Homework #3

ECE661

Name:Chengzhang Zhong

Email:zhongc@purdue.edu

Method introduction

Two step method:

1. Remove projective distortion

To compute the homogenous to remove projective distortion, we need to find the vanishing line first. Thus, we first need to find two pairs of lines in the image, which suppose to be parallel in the world coordinate. For example, the four points can be represented as $p_1, p_{2,} p_3, p_{4.}$ Two pairs of parallel lines are $l_1 = p_1 X p_2$,$l_2 = p_3 X p_4$, $l_3 = p_1 X p_3$ and $l_4 = p_2 X p_4$.Then, the two points A and B on the vanishing line are $A = l_1 X l_2$ and $B = l_3 X l_4$. Now the vanishing line can be written as:

$$V = A X B$$

Where $V = [k_1 \ k_2 \ k_3]^T$. Now the Homography that maps this vanishing line to infinity can be written as:

$$H_p = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ k_1 & k_2 & k_3 \end{bmatrix}$$

Apply this Homography to the image, we can get rid of the projective distortion from the image.

2. Remove affine distortion

To remove affine distortion, two pairs of orthogonal lines are required.

For example, if we have orthogonal lines $l = [l_1 \ l_2 \ l_3]^T$ and $m = [m_1 \ m_2 \ m_3]^T$

The angle θ between them can be represent as:

$$\cos(\theta) = \frac{l_1 m_1 + l_2 m_2}{\sqrt{(l_1^2 + l_2^2)(m_1^2 + m_2^2)}}$$

Now, represent it by the dual degenerate conic, where:

$$C_\infty^* = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$$\cos(\theta) = \frac{l^T C_\infty^* m}{\sqrt{(l^T C_\infty^* l)(m^T C_\infty^* m)}}$$

After further analysis, we have:

$$l^T H_a C_\infty^* H_a^T m = [l_1 \;\; l_2 \;\; l_3] \begin{bmatrix} AA^T & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} m_1 \\ m_2 \\ m_3 \end{bmatrix} = 0$$

where H_a is the homography we want to use:

$$H_a = \begin{bmatrix} A & 0 \\ 0 & 1 \end{bmatrix}$$

Now, we define $S = AA^T$, and the equation equals to:

$$[l_1 \quad l_2] \begin{bmatrix} s_{11} & s_{12} \\ s_{12} & s_{22} \end{bmatrix} \begin{bmatrix} m_1 \\ m_2 \end{bmatrix} = 0$$

We assume $s_{22} = 1$, and then:

$$H_a (l_1 m_1 \quad l_1 m_2 + l_2 m_1) \begin{bmatrix} s_{11} \\ s_{12} \end{bmatrix} = -l_2 m_2$$

The relation between S and A are:

$$A = VDV^T$$

$$S = VD^2V^T$$

Finally, we can use ${H_a}^{-1} H_p$ to remove both projective and affine distortion.

One step method:

This method is based on orthogonal lines. For example, two orthogonal lines $l = [l_1 \;\; l_2 \; l_3]^T$ and $m = [m_1 \;\; m_2 \; m_3]^T$. The format can be written as:

$$[l_1 \;\; l_2 \;\; l_3] \begin{bmatrix} a & b/2 & d/2 \\ b/2 & c & e/2 \\ d/2 & e/2 & f \end{bmatrix} \begin{bmatrix} m_1 \\ m_2 \\ m_3 \end{bmatrix} = 0$$

where:

$$C_\infty^* = \begin{bmatrix} a & b/2 & d/2 \\ b/2 & c & e/2 \\ d/2 & e/2 & f \end{bmatrix}$$

In here, we assume $f = 1$ and there left a, b, c, d, e five unknowns, which requires 5 pairs of orthogonal lines to solve this problem.

After solving C_∞^*, which is also:

$$C_\infty^* = \begin{bmatrix} AA^T & Av \\ v^T A^T & v^T v \end{bmatrix}$$

The Homography needs to use here is:

$$H = \begin{bmatrix} A & 0 \\ v^T & 1 \end{bmatrix}$$

Now, we define S = $\begin{bmatrix} a & b/2 \\ b/2 & c \end{bmatrix}$:

$$A = VDV^T$$

$$S = VD^2V^T$$

and $v^T A^T = [d/2 \quad e/2]$ to solve v^T.

Compare one step method and two step method:

For both one step and two step methods, it is hard to implement the procedure of selecting points for orthogonal lines. However, one step method requires more attempts than two step method to generate undistorted image. Thus, I consider two step method is more robust than one step method.

Also, two step methods requires only 4 points as input, and I picked 7 points for the one point method to work. From the size of input perspective, two step method is better than one step method.

Although one step method is less robust than two step method, sometimes it gives a better result. For example, the flatiron image, two step method's result looks more decline than

one step method.

Basic result demonstration(no lines illustration):

