## A Relaxed Symmetry-Constrained Non-negative Model for Large-Scale Undirected Weighted Network Representation: Supplementary File

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This is the supplementary file for the paper entitled *A Relaxed Symmetry-Constrained Non-negative Model for Large-Scale Undirected Weighted Network Representation*. Some supplementary tables and figures illustrating the experimental results are put in this file and cited in the manuscript.

## I. SUPPLEMENTARY TABLES

TARIE C1	DMSF of M1 8 on 1	D1 6 where A indicates	M8 has lower RMSE than	the rivel model

No.	M1	M2	М3	M4	M5	M6	M7	M8 <b>⊘</b>
D1	0.1319±3.8E-4 <b>❖</b>	0.1333 ±2.4E-4 <b>❖</b>	0.1329±9.8E-5 <b>❖</b>	0.1544±2.6E-4 <b>❖</b>	0.1574±1.2E-4 <b>❖</b>	0.1325±2.6E-4 <b>❖</b>	0.1289±2.4E-4 <b>♦</b>	0.1280±1.8E-4
D2	0.1290±9.4E-5 <b>❖</b>	0.1330±2.0E-4 <b>◆</b>	0.1324±4.5E-4 <b>◆</b>	0.1649±2.7E-4 <b>◆</b>	0.1813±1.6E-4 <b>◆</b>	0.1291±1.7E-4 <b>◆</b>	0.1272±1.0E-4 <b>む</b>	0.1261±3.5E-4
D3	0.1431±1.5E-3 <b>♦</b>	0.1437±2.0E-4 <b>◆</b>	0.1500±1.3E-3 <b>♦</b>	0.1806±4.1E-4 <b>◆</b>	0.1972±1.4E-4 <b>♦</b>	0.1412±2.5E-4 <b>♦</b>	0.1399±2.3E-4 <b>♦</b>	0.1389 ±2.3E-4
D4	0.2301 ±8.6E-4 <b>❖</b>	0.2559±5.5E-4 <b>◆</b>	0.3148±8.3E-4 <b>◆</b>	0.2297±1.3E-4 <b>◆</b>	0.2369±6.8E-4 <b>◆</b>	0.2233±9.7E-4 <b>♦</b>	0.2269±2.2E-3 <b>♦</b>	0.2215±1.3E-4
D5	0.0756±3.6E-4 <b>❖</b>	0.0821±1.7E-5 <b>❖</b>	0.1561 ±8.0E-4 <b>◆</b>	0.0740±1.2E-5 <b>◆</b>	0.0764±8.1E-5 <b>❖</b>	0.0741±2.6E-4 <b>♦</b>	0.0739±3.7E-4 <b>♦</b>	0.0728±3.6E-4
D6	0.0423 ±7.7E-5 <b>❖</b>	0.0464±1.3E-5 <b>♦</b>	0.0931 ±4.9E-4 <b>◆</b>	0.0415±8.5E-5 <b>❖</b>	0.0430±5.3E-5 <b>♦</b>	0.0413±7.2E-5 <b>♦</b>	0.0412±8.1E-5 <b>♦</b>	0.0409 ±1.9E-4
<b>⊘</b> Win/ Loss	6/0	6/0	6/0	6/0	6/0	6/0	6/0	-
Friedman Rank	4.167	6.333	6.667	5.333	7	3.333	2.167	1

TABLE S2. Converging Iteration Count of M1-8 on D1-6, where 🕈 indicates M8 has less Converging Iteration Count than the rival model.

No.	M1	M2	М3	M4	M5	M6	M7	M8 <b>≎</b>
D1	341 ±2.87�	187 ±4.0�	199±7.41�	685±16.71 <b>♦</b>	1000�	349 ±2.22€	87±15.87	$108\pm7.97$
D2	192±6.45♥	311±5.20€	175±17.90 <b>€</b>	975 ±42.01�	1000€	173 ±3.41�	93±14.46	101±17.86
D3	122±5.79 <b>♦</b>	284±3.24�	205 ±48.28€	959±29.28�	674±4.28 <b>€</b>	118±5.12 <b>◆</b>	95±11.99	118±4.88
D4	967±4.76€	40±2.49	88±4.15	33±0.32	69±0.89	1000€	39±4.42	205±10.35
D5	568±5.13 <b>♀</b>	251 ±1.22 <b>♦</b>	77±2.79	52±0.55	103	1000♥	44±4.48	164±3.42
D6	582±2.13 <b>♥</b>	227 ±1.26 <b>♦</b>	51±1.37	20±0.45	357 <b>↔</b>	1000❖	91±1.67 <b>♦</b>	81±1.33
<b>⊘</b> Win/ Loss	6/0	5/1	3/3	3/3	4/2	6/0	1/5	-
Friedman Rank	5.833	4.833	3.833	4.333	6.167	5.917	1.667	3.417

TABLE S3. Time Cost (Sec.) of M1-8 on D1-6, where ② indicates M8 has less Time Cost than the rival model.

