

Quiz on Eigendecomposition and Linear Recurrence Relations

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1

Calculate the form of the eigendecomposition of the following matrix:

$$\begin{pmatrix} 2 & 2 \\ 1 & 0 \end{pmatrix}.$$

2

Calculate the following vector:

$$\begin{pmatrix} 2 & 2 \\ 1 & 0 \end{pmatrix}^{2025} \begin{pmatrix} 1 \\ 1 \end{pmatrix}.$$

3

Suppose $\{T_n\}$ is a sequence generated from the following linear recurrence relation:

$$\begin{cases} T_0 = T_1 = 1, \\ T_{n+1} = 2T_n + 2T_{n-1}, \quad n \geq 1. \end{cases}$$

Derive the explicit formula for T_n .