See images with lines marked in the next session

1. flatiron.jpg

Figure 1.a original image

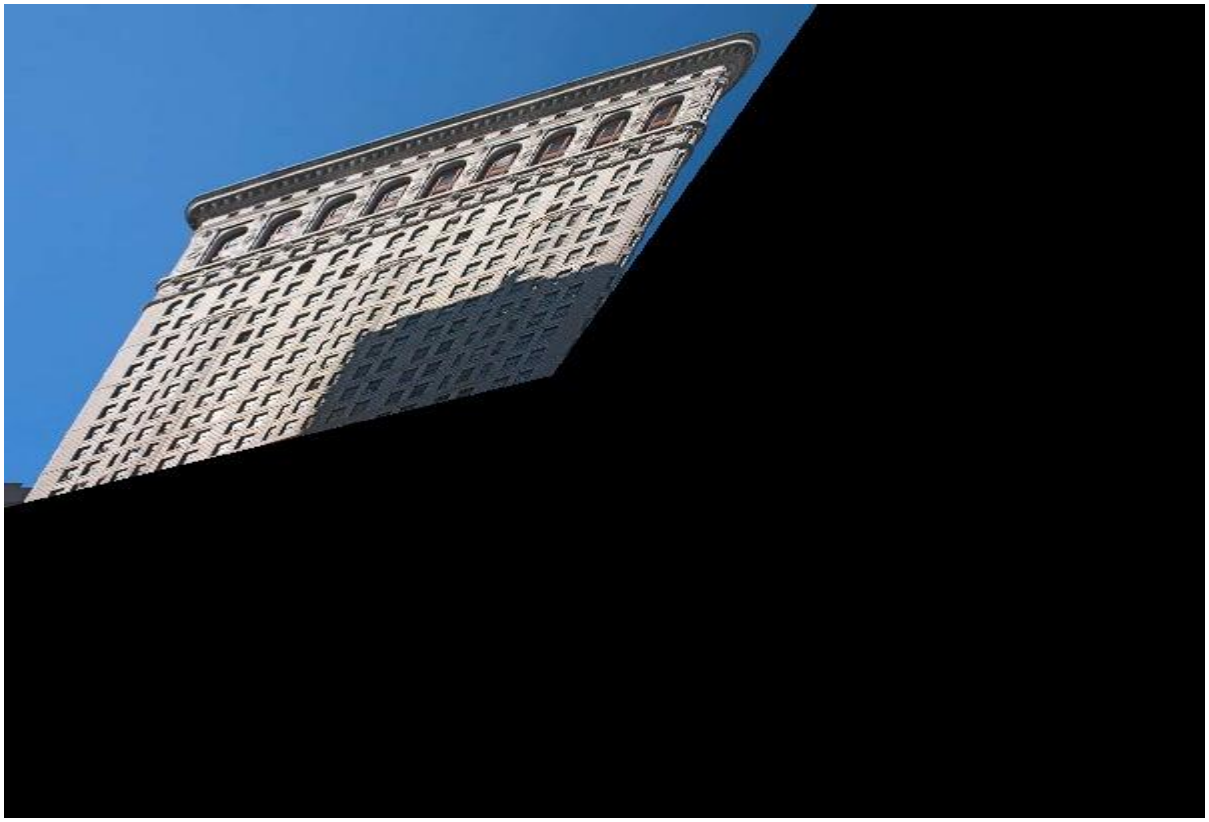

Figure 1.b two-step method, projective distortion removed

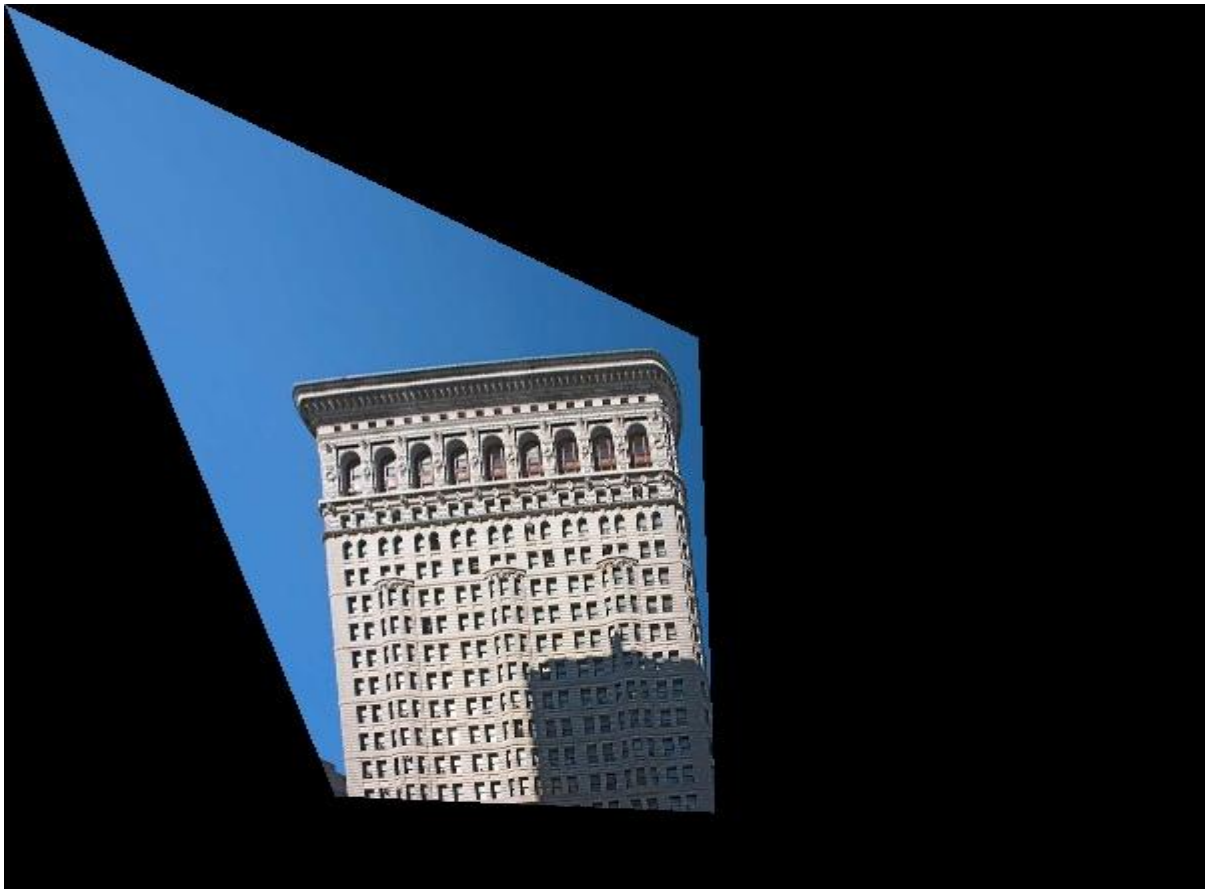

Figure 1.c two-step method, affine distortion removed

Figure 1.d one-step method

2. monalisa.jpg

Figure 2.a original image

Figure 2.b two-step method, projective distortion removed

Figure 2.c two-step method, affine distortion removed

Figure 2.d one-step method

3. wideangle.jpg

Figure 3.a original image

Figure 3.b two-step method, projective distortion removed

