# Pricing and Waging in Three-Sided Food Delivery Markets

Online food delivery platforms are typically three-sided markets in the sharing economy that connect customers, freelance drivers, and restaurants, which provide an alternative online channel for restaurants to serve customers besides the traditional dine-in offline channel. Different online pricing and wage-setting strategies of platforms affect the matching of online demands and delivery services that drivers provide, thus affecting all parties' profitability. In this paper, we develop a game-theoretic model to investigate the online pricing and waging strategies of the platform and the restaurant under four prevalent contracts: platform-pricing/no wage-commitment, restaurant-pricing/no wage-commitment, platform-pricing/wage-commitment and restaurant-pricing/wage-commitment contracts. We show that the cross-channel price competition between the online and dine-in offline channels is more fierce in the sharing economy compared to the traditional economy (with a fixed labor supply) if and only if the fixed labor supply is more than that in the sharing economy. The platform prefers to relegate online channel pricing to the restaurant, while other parties in the food delivery market usually prefer the platform-pricing/wage-commitment contract. Despite its prevalence, the platform-pricing/no wage-commitment contract typically results in the poorest performance for the platform and moderate performance for the restaurant. We design a modified platform-pricing/wage-commitment contract with a transfer payment can benefit all parties in the food delivery market, including the drivers and customers, unless the labor supply of drivers is excessively costly. Our findings also provide guidance to policymakers in balancing the interests of gig workers and society.

*Key words*: three-sided markets; online food delivery platforms; the sharing economy

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## 1. Introduction

Online food delivery platforms have surged in prominence and growth in recent years, particularly during the COVID-19 pandemic. According to the latest report by IMARC Group (2024), the global online food delivery market reached $134.9 billion in 2023 and is projected to expand at a compound annual growth rate of 9.7% from 2024 to 2032. Platforms, such as Uber Eats and Postmates in the US and Meituan and Ele.me in China, operate as three-sided platforms that connect customers, delivery drivers, and restaurants. Customers now can order food online without traveling to restaurants, while freelance drivers, compensated by these platforms, handle the delivery orders. This setup provides restaurants an alternative online sales channel, allowing them to reach a broader customer base and expand their market presence without incurring significant additional operating costs.

Online food delivery markets are part of the sharing economy, where platforms are compensated only when they successfully match customer demands with service providers (Hu and Liu 2023). Thus, managing this match is crucial for platforms. Unlike in the traditional economy, where the service supply is fixed, service providers in the sharing economy are freelance workers with flexible working options who can decide whether and when to work for a platform based on the offered compensation (Lin et al. 2025). In the online food delivery markets, the delivery services are provided by self-scheduling drivers, whose availability is influenced by wages set by platforms. Consequently, platforms can control the labor supply of delivery drivers on the service supply side through wage adjustments (Zhang et al. 2022a). On the demand side of online food delivery platforms, customer demands are determined through cross-channel competitions between online and offline channel pricing, which are established through strategic interactions between the platforms and restaurants. Therefore, online food delivery platforms must effectively manage their relationship with restaurants to control online customer demands and match them with the labor supply of delivery drivers.

This paper examines a three-sided food delivery market operating within the sharing economy. We analyze the cross-channel competition between an online channel, where a restaurant sells food via an online food delivery platform, and an offline channel, where the restaurant serves dine-in customers directly. The platform sets wages for self-scheduling delivery drivers on the delivery service supply side and collaborates with the restaurant to set the online channel prices. A range of contractual agreements between them governs the collaboration between the platform and the restaurant. These contracts typically involve the platform charging a commission fee to the restaurant, with authorities over the final online price decisions varying across different contracts. Specifically, while some platforms retain control over the final online channel price (i.e., platform pricing), others might relegate the online pricing to restaurants (i.e., restaurant pricing). For instance, the two major food delivery platforms in North America, DoorDash and Uber Eats, are among the platforms that determine the final prices for online customers. These platforms charge a delivery fee or provide a discount to online customers on top of the food prices set by restaurants, allowing them to control the final channel price (Christopher 2023). In contrast, China's two leading food delivery platforms, Meituan and Ele.me, grant restaurants the authority to set the final price for online customers. Under this mechanism, restaurants can independently determine the offline and online prices for customers (Eleme 2024).

