A Relaxed Symmetry-Constrained Non-negative Model for Large-Scale Undirected Weighted Network Representation: Supplementary File

Yurong Zhong, Xin Luo, *Senior Member*, *IEEE*, and Zidong Wang, *Fellow*, *IEEE*

This is the supplementary file for the paper entitled *A Relaxed Symmetry-Constrained Non-negative Model for Large-Scale Undirected Weighted Network Representation.* Some supplementary tables and figures illustrating the experimental results are put in this file and cited in the manuscript.

I. Supplementary Tables

TABLE S1. All competitors' achieved hyper-parameter settings on D1-6.

No.	D1	D2	D3	D4	D5	D6
M1	$\lambda=2^{-9}$	$\lambda=2^{-6}$	$\lambda=2^{-8}$	$\lambda=2^{-11}$	$\lambda=2^{-12}$	$\lambda=2^{-13}$
M2	α=10, β=100, η=1	α=10, β=100, η=1	α=10, β=100, η=1,	α=1, β=1, η=1	α=1, β=1000, η=1	α=1, β=10, η=2
M3	λ=0.5, $\eta=2^{-4}$, ^{1}bs=512	λ=0.1, $\eta=2^{-4}$, ^{1}bs=512	λ=0.1, $\eta=2^{-4}$, ^{1}bs=512	$\lambda=2^{-2}$, $\eta=2^{-4}$, ^{1}bs=128	$\lambda=2^{-3}$, $\eta=2^{-4}$, ^{1}bs=256	$\lambda=2^{-2}$, $\eta=2^{-4}$, ^{1}bs=256
M4	β=0.8	β=0.8	β=0.6	$\beta=2^{-5}$	β=0.5	β=0.5
M5	$\lambda=2^{-6}$	$\lambda=2^{-4}$	$\lambda=2^{-4}$	λ=1.2	λ=0.5	$\lambda=2^{-3}$
M6	$\lambda=2^{-4}$, $\eta=2^{-3}$, ^{1}bs=2048, ^{2}n=3	$\lambda=2^{-4}$, $\eta=2^{-4}$, ^{1}bs=1024, ^{2}n=3	$\lambda=2^{-4}$, $\eta=2^{-3}$, ^{1}bs=2048, ^{2}n=3	$\lambda=2^{-4}$, $\eta=2^{-3}$, ^{1}bs=2048, ^{2}n=3	$\lambda=2^{-4}$, $\eta=2^{-3}$, ^{1}bs=2048, ^{2}n=3	$\lambda=2^{-6}$, $\eta=2^{-3}$, ^{1}bs=2048, ^{2}n=3
M7	λ=1, η=0.6, $\theta=2^{-6}$	λ=0.8, η=1, $\theta=2^{-6}$	λ=1, η=1, $\theta=2^{-6}$	$\lambda=2^{-12}$, η=1.2, $\theta=2^{-6}$	$\lambda=2^{-12}$, η=0.8, $\theta=2^{-8}$	$\lambda=2^{-14}$, η=1, $\theta=2^{-8}$
M8	$\lambda_1=2^{-3}$, λ_2=0.005, $\eta=2^{-4}$, τ=0.2, ^{1}bs=2048, ^{2}n=2	$\lambda_1=2^{-3}$, λ_2=0.005, $\eta=2^{-4}$, τ=0.2, ^{1}bs=2048, ^{2}n=2	$\lambda_1=2^{-3}$, λ_2=0.005, $\eta=2^{-4}$, τ=0.2, ^{1}bs=2048, ^{2}n=2	$\lambda_1=2^{-4}$, λ_2=0.005, $\eta=2^{-3}$, τ=0.2, ^{1}bs=2048, ^{2}n=3	$\lambda_1=2^{-4}$, λ_2=0.005, $\eta=2^{-3}$, τ=0.2, ^{1}bs=2048, ^{2}n=3	$\lambda_1=2^{-4}$, λ_2=0.005, $\eta=2^{-3}$, τ=0.2, ^{1}bs=2048, ^{2}n=3
M9	$\lambda=2^{-4}$, $\eta=2^{-3}$, r=0.75, ^{1}bs=2048, ^{2}n=3	$\lambda=2^{-4}$, $\eta=2^{-4}$, r=0.4, ^{1}bs=1024, ^{2}n=3	$\lambda=2^{-4}$, $\eta=2^{-3}$, r=0.75, ^{1}bs=2048, ^{2}n=3	$\lambda=2^{-6}$, $\eta=2^{-3}$, r=0.8, ^{1}bs=512, ^{2}n=3	$\lambda=2^{-6}$, $\eta=2^{-3}$, r=0.8, ^{1}bs=2048, ^{2}n=3	$\lambda=2^{-6}$, $\eta=2^{-3}$, r=0.8, ^{1}bs=128, ^{2}n=3