Figure 3.c two-step method, affine distortion removed

Figure 3.d one-step method

4. my first image

Figure 4.a original image

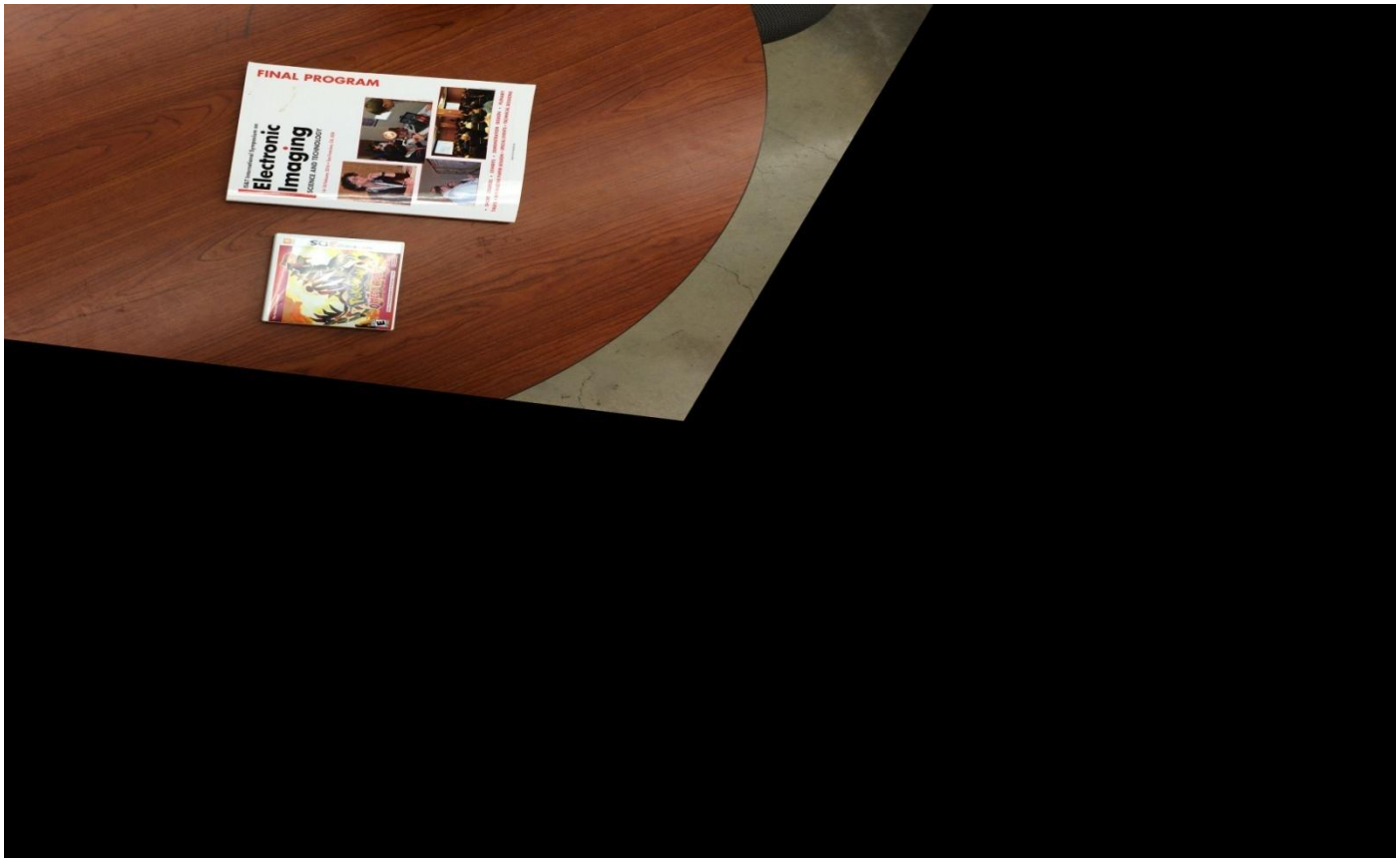

Figure 4.b two step method, projective distortion removed

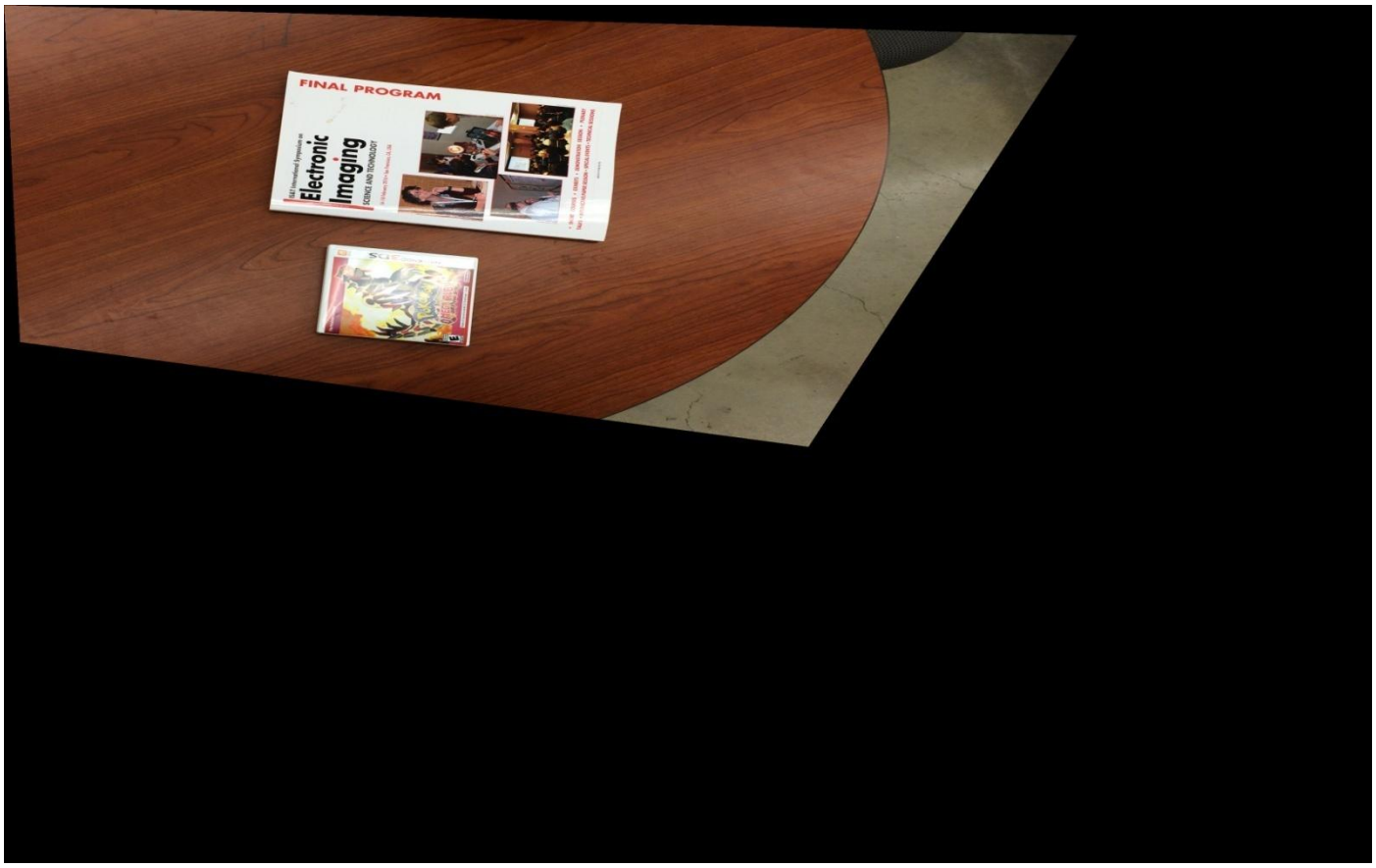

Figure 4.c two step method, affine distortion removed

Figure 4.d one-step method

5. my second image

Figure 5.a original image

Figure 5.b two-step method, projective distortion removed

Figure 5.c two-step method, affine distortion removed

Figure 5.d one-step method

Result demonstration:

