# I. SUPPLEMENTARY MATERIALS

## A. Calculation process of Equation (8)

By differentiating the Lagrangian function with respect to $\boldsymbol{w}$, $\boldsymbol{b}$, $\boldsymbol{\xi}$, $\boldsymbol{\xi^*}$ and $\eta$, we can obtain the following formulas.

$$\frac{\partial L}{\partial \boldsymbol{w}} = \boldsymbol{w} + \sum_{j=1}^{r}\sum_{i=1}^{n^j}\left(\alpha_i^j + \beta_i^j - \alpha_i^{*j} - \beta_i^{*j}\right)\boldsymbol{x}_i^j = 0, \tag{1}$$

$$\begin{aligned}\frac{\partial L}{\partial b_j} = &\sum_{i=1}^{n^j}\left(-\frac{1}{2}\alpha_i^j + \frac{1}{2}\alpha_i^{*j} - \left(1-\frac{\tau}{2}\right)\beta_i^j + \frac{\tau}{2}\beta_i^{*j}\right)\\ &+ \sum_{i=1}^{n^{j+1}}\left(-\frac{1}{2}\alpha_i^{j+1} + \frac{1}{2}\alpha_i^{*j+1} - \frac{\tau}{2}\beta_i^{j+1} + \left(1-\frac{\tau}{2}\right)\beta_i^{*j+1}\right)\\ &+ \lambda_{j+1} - \lambda_j = 0,\end{aligned} \tag{2}$$

$$\begin{aligned}\frac{\partial L}{\partial b_0} = &\sum_{i=1}^{n^1}\left(-\frac{1}{2}\alpha_i^1 + \frac{1}{2}\alpha_i^{*1} - \frac{\tau}{2}\beta_i^1 + \left(1-\frac{\tau}{2}\right)\beta_i^{*1}\right) + \lambda_1 - \gamma\\ &= 0,\end{aligned} \tag{3}$$

$$\begin{aligned}\frac{\partial L}{\partial b_r} = &\sum_{i=1}^{n^r}\left(-\frac{1}{2}\alpha_i^r + \frac{1}{2}\alpha_i^{*r} - \left(1-\frac{\tau}{2}\right)\beta_i^r + \frac{\tau}{2}\beta_i^{*r}\right) - \lambda_r + \gamma\\ &= 0,\end{aligned} \tag{4}$$

$$\frac{\partial L}{\partial \xi_i^j} = C_1 - \frac{1}{\tau}\alpha_i^j - \beta_i^j - \delta_i^j = 0, \tag{5}$$

$$\frac{\partial L}{\partial \xi_i^{*j}} = C_1 - \frac{1}{\tau}\alpha_i^{*j} - \beta_i^{*j} - \delta_i^{*j} = 0, \tag{6}$$

$$\frac{\partial L}{\partial \eta} = C_2 - \gamma = 0. \tag{7}$$

From above formulas, we can have

$$\boldsymbol{w} = \sum_{j=1}^{r}\sum_{i=1}^{n^j}\left(\alpha_i^{*j} - \alpha_i^j + \beta_i^{*j} - \beta_i^j\right)\boldsymbol{x}_i^j, \tag{8}$$

$$\alpha_i^j = \tau\left(C_1 - \beta_i^j - \delta_i^j\right), \tag{9}$$

$$\alpha_i^{*j} = \tau\left(C_1 - \beta_i^{*j} - \delta_i^{*j}\right), \tag{10}$$

$$C_2 = \gamma. \tag{11}$$

By substituting (9) and (10) into (8) and (2), we can have

$$\boldsymbol{w} = \sum_{j=1}^{r}\sum_{i=1}^{n^j}\left((1-\tau)\left(\beta_i^{*j} - \beta_i^j\right) + \tau\left(\delta_i^j - \delta_i^{*j}\right)\right)\boldsymbol{x}_i^j, \tag{12}$$

and

$$\begin{aligned}(1-\tau)&\left(\sum_{i=1}^{n^{j+1}}\beta_i^{*j+1} - \sum_{i=1}^{n^j}\beta_i^j\right) + \frac{\tau}{2}\sum_{i=1}^{n^j}\left(\delta_i^j - \delta_i^{*j}\right)\\ &+ \frac{\tau}{2}\sum_{i=1}^{n^{j+1}}\left(\delta_i^{j+1} - \delta_i^{*j+1}\right) + \lambda_{j+1} - \lambda_j = 0.\end{aligned} \tag{13}$$

Moreover, from (3), (4) and (11), we can know that when $j=0$ or $j=r$, it has

$$\sum_{i=1}^{n^1}(1-\tau)\beta_i^{*1} + \frac{\tau}{2}\sum_{i=1}^{n^1}\left(\delta_i^1 - \delta_i^{*1}\right) + \lambda_1 - C_2 = 0, \tag{14}$$

$$\sum_{i=1}^{n^r}(1-\tau)\beta_i^r + \frac{\tau}{2}\sum_{i=1}^{n^r}\left(\delta_i^{*r} - \delta_i^r\right) + \lambda_r - C_2 = 0. \tag{15}$$

