Alternating-direction-method of Multipliers-Based Adaptive Nonnegative Latent Factor Analysis: Supplementary Materials

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This is the supplementary file for the paper entitled *Alternating-direction-method of Multipliers-Based Adaptive Nonnegative Latent Factor Analysis*. Detailed convergence proof of A^{2}NLF and additional figures are presented here.

I. Convergence of A^{2}NLF

A. *Proof of Lemma* 1

Note that learning objective of A^{2}NLF-PSO/A^{2}NLF-TPE is non-convex. According to [48], any of its limit points where the gradient becomes zero can be a local/global optimum, or saddle point. Hence, such a limit point can be treated as a solution. Supposing that the optimal solution to $a_{u,k(q)}$ by (4c) is $a^t_{u,k(q)}$. Thus, it fulfills the following condition:

$$\lambda^t_{(q)}\left|\Lambda(u)\right|\left(p^t_{u,k(q)}-a^t_{u,k(q)}+\frac{h^{t-1}_{u,k(q)}}{\lambda^t_{(q)}\left|\Lambda(u)\right|}\right)=0. \tag{S1}$$

Following (4e) and (5c), by applying the update rule of $h_{u,k(q)}$ to (S1), we have:

$$h^t_{u,k(q)}=\left(\eta^t_{(q)}-1\right)\lambda^t_{(q)}\left|\Lambda(u)\right|\left(p^t_{u,k(q)}-a^t_{u,k(q)}\right). \tag{S2}$$

Then (22a) stands based on (S2). Following the same principle, we can derive the optimality condition of (5*b*) related to $x_{i,k(q)}$:

$$\lambda^t_{(q)}\left|\Lambda(i)\right|\left(z^t_{i,k(q)}-x^t_{i,k(q)}+\frac{w^{t-1}_{i,k(q)}}{\lambda^t_{(q)}\left|\Lambda(i)\right|}\right)=0,\ \Rightarrow w^t_{i,k(q)}=\left(\eta^t_{(q)}-1\right)\lambda^t_{(q)}\left|\Lambda(i)\right|\left(z^t_{i,k(q)}-x^t_{i,k(q)}\right). \tag{S3}$$

Then (22b) holds based on (S3). Hence, *Lemma* 1 holds, and **Step 1** is implemented.□

B. *Proof of Lemma* 2

Considering the difference between $g\left(p^t_{u,k(q)},z^t_{i,k(q)},a^{t-1}_{u,k(q)},x^{t-1}_{i,k(q)},\psi^{t-1}_{2(q)}\right)$ and $g\left(\psi^{t-1}_{1(q)},\psi^{t-1}_{2(q)}\right)$, we have:

$$\begin{aligned}
&g\left(p^t_{u,k(q)},z^t_{i,k(q)},a^{t-1}_{u,k(q)},x^{t-1}_{i,k(q)},\psi^{t-1}_{2(q)}\right)-g\left(\psi^{t-1}_{1(q)},\psi^{t-1}_{2(q)}\right)\\
&\overset{(*)}{=}\left(\sum_{i\in\Lambda(u)}z^{t-1}_{i,k(q)}\left(y_{u,i}-\sum_{l_1=1}^{k-1}p^t_{u,l_1(q)}z^t_{i,l_1(q)}-p^t_{u,k(q)}z^{t-1}_{i,k(q)}-\sum_{l_2=k+1}^{d}p^{t-1}_{u,l_2(q)}z^{t-1}_{i,l_2(q)}\right)-\lambda^t_{(q)}\left|\Lambda(u)\right|\left(p^t_{u,k(q)}-a^{t-1}_{u,k(q)}+\frac{h^{t-1}_{u,k(q)}}{\lambda^t_{(q)}\left|\Lambda(u)\right|}\right)\right)\left(p^{t-1}_{u,k(q)}-p^t_{u,k(q)}\right)\\
&\quad+\left(\sum_{u\in\Lambda(i)}p^{t-1}_{u,k(q)}\left(y_{u,i}-\sum_{l_1=1}^{k-1}p^t_{u,l_1(q)}z^t_{i,l_1(q)}-p^{t-1}_{u,k(q)}z^t_{i,k(q)}-\sum_{l_2=k+1}^{d}p^{t-1}_{u,l_2(q)}z^{t-1}_{i,l_2(q)}\right)-\lambda^t_{(q)}\left|\Lambda(i)\right|\left(z^t_{i,k(q)}-x^{t-1}_{i,k(q)}+\frac{w^{t-1}_{i,k(q)}}{\lambda^t_{(q)}\left|\Lambda(i)\right|}\right)\right)\left(z^{t-1}_{i,k(q)}-z^t_{i,k(q)}\right)\\
&\quad-\frac{1}{2}\left(\sum_{i\in\Lambda(u)}\left(z^{t-1}_{i,k(q)}\right)^2+\lambda^t_{(q)}\left|\Lambda(u)\right|\right)\left(p^t_{u,k(q)}-p^{t-1}_{u,k(q)}\right)^2-\frac{1}{2}\left(\sum_{u\in\Lambda(i)}\left(p^{t-1}_{u,k(q)}\right)^2+\lambda^t_{(q)}\left|\Lambda(i)\right|\right)\left(z^t_{i,k(q)}-z^{t-1}_{i,k(q)}\right)^2.
\end{aligned} \tag{S4}$$

where (*) performs the second-order Taylor expansion of the left term. Then, considering (5a)'s optimality condition, (S4) is transformed as:

$$g\left(p^t_{u,k(q)},z^t_{i,k(q)},a^{t-1}_{u,k(q)},x^{t-1}_{i,k(q)},\psi^{t-1}_{2(q)}\right)-g\left(\psi^{t-1}_{1(q)},\psi^{t-1}_{2(q)}\right)$$
$$=-\frac{1}{2}\left(\sum_{i\in\Lambda(u)}\left(z^{t-1}_{i,k(q)}\right)^2+\lambda^t_{(q)}\left|\Lambda(u)\right|\right)\left(p^t_{u,k(q)}-p^{t-1}_{u,k(q)}\right)^2-\frac{1}{2}\left(\sum_{u\in\Lambda(i)}\left(p^{t-1}_{u,k(q)}\right)^2+\lambda^t_{(q)}\left|\Lambda(i)\right|\right)\left(z^t_{i,k(q)}-z^{t-1}_{i,k(q)}\right)^2. \tag{S5}$$

Thus, the difference between $g\left(\psi^t_{1(q)},\psi^{t-1}_{2(q)}\right)$ and $g\left(p^t_{u,k(q)},z^t_{i,k(q)},a^{t-1}_{u,k(q)},x^{t-1}_{i,k(q)},\psi^{t-1}_{2(q)}\right)$ is:

$$g\left(\psi^t_{1(q)},\psi^{t-1}_{2(q)}\right)-g\left(p^t_{u,k(q)},z^t_{i,k(q)},a^{t-1}_{u,k(q)},x^{t-1}_{i,k(q)},\psi^{t-1}_{2(q)}\right)=-\left(\lambda^t_{(q)}\left|\Lambda(u)\right|/2\right)\left(a^t_{u,k(q)}-a^{t-1}_{u,k(q)}\right)^2-\left(\lambda^t_{(q)}\left|\Lambda(i)\right|/2\right)\left(x^t_{i,k(q)}-x^{t-1}_{i,k(q)}\right)^2. \tag{S6}$$

Moreover, $g\left(\psi^t_{1(q)},\psi^t_{2(q)}\right)$ and $g\left(\psi^t_{1(q)},\psi^{t-1}_{2(q)}\right)$ yields:

$$\begin{aligned}
&g\left(\psi^t_{1(q)},\psi^t_{2(q)}\right)-g\left(\psi^t_{1(q)},\psi^{t-1}_{2(q)}\right)\\
&=\left(p^t_{u,k(q)}-a^t_{u,k(q)}\right)\left(h^t_{u,k(q)}-h^{t-1}_{u,k(q)}\right)+\left(z^t_{i,k(q)}-x^t_{i,k(q)}\right)\left(w^t_{i,k(q)}-w^{t-1}_{i,k(q)}\right)\\
&\overset{\text{(I)}}{=}\frac{1}{\eta^t_{(q)}\lambda^t_{(q)}\left|\Lambda(u)\right|}\left(h^t_{u,k(q)}-h^{t-1}_{u,k(q)}\right)^2+\frac{1}{\eta^t_{(q)}\lambda^t_{(q)}\left|\Lambda(i)\right|}\left(w^t_{i,k(q)}-w^{t-1}_{i,k(q)}\right)^2\\
&\overset{\text{(II)}}{\le}\frac{2\left|\Lambda(u)\right|}{\eta^t_{(q)}\lambda^t_{(q)}}\left(\left(\left(\eta^t_{(q)}-1\right)\lambda^t_{(q)}p^t_{u,k(q)}-\left(\eta^{t-1}_{(q)}-1\right)\lambda^{t-1}_{(q)}p^{t-1}_{u,k(q)}\right)^2+\left(\left(\eta^t_{(q)}-1\right)\lambda^t_{(q)}a^t_{u,k(q)}-\left(\eta^{t-1}_{(q)}-1\right)\lambda^{t-1}_{(q)}a^{t-1}_{u,k(q)}\right)^2\right)\\
&+\frac{2\left|\Lambda(i)\right|}{\eta^t_{(q)}\lambda^t_{(q)}}\left(\left(\left(\eta^t_{(q)}-1\right)\lambda^t_{(q)}z^t_{i,k(q)}-\left(\eta^{t-1}_{(q)}-1\right)\lambda^{t-1}_{(q)}z^{t-1}_{i,k(q)}\right)^2+\left(\left(\eta^t_{(q)}-1\right)\lambda^t_{(q)}x^t_{i,k(q)}-\left(\eta^{t-1}_{(q)}-1\right)\lambda^{t-1}_{(q)}x^{t-1}_{i,k(q)}\right)^2\right),
\end{aligned} \tag{S7}$$

where (I) is based on the update rules of ($h_{u,k(q)}$, $w_{i,k(q)}$) given in (4e), (4f) and (5c), and (II) is achieved with *Lemma* 1.With (S5)-(S7), we have the following deduction:

$$\begin{aligned}
&g\left(\psi^t_{1(q)},\psi^t_{2(q)}\right)-g\left(\psi^{t-1}_{1(q)},\psi^{t-1}_{2(q)}\right)\\
&\le-\frac{1}{2}\left(\sum_{i\in\Lambda(u)}\left(z^{t-1}_{i,k(q)}\right)^2+\lambda^t_{(q)}\left|\Lambda(u)\right|\right)\left(p^t_{u,k(q)}-p^{t-1}_{u,k(q)}\right)^2-\frac{\lambda^t_{(q)}\left|\Lambda(u)\right|}{2}\left(a^t_{u,k(q)}-a^{t-1}_{u,k(q)}\right)^2\\
&-\frac{1}{2}\left(\sum_{u\in\Lambda(i)}\left(p^{t-1}_{u,k(q)}\right)^2+\lambda^t_{(q)}\left|\Lambda(i)\right|\right)\left(z^t_{i,k(q)}-z^{t-1}_{i,k(q)}\right)^2-\frac{\lambda^t_{(q)}\left|\Lambda(i)\right|}{2}\left(x^t_{i,k(q)}-x^{t-1}_{i,k(q)}\right)^2\\
&+\frac{2\left|\Lambda(u)\right|}{\eta^t_{(q)}\lambda^t_{(q)}}\left(\left(\left(\eta^t_{(q)}-1\right)\lambda^t_{(q)}p^t_{u,k(q)}-\left(\eta^{t-1}_{(q)}-1\right)\lambda^{t-1}_{(q)}p^{t-1}_{u,k(q)}\right)^2+\left(\left(\eta^t_{(q)}-1\right)\lambda^t_{(q)}a^t_{u,k(q)}-\left(\eta^{t-1}_{(q)}-1\right)\lambda^{t-1}_{(q)}a^{t-1}_{u,k(q)}\right)^2\right)\\
&+\frac{2\left|\Lambda(i)\right|}{\eta^t_{(q)}\lambda^t_{(q)}}\left(\left(\left(\eta^t_{(q)}-1\right)\lambda^t_{(q)}z^t_{i,k(q)}-\left(\eta^{t-1}_{(q)}-1\right)\lambda^{t-1}_{(q)}z^{t-1}_{i,k(q)}\right)^2+\left(\left(\eta^t_{(q)}-1\right)\lambda^t_{(q)}x^t_{i,k(q)}-\left(\eta^{t-1}_{(q)}-1\right)\lambda^{t-1}_{(q)}x^{t-1}_{i,k(q)}\right)^2\right).
\end{aligned} \tag{S8}$$