^{1}bs denotes batch size; ^{2}n denotes the count of layers;

TABLE S2. RMSE of M1-10 on D1-6.

No.	D1	D2	D3	D4	D5	D6	Win/Loss	Friedman Rank
M1	0.1319±3.8E-4	0.1290±9.4E-5	0.1431±1.5E-3	0.2301±8.6E-4	0.0756±3.6E-4	0.0423±7.7E-5	6/0	4.833
M2	0.1333±2.4E-4	0.1330±2.0E-4	0.1437±2.0E-4	0.2559±5.5E-4	0.0821±1.7E-5	0.0464±1.3E-5	6/0	6.333
M3	0.1334±2.6E-4	0.1285±1.9E-4	0.1413±3.0E-4	0.3147±1.3E-4	0.1477±1.3E-5	0.0853±2.5E-4	6/0	7.5
M4	0.1544±2.6E-4	0.1649±2.7E-4	0.1806±4.1E-4	0.2297±1.3E-4	0.0740±1.2E-5	0.0415±8.5E-5	6/0	5.833
M5	0.1574±1.2E-4	0.1813±1.6E-4	0.1972±1.4E-4	0.2369±6.8E-4	0.0764±8.1E-5	0.0430±5.3E-5	6/0	7.5
M6	0.1318±1.6E-4	0.1286±2.7E-4	0.1415±1.3E-4	0.3079±7.2E-4	0.1037±2.2E-4	0.0601±5.0E-4	6/0	6.833
M7	0.1289±2.4E-4	0.1272±1.0E-4	0.1399±2.3E-4	0.2269±2.2E-3	0.0735±3.7E-4	0.0412±8.1E-5	6/0	2.5
M8	0.1548±3.2E-4	0.1499±1.4E-4	0.1610±6.2E-4	0.3052±1.4E-4	0.0965±3.8E-5	0.0540±2.3E-4	6/0	8.167
M9	0.1286±9.7E-5	0.1266±1.7E-4	0.1392±1.6E-4	0.3007±7.4E-5	0.0928±8.8E-5	0.0519±3.0E-5	6/0	4.5
M10	**0.1280±1.8E-4**	**0.1261±3.5E-4**	**0.1389±2.3E-4**	**0.2215±1.3E-4**	**0.0728±3.6E-4**	**0.0409±1.9E-4**	-	**1**

TABLE S3. Converging Iteration Count of M1-10 on D1-6, where ✪ indicates M10 has more converging iteration count than the rival model.

