## A Relaxed Symmetry-Constrained Non-negative Model for Large-Scale Undirected Weighted Network Representation: Supplementary File

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This is the supplementary file for the paper entitled *A Relaxed Symmetry-Constrained Non-negative Model for Large-Scale Undirected Weighted Network Representation*. Some supplementary tables and figures illustrating the experimental results are put in this file and cited in the manuscript.

## I. SUPPLEMENTARY TABLES

TABLE S1. RMSE of M1-8 on D1-6, where ② indicates M8 has lower RMSE than the rival model.

No.	M1	M2	М3	M4	M5	M6	M7	M8 <b>©</b>
D1	0.1319±3.8E-4 <b>❖</b>	0.1333 ±2.4E-4 <b>❖</b>	0.1329±9.8E-5 <b>❖</b>	0.1544±2.6E-4 <b>❖</b>	0.1574±1.2E-4 <b>❖</b>	0.1318±1.6E-4 <b>❖</b>	0.1289±2.4E-4 <b>♦</b>	0.1280±1.8E-4
D2	0.1290±9.4E-5 <b>❖</b>	0.1330±2.0E-4 <b>◆</b>	0.1324±4.5E-4 <b>◆</b>	0.1649±2.7E-4 <b>◆</b>	0.1813±1.6E-4 <b>◆</b>	0.1297±1.4E-4 <b>♦</b>	0.1272±1.0E-4 <b>む</b>	0.1261±3.5E-4
D3	0.1431±1.5E-3 <b>♦</b>	0.1437±2.0E-4 <b>◆</b>	0.1500±1.3E-3 <b>♦</b>	0.1806±4.1E-4 <b>◆</b>	0.1972±1.4E-4 <b>♦</b>	0.1415±1.3E-4 <b>♦</b>	0.1399±2.3E-4 <b>♦</b>	0.1389 ±2.3E-4
D4	0.2301 ±8.6E-4 <b>❖</b>	0.2559±5.5E-4 <b>◆</b>	0.3148±8.3E-4 <b>◆</b>	0.2297±1.3E-4 <b>◆</b>	0.2369±6.8E-4 <b>❖</b>	0.3079±7.2E-4 <b>♦</b>	0.2269±2.2E-3 <b>♦</b>	0.2215±1.3E-4
D5	0.0756±3.6E-4 <b>❖</b>	0.0821±1.7E-5 <b>❖</b>	0.1561 ±8.0E-4 <b>❖</b>	0.0740±1.2E-5 <b>◆</b>	0.0764±8.1E-5 <b>❖</b>	0.1037±2.2E-4 <b>♦</b>	0.0735±3.7E-4 <b>♦</b>	0.0728±3.6E-4
D6	0.0423 ±7.7E-5 <b>❖</b>	0.0464±1.3E-5 <b>♦</b>	0.0931 ±4.9E-4 <b>♦</b>	0.0415±8.5E-5 <b>❖</b>	0.0430±5.3E-5 <b>♦</b>	0.0631±6.9E-5 <b>♦</b>	0.0412±8.1E-5 <b>♦</b>	0.0409 ±1.9E-4
<b>⊘</b> Win/ Loss	6/0	6/0	6/0	6/0	6/0	6/0	6/0	-
Friedman Rank	3.833	5.833	6.667	5	6.5	5.167	2	1

TABLE S2. Converging Iteration Count of M1-8 on D1-6, where ❖ indicates M8 has less converging iteration count than the rival model.

No.	M1	M2	М3	M4	M5	M6	M7	M8 <b>②</b>	
D1	341 ±2.87₺	187 ±4.0�	199±7.41 <b>♦</b>	685±16.71 <b>€</b>	1000✿	89±1.78	87±15.87	108±7.97	
D2	192±6.45�	311±5.20€	175 ±17.90 <b>♦</b>	975 ±42.01�	1000✿	79±2.39	93±14.46	101±17.86	
D3	122±5.79 <b>♦</b>	284±3.24€	205 ±48.28❖	959±29.28�	674±4.28 <b>◆</b>	86±1.61	95±11.99	118±4.88	
D4	967±4.76�	40±2.49	88±4.15	33±0.32	69±0.89	$135\pm1.0$	39±4.42	205±10.35	
D5	568±5.13 <b>❖</b>	251 ±1.22 <b>۞</b>	77±2.79	52±0.55	103	42±0.71	44±4.48	164±3.42	
D6	582±2.13 <b>©</b>	227 ±1.26 <b>♦</b>	51±1.37	20±0.45	357❖	30±0.91	91±1.67 <b>�</b>	81±1.33	
<b>⊘</b> Win/ Loss	6/0	5/1	3/3	3/3	4/2	0/6	1/5	-	
Friedman Rank	6.5	5.333	4.333	4.5	6.5	2.167	2.333	4.333	

TABLE S3. Time Cost (Sec.) of M1-8 on D1-6, where ② indicates M8 has less Time Cost than the rival model.