Owing to (24a), (25a) stands, which indicates that the augmented Lagrangian function (3) related to $s_{(q)}$ or τ_q is non-increasing as $a^t_{u,k(q)}>0$ and $x^t_{i,k(q)}>0$. Then after the *t*-th iteration, the partial objective from (3) related to $s_{(q)}$ or τ_q is formulated as:

$$\begin{aligned}
&g\left(\psi^t_{1(q)},\psi^t_{2(q)}\right)\\
&=\frac{1}{2}\sum_{y_{u,i}\in\Lambda}\left(y_{u,i}-\sum_{l_1=1}^{k}p^t_{u,l_1(q)}z^t_{i,l_1(q)}-\sum_{l_2=k+1}^{d}p^{t-1}_{u,l_2(q)}z^{t-1}_{i,l_2(q)}\right)^2\\
&+\sum_u\left(\left(\sum_{l_1=1}^{k}h^t_{u,l_1(q)}\left(p^t_{u,l_1(q)}-a^t_{u,l_1(q)}\right)\right)+\left(\sum_{l_2=k+1}^{d}h^{t-1}_{u,l_2(q)}\left(p^{t-1}_{u,l_2(q)}-a^{t-1}_{u,l_2(q)}\right)\right)\right)+\sum_i\left(\left(\sum_{l_1=1}^{k}w^t_{i,l_1(q)}\left(z^t_{i,l_1(q)}-x^t_{i,l_1(q)}\right)\right)+\left(\sum_{l_2=k+1}^{d}w^{t-1}_{i,l_2(q)}\left(z^{t-1}_{i,l_2(q)}-x^{t-1}_{i,l_2(q)}\right)\right)\right)\\
&+\sum_u\frac{\lambda^t_{(q)}\left|\Lambda(u)\right|}{2}\left(\sum_{l_1=1}^{k}\left(p^t_{u,l_1(q)}-a^t_{u,l_1(q)}\right)^2+\sum_{l_2=k+1}^{d}\left(p^{t-1}_{u,l_2(q)}-a^{t-1}_{u,l_2(q)}\right)^2\right)+\sum_i\frac{\lambda^t_{(q)}\left|\Lambda(i)\right|}{2}\left(\sum_{l_1=1}^{k}\left(z^t_{i,l_1(q)}-x^t_{i,l_1(q)}\right)^2+\sum_{l_2=k+1}^{d}\left(z^{t-1}_{i,l_2(q)}-x^{t-1}_{i,l_2(q)}\right)^2\right).
\end{aligned}$$

(S9)

By substituting (S2) and (S3) into (S9), we have:

$$
\begin{aligned}
& g\left(\psi_{1(q)}^{t},\psi_{2(q)}^{t}\right) \\
& =\frac{1}{2}\sum_{y_{u,i}\in\Lambda}\left(y_{u,i}-\sum_{l_1=1}^{k}p_{u,l_1(q)}^{t}z_{i,l_1(q)}^{t}-\sum_{l_2=k+1}^{d}p_{u,l_2(q)}^{t-1}z_{i,l_2(q)}^{t-1}\right)^2 \\
& +\sum_{u}\left|\Lambda(u)\right|\left(\left(\eta_{(q)}^{t}-1\right)\lambda_{(q)}^{t}\sum_{l_1=1}^{k}\left(p_{u,l_1(q)}^{t}-a_{u,l_1(q)}^{t}\right)^2+\left(\eta_{(q)}^{t-1}-1\right)\lambda_{(q)}^{t-1}\sum_{l_2=k+1}^{d}\left(p_{u,l_2(q)}^{t-1}-a_{u,l_2(q)}^{t-1}\right)^2\right) \\
& +\sum_{i}\left|\Lambda(i)\right|\left(\left(\eta_{(q)}^{t}-1\right)\lambda_{(q)}^{t}\sum_{l_1=1}^{k}\left(z_{i,l_1(q)}^{t}-x_{i,l_1(q)}^{t}\right)^2+\left(\eta_{(q)}^{t-1}-1\right)\lambda_{(q)}^{t-1}\sum_{l_2=k+1}^{d}w_{i,l_2(q)}^{t}\left(z_{i,l_2(q)}^{t-1}-x_{i,l_2(q)}^{t-1}\right)^2\right) \\
& +\sum_{u}\frac{\lambda_{(q)}^{t}\left|\Lambda(u)\right|}{2}\left(\sum_{l_1=1}^{k}\left(p_{u,l_1(q)}^{t}-a_{u,l_1(q)}^{t}\right)^2+\sum_{l_2=k+1}^{d}\left(p_{u,l_2(q)}^{t-1}-a_{u,l_2(q)}^{t-1}\right)^2\right)+\sum_{i}\frac{\lambda_{(q)}^{t}\left|\Lambda(i)\right|}{2}\left(\sum_{l_1=1}^{k}\left(z_{i,l_1(q)}^{t}-x_{i,l_1(q)}^{t}\right)^2+\sum_{l_2=k+1}^{d}\left(z_{i,l_2(q)}^{t-1}-x_{i,l_2(q)}^{t-1}\right)^2\right).
\end{aligned}
\tag{S10}
$$

(S10) indicates that if (24b) is fulfilled, (25b) holds, thereby making (3) related to $s_{(q)}$ or τ_q lower-bounded as $a_{u,k(q)}^{t}>0$ and $x_{i,k(q)}^{t}>0$. Based on the above inferences, *Lemma* 2 stands, and **Step 2** is implemented.□

C. Proof of Theorem 1

Part a. Following *Lemma* 2, $g\left(\psi_{1(q)}^{t},\psi_{2(q)}^{t}\right)$ converges as $t\rightarrow\infty$, indicating that:

$$\lim_{t\rightarrow\infty}g\left(\psi_{1(q)}^{t},\psi_{2(q)}^{t}\right)-g\left(\psi_{1(q)}^{t-1},\psi_{2(q)}^{t-1}\right)\rightarrow 0. \tag{S11}$$

With (24), when (S1) is fulfilled, the upper-bound of $g\left(\psi_{1(q)}^{t},\psi_{2(q)}^{t}\right)-g\left(\psi_{1(q)}^{t-1},\psi_{2(q)}^{t-1}\right)$ is zero as $t\rightarrow\infty$, thereby achieving (26). Following (S8) and (26), we have [24]:

$$\lim_{t\rightarrow\infty}\left(p_{u,k(q)}^{t}-p_{u,k(q)}^{t-1}\right)\rightarrow 0, \tag{S12a}$$

$$\lim_{t\rightarrow\infty}\left(z_{i,k(q)}^{t}-z_{i,k(q)}^{t-1}\right)\rightarrow 0, \tag{S12b}$$

$$\lim_{t\rightarrow\infty}\left(a_{u,k(q)}^{t}-a_{u,k(q)}^{t-1}\right)\rightarrow 0, \tag{S12c}$$

$$\lim_{t\rightarrow\infty}\left(x_{i,k(q)}^{t}-x_{i,k(q)}^{t-1}\right)\rightarrow 0, \tag{S12d}$$

$$\lim_{t\rightarrow\infty}\left(\left(\left(\eta_{(q)}^{t}-1\right)\lambda_{(q)}^{t}p_{u,k(q)}^{t}-\left(\eta_{(q)}^{t-1}-1\right)\lambda_{(q)}^{t-1}p_{u,k(q)}^{t-1}\right)^2+\left(\left(\eta_{(q)}^{t}-1\right)\lambda_{(q)}^{t}a_{u,k(q)}^{t}-\left(\eta_{(q)}^{t-1}-1\right)\lambda_{(q)}^{t-1}a_{u,k(q)}^{t-1}\right)^2\right)\rightarrow 0, \tag{S12e}$$

$$\lim_{t\rightarrow\infty}\left(\left(\left(\eta_{(q)}^{t}-1\right)\lambda_{(q)}^{t}z_{i,k(q)}^{t}-\left(\eta_{(q)}^{t-1}-1\right)\lambda_{(q)}^{t-1}z_{i,k(q)}^{t-1}\right)^2+\left(\left(\eta_{(q)}^{t}-1\right)\lambda_{(q)}^{t}x_{i,k(q)}^{t}-\left(\eta_{(q)}^{t-1}-1\right)\lambda_{(q)}^{t-1}x_{i,k(q)}^{t-1}\right)^2\right)\rightarrow 0. \tag{S12f}$$

Based on (22) and (S12), we have the following inferences:

$$\lim_{t\rightarrow\infty}\left(h_{u,k(q)}^{t}-h_{u,k(q)}^{t-1}\right)\rightarrow 0, \tag{S13a}$$

$$\lim_{t\rightarrow\infty}\left(w_{i,k(q)}^{t}-w_{i,k(q)}^{t-1}\right)\rightarrow 0. \tag{S13b}$$

Based on (4e), (4f) and (S13), we conclude that (27) is fulfilled.