No.	D1	D2	D3	D4	D5	D6	✪Win/Loss	Friedman Rank
M1	341±2.87	192±6.45	122±5.79	967±4.76	568±5.13	582±2.13	6/0	7.667
M2	187±4.0	311±5.20	284±3.24	40±2.49✪	251±1.22	227±1.26	5/1	6.333
M3	772±11.16	578±14.84	743±20.49	93±2.35✪	31±1.48✪	**15±4.24**✪	3/3	5.667
M4	685±16.71	975±42.01	959±29.28	**33±0.32**✪	52±0.55✪	20±0.45✪	3/3	5.833
M5	1000	1000	674±4.28	69±0.89✪	103✪	357	4/2	8
M6	89±1.78✪	354±5.53	86±1.61✪	135±1.0✪	42±0.71✪	27±7.68✪	1/5	4.167
M7	87±15.87✪	93±14.46✪	95±11.99✪	39±4.42✪	44±4.48✪	91±1.67✪	1/5	**3.333**
M8	192±14.79	205±11.29	196±18.7	698±6.66	**17±3.69**✪	27±7.4✪	4/2	5.167
M9	**72±2.86**✪	**55±2.79**✪	**75±2.77**✪	358±8.77	57±23.38✪	27±3.74✪	1/5	3.5
M10	108±7.97	101±17.86	118±4.88	205±10.35	164±3.42	81±1.33	-	5.333

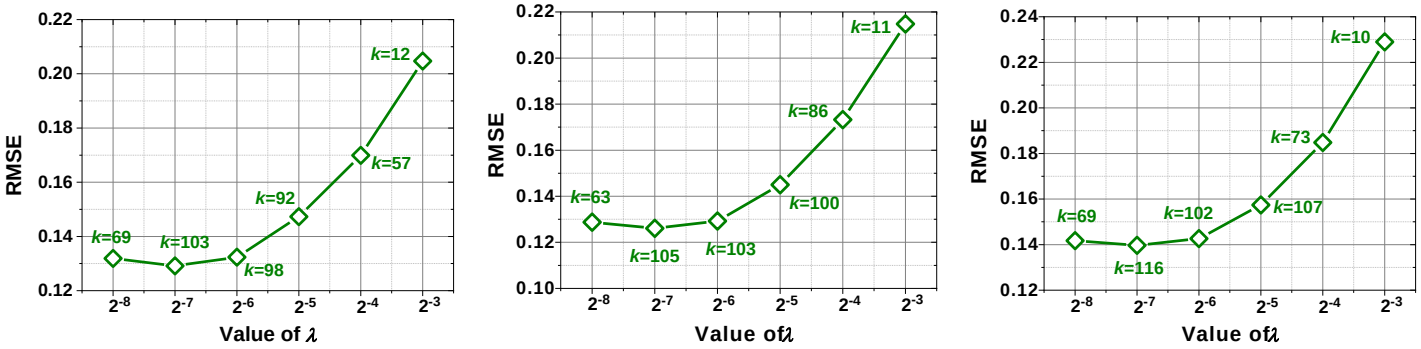
TABLE S4. Time Cost (Sec.) of M1-10 on D1-6, where ✪ indicates M10 has higher Time Cost than the rival model.

No.	D1	D2	D3	D4	D5	D6	✪Win/Loss	Friedman Rank
M1	**64±2.01**✪	**53±4.05**✪	**40±9.7**✪	19±2.51✪	41±7.21✪	74±6.61	2/4	**2.167**
M2	349±17.29	2272±38.06	1103±74.94	109±7.98	5879±46.83	18171±115.57	6/0	7.333
M3	65±5.98✪	130±14.2	67±7.37✪	20±2.11	20±1.57✪	**27±1.84**✪	2/4	2.5
M4	1043±30.48	5759±504.90	2101±247.69	80±1.91	872±26.98	1074±26.41	6/0	7.333
M5	2287±18.82	9542±980.55	2477±131.54	215±3.78	2631±6.11	30074±128.59	6/0	9.167
M6	819±36.38	6998±328.87	1048±79.63	28±0.42	61±2.03	1436±405.71	6/0	6.5
M7	67±13.89✪	94±16.25✪	120±29.15✪	**2±0.31**✪	**19±0.99**✪	44±2.92	1/5	**2.167**
M8	3267±494.54	4994±876.94	4254±608.28	388±20.72	107±23.03	448±104.92	6/0	8.167
M9	652±28.65	555±167.95	822±75.48	227±19.85	68±25.6	1480±235.71	6/0	6.5
M10	88±6.55	112±19.04	178±14.04	11±1.02	48±1.26	42±1.44	-	3.167