No.	M1	M2	M3	M4	M5	M6	M7	M8 <b>⊘</b>
D1	$64\pm2.01$	349±17.29 <b>♦</b>	6569±152.57 <b>♦</b>	1043 ±30.48�	2287±18.82 <b>♦</b>	34±2.82	67±13.89	88±6.55
D2	53±4.05	2272±38.06 <b>€</b>	12197±814.95 <b>♥</b>	5759±504.90 <b>�</b>	9542±980.55♥	22±3.41	94±16.25	112±19.04
D3	40±9.7	1103 ±74.94 <b>◆</b>	48±11.67 <b>℃</b>	2101±247.69�	2477 ±131.54 <b>♦</b>	19±2.91	120±29.15	$178\pm14.04$
D4	19±2.51✿	109±7.98 <b>♦</b>	2013±78.62 <b>♦</b>	80±1.91 <b>≎</b>	215±3.78 <b>♥</b>	11 ±2.09�	2±0.31	11±1.02
D5	41 ±7.21 <b>♥</b>	5879 ±46.83 <b>♥</b>	12527±389.87 <b>❖</b>	872±26.98 <b>℃</b>	2631 ±6.11♥	50±2.48 <b>℃</b>	19±0.99	48±1.26
D6	74±6.61 <b>♥</b>	18171±115.57 <b>♦</b>	30029±651.33 <b>♦</b>	1074 ±26.41�	30074±128.59 <b>♦</b>	81±3.54 <b>℃</b>	44±2.92 <b>♦</b>	42±1.44
<b>⊘</b> Win/ Loss	3/3	6/0	6/0	6/0	6/0	3/3	1/5	-
Friedman Rank	2.5	5.833	7	5.667	7.167	2.25	2.333	3.25

TABLE S4. Results of Wilcoxon Signed-Ranks Test.

Comparison	RMSE			Converging Iteration Count				Time C	Time Cost	
	<b>R</b> +	R-	<i>p</i> -value	<b>R</b> +	R-	<i>p</i> -value	<b>R</b> +	R-	<i>p</i> -value	
M1 vs. M8	21	0	0.0156	21	0	0.0156	15	6	0.2188	
M2 vs. M8	21	0	0.0156	17	4	0.1094	21	0	0.0156	
M3 vs. M8	21	0	0.0156	10.5	10.5	0.5313	20	1	0.0313	
M4 vs. M8	21	0	0.0156	15	6	0.2188	21	0	0.0156	
M5 vs. M8	21	0	0.0156	18	3	0.0781	21	0	0.0156	
M6 vs. M8	21	0	0.0156	15	0	0.0313	12	3	0.1563	
M7 vs. M8	21	0	0.0156	19	2	0.0469	20	1	0.0313	

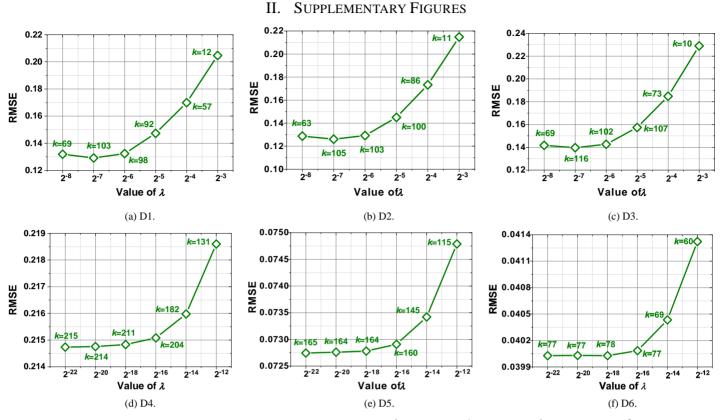


Fig. S1. Sensitive experiments of  $\lambda$  in RSCN with fixed  $\theta$  and  $\eta$ . (a)-(c)  $\theta = 2^{-5}$ ,  $\eta = 0.2$ , (d)  $\theta = 2^{-4}$ ,  $\eta = 0.6$ , (e)  $\theta = 2^{-5}$ ,  $\eta = 0.2$ , and (f)  $\theta = 2^{-7}$ ,  $\eta = 0.2$ .