Part b. Firstly, following (4a), (4b) and (5a), the update rules of ($p_{u,k(q)}$, $z_{i,k(q)}$) can be rearranged as:

$$\begin{aligned}&\left(p_{u,k(q)}^{t-1}-p_{u,k(q)}^{t}\right)\left(\sum_{i\in\Lambda(u)}\left(z_{i,k(q)}^{t-1}\right)^2+\lambda_{(q)}^{t}\left|\Lambda(u)\right|\right)\\&=\sum_{i\in\Lambda(u)}z_{i,k(q)}^{t-1}\left(\sum_{l_1=1}^{k}p_{u,l_1(q)}^{t}z_{i,l_1(q)}^{t}+\sum_{l_2=k+1}^{d}p_{u,l_2(q)}^{t-1}z_{i,l_2(q)}^{t-1}-y_{u,i}\right)+\lambda_{(q)}^{t}\left|\Lambda(u)\right|\left(p_{u,k(q)}^{t-1}-a_{u,k(q)}^{t-1}\right)+h_{u,k(q)}^{t-1},\end{aligned}\tag{S14a}$$

$$\begin{aligned}&\left(z_{i,k(q)}^{t-1}-z_{i,k(q)}^{t}\right)\left(\sum_{u\in\Lambda(i)}\left(p_{i,k(q)}^{t-1}\right)^2+\lambda_{(q)}^{t}\left|\Lambda(i)\right|\right)\\&=\sum_{u\in\Lambda(i)}p_{u,k(q)}^{t-1}\left(\sum_{l_1=1}^{k}p_{u,l_1(q)}^{t}z_{i,l_1(q)}^{t}+\sum_{l_2=k+1}^{d}p_{u,l_2(q)}^{t-1}z_{i,l_2(q)}^{t-1}-y_{u,i}\right)+\lambda_{(q)}^{t}\left|\Lambda(i)\right|\left(z_{i,k(q)}^{t-1}-x_{i,k(q)}^{t-1}\right)+w_{i,k(q)}^{t-1}.\end{aligned}\tag{S14b}$$

Then by substituting (27) and (S12) into (S14), we have:

$$\sum_{i\in\Lambda(u)}z_{i,k(q)}^{t-1}\left(\sum_{l_1=1}^{k}p_{u,l_1(q)}^{t}z_{i,l_1(q)}^{t}+\sum_{l_2=k+1}^{d}p_{u,l_2(q)}^{t-1}z_{i,l_2(q)}^{t-1}-y_{u,i}\right)+h_{u,k(q)}^{t-1}\to 0,\tag{S15a}$$

$$\sum_{u\in\Lambda(i)}p_{u,k(q)}^{t-1}\left(\sum_{l_1=1}^{k}p_{u,l_1(q)}^{t}z_{i,l_1(q)}^{t}+\sum_{l_2=k+1}^{d}p_{u,l_2(q)}^{t-1}z_{i,l_2(q)}^{t-1}-y_{u,i}\right)+w_{i,k(q)}^{t-1}\to 0.\tag{S15b}$$

Hence, considering a limit point $\left\{\psi_{1(q)}^{*},\psi_{2(q)}^{*}\right\}$ of a sequence $\left\{\psi_{1(q)}^{t},\psi_{2(q)}^{t}\right\}$ generated by the update rules of $\{\psi_{1(q)}, \psi_{2(q)}\}$ based on (4) and (5), the following KKT conditions are satisfied with (27) and (S15):

$$\sum_{i\in\Lambda(u)}z_{i,k(q)}^{*}\left(\sum_{k=1}^{d}p_{u,k(q)}^{*}z_{i,k(q)}^{*}-y_{u,i}\right)+h_{u,k(q)}^{*}\to 0,\tag{S16a}$$

$$\sum_{u\in\Lambda(i)}p_{u,k(q)}^{*}\left(\sum_{k=1}^{d}p_{u,k(q)}^{*}z_{i,k(q)}^{*}-y_{u,i}\right)+w_{i,k(q)}^{*}\to 0,\tag{S16b}$$

$$p_{u,k}^{*}-a_{u,k}^{*}\to 0,\tag{S16c}$$

$$z_{i,k}^{*}-x_{i,k}^{*}\to 0.\tag{S16d}$$

Afterwards, considering the remaining KKT conditions regarding constraints $a_{u,k(q)}>0$ and $x_{i,k(q)}>0$, we extend the original augmented Lagrangian g:

$$g_{(q)}^{\#}=g_{(q)}-Tr\left(M_{(q)}\left(A_{(q)}\right)^{T}\right)-Tr\left(N_{(q)}\left(X_{(q)}\right)^{T}\right)=g_{(q)}-\sum_{(u,k)}m_{u,k(q)}a_{u,k(q)}-\sum_{(i,k)}n_{i,k(q)}x_{i,k(q)},\tag{S17}$$

where the operator $Tr(\cdot)$ computes the trace of an enclosed matrix, and the definition of $g_{(q)}$ is given by:

$$\begin{aligned}g_{(q)}=&\frac{1}{2}\sum_{y_{u,i}\in\Lambda}\left(y_{u,i}-\sum_{k=1}^{d}p_{u,k(q)}z_{i,k(q)}\right)^2+\sum_{(u,k)}h_{u,k(q)}\left(p_{u,k(q)}-a_{u,k(q)}\right)+\sum_{(u,k)}\frac{\lambda_{(q)}\left|\Lambda(u)\right|}{2}\left(p_{u,k(q)}-a_{u,k(q)}\right)^2\\&+\sum_{(i,k)}w_{i,k(q)}\left(z_{i,k(q)}-x_{i,k(q)}\right)+\sum_{(i,k)}\frac{\lambda_{(q)}\left|\Lambda(i)\right|}{2}\left(z_{i,k(q)}-x_{i,k(q)}\right)^2.\end{aligned}\tag{S18}$$

For the partial derivatives of $g_{(q)}^{\#}$ with $a_{u,k(q)}$ and $x_{i,k(q)}$, we have:

$$\begin{cases} \dfrac{\partial g_{(q)}^{\#}}{\partial a_{u,k}} = -\lambda_{(q)} \left|\Lambda(u)\right| \left(p_{u,k(q)} - a_{u,k(q)} + \dfrac{h_{u,k(q)}}{\lambda_{(q)} \left|\Lambda(u)\right|} \right) - m_{u,k(q)} = 0, \\ \dfrac{\partial g_{(q)}^{\#}}{\partial x_{i,k}} = -\lambda_{(q)} \left|\Lambda(i)\right| \left(z_{i,k(q)} - x_{i,k(q)} + \dfrac{w_{i,k(q)}}{\lambda_{(q)} \left|\Lambda(i)\right|} \right) - n_{i,k(q)} = 0, \end{cases} \Rightarrow \begin{cases} m_{u,k} = -\lambda_{(q)} \left|\Lambda(u)\right| \left(p_{u,k(q)} - a_{u,k(q)} + \dfrac{h_{u,k(q)}}{\lambda_{(q)} \left|\Lambda(u)\right|} \right), \\ n_{i,k} = -\lambda_{(q)} \left|\Lambda(i)\right| \left(z_{i,k(q)} - x_{i,k(q)} + \dfrac{w_{i,k(q)}}{\lambda_{(q)} \left|\Lambda(i)\right|} \right). \end{cases} \tag{S19}$$

Then, with the KKT conditions of $\forall m_{u,k(q)}$, $a_{u,k(q)}$: $m_{u,k(q)}a_{u,k(q)}$=0 and $\forall n_{i,k(q)}$, $x_{i,k(q)}$: $n_{i,k(q)}x_{i,k(q)}$=0 for (S17), we achieve the following equations based on (S19) [19, 21, 24, 39]:

$$\begin{cases} a_{u,k(q)} \left(-\lambda_{(q)} \left|\Lambda(u)\right| \left(p_{u,k(q)} - a_{u,k(q)} + \dfrac{h_{u,k(q)}}{\lambda_{(q)} \left|\Lambda(u)\right|} \right) \right) = 0, \\ x_{i,k(q)} \left(-\lambda_{(q)} \left|\Lambda(i)\right| \left(z_{i,k(q)} - x_{i,k(q)} + \dfrac{w_{i,k(q)}}{\lambda_{(q)} \left|\Lambda(i)\right|} \right) \right) = 0, \end{cases} \Rightarrow \begin{cases} a_{u,k(q)} = p_{u,k(q)} + \dfrac{h_{u,k(q)}}{\lambda_{(q)} \left|\Lambda(u)\right|}, \\ x_{i,k(q)} = z_{i,k(q)} + \dfrac{w_{i,k(q)}}{\lambda_{(q)} \left|\Lambda(i)\right|}. \end{cases} \tag{S20}$$

To satisfy the nonnegativity of output LFs $a_{u,k(q)}$ and $x_{i,k(q)}$, (S20) can be rewritten as:

$$\begin{cases} a_{u,k(q)} = \max\left(0, p_{u,k(q)} + \dfrac{h_{u,k(q)}}{\lambda_{(q)} \left|\Lambda(u)\right|} \right), \\ x_{i,k(q)} = \max\left(0, z_{i,k(q)} + \dfrac{w_{i,k(q)}}{\lambda_{(q)} \left|\Lambda(i)\right|} \right). \end{cases} \tag{S21}$$

Note that (S21) is consistent with the update rules of $a_{u,k(q)}$ and $x_{i,k(q)}$ based on (4c) and (4d). Therefore, (S17)-(S21) show that learning rules of A^{2}NLF-PSO/A^{2}NLF-TPE are closely connected with the KKT conditions of its learning objective.

Then considering the KKT conditions related to $a_{u,k(q)}$:

$$\left.\frac{\partial g_{(q)}^{\#}}{\partial a_{u,k(q)}}\right|_{a_{u,k(q)}=a_{u,k(q)}^{*}} = -\lambda_{(q)}^{*} \left|\Lambda(u)\right| \left(p_{u,k(q)}^{*} - a_{u,k(q)}^{*} + \frac{h_{u,k(q)}^{*}}{\lambda_{(q)}^{*} \left|\Lambda(u)\right|} \right) - m_{u,k(q)}^{*} = 0, \tag{S22a}$$

$$m_{u,k(q)}^{*} a_{u,k(q)}^{*} = 0, \tag{S22b}$$

$$a_{u,k(q)}^{*} \ge 0, \tag{S22c}$$

$$m_{u,k(q)}^{*} \ge 0, \tag{S22d}$$

where $a_{u,k(q)}^{*}$ is a KKT stationary point of $a_{u,k(q)}$, and $m_{u,k(q)}^{*}$ is a limit point of the sequence $\{m_{u,k(q)}^{t}\}$ generated by the update rules of $m_{u,k}$ based on (S19). According to (S17)-(S21) and $a_{u,k(q)}^{t}$=0, conditions (S22a)-(S22c) are satisfied. Thus, we have:

$$m_{u,k(q)}^{*} = -\lambda_{(q)}^{*} \left|\Lambda(u)\right| \left(p_{u,k(q)}^{*} - a_{u,k(q)}^{*} + \frac{h_{u,k(q)}^{*}}{\lambda_{(q)}^{*} \left|\Lambda(u)\right|} \right). \tag{S23}$$

Thus, we focus on condition (S22d). Since $a_{u,k(q)}^{t}$>0 in this case, the update rule for $a_{u,k(q)}$ is given as:

$$a_{u,k(q)}^{*} \leftarrow p_{u,k(q)}^{*} + \frac{h_{u,k(q)}^{*}}{\lambda_{(q)}^{*} \left|\Lambda(u)\right|}. \tag{S24}$$

By substituting (S24) into (S23), we have $m_{u,k(q)}^{*}$=0. Hence, conditions (S22c) and (S22d) are fulfilled. Note that as $x_{i,k(q)}^{t}$>0 in this case, the proof regarding the KKT conditions of $x_{u,k(q)}$ can be achieved similarly. *Theorem* 1 stands, and **Step 3** is implemented. □

D. Proof of Lemma 3

The difference between $g\left(p_{u,k(q)}^{t}, z_{i,k(q)}^{t}, a_{u,k(q)}^{t-1}, x_{i,k(q)}^{t-1}, \psi_{2(q)}^{t-1}\right)$ and $g\left(\psi_{1(q)}^{t-1}, \psi_{2(q)}^{t-1}\right)$ in this case is also given by (S5). Considering the fact of $a_{u,k(q)}^{t}$=0 and, $x_{i,k(q)}^{t}$>0 the difference between $g\left(\psi_{1(q)}^{t}, \psi_{2(q)}^{t-1}\right)$ and $g\left(p_{u,k(q)}^{t}, z_{i,k(q)}^{t}, a_{u,k(q)}^{t-1}, x_{i,k(q)}^{t-1}, \psi_{2(q)}^{t-1}\right)$ is:

$$g\left(\psi_{1(q)}^{t}, \psi_{2(q)}^{t-1}\right) - g\left(p_{u,k(q)}^{t}, z_{i,k(q)}^{t}, a_{u,k(q)}^{t-1}, x_{i,k(q)}^{t-1}, \psi_{2(q)}^{t-1}\right) = -\frac{\lambda_{(q)}^{t}\left|\Lambda(u)\right|}{2}\left(a_{u,k(q)}^{t-1}\right)^{2} - \frac{\lambda_{(q)}^{t}\left|\Lambda(i)\right|}{2}\left(x_{i,k(q)}^{t} - x_{i,k(q)}^{t-1}\right)^{2}. \tag{S25}$$