1. flatiron.jpg

Figure 1.a original image

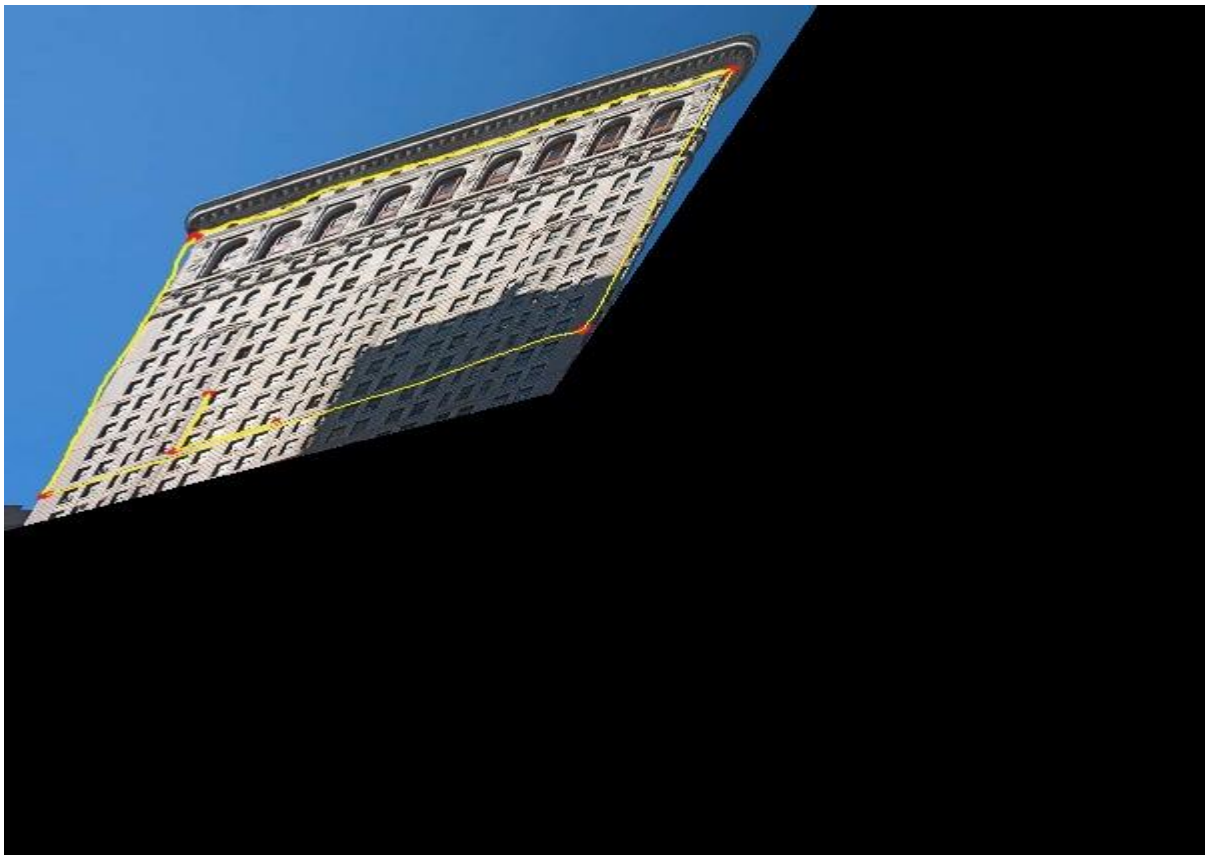

Figure 1.b two-step method, projective distortion removed

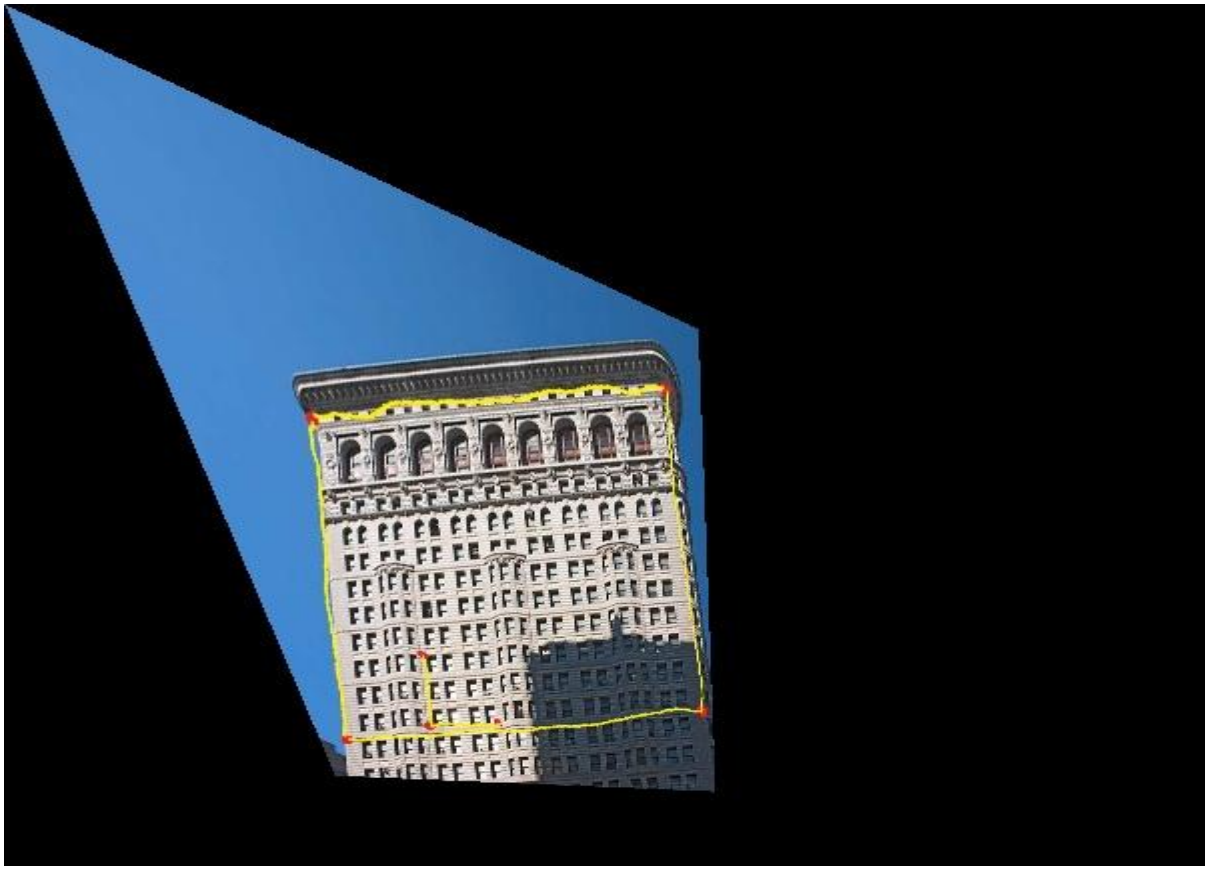

Figure 1.c two-step method, affine distortion removed

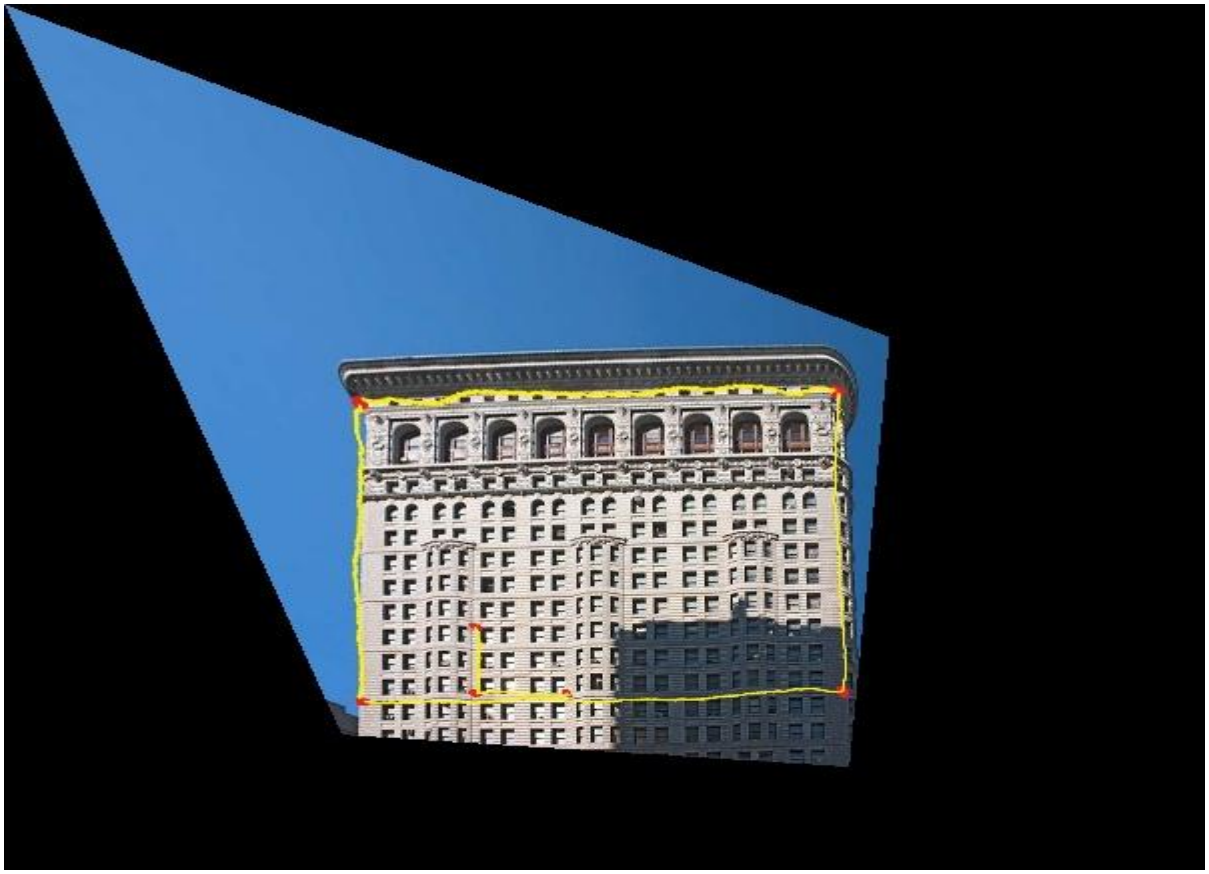