## B. Calculation process of Equation (16)

The detail calculation process of (16) is presented as follows.

$$\begin{aligned}&\min_{\boldsymbol{w},\boldsymbol{b}} \frac{1}{2}\|\boldsymbol{w}\|^2 + C_a\sum_{j=1}^{r}\sum_{i=1}^{n^j}|\boldsymbol{w}\cdot\boldsymbol{x}_i^j - \frac{b_{j-1}+b_j}{2}| + C_bQ_H\\
&= \min_{\boldsymbol{w},\boldsymbol{b}} \frac{1}{2}\|\boldsymbol{w}\|^2 + C_a\sum_{j=1}^{r}\sum_{i=1}^{n^j}|\boldsymbol{w}\cdot\boldsymbol{x}_i^j - \frac{b_{j-1}+b_j}{2}|\\
&+ C_b\sum_{j=1}^{r}\sum_{i=1}^{n^j}\max\{\boldsymbol{w}\cdot\boldsymbol{x}_i^j - (b_j-1), (b_{j-1}+1) - \boldsymbol{w}\cdot\boldsymbol{x}_i^j, 0\}\\
&= \min_{\boldsymbol{w},\boldsymbol{b}} \frac{1}{2}\|\boldsymbol{w}\|^2 + C_a\sum_{j=1}^{r}\sum_{i\in S_{in}}^{n^j}|\boldsymbol{w}\cdot\boldsymbol{x}_i^j - \frac{b_{j-1}+b_j}{2}|\\
&+ \sum_{j=1}^{r}\sum_{i\in S_{out}}^{n^j}\max\{C_a\big(\boldsymbol{w}\cdot\boldsymbol{x}_i^j - \frac{b_{j-1}+b_j}{2}\big) + C_b\big(\boldsymbol{w}\cdot\boldsymbol{x}_i^j - (b_j-1)\big),\\
&C_a\big(\frac{b_{j-1}+b_j}{2} - \boldsymbol{w}\cdot\boldsymbol{x}_i^j\big) + C_b\big((b_{j-1}+1) - \boldsymbol{w}\cdot\boldsymbol{x}_i^j\big), 0\}\\
&= \min_{\boldsymbol{w},\boldsymbol{b}} \frac{1}{2}\|\boldsymbol{w}\|^2 + C_a\sum_{j=1}^{r}\sum_{i\in S_{in}}^{n^j}|\boldsymbol{w}\cdot\boldsymbol{x}_i^j - \frac{b_{j-1}+b_j}{2}|\\
&+ \sum_{j=1}^{r}\sum_{i\in S_{out}}^{n^j}\max\{(C_a+C_b)\big(\boldsymbol{w}\cdot\boldsymbol{x}_i^j - b_j + 1\big) + C_a\big(b_j - 1 - \frac{b_{j-1}+b_j}{2}\big),\\
&(C_a+C_b)\big(b_{j-1} + 1 - \boldsymbol{w}\cdot\boldsymbol{x}_i^j\big) + C_a\big(\frac{b_{j-1}+b_j}{2} - b_{j-1} - 1\big), 0\}\\
&= \min_{\boldsymbol{w},\boldsymbol{b}} \frac{1}{2}\|\boldsymbol{w}\|^2 + (C_a+C_b)\Big(\frac{C_a}{C_a+C_b}\sum_{j=1}^{r}\sum_{i\in S_{in}}^{n^j}|\boldsymbol{w}\cdot\boldsymbol{x}_i^j - \frac{b_{j-1}+b_j}{2}|\\
&+ \sum_{j=1}^{r}\sum_{i\in S_{out}}^{n^j}\max\Big\{\boldsymbol{w}\cdot\boldsymbol{x}_i^j - b_j + 1 + \frac{C_a}{C_a+C_b}\big(b_j - 1 - \frac{b_{j-1}+b_j}{2}\big),\\
&b_{j-1} + 1 - \boldsymbol{w}\cdot\boldsymbol{x}_i^j + \frac{C_a}{C_a+C_b}\big(\frac{b_{j-1}+b_j}{2} - b_{j-1} - 1\big), 0\Big\}\Big).\end{aligned} \tag{16}$$