Moreover, $g\left(\psi_{1(q)}^{t}, \psi_{2(q)}^{t}\right)$ and $g\left(\psi_{1(q)}^{t}, \psi_{2(q)}^{t-1}\right)$ yields:

$$\begin{aligned}
&g\left(\psi_{1(q)}^{t}, \psi_{2(q)}^{t}\right) - g\left(\psi_{1(q)}^{t}, \psi_{2(q)}^{t-1}\right) \\
&= \left(p_{u,k(q)}^{t} - a_{u,k(q)}^{t}\right)\left(h_{u,k(q)}^{t} - h_{u,k(q)}^{t-1}\right) + \left(z_{i,k(q)}^{t} - x_{i,k(q)}^{t}\right)\left(w_{i,k(q)}^{t} - w_{i,k(q)}^{t-1}\right) \\
&\overset{\text{(I)}}{=} \eta_{(q)}^{t}\lambda_{(q)}^{t}\left|\Lambda(u)\right|\left(p_{u,k(q)}^{t} - a_{u,k(q)}^{t}\right)^{2} + \frac{1}{\eta_{(q)}^{t}\lambda_{(q)}^{t}\left|\Lambda(i)\right|}\left(w_{i,k(q)}^{t} - w_{i,k(q)}^{t-1}\right)^{2} \\
&\overset{\text{(II)}}{\leq} \eta_{(q)}^{t}\lambda_{(q)}^{t}\left|\Lambda(u)\right|\left(p_{u,k(q)}^{t}\right)^{2} + \frac{2\left|\Lambda(i)\right|}{\eta_{(q)}^{t}\lambda_{(q)}^{t}}\left(\left(\left(\eta_{(q)}^{t}-1\right)\lambda_{(q)}^{t} z_{i,k(q)}^{t} - \left(\eta_{(q)}^{t-1}-1\right)\lambda_{(q)}^{t-1} z_{i,k(q)}^{t-1}\right)^{2} + \left(\left(\eta_{(q)}^{t}-1\right)\lambda_{(q)}^{t} x_{i,k(q)}^{t} - \left(\eta_{(q)}^{t-1}-1\right)\lambda_{(q)}^{t-1} x_{i,k(q)}^{t-1}\right)^{2}\right),
\end{aligned} \tag{S26}$$

where (I) is based on the update rules of ($h_{u,k(q)}$, $w_{i,k(q)}$) given in (4e), (4f) and (5c), and (II) is achieved with (22b) and $a_{u,k(q)}^{t}$=0. With (S5), (S25) and (S26), we have the following deduction:

$$\begin{aligned}
&g\left(\psi_{1(q)}^{t}, \psi_{2(q)}^{t}\right) - g\left(\psi_{1(q)}^{t-1}, \psi_{2(q)}^{t-1}\right) \\
&\leq -\frac{1}{2}\left(\sum_{i\in\Lambda(u)}\left(z_{i,k(q)}^{t-1}\right)^{2} + \lambda_{(q)}^{t}\left|\Lambda(u)\right|\right)\left(p_{u,k(q)}^{t} - p_{u,k(q)}^{t-1}\right)^{2} - \frac{\lambda_{(q)}^{t}\left|\Lambda(u)\right|}{2}\left(a_{u,k(q)}^{t-1}\right)^{2} \\
&\quad -\frac{1}{2}\left(\sum_{u\in\Lambda(i)}\left(p_{u,k(q)}^{t-1}\right)^{2} + \lambda_{(q)}^{t}\left|\Lambda(i)\right|\right)\left(z_{i,k(q)}^{t} - z_{i,k(q)}^{t-1}\right)^{2} - \frac{\lambda_{(q)}^{t}\left|\Lambda(i)\right|}{2}\left(x_{i,k(q)}^{t} - x_{i,k(q)}^{t-1}\right)^{2} \\
&\quad + \eta_{(q)}^{t}\lambda_{(q)}^{t}\left|\Lambda(u)\right|\left(p_{u,k(q)}^{t}\right)^{2} + \frac{2\left|\Lambda(i)\right|}{\eta_{(q)}^{t}\lambda_{(q)}^{t}}\left(\left(\left(\eta_{(q)}^{t}-1\right)\lambda_{(q)}^{t} z_{i,k(q)}^{t} - \left(\eta_{(q)}^{t-1}-1\right)\lambda_{(q)}^{t-1} z_{i,k(q)}^{t-1}\right)^{2} + \left(\left(\eta_{(q)}^{t}-1\right)\lambda_{(q)}^{t} x_{i,k(q)}^{t} - \left(\eta_{(q)}^{t-1}-1\right)\lambda_{(q)}^{t-1} x_{i,k(q)}^{t-1}\right)^{2}\right).
\end{aligned} \tag{S27}$$

Owing to (29), (25a) stands, which indicates that the augmented Lagrangian function (3) related to $s_{(q)}$ or τ_q is non-increasing as $a_{u,k(q)}^{t}$=0 and $x_{i,k(q)}^{t}$>0 in this case. Then after the *t*-th iteration, we substitute $a_{u,k(q)}^{t}$=0 into (S10):

$$\begin{aligned}
g\left(\psi_{1(q)}^{t}, \psi_{2(q)}^{t}\right) = &\frac{1}{2}\sum_{y_{u,i}\in\Lambda}\left(y_{u,i} - \sum_{l_1=1}^{k} p_{u,l_1(q)}^{t} z_{i,l_1(q)}^{t} - \sum_{l_2=k+1}^{d} p_{u,l_2(q)}^{t-1} z_{i,l_2(q)}^{t-1}\right)^{2} \\
&+\sum_{u}\left|\Lambda(u)\right|\left(\left(\eta_{(q)}^{t}-1\right)\lambda_{(q)}^{t}\left(\sum_{l_1=1}^{k-1}\left(p_{u,l_1(q)}^{t} - a_{u,l_1(q)}^{t}\right)^{2} + \left(p_{u,k(q)}^{t}\right)^{2}\right) + \left(\eta_{(q)}^{t-1}-1\right)\lambda_{(q)}^{t-1}\sum_{l_2=k+1}^{d}\left(p_{u,l_2(q)}^{t-1} - a_{u,l_2(q)}^{t-1}\right)^{2}\right) \\
&+\sum_{i}\left|\Lambda(i)\right|\left(\left(\eta_{(q)}^{t}-1\right)\lambda_{(q)}^{t}\sum_{l_1=1}^{k}\left(z_{i,l_1(q)}^{t} - x_{i,l_1(q)}^{t}\right)^{2} + \left(\eta_{(q)}^{t-1}-1\right)\lambda_{(q)}^{t-1}\sum_{l_2=k+1}^{d} w_{i,l_2(q)}^{t}\left(z_{i,l_2(q)}^{t-1} - x_{i,l_2(q)}^{t-1}\right)^{2}\right) \\
&+\sum_{u}\frac{\lambda_{(q)}^{t}\left|\Lambda(u)\right|}{2}\left(\sum_{l_1=1}^{k-1}\left(p_{u,l_1(q)}^{t} - a_{u,l_1(q)}^{t}\right)^{2} + \left(p_{u,k(q)}^{t}\right)^{2} + \sum_{l_2=k+1}^{d}\left(p_{u,l_2(q)}^{t-1} - a_{u,l_2(q)}^{t-1}\right)^{2}\right) \\
&+\sum_{i}\frac{\lambda_{(q)}^{t}\left|\Lambda(i)\right|}{2}\left(\sum_{l_1=1}^{k}\left(z_{i,l_1(q)}^{t} - x_{i,l_1(q)}^{t}\right)^{2} + \sum_{l_2=k+1}^{d}\left(z_{i,l_2(q)}^{t-1} - x_{i,l_2(q)}^{t-1}\right)^{2}\right).
\end{aligned} \tag{S28}$$

(S28) indicates that if (24b) is fulfilled, (25b) holds, thereby making (3) related to $s_{(q)}$ or τ_q lower-bounded as $a_{u,k(q)}^{t}$=0, and $x_{i,k(q)}^{t}$>0 in this case. Based on the above inferences, *Lemma* 2 stands, and **Step 4** is implemented.□

E. Proof of Theorem 2

Part a. Following *Lemma* 3, $g\left(\psi_{1(q)}^{t},\psi_{2(q)}^{t}\right)$ converges as $t\rightarrow\infty$, indicating that (S11) is fulfilled. With (26), (29) and (S27), we have (S12a), (S12b), (S12d), (S12f) and the following inferences:

$$\lim_{t\rightarrow\infty} a_{u,k(q)}^{t-1} \rightarrow 0, \tag{S29a}$$

$$\lim_{t\rightarrow\infty} p_{u,k(q)}^{t} \rightarrow 0. \tag{S29b}$$

Then according to (S12f), (S13b) is fulfilled. Hence, based on (S13b), (S29b) and $a_{u,k(q)}^{t}$=0, (27) is fulfilled.

Part b. Firstly, considering a limit point $\left\{\psi_{1(q)}^{*},\psi_{2(q)}^{*}\right\}$ of a sequence $\left\{\psi_{1(q)}^{t},\psi_{2(q)}^{t}\right\}$ generated by the update rules of $\{\psi_{1(q)}, \psi_{2(q)}\}$ based on (4) and (5), according to (27) and (S15), (S16) holds when $a_{u,k(q)}^{t}$=0, and $x_{i,k(q)}^{t}$>0 in this case. Then considering the KKT conditions related to $a_{u,k(q)}$, i.e., (S22).