Figure 1.d one-step method

2. monalisa.jpg

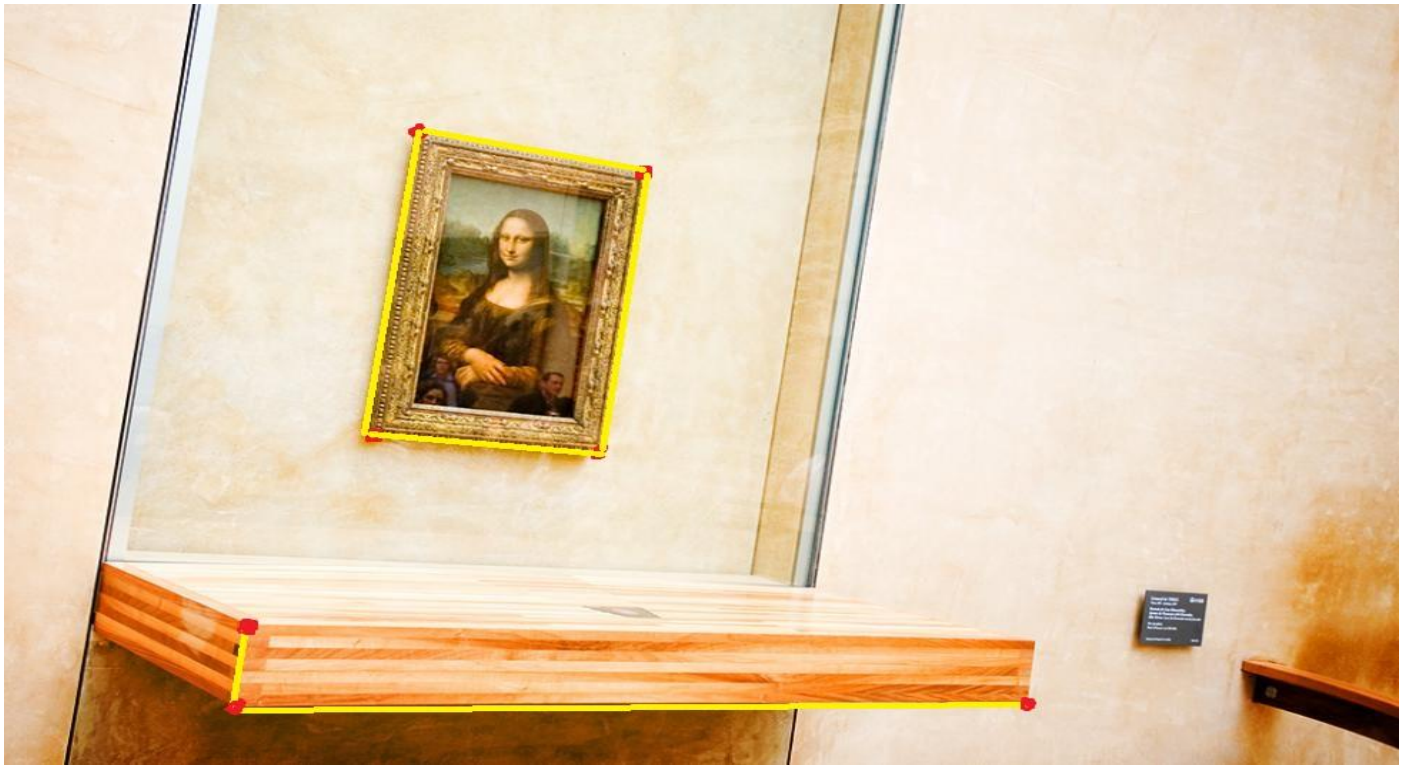

Figure 2.a original image

Figure 2.b two-step method, projective distortion removed

Figure 2.c two-step method, affine distortion removed

Figure 2.d one-step method

3. wideangle.jpg