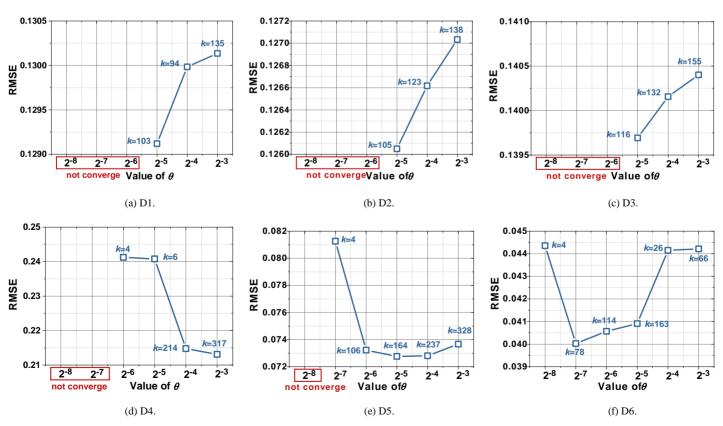


Fig. S2. Sensitive experiments of  $\theta$  in RSCN with fixed  $\lambda$  and  $\eta$ . (a)-(c)  $\lambda = 2^{-7}$ ,  $\eta = 0.2$ , (d)  $\lambda = 2^{-20}$ ,  $\eta = 0.6$ , (e)  $\lambda = 2^{-20}$ ,  $\eta = 0.2$ , and (f)  $\lambda = 2^{-18}$ ,  $\eta = 0.2$ .

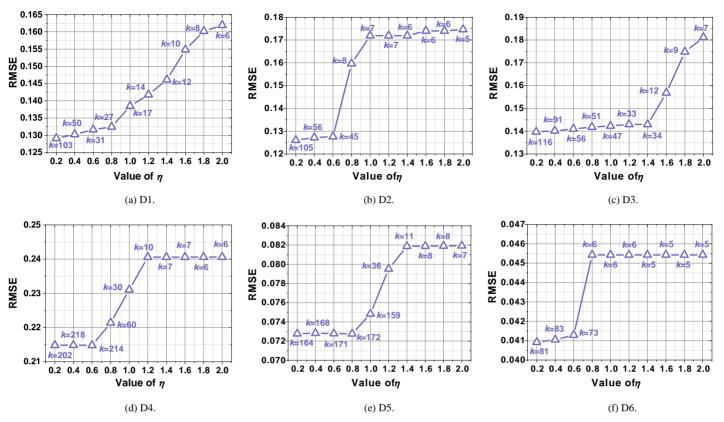


Fig. S3. Sensitive experiments of  $\eta$  in RSCN with fixed  $\lambda$  and  $\theta$ . (a)-(c)  $\lambda = 2^{-7}$ ,  $\theta = 2^{-5}$ , (d)  $\lambda = 2^{-20}$ ,  $\theta = 2^{-4}$ , (e)  $\lambda = 2^{-20}$ ,  $\theta = 2^{-5}$ , and (f)  $\lambda = 2^{-18}$ ,  $\theta = 2^{-7}$ .

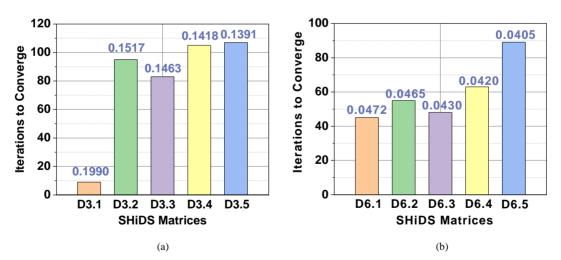


Fig. S4. RSCN's Performance on D3 and D6 as data density varies.

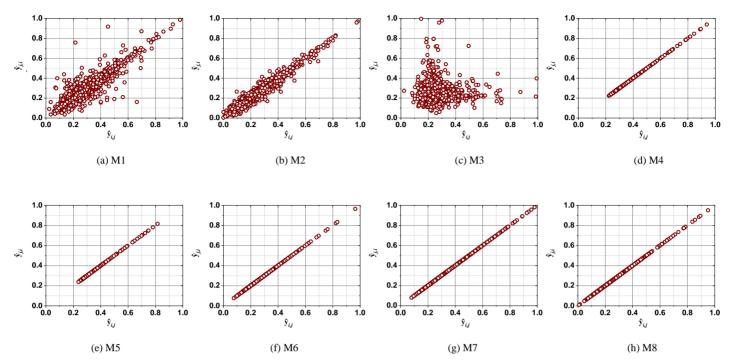


Fig. S5. Symmetric representation of M1-8 on D1.