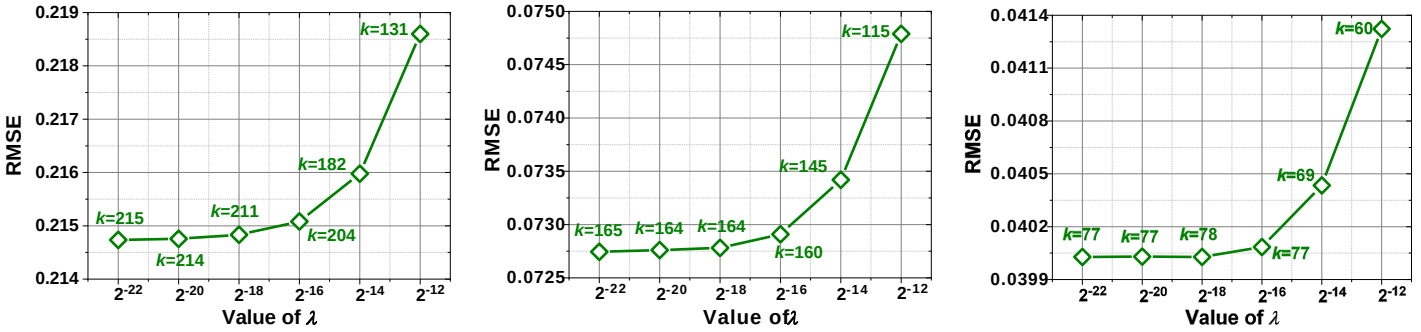
TABLE S5. Results of Wilcoxon Signed-Ranks Test.

Comparison	RMSE			Converging Iteration Count			Time Cost		
	R+	***R-***	***p*-value**	***R+***	***R-***	***p*-value**	***R+***	***R-***	***p*-value**
M1 vs. M10	21	0	**0.0156**	21	0	**0.0156**	15	6	0.2188
M2 vs. M10	21	0	**0.0156**	17	4	0.1094	21	0	**0.0156**
M3 vs. M10	21	0	**0.0156**	15	6	0.2188	17	4	0.1094
M4 vs. M10	21	0	**0.0156**	15	6	0.2188	21	0	**0.0156**
M5 vs. M10	21	0	**0.0156**	18	3	0.0781	21	0	**0.0156**
M6 vs. M10	21	0	**0.0156**	15	6	0.2188	21	0	**0.0156**
M7 vs. M10	21	0	**0.0156**	19	2	**0.0469**	20	1	**0.0313**
M8 vs. M10	21	0	**0.0156**	15	6	0.2188	21	0	**0.0156**
M9 vs. M10	21	0	**0.0156**	15	6	0.2188	21	0	**0.0156**

II. SUPPLEMENTARY FIGURES

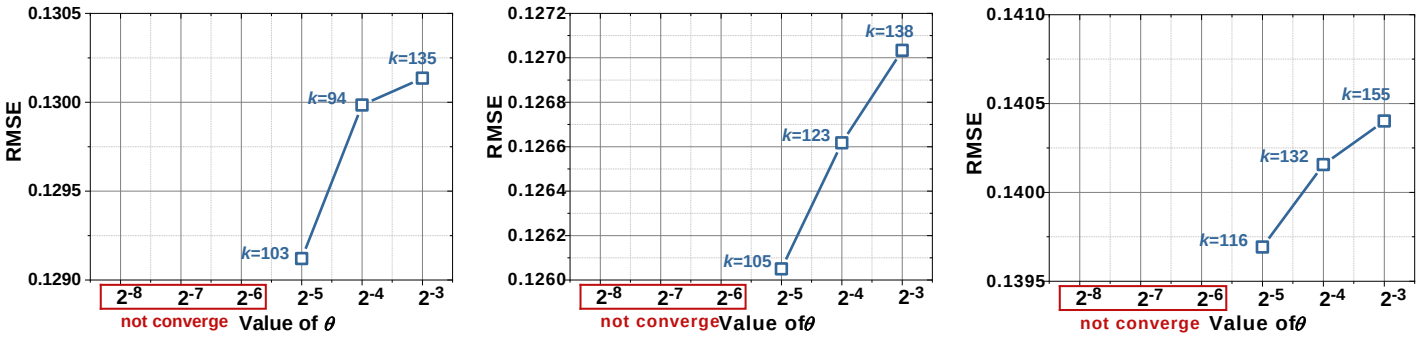


(a) D1. (b) D2. (c) D3.

