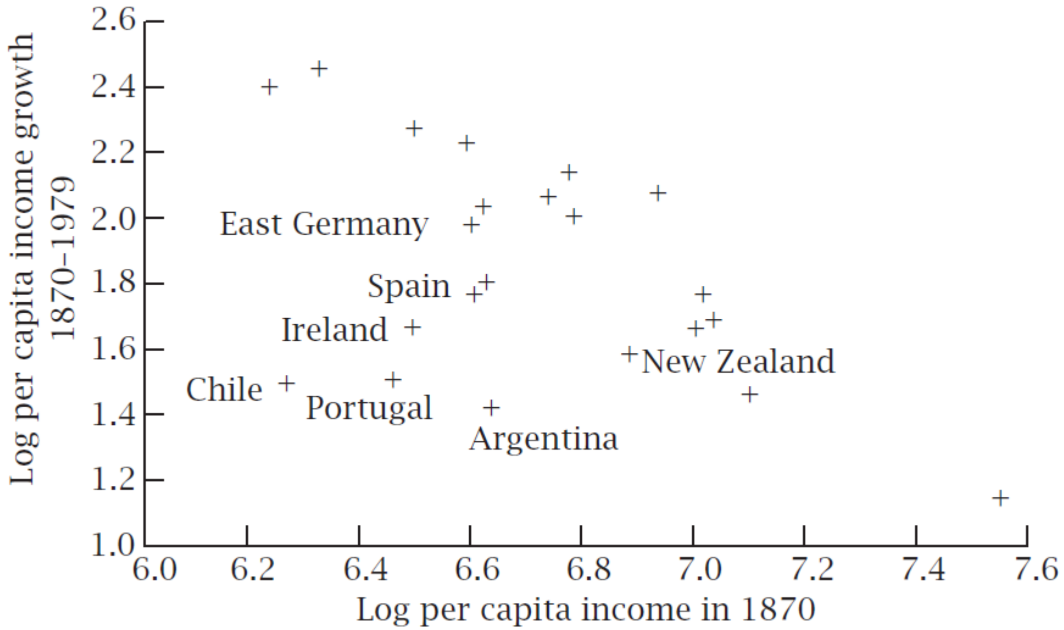


FIGURE 1.8 **Initial income and subsequent growth in the expanded sample (from DeLong, 1988; used with permission)**

Convergence After 1970

- Evidence from 1970-2003 shows little sign of convergence across the non-Communist world.
- Suggests that global convergence remains limited in scope.

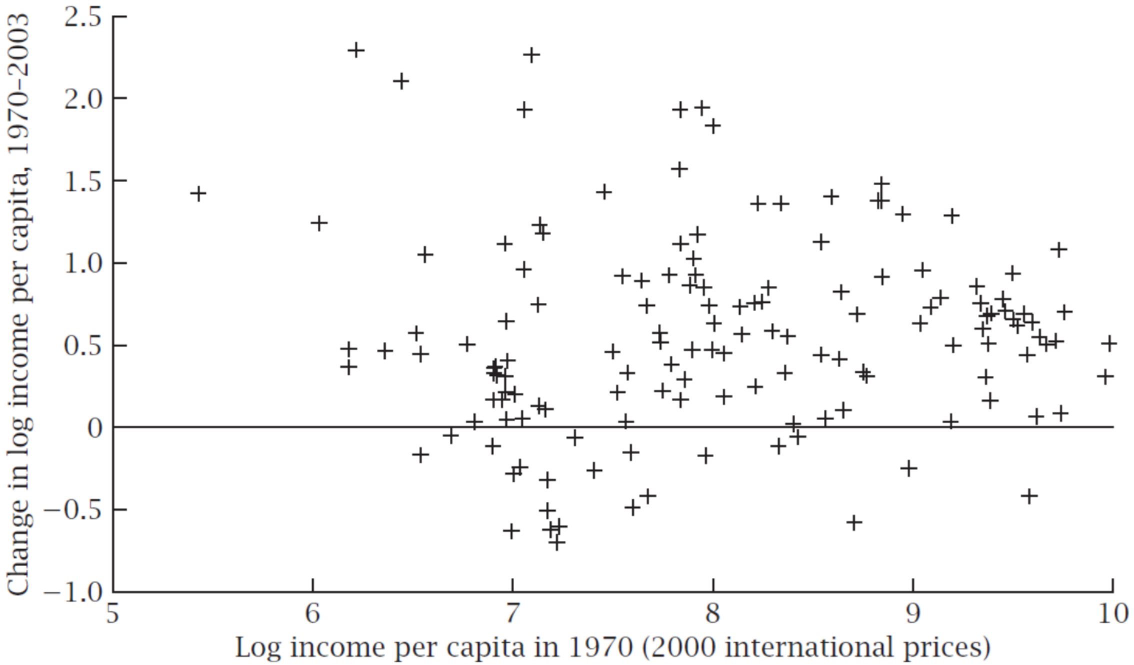


FIGURE 1.9 Initial income and subsequent growth in a large sample

Conclusion on Convergence

- While theory suggests convergence, empirical evidence is mixed.
- Challenges include sample selection and measurement errors.
- More recent data shows limited convergence, raising questions about global economic equality.

Saving and Investment

- According to the Solow Model, savings in one country does not necessarily imply domestic investment.

- The model predicts that savings could flow abroad in the absence of barriers to capital movement.

Key Assumption in Solow Model

1: An increase in savings reduces the marginal product of capital in that country.

2: This creates incentives for capital to flow abroad, assuming no barriers to capital movement.

3: Therefore, there should not be a direct correlation between high savings and high investment.

Feldstein and Horioka (1980) Findings

- Main Point: Contrary to the Solow Model, Feldstein and Horioka found a strong correlation between savings and investment rates.
- Regression Equation:

$$\left(\frac{I}{Y}\right)_i = \underset{(0.018)}{0.035} + \underset{(0.074)}{0.887}\left(\frac{S}{Y}\right)_i, \qquad R^2 = 0.91,$$

- The close correlation suggests barriers to capital mobility or other factors at play.

Explanations for the Strong Correlation

- Explanation 1: Barriers to Capital Mobility
- Despite global integration, significant barriers to capital movement still exist.
- However, there is little evidence of large differences in the rate of return across countries.
- Explanation 2: Underlying Variables
- Other factors like tax rates, discount rates, and saving behavior may influence both saving and investment.

Government Policy as an Explanation

- Government policies may prevent large gaps between saving and investment.
- Example: Large saving-investment imbalances could lead to trade deficits or surpluses, leading governments to adjust fiscal policy.
- Supporting Evidence: Helliwell (1998) found that the saving-investment correlation is weaker across regions than across countries.