# Graphon Trading Networks

## Static Properties

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# Outline

# Motivation

# Research Motivation





## Motivation

This research is motivated by the paper Graphon Games: A Statistical Framework for Network Games and Interventions.

## Literature Review

In this paper, they present a unifying framework for analyzing equilibria and designing interventions for large network games represented by a graphon. They gave 3 examples:

1. Linear quadratic network games
2. Community structure for discrete agent types
3. Community structure for continuous agent types

They used real world data on households across villages in India, and showed that initial investigation of graphon can be used as a tool to model strategic behavior.

# Introduction

# Data Description

We start with a fixed aggregate amount of each commodity, and partition them randomly among the agents as initial endowment.

- 20,40,60 agents
- Two commodities A,B
- pairs(x,y)

# Network:Graphon

The graphon, in its simplest form, is a function $W : [0,1)^2 \mapsto [0,1]$ that gives the probability that two agents, indexed by their location on the unit segment, have a trading relationship.

## Community Structure

The unit segment can be partitioned into $k = 2$ subsegments $\{\mathcal{S}_k\}_{k=1,\ldots,K}$, defining "communities". Fix $g_{\text{in}}, g_{\text{out}} \in [0,1]$, with typically $g_{\text{in}} > g_{\text{out}}$, and define

$$(x, y) = \begin{cases} g_{\text{in}} = 0.8 & \exists\ k \ni x, y \in \mathcal{S}_k \\ g_{\text{out}} = 0.1 & \text{otherwise} \end{cases}$$

That is, members of a community are more likely to trade with each other than with outsiders.


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# Network: Graphon

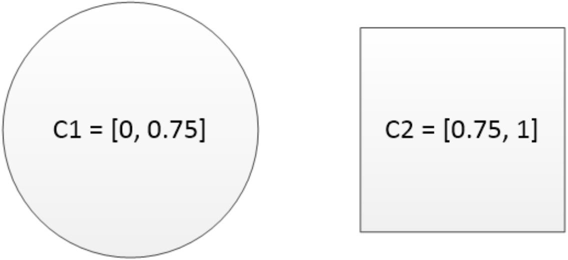


Figure: Graphon network of 2 communities

# Network: Graphon

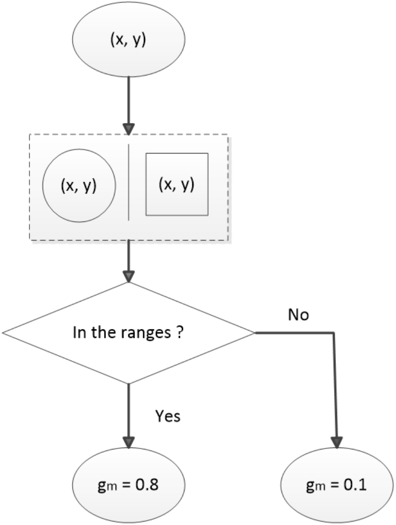


Figure: Graphon relation logic

# Leontief Potential

Consider a simple economy with two complementary commodities, $A$ and $B$.

- Agents ascribe no utility to any excess in $A$ or $B$,
- but are willing to trade that excess with each other in order to increase $\min(A, B)$.

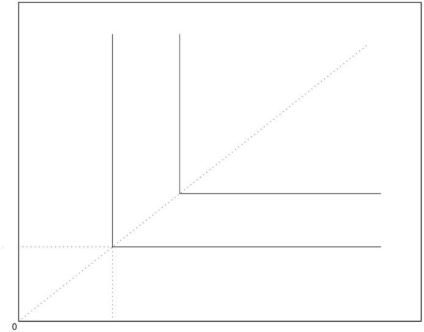


Figure: Leonteif Production Function

# Methodology

# Trading strategy

- Start with a fixed, finite realization of the graphon, and let graphon determined the list of potential trading pairs.
- Calculate $\Delta$ for each pair in turn to determine if a trade can take place.
- If the $\Delta \neq 0$, uniformly random select $(\Delta A, \Delta B)$ from within the feasible region and adjust the quantities of commodities A, B accordingly.
- We would expect the exchange rate $(\Delta A/\Delta B)$ to converge to utility in a finite number of rounds.

# Initialization

- Fix the aggregate amount of each commodity A, B to one unit.
- Draw independent random Gamma variates and allocate in proportion for each commodity as initial endowments.
- If agents have opportunity to trade, the trade should increase the utility of at least one and not decrease the utility for either.
- Agents will only trade surpluses with each other.

# Perturbation

- Consider a second endowment, draw another independent random Gamma variates and allocate to each commodity.
- We want to explore the nature of re-convergence in response to perturbations, as functions of the graphon and the nature of the perturbation.