## C. Proof of Equation (16)

Firstly, we let $C_1 = C_a + C_b$ and $\tau = \frac{C_a}{C_a+C_b}$, the formula (16) can be transformed into

$$\begin{aligned}&\min_{\boldsymbol{w},\boldsymbol{b}} \frac{1}{2}\|\boldsymbol{w}\|^2 + (C_a+C_b)\Big(\frac{C_a}{C_a+C_b}\sum_{j=1}^{r}\sum_{i\in S_{in}}^{n^j}|\boldsymbol{w}\cdot\boldsymbol{x}_i^j - \frac{b_{j-1}+b_j}{2}|\\
&+ \sum_{j=1}^{r}\sum_{i\in S_{out}}^{n^j}\max\Big\{\boldsymbol{w}\cdot\boldsymbol{x}_i^j - b_j + 1 + \frac{C_a}{C_a+C_b}\big(b_j - 1 - \frac{b_{j-1}+b_j}{2}\big),\\
&(b_{j-1}+1) - \boldsymbol{w}\cdot\boldsymbol{x}_i^j + \frac{C_a}{C_a+C_b}\big(\frac{b_{j-1}+b_j}{2} - b_{j-1} - 1\big), 0\Big\}\Big)\\
&= \min_{\boldsymbol{w},\boldsymbol{b}} \frac{1}{2}\|\boldsymbol{w}\|^2 + C_1\Big(\tau\sum_{j=1}^{r}\sum_{i\in S_{in}}^{n^j}|\boldsymbol{w}\cdot\boldsymbol{x}_i^j - \frac{b_{j-1}+b_j}{2}|\\
&+ \sum_{j=1}^{r}\sum_{i\in S_{out}}^{n^j}\max\Big\{\boldsymbol{w}\cdot\boldsymbol{x}_i^j - (b_j-1) + \tau\big((b_j-1) - \frac{b_{j-1}+b_j}{2}\big),\\
&(b_{j-1}+1) - \boldsymbol{w}\cdot\boldsymbol{x}_i^j + \tau\big(\frac{b_{j-1}+b_j}{2} - (b_{j-1}+1)\big), 0\Big\}\Big).\end{aligned} \tag{17}$$

The formula (17) can be written as a formula with constrains, as follows.

$$\begin{aligned}
\min_{\boldsymbol{w},\boldsymbol{b},\boldsymbol{\xi},\boldsymbol{\xi}^*,\eta} \ & \frac{1}{2}\|\boldsymbol{w}\|^2 + C_1 \sum_{j=1}^{r}\sum_{i=1}^{n^j}\left(\xi_i^j + \xi_i^{*j}\right) + C_2\eta \\
\text{s.t.} \ & \tau\left(\boldsymbol{w}\cdot\boldsymbol{x}_i^j - \frac{b_{j-1}+b_j}{2}\right) \le \xi_i^j, \\
& \tau\left(\frac{b_{j-1}+b_j}{2} - \boldsymbol{w}\cdot\boldsymbol{x}_i^j\right) \le \xi_i^{*j}, \\
& \boldsymbol{w}\cdot\boldsymbol{x}_i^j - (b_j - 1) + \tau\left((b_j-1) - \frac{b_{j-1}+b_j}{2}\right) \le \xi_i^j, \\
& (b_{j-1}+1) - \boldsymbol{w}\cdot\boldsymbol{x}_i^j + \tau\left(\frac{b_{j-1}+b_j}{2} - (b_{j-1}+1)\right) \le \xi_i^{*j}, \\
& \xi_i^j \ge 0, \quad \xi_i^{*j} \ge 0, \\
& i = 1,2,\ldots,n^j, \quad j = 1,2,\ldots,r.
\end{aligned} \tag{18}$$

After adding the constraints $b_r - b_0 \le \eta$ and $b_{j-1} \le b_j$, the formulation of pin-SVOR (7) is obtained after simplifying.

$$\begin{aligned}
\min_{\boldsymbol{w},\boldsymbol{b},\boldsymbol{\xi},\boldsymbol{\xi}^*,\eta} \ & \frac{1}{2}\|\boldsymbol{w}\|^2 + C_1 \sum_{j=1}^{r}\sum_{i=1}^{n^j}\left(\xi_i^j + \xi_i^{*j}\right) + C_2\eta \\
\text{s.t.} \ & \boldsymbol{w}\cdot\boldsymbol{x}_i^j - \frac{b_{j-1}+b_j}{2} \le \frac{1}{\tau}\xi_i^j, \\
& \boldsymbol{w}\cdot\boldsymbol{x}_i^j - \frac{b_{j-1}+b_j}{2} \ge -\frac{1}{\tau}\xi_i^{*j}, \\
& \boldsymbol{w}\cdot\boldsymbol{x}_i^j - \left(1-\frac{\tau}{2}\right)b_j - \frac{\tau}{2}b_{j-1} + 1 - \tau \le \xi_i^j, \\
& \boldsymbol{w}\cdot\boldsymbol{x}_i^j - \left(1-\frac{\tau}{2}\right)b_{j-1} - \frac{\tau}{2}b_j - 1 + \tau \ge -\xi_i^{*j}, \\
& b_{j-1} \le b_j, \quad b_r - b_0 \le \eta, \quad \xi_i^j \ge 0, \quad \xi_i^{*j} \ge 0, \\
& i = 1,2,\ldots,n^j, \quad j = 1,2,\ldots,r.
\end{aligned} \tag{19}$$