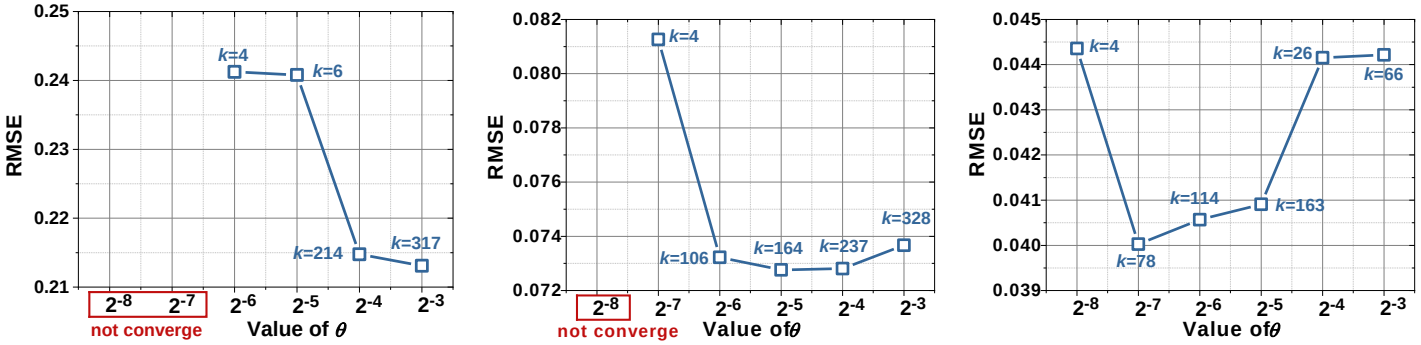


(d) D4. (e) D5. (f) D6.

Fig. S1. Sensitive experiments of λ in RSCN with fixed θ and η. (a)-(c) θ=2^{-5}, η=0.2, (d) θ=2^{-4}, η=0.6, (e) θ=2^{-5}, η=0.2, and (f) θ=2^{-7}, η=0.2.

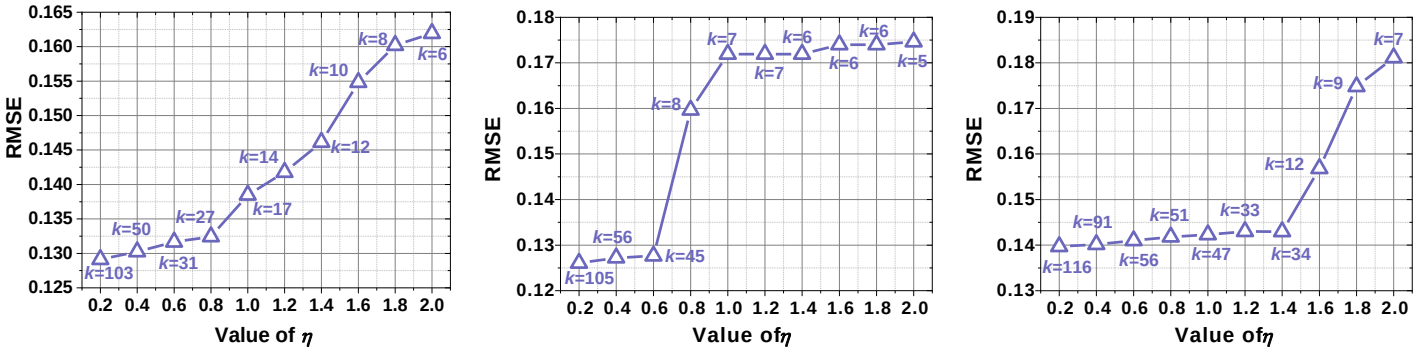


(a) D1. (b) D2. (c) D3.