# Preliminary Data Results

# Static Properties

Convergence of a sample trading network of 20 agents

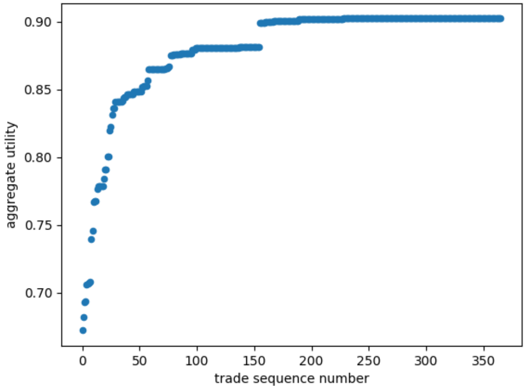


Figure: Convergence of 20 agents

# Static Properties

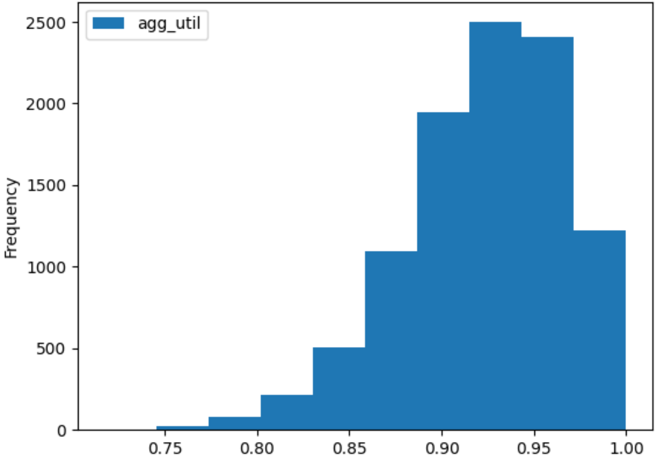


Figure: Aggregate utility distribution of 20 agents

# Static Properties

Convergence of a sample trading network of 40 agents

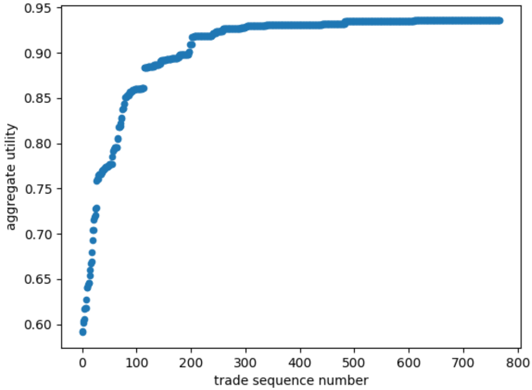


Figure: Convergence of 40 agents

# Static Properties

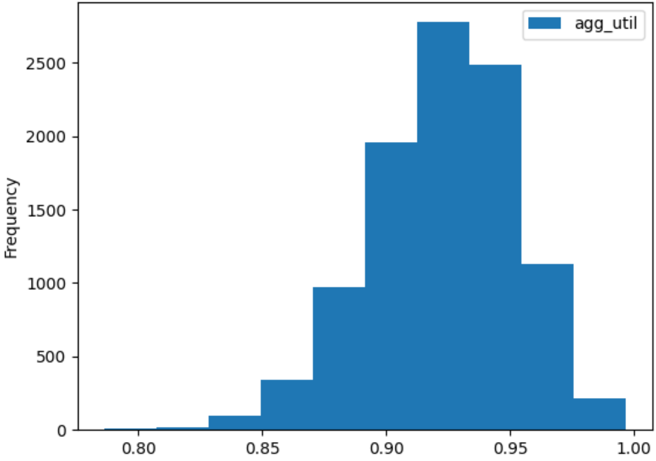


Figure: Aggregate utility distribution of 40 agents

# Static Properties

Convergence of a sample trading network of 60 agents

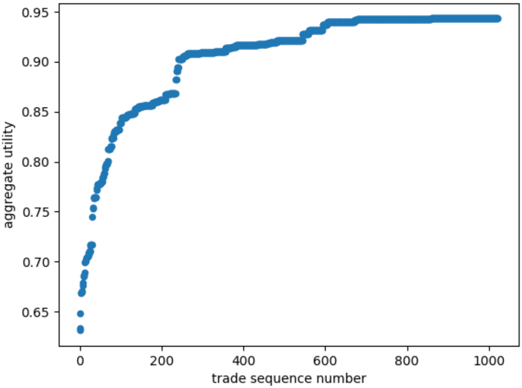


Figure: Convergence of 60 agents

# Static Properties

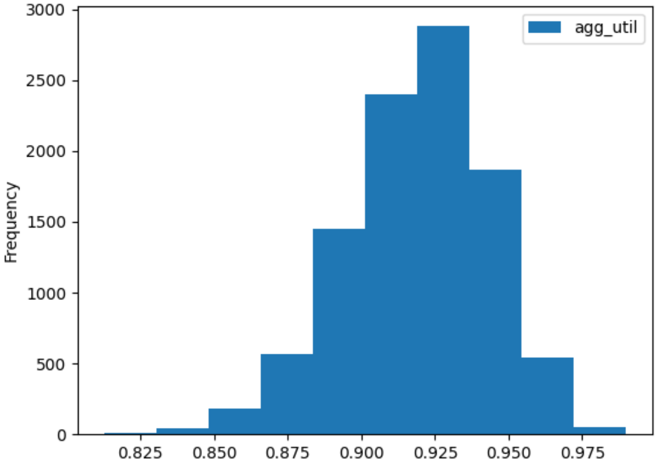


Figure: Aggregate utility distribution of 60 agents

# Question Remained

# Question Remained

- We want to see that after a tremendous amount of trading,the exchange rate (aggregate utility) should be converged to 1.
- What's interrupting the convergence here?
- One hypothesis is that some pairs of agents were not able to trade by graphon and seemed to be stuck at the first round. These agents have significant surplus and they are not interested in trading with anybody, so the convergence to 1 is interrupted.
- Another setup may solve this.

# References

# Reference

Parise, Francesca, and Asuman Ozdaglar. Graphon Games: A Statistical Framework for Network Games and Interventions.

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