No.	M1	M2	M3	M4	M5	M6	M7	M8 <b>©</b>
D1	64±2.01	349±17.29 <b>♦</b>	6569±152.57 <b>♥</b>	1043 ±30.48 <b>♦</b>	2287±18.82 <b>❖</b>	1086±165 <b>❖</b>	67±13.89	88±6.55
D2	53±4.05	2272±38.06 <b>❖</b>	12197±814.95 <b>♦</b>	5759±504.90 <b>♦</b>	9542±980.55€	1106±188 <b>≎</b>	94±16.25	112±19.04
D3	40±9.7	1103 ±74.94 <b>�</b>	48±11.67	2101 ±247.69❖	2477 ±131.54 <b>♦</b>	1298±214✿	120 ±29.15	178±14.04
D4	19±2.51 <b>♥</b>	109 ±7.98�	2013 ±78.62 <b>♦</b>	80±1.91 <b>⊘</b>	215±3.78 <b>♀</b>	47±6.83€	2±0.31	11±1.02
D5	41±7.21	5879 ±46.83❖	12527±389.87 <b>♦</b>	872±26.98 <b>♦</b>	2631 ±6.11 <b>♦</b>	94±8.92✿	19±0.99	48±1.26
D6	74±6.61 <b>♥</b>	18171±115.57 <b>♀</b>	30029±651.33❖	1074 ±26.41�	30074±128.59 <b>3</b>	162±15.37 <b>♥</b>	44±2.92 <b>❖</b>	42±1.44
<b>Q</b> Win/ Loss	3/3	6/0	6/0	6/0	6/0	6/0	1/5	-
Friedman Rank	1.833	5.5	6.833	5.5	7.167	4.667	1.833	2.667

TABLE S4. Results of Wilcoxon Signed-Ranks Test.

Comparison	RMSE			Converging Iteration Count				Time C	Time Cost	
	R+	R-	<i>p</i> -value	<b>R</b> +	R-	<i>p</i> -value	R+	R-	<i>p</i> -value	
M1 vs. M8	21	0	0.0156	21	0	0.0156	15	6	0.2188	
M2 vs. M8	21	0	0.0156	17	4	0.1094	21	0	0.0156	
M3 vs. M8	21	0	0.0156	10.5	10.5	0.5313	20	1	0.0313	
M4 vs. M8	21	0	0.0156	15	6	0.2188	21	0	0.0156	
M5 vs. M8	21	0	0.0156	18	3	0.0781	21	0	0.0156	
M6 vs. M8	21	0	0.0156	21	0	0.0156	21	0	0.0156	
M7 vs. M8	21	0	0.0156	19	2	0.0469	20	1	0.0313	

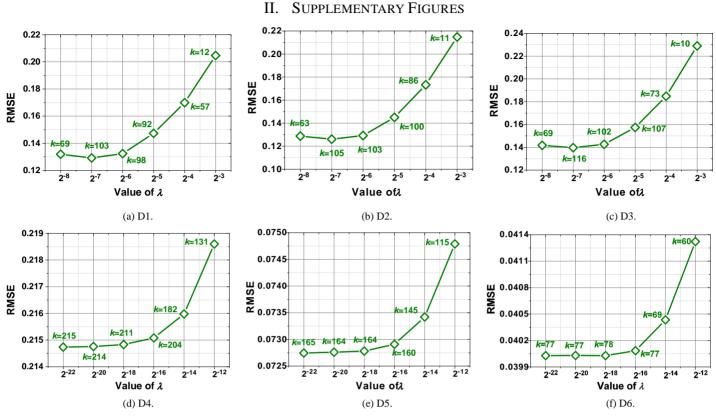


Fig. S1. Sensitive experiments of  $\lambda$  in RSCN with fixed  $\theta$  and  $\eta$ . (a)-(c)  $\theta = 2^{-5}$ ,  $\eta = 0.2$ , (d)  $\theta = 2^{-4}$ ,  $\eta = 0.6$ , (e)  $\theta = 2^{-5}$ ,  $\eta = 0.2$ , and (f)  $\theta = 2^{-7}$ ,  $\eta = 0.2$ .