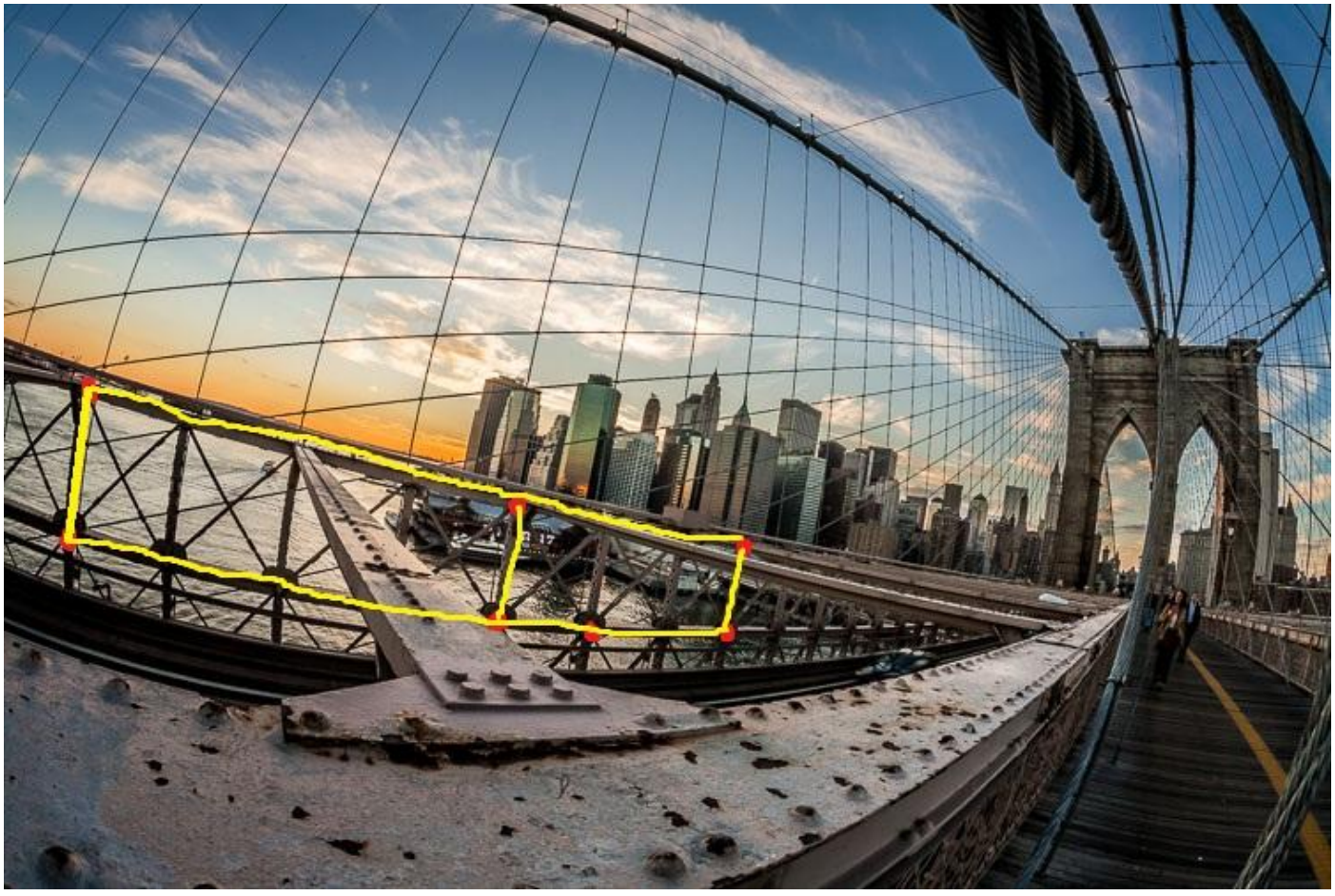

Figure 3.a original image

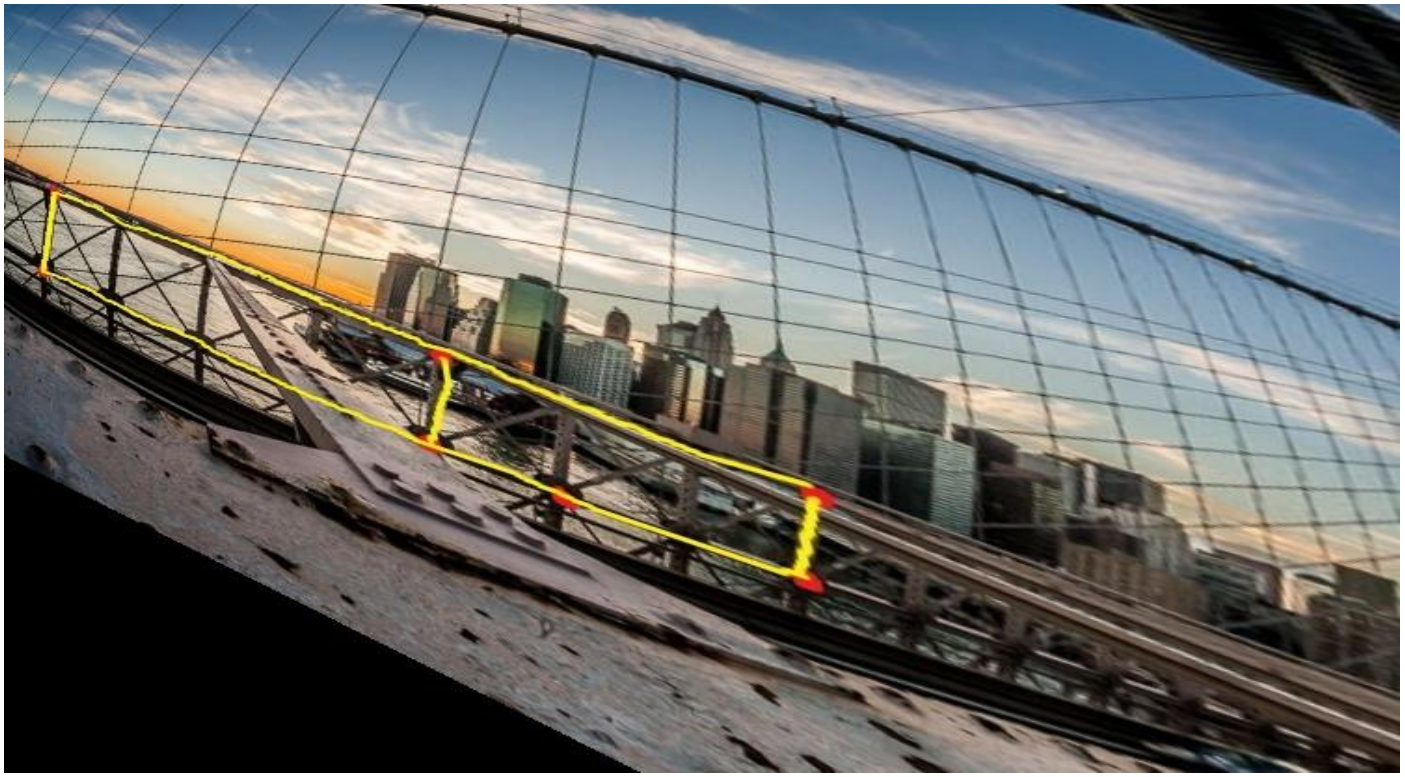

Figure 3.b two-step method, projective distortion removed

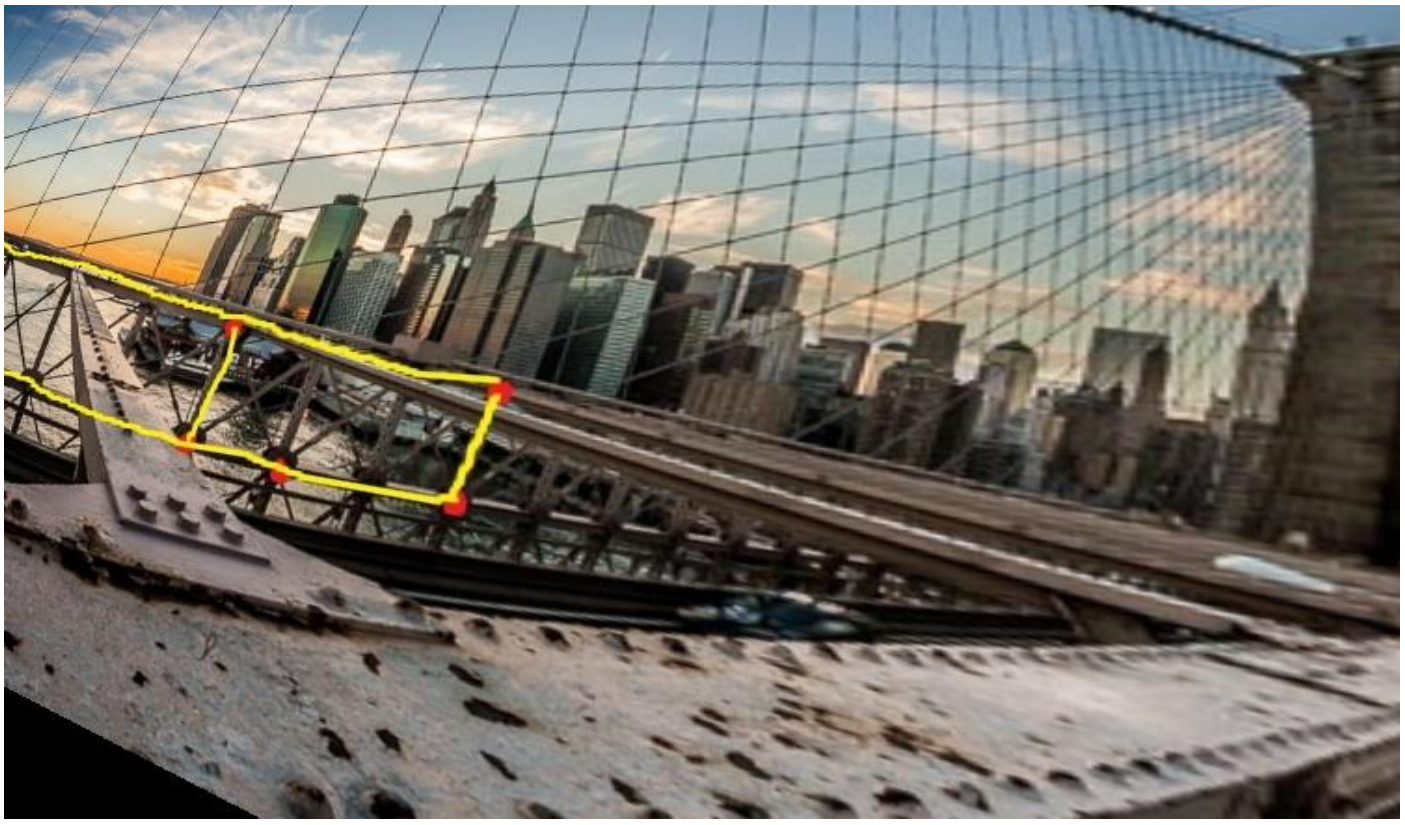

Figure 3.c two-step method, affine distortion removed

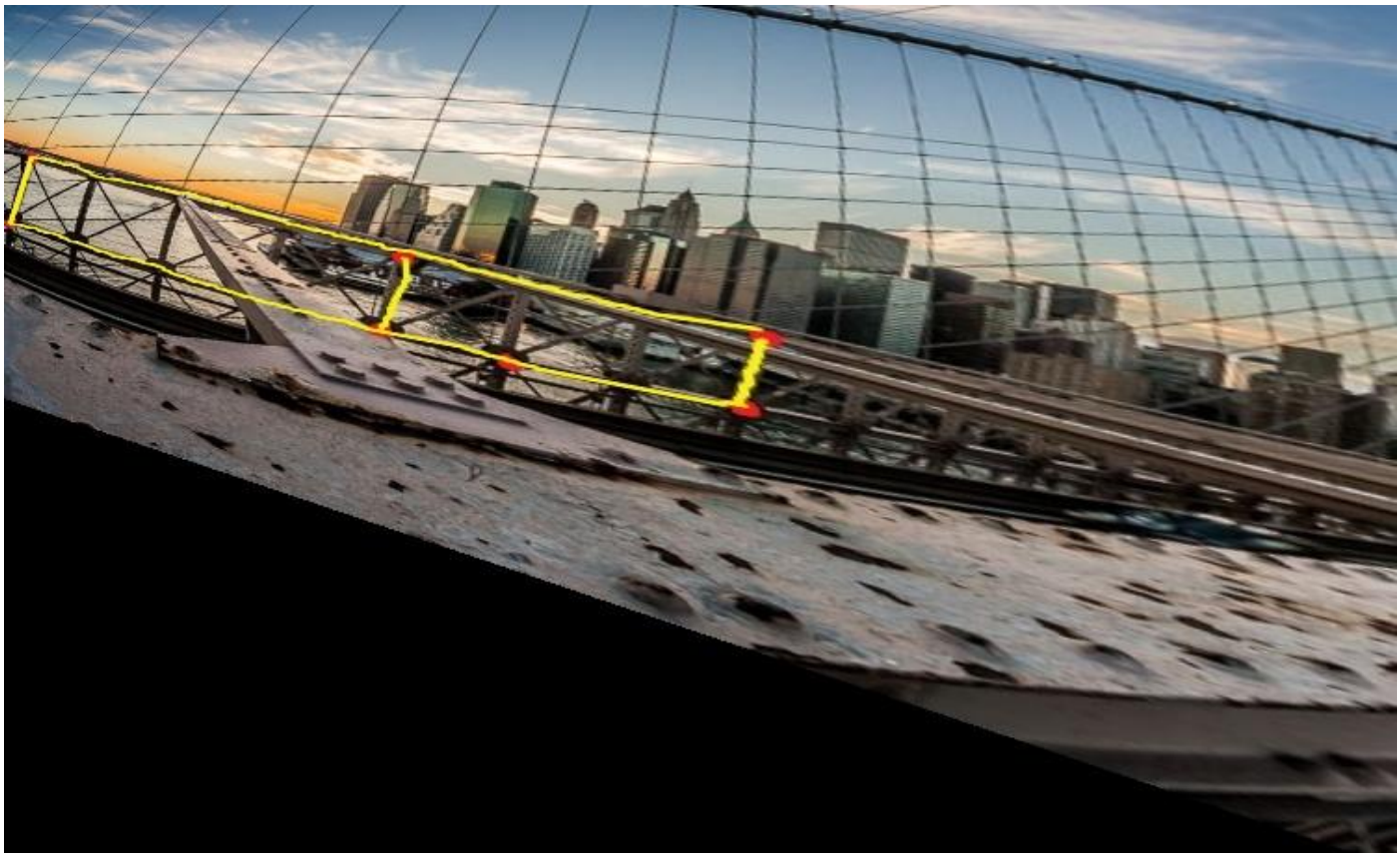