(d) D4. (e) D5. (f) D6.

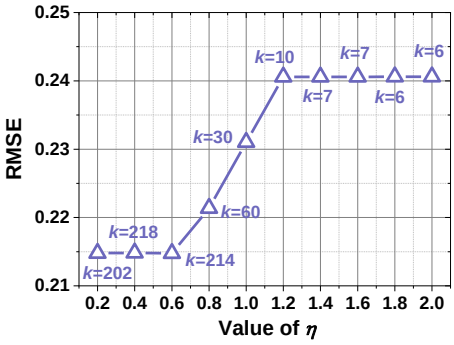
Fig. S2. Sensitive experiments of θ in RSCN with fixed λ and η. (a)-(c) λ=2^{-7}, η=0.2, (d) λ=2^{-20}, η=0.6, (e) λ=2^{-20}, η=0.2, and (f) λ=2^{-18}, η=0.2.



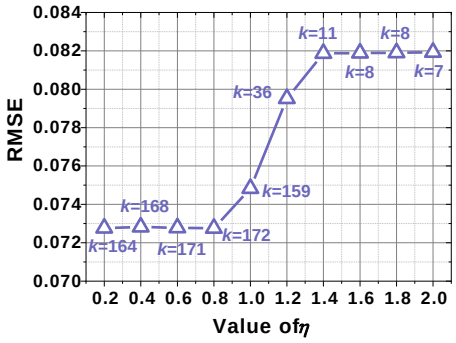
(a) D1.

(b) D2.

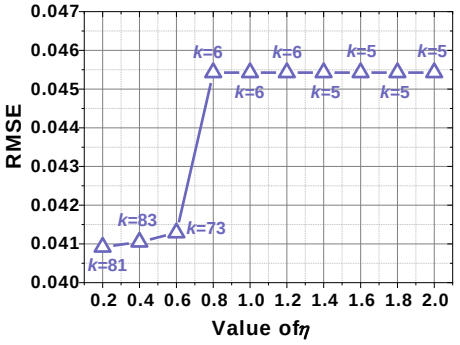
(c) D3.



(d) D4.

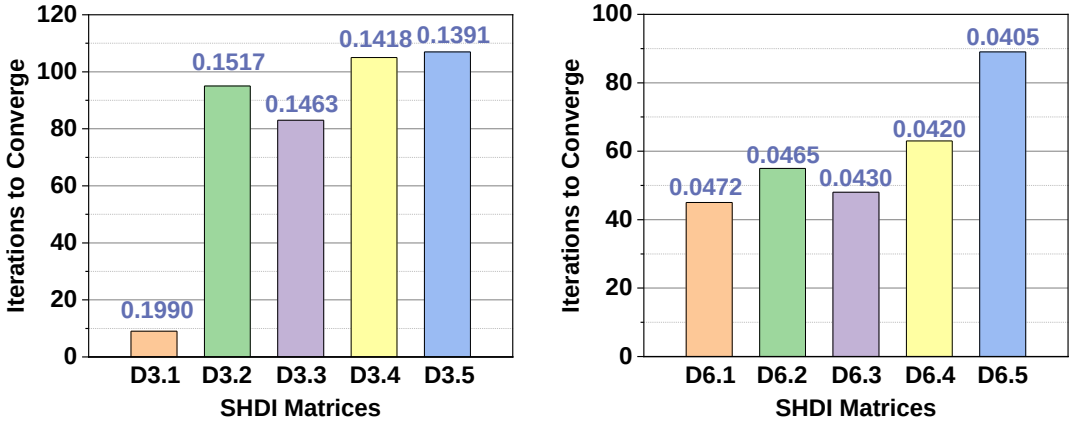


(e) D5.