According to (S17)-(S21) and with $a_{u,k(q)}^{t}$=0, conditions (S22a)-(S22c) are naturally satisfied. Thus, we focus on analyzing condition (S22d). Since we have $a_{u,k(q)}^{t}$=0 in this case, the following inequality holds:

$$p_{u,k(q)}^{*} + \frac{h_{u,k(q)}^{*}}{\lambda_{(q)}^{*}\left|\Lambda(u)\right|} \leq 0. \tag{S30}$$

Note that (S30) indicates that:

$$m_{u,k(q)}^{*} = -\lambda_{(q)}^{*}\left|\Lambda(u)\right|\left(p_{u,k(q)}^{*} + \frac{h_{u,k(q)}^{*}}{\lambda_{(q)}^{*}\left|\Lambda(u)\right|}\right) \geq 0. \tag{S31}$$

Thus, condition (S22) are all fulfilled in this case. Note that as $x_{i,k(q)}^{t}$>0 in this case, the proof regarding the KKT conditions of $x_{u,k(q)}$ can be achieved similarly. *Theorem* 2 stands, and **Step 5** is implemented. ■

II. ADDITIONAL FIGURES

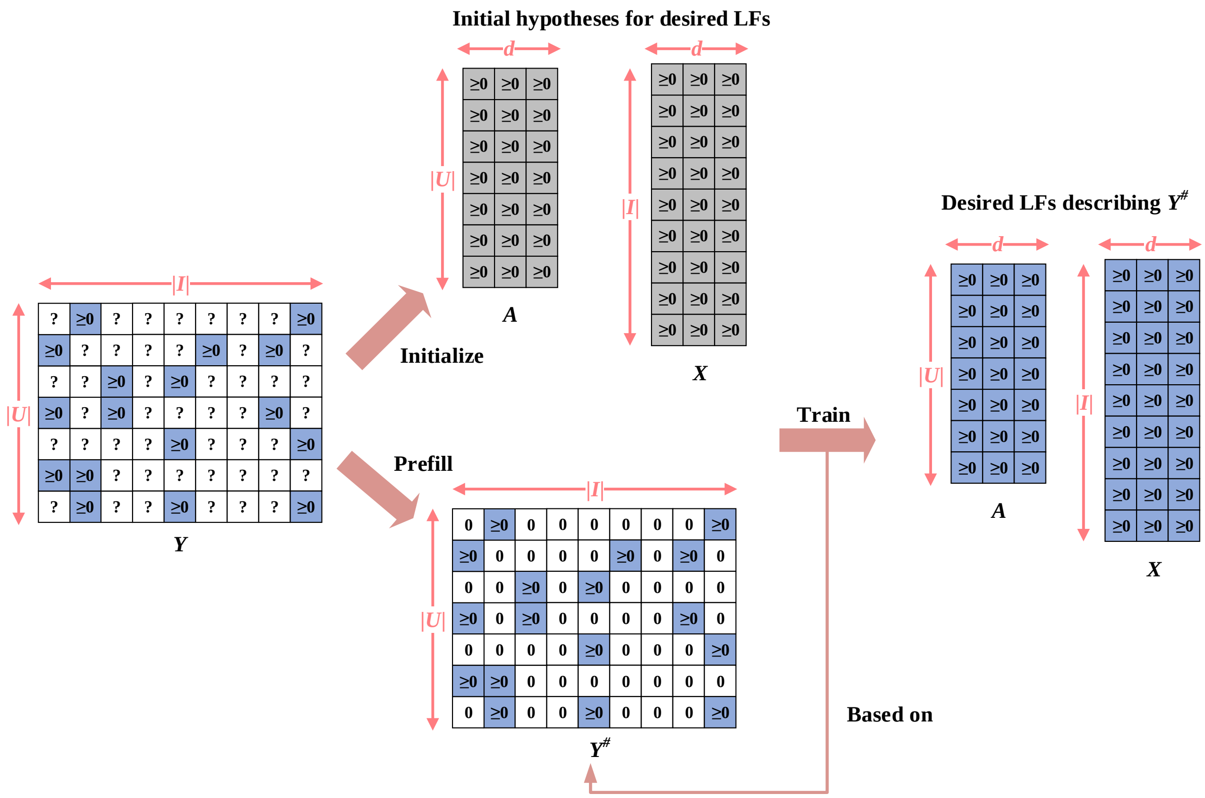

Fig. S1. Processing flow of an NMF model.

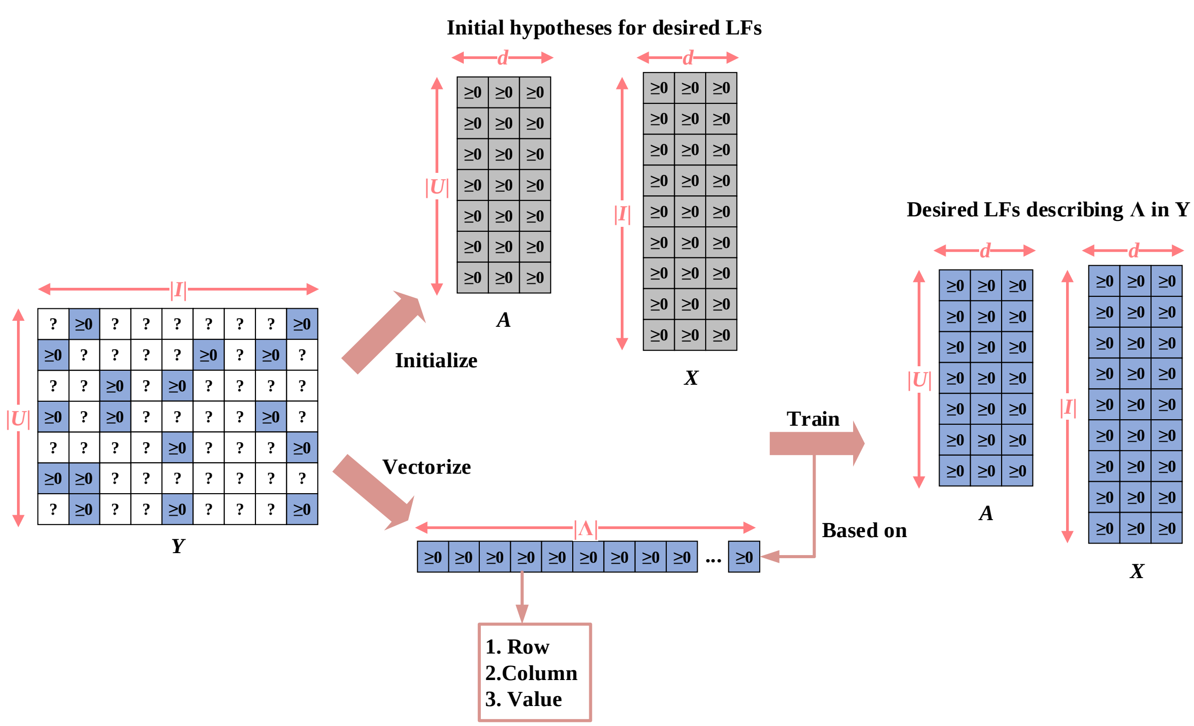

Fig. S2. Processing flow of an NLFA model.

III. ADDITIONAL TABLES

TABLE S.I. Optimal Hyper-parameters during M1's ten times' training process on D1-4

No.	Type	1	2	3	4	5	6	7	8	9	10
D1	**RMSE**	$\lambda=2^{-4}$, $\eta=2$	$\lambda=2^{-4}$, $\eta=2$	$\lambda=2^{-4}$, $\eta=2$	$\lambda=2^{-2}$, $\eta=1$	$\lambda=2^{-4}$, $\eta=2$	$\lambda=2^{-4}$, $\eta=2$	$\lambda=2^{-4}$, $\eta=2$	$\lambda=2^{-4}$, $\eta=2$	$\lambda=2^{-2}$, $\eta=1$	$\lambda=2^{-4}$, $\eta=2$
	MAE	$\lambda=2^{-2}$, $\eta=1$	$\lambda=2^{-4}$, $\eta=1$	$\lambda=2^{-4}$, $\eta=1$	$\lambda=2^{-2}$, $\eta=1$	$\lambda=2^{-4}$, $\eta=1$	$\lambda=2^{-4}$, $\eta=1$	$\lambda=2^{-4}$, $\eta=1$	$\lambda=2^{-2}$, $\eta=1$	$\lambda=2^{-4}$, $\eta=1$	$\lambda=2^{-4}$, $\eta=1$
D2	**RMSE**	$\lambda=1$, $\eta=1$	$\lambda=1$, $\eta=2^{-1}$	$\lambda=1$, $\eta=2^{-1}$	$\lambda=1$, $\eta=1$	$\lambda=1$, $\eta=2$	$\lambda=1$, $\eta=2$	$\lambda=1$, $\eta=1$	$\lambda=1$, $\eta=2^{-1}$	$\lambda=1$, $\eta=1$	$\lambda=1$, $\eta=2$
	MAE	$\lambda=1$, $\eta=2$	$\lambda=1$, $\eta=2$	$\lambda=1$, $\eta=2^{-1}$	$\lambda=1$, $\eta=1$	$\lambda=1$, $\eta=2^{-1}$	$\lambda=1$, $\eta=1$	$\lambda=1$, $\eta=2^{-1}$	$\lambda=1$, $\eta=1$	$\lambda=1$, $\eta=2$	$\lambda=1$, $\eta=2$
D3	**RMSE**	$\lambda=1$, $\eta=1$	$\lambda=1$, $\eta=2$	$\lambda=1$, $\eta=2^{-1}$	$\lambda=1$, $\eta=2$	$\lambda=1$, $\eta=1$	$\lambda=1$, $\eta=1$	$\lambda=1$, $\eta=2^{-1}$	$\lambda=1$, $\eta=1$	$\lambda=1$, $\eta=1$	$\lambda=1$, $\eta=2$
	MAE	$\lambda=1$, $\eta=1$	$\lambda=1$, $\eta=2^{-1}$	$\lambda=1$, $\eta=2^{-2}$	$\lambda=1$, $\eta=2$	$\lambda=1$, $\eta=2^{-1}$	$\lambda=1$, $\eta=1$	$\lambda=1$, $\eta=2^{-2}$	$\lambda=1$, $\eta=1$	$\lambda=1$, $\eta=2$	$\lambda=1$, $\eta=2$
D4	**RMSE**	$\lambda=1$, $\eta=2^{-1}$	$\lambda=1$, $\eta=2^{-1}$	$\lambda=1$, $\eta=2^{-1}$	$\lambda=1$, $\eta=2^{-1}$	$\lambda=1$, $\eta=2$	$\lambda=1$, $\eta=1$	$\lambda=1$, $\eta=2$	$\lambda=1$, $\eta=1$	$\lambda=1$, $\eta=2$	$\lambda=1$, $\eta=1$
	MAE	$\lambda=1$, $\eta=2$	$\lambda=1$, $\eta=1$	$\lambda=1$, $\eta=2^{-1}$	$\lambda=1$, $\eta=1$	$\lambda=1$, $\eta=2$	$\lambda=1$, $\eta=2$	$\lambda=1$, $\eta=1$	$\lambda=1$, $\eta=2^{-1}$	$\lambda=1$, $\eta=2^{-1}$	$\lambda=1$, $\eta=2^{-1}$

TABLE S.II. Optimal Hyper-parameters during M2's ten times' training process on D1-4

No.	Type	1	2	3	4	5	6	7	8	9	10
D1	**RMSE**	$\lambda=1.6$	$\lambda=1.6$	$\lambda=1.6$	$\lambda=1.8$	$\lambda=1.6$	$\lambda=1.8$	$\lambda=1.6$	$\lambda=1.8$	$\lambda=1.6$	$\lambda=1.6$
	MAE	$\lambda=1.8$	$\lambda=1.8$	$\lambda=1.8$	$\lambda=1.8$	$\lambda=1.8$	$\lambda=1.8$	$\lambda=1.2$	$\lambda=1.8$	$\lambda=1.8$	$\lambda=1.2$
D2	**RMSE**	$\lambda=1.8$	$\lambda=1.8$	$\lambda=1.8$	$\lambda=1.8$	$\lambda=1.8$	$\lambda=1.8$	$\lambda=1.8$	$\lambda=1.8$	$\lambda=1.8$	$\lambda=1.8$
	MAE	$\lambda=1.8$	$\lambda=1.8$	$\lambda=1.8$	$\lambda=1.8$	$\lambda=1.8$	$\lambda=1.8$	$\lambda=1.8$	$\lambda=1.8$	$\lambda=1.8$	$\lambda=1.8$
D3	**RMSE**	$\lambda=1.8$	$\lambda=1.8$	$\lambda=1.8$	$\lambda=1.8$	$\lambda=1.8$	$\lambda=1.8$	$\lambda=1.8$	$\lambda=1.8$	$\lambda=1.8$	$\lambda=1.8$
	MAE	$\lambda=1.8$	$\lambda=1.8$	$\lambda=1.8$	$\lambda=1.8$	$\lambda=1.8$	$\lambda=1.8$	$\lambda=1.8$	$\lambda=1.8$	$\lambda=1.8$	$\lambda=1.8$
D4	**RMSE**	[1]Failure	[1]Failure	[1]Failure	[1]Failure	[1]Failure	[1]Failure	[1]Failure	[1]Failure	[1]Failure	[1]Failure
	MAE	[1]Failure	[1]Failure	[1]Failure	[1]Failure	[1]Failure	[1]Failure	[1]Failure	[1]Failure	[1]Failure	[1]Failure

[1]M2 fails to obtain the final results on D4: their memory requirements are too large to meet on our experimental environment as shown in Table VI.