Figure 3.d one-step method

4. my first image

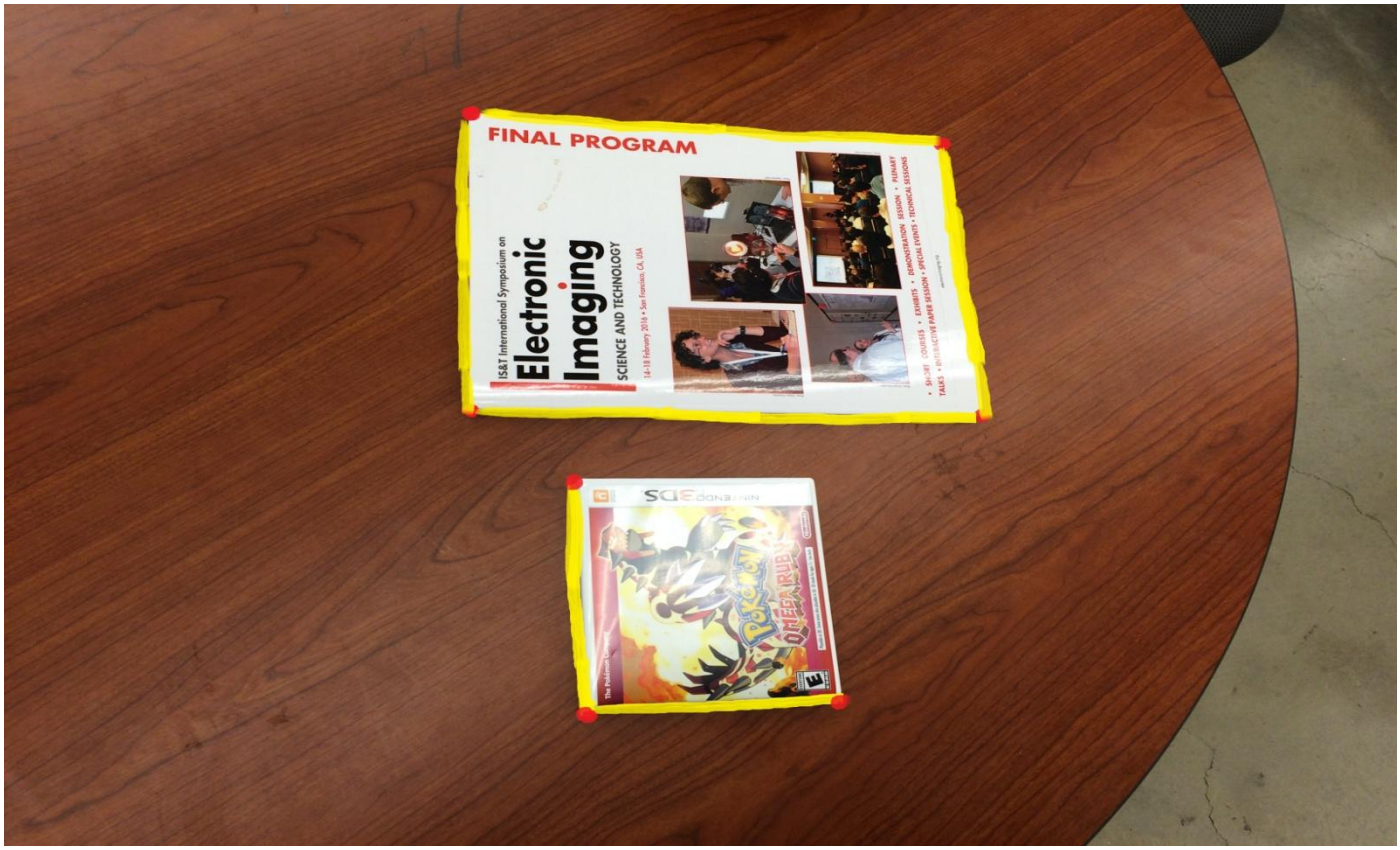

Figure 4.a original image

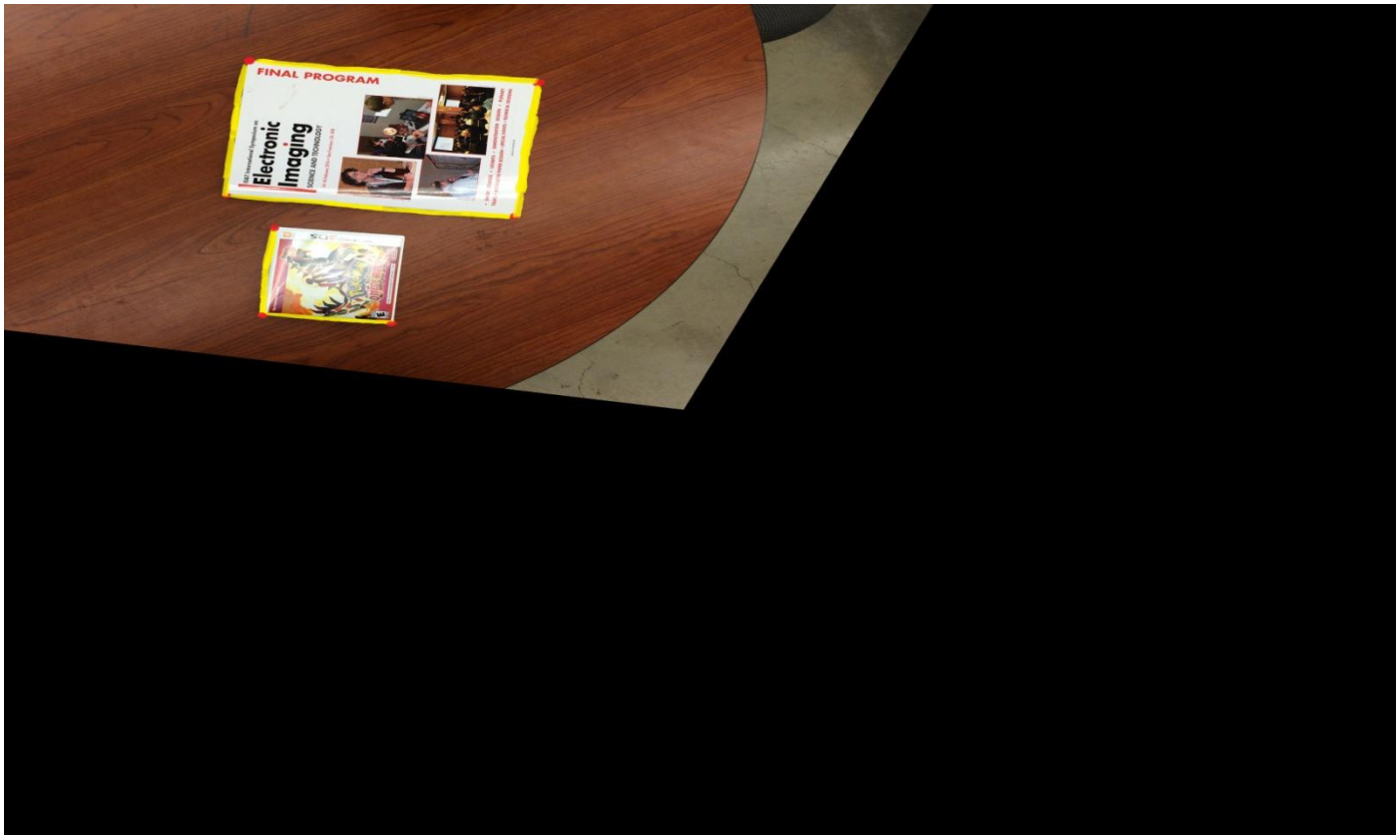

Figure 4.b two step method, projective distortion removed

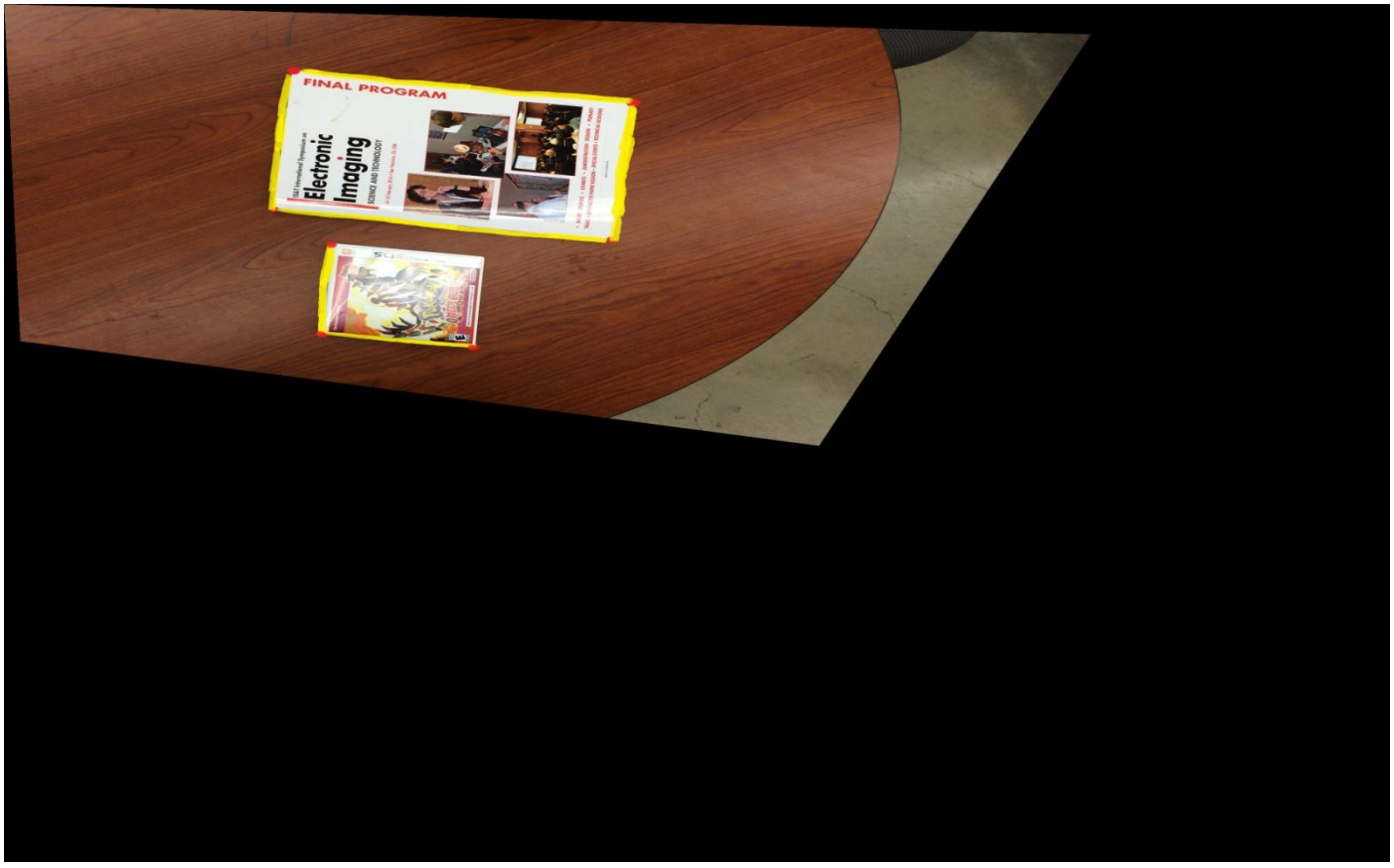