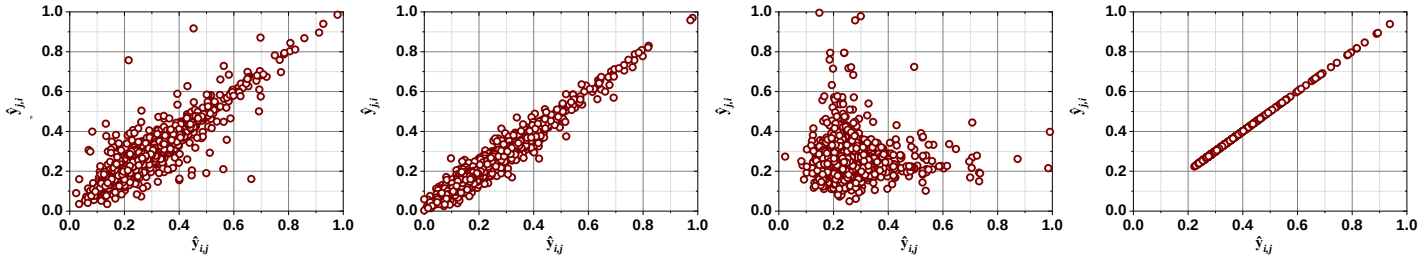
(f) D6.

Fig. S3. Sensitive experiments of η in RSCN with fixed λ and θ. (a)-(c) λ=2^{-7}, θ=2^{-5}, (d) λ=2^{-20}, θ=2^{-4}, (e) λ=2^{-20}, θ=2^{-5}, and (f) λ=2^{-18}, θ=2^{-7}.

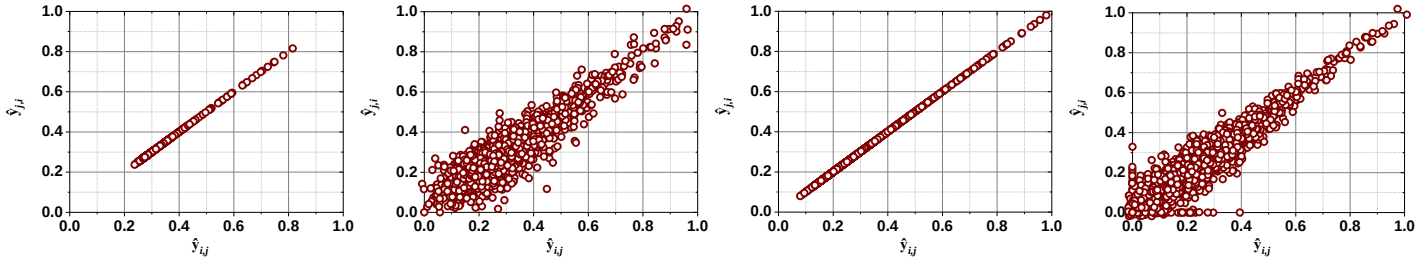


(a) (b)

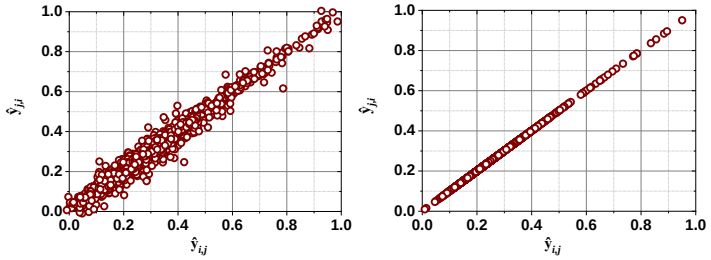
Fig. S4. RSCN's Performance on D3 and D6 as data density varies.



(a) M1 (b) M2 (c) M3 (d) M4



(e) M5 (f) M6 (g) M7 (h) M8



(e) M9 (f) M10

Fig. S5. Symmetric representation of M1-10 on D1.