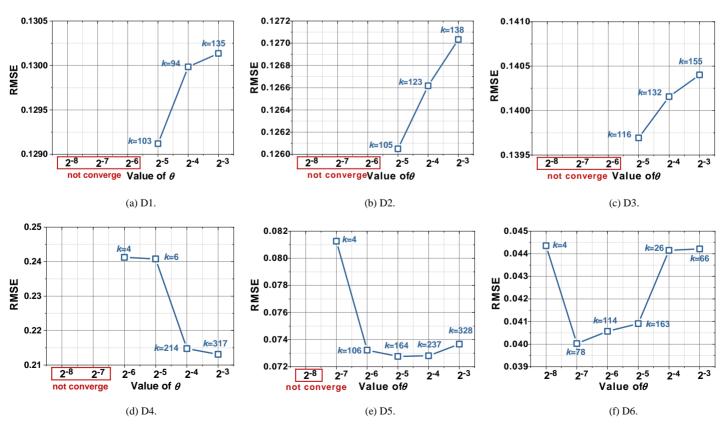


Fig. S2. Sensitive experiments of  $\theta$  in RSCN with fixed  $\lambda$  and  $\eta$ . (a)-(c)  $\lambda = 2^{-7}$ ,  $\eta = 0.2$ , (d)  $\lambda = 2^{-20}$ ,  $\eta = 0.6$ , (e)  $\lambda = 2^{-20}$ ,  $\eta = 0.2$ , and (f)  $\lambda = 2^{-18}$ ,  $\eta = 0.2$ .

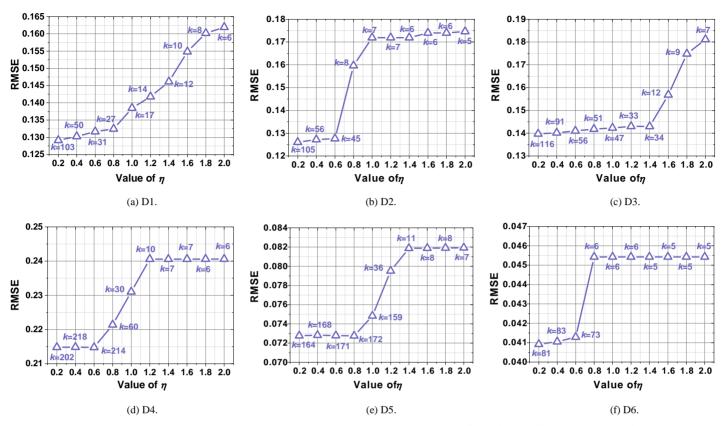


Fig. S3. Sensitive experiments of  $\eta$  in RSCN with fixed  $\lambda$  and  $\theta$ . (a)-(c)  $\lambda = 2^{-7}$ ,  $\theta = 2^{-5}$ , (d)  $\lambda = 2^{-20}$ ,  $\theta = 2^{-4}$ , (e)  $\lambda = 2^{-20}$ ,  $\theta = 2^{-5}$ , and (f)  $\lambda = 2^{-18}$ ,  $\theta = 2^{-7}$ .

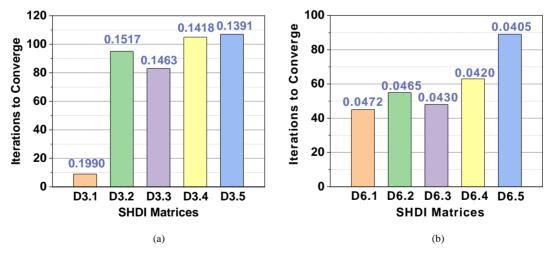


Fig. S4. RSCN's Performance on D3 and D6 as data density varies.

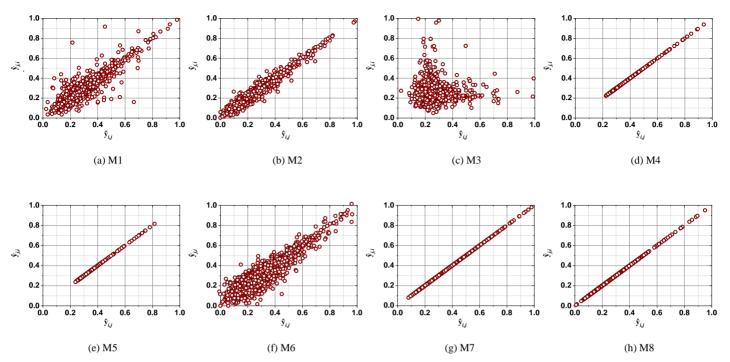


Fig. S5. Symmetric representation of M1-8 on D1.