TABLE S.III. Optimal Hyper-parameters during M3's ten times' training process on D1-4

No.	Type	1	2	3	4	5	6	7	8	9	10
D1	**RMSE**	$\mu=2^{-1}$, $p=1.9$	$\mu=2^{-1}$, $p=1.8$	$\mu=2^{-1}$, $p=1.7$	$\mu=2^{-1}$, $p=1.8$	$\mu=2^{-1}$, $p=1.8$	$\mu=2^{-1}$, $p=1.7$	$\mu=2^{-1}$, $p=1.8$	$\mu=2^{-1}$, $p=1.9$	$\mu=2^{-1}$, $p=1.8$	$\mu=2^{-1}$, $p=1.8$
	MAE	$\mu=2^{-1}$, $p=1.8$	$\mu=2^{-1}$, $p=1.9$	$\mu=2^{-1}$, $p=1.8$	$\mu=2^{-1}$, $p=1.7$	$\mu=2^{-1}$, $p=1.8$	$\mu=2^{-1}$, $p=1.7$	$\mu=2^{-1}$, $p=1.8$	$\mu=2^{-1}$, $p=1.8$	$\mu=2^{-1}$, $p=1.9$	$\mu=2^{-1}$, $p=1.9$
D2	**RMSE**	$\mu=2$, $p=1.9$	$\mu=4$, $p=1.9$	$\mu=4$ $p=1.9$	$\mu=2^{-1}$, $p=1.8$	$\mu=2^{-1}$, $p=1.9$	$\mu=2$, $p=1.9$	$\mu=2^{-1}$, $p=1.8$	$\mu=2$, $p=1.9$	$\mu=2$, $p=1.8$	$\mu=2$, $p=1.8$
	MAE	$\mu=4$, $p=1.9$	$\mu=2$, $p=1.9$	$\mu=2$, $p=1.8$	$\mu=4$, $p=1.9$	$\mu=4$, $p=1.9$	$\mu=2$, $p=1.9$	$\mu=1$, $p=1.9$	$\mu=2$, $p=1.8$	$\mu=2$, $p=1.8$	$\mu=2$, $p=1.9$
D3	**RMSE**	$\mu=4$, $p=1.9$	$\mu=4$, $p=1.9$	$\mu=2$, $p=1.8$	$\mu=2$, $p=1.8$	$\mu=4$, $p=1.9$	$\mu=4$, $p=1.9$	$\mu=2$, $p=1.9$	$\mu=2$, $p=1.9$	$\mu=2$, $p=1.9$	$\mu=2$, $p=1.9$
	MAE	$\mu=2$, $p=1.9$	$\mu=2$, $p=1.8$	$\mu=2$, $p=1.9$	$\mu=2$, $p=1.8$	$\mu=4$, $p=1.9$	$\mu=4$, $p=1.9$	$\mu=4$, $p=1.9$	$\mu=2$, $p=1.8$	$\mu=2$, $p=1.9$	$\mu=2$, $p=1.9$
D4	**RMSE**	[1]Failure	[1]Failure	[1]Failure	[1]Failure	[1]Failure	[1]Failure	[1]Failure	[1]Failure	[1]Failure	[1]Failure
	MAE	[1]Failure	[1]Failure	[1]Failure	[1]Failure	[1]Failure	[1]Failure	[1]Failure	[1]Failure	[1]Failure	[1]Failure

[1]M3 fails to obtain the final results on D4: their memory requirements are too large to meet on our experimental environment as shown in Table VI.

TABLE S.IV. Optimal Hyper-parameters during M4's ten times' training process on D1-4

No.	Type	1	2	3	4	5	6	7	8	9	10
D1	**RMSE**	$\alpha=10^4$, $\beta=10^3$	$\alpha=10^4$, $\beta=10^3$	$\alpha=10^4$, $\beta=10^3$	$\alpha=10^3$, $\beta=10^4$	$\alpha=10^4$, $\beta=10^3$	$\alpha=10^3$, $\beta=10^4$	$\alpha=10^4$, $\beta=10^3$	$\alpha=10^4$, $\beta=10^3$	$\alpha=10^4$, $\beta=10^3$	$\alpha=10^4$, $\beta=10^3$
	MAE	$\alpha=10^4$, $\beta=10^3$	$\alpha=10^3$, $\beta=10^4$	$\alpha=10^4$, $\beta=10^3$	$\alpha=10^4$, $\beta=10^3$	$\alpha=10^4$, $\beta=10^3$	$\alpha=10^4$, $\beta=10^3$	$\alpha=10^4$, $\beta=10^3$	$\alpha=10^4$, $\beta=10^3$	$\alpha=10^3$, $\beta=10^4$	$\alpha=10^3$, $\beta=10^4$
D2	**RMSE**	$\alpha=10^3$, $\beta=10^4$	$\alpha=10^4$, $\beta=10^2$	$\alpha=10^3$, $\beta=10^3$	$\alpha=10^3$, $\beta=10^4$	$\alpha=10^3$, $\beta=10^3$	$\alpha=10^4$, $\beta=10^2$	$\alpha=10^3$, $\beta=10^3$	$\alpha=10^3$, $\beta=10^4$	$\alpha=10^4$, $\beta=10^2$	$\alpha=10^3$, $\beta=10^3$
	MAE	$\alpha=10^3$, $\beta=10^4$	$\alpha=10^3$, $\beta=10^4$	$\alpha=10^3$, $\beta=10^3$	$\alpha=10^3$, $\beta=10^4$	$\alpha=10^3$, $\beta=10^3$	$\alpha=10^4$, $\beta=10^3$	$\alpha=10^3$, $\beta=10^3$	$\alpha=10^3$, $\beta=10^4$	$\alpha=10^3$, $\beta=10^4$	$\alpha=10^3$, $\beta=10^3$
D3	**RMSE**	$\alpha=10^4$, $\beta=10^3$	$\alpha=10^3$, $\beta=10^4$	$\alpha=10^4$, $\beta=10^3$	$\alpha=10^3$, $\beta=10^4$	$\alpha=10^3$, $\beta=10^4$	$\alpha=10^4$, $\beta=10^3$	$\alpha=10^4$, $\beta=10^3$	$\alpha=10^3$, $\beta=10^4$	$\alpha=10^3$, $\beta=10^4$	$\alpha=10^4$, $\beta=10^3$
	MAE	$\alpha=10^3$, $\beta=10^4$	$\alpha=10^3$, $\beta=10^4$	$\alpha=10^4$, $\beta=10^3$	$\alpha=10^4$, $\beta=10^3$	$\alpha=10^4$, $\beta=10^3$	$\alpha=10^3$, $\beta=10^4$	$\alpha=10^3$, $\beta=10^4$	$\alpha=10^3$, $\beta=10^4$	$\alpha=10^4$, $\beta=10^3$	$\alpha=10^3$, $\beta=10^4$
D4	**RMSE**	[1]Failure	[1]Failure	[1]Failure	[1]Failure	[1]Failure	[1]Failure	[1]Failure	[1]Failure	[1]Failure	[1]Failure
	MAE	[1]Failure	[1]Failure	[1]Failure	[1]Failure	[1]Failure	[1]Failure	[1]Failure	[1]Failure	[1]Failure	[1]Failure

[1]M4 fails to obtain the final results on D4: their memory requirements are too large to meet on our experimental environment as shown in Table VI.

TABLE S.V. Optimal Hyper-parameters during M5's ten times' training process on D1-4

No.	Type	1	2	3	4	5	6	7	8	9	10
D1	RMSE	$\rho=2^{-5}$	$\rho=2^{-5}$	$\rho=2^{-5}$	$\rho=2^{-5}$	$\rho=2^{-5}$	$\rho=2^{-5}$	$\rho=2^{-5}$	$\rho=2^{-5}$	$\rho=2^{-5}$	$\rho=2^{-5}$
	MAE	$\rho=2^{-7}$	$\rho=2^{-5}$	$\rho=2^{-7}$	$\rho=2^{-5}$	$\rho=2^{-5}$	$\rho=2^{-7}$	$\rho=2^{-5}$	$\rho=2^{-5}$	$\rho=2^{-7}$	$\rho=2^{-5}$
D2	RMSE	$\rho=2^{-15}$	$\rho=2^{-13}$	$\rho=2^{-15}$	$\rho=2^{-13}$	$\rho=2^{-15}$	$\rho=2^{-13}$	$\rho=2^{-15}$	$\rho=2^{-15}$	$\rho=2^{-15}$	$\rho=2^{-13}$
	MAE	$\rho=2^{-13}$	$\rho=2^{-15}$	$\rho=2^{-15}$	$\rho=2^{-15}$	$\rho=2^{-13}$	$\rho=2^{-15}$	$\rho=2^{-13}$	$\rho=2^{-13}$	$\rho=2^{-15}$	$\rho=2^{-15}$
D3	RMSE	$\rho=2^{-13}$	$\rho=2^{-15}$	$\rho=2^{-13}$	$\rho=2^{-15}$	$\rho=2^{-13}$	$\rho=2^{-15}$	$\rho=2^{-13}$	$\rho=2^{-13}$	$\rho=2^{-15}$	$\rho=2^{-13}$
	MAE	$\rho=2^{-13}$	$\rho=2^{-15}$	$\rho=2^{-13}$	$\rho=2^{-13}$	$\rho=2^{-15}$	$\rho=2^{-13}$	$\rho=2^{-13}$	$\rho=2^{-15}$	$\rho=2^{-15}$	$\rho=2^{-13}$
D4	RMSE	[1]Failure	[1]Failure	[1]Failure	[1]Failure	[1]Failure	[1]Failure	[1]Failure	[1]Failure	[1]Failure	[1]Failure
	MAE	[1]Failure	[1]Failure	[1]Failure	[1]Failure	[1]Failure	[1]Failure	[1]Failure	[1]Failure	[1]Failure	[1]Failure

[1]M5 fails to obtain the final results on D4: their memory requirements are too large to meet on our experimental environment as shown in Table VI.