Figure 4.c two step method, affine distortion removed

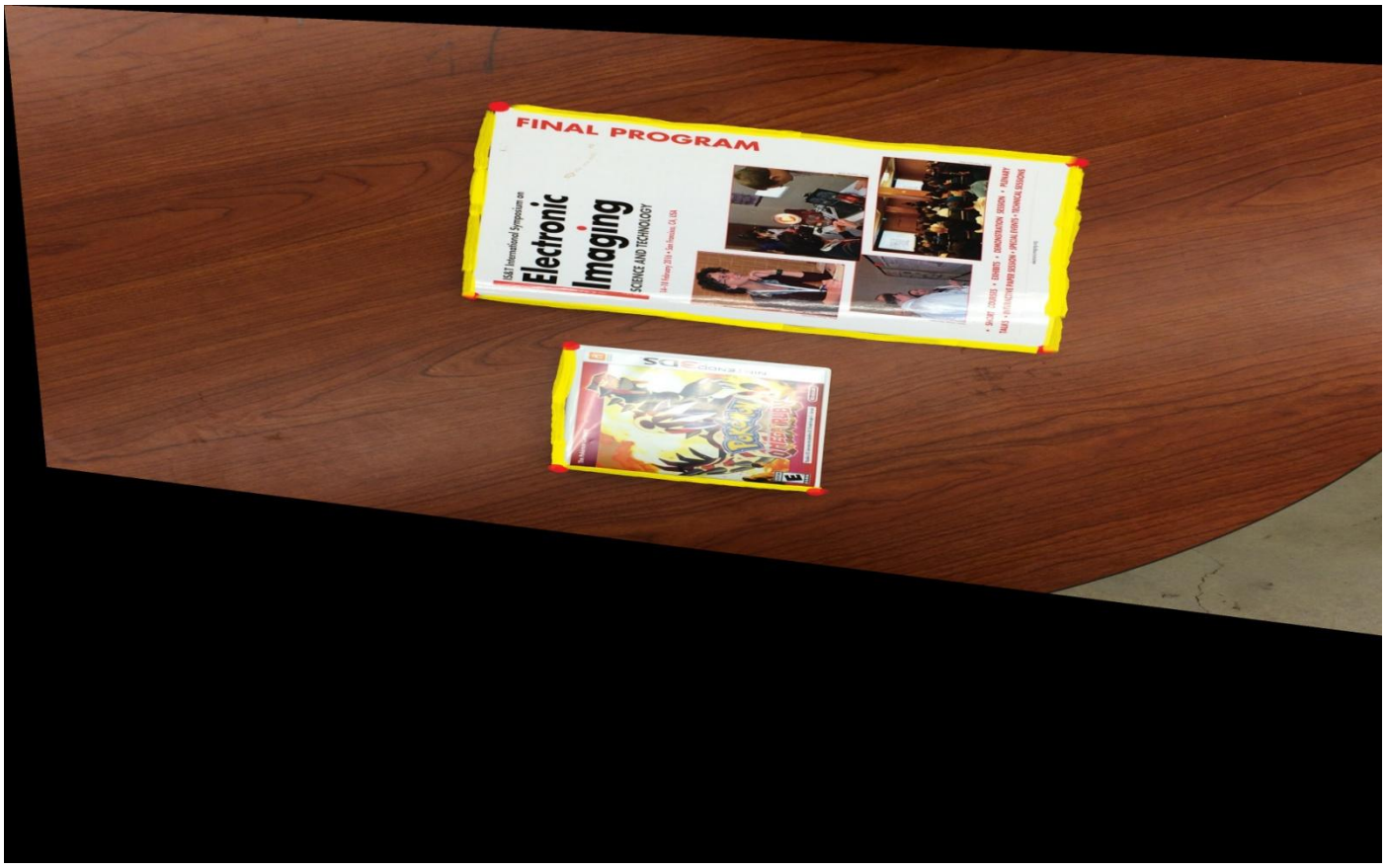

Figure 4.d one-step method

5. my second image

Figure 5.a original image

Figure 5.b two-step method, projective distortion removed

Figure 5.c two-step method, affine distortion removed

Figure 5.d one-step method