TABLE S.VI. Optimal Hyper-parameters during M6's ten times' training process on D1-4

No.	Type	1	2	3	4	5	6	7	8	9	10
D1	RMSE	$\lambda=10^{-4}$, $\eta=5\times10^{-3}$, [1]bs=512	$\lambda=10^{-3}$, $\eta=5\times10^{-3}$, bs=512	$\lambda=10^{-5}$, $\eta=10^{-3}$, bs=512	$\lambda=10^{-3}$, $\eta=5\times10^{-3}$, bs=512	$\lambda=10^{-4}$, $\eta=5\times10^{-3}$, bs=512	$\lambda=10^{-5}$, $\eta=10^{-3}$, bs=512	$\lambda=10^{-3}$, $\eta=5\times10^{-3}$, bs=512	$\lambda=10^{-5}$, $\eta=10^{-3}$, bs=512	$\lambda=10^{-4}$, $\eta=5\times10^{-3}$, bs=512	$\lambda=10^{-3}$, $\eta=5\times10^{-3}$, bs=512
	MAE	$\lambda=10^{-5}$, $\eta=10^{-3}$, bs=512	$\lambda=10^{-5}$, $\eta=10^{-3}$, bs=512	$\lambda=10^{-2}$, $\eta=5\times10^{-3}$, bs=512	$\lambda=10^{-2}$, $\eta=5\times10^{-3}$, bs=512	$\lambda=10^{-2}$, $\eta=5\times10^{-3}$, bs=512	$\lambda=10^{-5}$, $\eta=10^{-3}$, bs=512	$\lambda=10^{-5}$, $\eta=10^{-3}$, bs=512	$\lambda=10^{-5}$, $\eta=10^{-3}$, bs=512	$\lambda=10^{-5}$, $\eta=10^{-3}$, bs=512	$\lambda=10^{-2}$, $\eta=5\times10^{-3}$, bs=512
D2	RMSE	$\lambda=10^{-4}$, $\eta=10^{-3}$, bs=64	$\lambda=10^{-4}$, $\eta=10^{-3}$, bs=64	$\lambda=10^{-1}$, $\eta=5\times10^{-3}$, bs=64	$\lambda=10^{-1}$, $\eta=5\times10^{-3}$, bs=64	$\lambda=10^{-2}$, $\eta=10^{-3}$, bs=64	$\lambda=10^{-1}$, $\eta=5\times10^{-3}$, bs=64	$\lambda=10^{-2}$, $\eta=10^{-3}$, bs=64	$\lambda=10^{-2}$, $\eta=10^{-3}$, bs=64	$\lambda=10^{-1}$, $\eta=5\times10^{-3}$, bs=64	$\lambda=10^{-4}$, $\eta=10^{-3}$, bs=64
	MAE	$\lambda=10^{-4}$, $\eta=10^{-2}$, bs=64	$\lambda=10^{-3}$, $\eta=10^{-2}$, bs=64	$\lambda=10^{-4}$, $\eta=10^{-2}$, bs=64	$\lambda=10^{-3}$, $\eta=10^{-2}$, bs=64	$\lambda=10^{-4}$, $\eta=10^{-2}$, bs=64	$\lambda=10^{-2}$, $\eta=10^{-3}$, bs=64	$\lambda=10^{-4}$, $\eta=10^{-2}$, bs=64	$\lambda=10^{-2}$, $\eta=10^{-3}$, bs=64	$\lambda=10^{-2}$, $\eta=10^{-3}$, bs=64	$\lambda=10^{-2}$, $\eta=10^{-3}$, bs=64
D3	RMSE	$\lambda=10^{-4}$, $\eta=5\times10^{-4}$, bs=128	$\lambda=10^{-4}$, $\eta=5\times10^{-4}$, bs=128	$\lambda=10^{-4}$, $\eta=5\times10^{-4}$, bs=128	$\lambda=10^{-4}$, $\eta=5\times10^{-4}$, bs=128	$\lambda=10^{-4}$, $\eta=5\times10^{-4}$, bs=128	$\lambda=10^{-5}$, $\eta=10^{-3}$, bs=128	$\lambda=10^{-5}$, $\eta=10^{-3}$, bs=128	$\lambda=10^{-4}$, $\eta=5\times10^{-4}$, bs=128	$\lambda=10^{-4}$, $\eta=5\times10^{-4}$, bs=128	$\lambda=10^{-4}$, $\eta=5\times10^{-4}$, bs=128
	MAE	$\lambda=10^{-5}$, $\eta=10^{-3}$, bs=128	$\lambda=10^{-4}$, $\eta=10^{-3}$, bs=128	$\lambda=10^{-5}$, $\eta=10^{-3}$, bs=128	$\lambda=10^{-5}$, $\eta=10^{-3}$, bs=128	$\lambda=10^{-4}$, $\eta=10^{-3}$, bs=128	$\lambda=10^{-4}$, $\eta=10^{-3}$, bs=128	$\lambda=10^{-4}$, $\eta=10^{-3}$, bs=128	$\lambda=10^{-4}$, $\eta=10^{-3}$, bs=128	$\lambda=10^{-4}$, $\eta=10^{-3}$, bs=128	$\lambda=10^{-4}$, $\eta=10^{-3}$, bs=128
D4	RMSE	$\lambda=10^{-3}$, $\eta=10^{-4}$, bs=512	$\lambda=10^{-4}$, $\eta=10^{-3}$, bs=512	$\lambda=10^{-5}$, $\eta=5\times10^{-3}$, bs=512	$\lambda=10^{-4}$, $\eta=10^{-3}$, bs=512	$\lambda=10^{-4}$, $\eta=10^{-3}$, bs=512	$\lambda=10^{-3}$, $\eta=10^{-4}$, bs=512	$\lambda=10^{-5}$, $\eta=5\times10^{-3}$, bs=512	$\lambda=10^{-3}$, $\eta=10^{-4}$, bs=512	$\lambda=10^{-5}$, $\eta=5\times10^{-3}$, bs=512	$\lambda=10^{-3}$, $\eta=10^{-4}$, bs=512
	MAE	$\lambda=10^{-2}$, $\eta=10^{-3}$, bs=512	$\lambda=10^{-2}$, $\eta=10^{-3}$, bs=512	$\lambda=10^{-2}$, $\eta=5\times10^{-3}$, bs=512	$\lambda=10^{-2}$, $\eta=5\times10^{-3}$, bs=512	$\lambda=10^{-5}$, $\eta=5\times10^{-3}$, bs=512	$\lambda=10^{-2}$, $\eta=5\times10^{-3}$, bs=512	$\lambda=10^{-5}$, $\eta=5\times10^{-3}$, bs=512	$\lambda=10^{-2}$, $\eta=5\times10^{-3}$, bs=512	$\lambda=10^{-2}$, $\eta=10^{-3}$, bs=512	$\lambda=10^{-2}$, $\eta=5\times10^{-3}$, bs=512

[1]The abbreviation 'bs' denotes the batch size adopted by M6 on an HDI matrix.

TABLE S.VII. Optimal Hyper-parameters during M7's ten times' training process on D1-4

No.	Type	1	2	3	4	5	6	7	8	9	10
D1	RMSE	$\lambda=10^{-3}$, $\eta=10^{-3}$, [1]bs=2048	$\lambda=10^{-3}$, $\eta=5\times10^{-3}$, bs=2048	$\lambda=10^{-3}$, $\eta=5\times10^{-3}$, bs=2048	$\lambda=10^{-3}$, $\eta=5\times10^{-2}$, bs=2048	$\lambda=10^{-3}$, $\eta=5\times10^{-3}$, bs=2048	$\lambda=5\times10^{-3}$, $\eta=5\times10^{-2}$, bs=2048	$\lambda=10^{-3}$, $\eta=5\times10^{-3}$, bs=2048	$\lambda=5\times10^{-3}$, $\eta=5\times10^{-2}$, bs=2048	$\lambda=10^{-3}$, $\eta=5\times10^{-2}$, bs=2048	$\lambda=10^{-3}$, $\eta=10^{-3}$, bs=2048
	MAE	$\lambda=10^{-3}$, $\eta=10^{-3}$, bs=2048	$\lambda=10^{-3}$, $\eta=10^{-3}$, bs=2048	$\lambda=5\times10^{-3}$, $\eta=10^{-3}$, bs=2048	$\lambda=10^{-3}$ $\eta=5\times10^{-3}$, bs=2048	$\lambda=10^{-3}$, $\eta=5\times10^{-3}$, bs=2048	$\lambda=5\times10^{-3}$, $\eta=10^{-3}$, bs=2048	$\lambda=10^{-3}$, $\eta=10^{-3}$, bs=2048	$\lambda=10^{-3}$, $\eta=10^{-3}$, bs=2048	$\lambda=10^{-3}$, $\eta=10^{-3}$, bs=2048	$\lambda=10^{-3}$, $\eta=5\times10^{-2}$, bs=2048
D2	RMSE	$\lambda=10^{-3}$, $\eta=5\times10^{-3}$, bs=1024	$\lambda=10^{-3}$, $\eta=10^{-3}$, bs=1024	$\lambda=10^{-3}$, $\eta=10^{-3}$, bs=1024	$\lambda=10^{-3}$, $\eta=5\times10^{-3}$, bs=1024	$\lambda=10^{-3}$, $\eta=5\times10^{-3}$, bs=1024	$\lambda=10^{-3}$, $\eta=10^{-3}$, bs=1024	$\lambda=10^{-3}$, $\eta=5\times10^{-3}$, bs=1024	$\lambda=10^{-3}$, $\eta=10^{-3}$, bs=1024	$\lambda=10^{-3}$, $\eta=5\times10^{-3}$, bs=1024	$\lambda=10^{-3}$, $\eta=10^{-3}$, bs=1024
	MAE	$\lambda=10^{-3}$, $\eta=5\times10^{-3}$, bs=1024	$\lambda=10^{-3}$, $\eta=10^{-3}$, bs=1024	$\lambda=10^{-3}$, $\eta=10^{-3}$, bs=1024	$\lambda=10^{-3}$, $\eta=5\times10^{-3}$, bs=1024	$\lambda=10^{-3}$, $\eta=10^{-3}$, bs=1024	$\lambda=10^{-3}$, $\eta=5\times10^{-3}$, bs=1024	$\lambda=10^{-3}$, $\eta=10^{-3}$, bs=1024	$\lambda=5\times10^{-3}$, $\eta=10^{-3}$, bs=1024	$\lambda=10^{-3}$, $\eta=5\times10^{-3}$, bs=1024	$\lambda=10^{-3}$, $\eta=10^{-3}$, bs=1024
D3	RMSE	$\lambda=10^{-3}$, $\eta=10^{-3}$, bs=2048	$\lambda=10^{-3}$, $\eta=5\times10^{-3}$, bs=2048	$\lambda=5\times10^{-3}$, $\eta=5\times10^{-2}$, bs=2048	$\lambda=5\times10^{-3}$, $\eta=10^{-3}$, bs=2048	$\lambda=10^{-3}$, $\eta=10^{-3}$, bs=2048	$\lambda=10^{-3}$, $\eta=5\times10^{-3}$, bs=2048	$\lambda=10^{-3}$, $\eta=10^{-3}$, bs=2048	$\lambda=10^{-3}$, $\eta=10^{-3}$, bs=2048	$\lambda=10^{-3}$, $\eta=5\times10^{-2}$, bs=2048	$\lambda=10^{-3}$, $\eta=10^{-3}$, bs=2048
	MAE	$\lambda=10^{-3}$, $\eta=10^{-3}$, bs=2048	$\lambda=10^{-3}$, $\eta=5\times10^{-3}$, bs=2048	$\lambda=10^{-3}$, $\eta=5\times10^{-2}$, bs=2048	$\lambda=10^{-3}$, $\eta=5\times10^{-3}$, bs=2048	$\lambda=10^{-3}$, $\eta=5\times10^{-3}$, bs=2048	$\lambda=5\times10^{-3}$, $\eta=10^{-3}$, bs=2048	$\lambda=10^{-3}$, $\eta=10^{-3}$, bs=2048	$\lambda=10^{-3}$, $\eta=10^{-3}$, bs=2048	$\lambda=10^{-3}$, $\eta=10^{-3}$, bs=2048	$\lambda=10^{-3}$, $\eta=10^{-3}$, bs=2048
D4	RMSE	$\lambda=10^{-3}$, $\eta=10^{-3}$, bs=4096	$\lambda=10^{-3}$, $\eta=5\times10^{-3}$, bs=4096	$\lambda=10^{-3}$, $\eta=5\times10^{-3}$, bs=4096	$\lambda=10^{-3}$, $\eta=10^{-3}$, bs=4096	$\lambda=10^{-3}$, $\eta=5\times10^{-2}$, bs=4096	$\lambda=10^{-3}$, $\eta=10^{-3}$, bs=4096	$\lambda=10^{-3}$, $\eta=5\times10^{-3}$, bs=4096	$\lambda=10^{-3}$, $\eta=10^{-3}$, bs=4096	$\lambda=10^{-3}$, $\eta=5\times10^{-3}$, bs=4096	$\lambda=10^{-3}$, $\eta=10^{-3}$, bs=4096
	MAE	$\lambda=10^{-3}$, $\eta=10^{-3}$, bs=4096	$\lambda=10^{-3}$, $\eta=10^{-3}$, bs=4096	$\lambda=10^{-3}$, $\eta=5\times10^{-3}$, bs=4096	$\lambda=5\times10^{-3}$, $\eta=10^{-3}$, bs=4096	$\lambda=10^{-3}$, $\eta=10^{-3}$, bs=4096	$\lambda=10^{-3}$, $\eta=10^{-3}$, bs=4096	$\lambda=5\times10^{-3}$, $\eta=5\times10^{-3}$, bs=4096	$\lambda=10^{-3}$, $\eta=10^{-3}$, bs=4096	$\lambda=10^{-3}$, $\eta=5\times10^{-3}$, bs=4096	$\lambda=10^{-3}$, $\eta=10^{-3}$, bs=4096

[1]The abbreviation 'bs' denotes batch size adopted by M7 on an HDI matrix.

TABLE S.VIII. RMSE, MAE and Time Cost of M1, M8 and M9 on D1-4.

Dataset	Model	Case	Prediction Accuracy	[1]Tuning Time Cost (Secs)	[2]Testing Time Cost (Secs)	Total Time Cost (Secs)
D1	M1	RMSE	0.2373±2.2E-6	428±22.7	6±2.4	434±25
		MAE	0.1815±1.1E-6	439±25.4	6±2.8	445±28
	M8	RMSE	**0.2339±2.7E-4**	-	-	**46±4**
		MAE	**0.1793±3.3E-4**	-	-	**60±5**
	M9	RMSE	0.2573±1.4E-6	-	-	387±33
		MAE	0.2015±2.8E-6	-	-	396±46
D2	M1	RMSE	1.0187±1.1E-6	271±45.4	4±1.7	275±47
		MAE	0.8079±2.5E-6	305±38.3	3±0.9	308±39
	M8	RMSE	**1.0158±6.4E-4**	-	-	**25±5**
		MAE	**0.7848±9.3E-4**	-	-	**29±8**
	M9	RMSE	1.0209±2.0E-5	-	-	87±11
		MAE	0.7883±1.3E-5	-	-	85±9
D3	M1	RMSE	**0.8665±7.8E-4**	739±72.3	21±14.7	759±87
		MAE	0.6829±1.7E-6	756±53.8	3±0.5	763±54
	M8	RMSE	0.8673±1.3E-3	-	-	**26±6**
		MAE	**0.6785±9.6E-5**	-	-	**30±9**
	M9	RMSE	0.8684±4.7E-4	-	-	487±59
		MAE	0.6793±2.8E-5	-	-	268±40
D4	M1	RMSE	0.8096±2.9E-6	2,934±353.8	38±24.6	3,972±378
		MAE	0.6221±7.9E-7	9,203±828.9	33±20.4	9,236±849
	M8	RMSE	0.8091±3.0E-3	-	-	**334±46**
		MAE	**0.6191±1.7E-4**	-	-	**358±31**
	M9	RMSE	**0.8086±6.0E-5**	-	-	3731±311
		MAE	0.6199±7.1E-6	-	-	5532±566

[1]Time cost consumed by M1 for manually grid-searching optimal hyper-parameters; [2]Time cost consumed by M1 with obtained hyper-parameters.

TABLE S.IX. RMSE/MAE of M1-9 on D1-4, including Win/Loss counts and Friedman Rank, where ● indicates that both M8 and M9 have higher RMSE/MAE than the rival models

No.	Case	M1	M2	M3	M4	M5	M6	M7	M8	M9
D1	RMSE	$0.2373_{\pm2.2E-6}$	$0.3058_{\pm5.0E-4}$	$0.3047_{\pm4.3E-5}$	$0.2384_{\pm1.0E-4}$	$0.4913_{\pm1.0E-4}$	●$\mathbf{0.2302_{\pm2.6E-3}}$	$0.2504_{\pm4.7E-4}$	$0.2339_{\pm2.7E-4}$	$0.2573_{\pm1.4E-6}$
	MAE	$0.1815_{\pm1.1E-6}$	$0.2439_{\pm4.4E-5}$	$0.2422_{\pm3.2E-5}$	$0.1832_{\pm4.8E-5}$	$0.4111_{\pm1.6E-2}$	●$\mathbf{0.1792_{\pm7.9E-5}}$	$0.1953_{\pm8.9E-4}$	$0.1793_{\pm3.3E-4}$	$0.2015_{\pm2.8E-6}$
D2	RMSE	$1.0187_{\pm1.1E-6}$	$1.1281_{\pm7.2E-3}$	$1.1257_{\pm1.8E-4}$	$1.0787_{\pm8.6E-7}$	$1.8808_{\pm2.8E-2}$	$1.1494_{\pm4.1E-5}$	$1.0255_{\pm4.5E-4}$	$\mathbf{1.0158_{\pm6.4E-4}}$	$1.0209_{\pm2.0E-5}$
	MAE	$0.8079_{\pm2.5E-6}$	$0.9256_{\pm1.5E-3}$	$0.9229_{\pm1.9E-4}$	$0.8580_{\pm6.2E-6}$	$1.5403_{\pm2.6E-2}$	$0.9257_{\pm2.1E-4}$	$0.7945_{\pm1.6E-3}$	$\mathbf{0.7848_{\pm9.3E-4}}$	$0.7883_{\pm1.3E-5}$
D3	RMSE	●$0.8665_{\pm7.8E-4}$	$1.0336_{\pm5.4E-4}$	$1.0848_{\pm5.3E-5}$	$0.8713_{\pm3.7E-4}$	$2.2963_{\pm9.8E-3}$	$0.8845_{\pm1.1E-3}$	$0.8972_{\pm8.2E-3}$	$0.8673_{\pm1.3E-3}$	$0.8684_{\pm4.7E-4}$
	MAE	$0.6829_{\pm1.7E-6}$	$0.8832_{\pm4.0E-4}$	$0.9021_{\pm4.2E-5}$	$0.6802_{\pm3.3E-4}$	$1.9190_{\pm9.6E-3}$	$0.7021_{\pm5.5E-3}$	$0.7036_{\pm3.6E-3}$	$\mathbf{0.6785_{\pm9.6E-5}}$	$0.6793_{\pm2.8E-5}$
D4	RMSE	$0.8096_{\pm2.9E-6}$	[1]Failure	[1]Failure	[1]Failure	[1]Failure	$0.8436_{\pm1.2E-3}$	$0.8574_{\pm1.2E-3}$	$0.8091_{\pm3.0E-3}$	$\mathbf{0.8086_{\pm6.0E-5}}$
	MAE	$0.6221_{\pm7.9E-7}$	[1]Failure	[1]Failure	[1]Failure	[1]Failure	$0.6530_{\pm4.9E-3}$	$0.6647_{\pm3.3E-3}$	$\mathbf{0.6191_{\pm1.7E-4}}$	$0.6199_{\pm7.1E-6}$
Win/Loss		8/1	9/0	9/0	9/0	9/0	7/2	9/0	-	-
F-Rank		2.875	7.375	7.125	5	8.625	4.5	4.875	**1.5**	3.125

[1]M2-M5 fails to obtain the final results on D4 on our experimental environment as shown in Table VI.

TABLE S.X. Total time cost of M1-9 in RMSE/MAE on D1-4 (Secs), including Win/Loss counts and Friedman Rank

No.	Case	M1	M2	M3	M4	M5	M6	M7	M8	M9
D1	RMSE	$434_{\pm25}$	$38{,}499_{\pm2{,}332}$	$43{,}271_{\pm2{,}962}$	$209{,}670_{\pm36{,}625}$	$3{,}856_{\pm578}$	$10{,}804_{\pm832}$	$2{,}011{,}581_{\pm481{,}806}$	$\mathbf{46_{\pm4}}$	$387_{\pm33}$
	MAE	$445_{\pm28}$	$51{,}443_{\pm2{,}643}$	$189{,}090_{\pm29{,}135}$	$210{,}745_{\pm27{,}190}$	$3{,}912_{\pm103}$	$13{,}432_{\pm2{,}219}$	$1{,}809{,}472_{\pm393{,}344}$	$\mathbf{60_{\pm5}}$	$396_{\pm46}$
D2	RMSE	$275_{\pm47}$	$236_{\pm35}$	$128_{\pm21}$	$138_{\pm39}$	$55_{\pm7}$	$2{,}109_{\pm295}$	$888{,}704_{\pm223{,}355}$	$\mathbf{25_{\pm5}}$	$87_{\pm11}$
	MAE	$308_{\pm39}$	$32_{\pm3}$	$128_{\pm27}$	$766_{\pm64}$	$56_{\pm6}$	$2{,}256_{\pm633}$	$2{,}015{,}970_{\pm861{,}492}$	$\mathbf{29_{\pm8}}$	$85_{\pm9}$
D3	RMSE	$759_{\pm87}$	$42{,}578_{\pm3{,}258}$	$71{,}927_{\pm2{,}718}$	$45{,}849_{\pm2{,}479}$	$717_{\pm22}$	$8{,}359_{\pm532}$	$883{,}008_{\pm63{,}588}$	$\mathbf{26_{\pm6}}$	$487_{\pm59}$
	MAE	$763_{\pm54}$	$1{,}906_{\pm291}$	$12{,}294_{\pm2{,}478}$	$48{,}821_{\pm2{,}363}$	$716_{\pm53}$	$7{,}700_{\pm219}$	$676{,}921_{\pm41{,}748}$	$\mathbf{30_{\pm9}}$	$268_{\pm40}$
D4	RMSE	$3{,}972_{\pm378}$	[1]Failure	[1]Failure	[1]Failure	[1]Failure	$266{,}885_{\pm12{,}775}$	$45{,}533{,}828_{\pm8{,}105{,}384}$	$\mathbf{334_{\pm46}}$	$3731_{\pm311}$
	MAE	$9{,}236_{\pm849}$	[1]Failure	[1]Failure	[1]Failure	[1]Failure	$232{,}974_{\pm13{,}157}$	$50{,}643{,}155_{\pm6{,}301{,}613}$	$\mathbf{358_{\pm31}}$	$5532_{\pm566}$
Win/Loss		9/0	9/0	9/0	9/0	9/0	9/0	9/0	-	-
F-Rank		4.125	5.75	6.625	7.25	4.25	5.625	8	**1**	2.375

[1]M2-M5 fails to obtain the final results on D4 on our experimental environment as shown in Table VI.