Online food delivery platforms have also introduced various strategies to manage the supply of delivery drivers on the service side. Specifically, these platforms may decide on the wages for delivery drivers either before or after contracting with restaurants. When wages are not predetermined (i.e., there is no wage commitment), the platforms gain greater flexibility to regulate the supply of drivers based on actual online demands. For instance, platforms like Uber Eats, DoorDash, Deliveroo, and Grubhub assign delivery orders to crowdsourced drivers, who are typically part-time workers and casual laborers from the local community[1]. Alternatively, platforms can create more stable relationships with drivers by committing to wages before they engage with restaurants. This means establishing a labor supply of delivery drivers ahead of demand for online services. For example, companies like Meituan and Ele.me employ full-time delivery drivers with fixed hourly wages, setting them apart from crowdsourced drivers. This practice is in line with recent legislation in many countries aimed at reclassifying gig workers as regular employees, and committed wage contracts between

[1] https://www.uber.com/us/en/deliver/

Table 1: Contracting Features of Well-Known Food Delivery Platforms

| | | Who sets the online channel price | |
|---|---|---|---|
| | | Platform pricing | Restaurant pricing |
| Whether platforms commit to wages for drivers | Yes | Ele.me, Meituan | Meituan |
| | No | Ubereats, DoorDash, Grubhub | Ele.me, Zomato, Deliveroo |

multi-sided platforms and service providers are anticipated to grow in prevalence in the future (Hu and Liu 2023). The rise of the sharing economy has led to significant debate worldwide regarding the employment status of gig workers. In response, some regulators are considering implementing a minimum wage (per hour) or a wage rate (per delivery) for drivers to improve their welfare. For instance, the New York State Supreme Court has ruled that DoorDash and Grubhub must pay their delivery drivers at least \$19.56 per hour or 50 cents per minute of delivery time. Minimum wage regulations can effectively serve as a wage commitment for delivery drivers, as industry reports suggest that platforms are often reluctant to pay more than these minimum wages[2]. Table 1 summarizes pricing/waging characteristics of some well-known food delivery platforms; more information about these platforms is provided in the Appendix B.

Given the various challenges faced by online food delivery platforms, such as the competition between the online and offline food delivery channels, matching online demands and labor supply of freelance workers under different pricing/waging contracts, and regulatory efforts to enforce minimum wage for drivers, platforms must strategically manage their interactions with customers, restaurants, and delivery drivers to achieve success. First, platforms should understand how the self-scheduling nature of delivery drivers affects the cross-channel competition between the online and offline channels, which determines demands in each channel. Different contractual relationships in these three-sided markets (i.e., platform vs. restaurant pricing and wage commitment vs. no wage commitment) raise questions about the relative performance of these contracts for players involved in such a market, particularly the restaurant, the platform, and the whole food delivery chain. Moreover, given the recent scrutiny of the government and the regulatory body to protect gig workers, it is interesting to study gig workers' welfare under different contracting schemes and to understand better how the regulatory body should deploy its regulations to protect not only gig workers but also the other players in the market and society as a whole.

To answer these questions, we develop a game-theoretic model in which a restaurant serves customers through two competing channels: the dine-in offline channel and the online channel through a food delivery platform. Customers can either visit the restaurant in person or place orders on the platform, which matches delivery demands with the labor supply of delivery drivers. We assume the

[2] https://www.nyc.gov/assets/dca/downloads/xlsx/Restaurant-Delivery-App-Data-Quarterly.xlsx

customers are sensitive to the prices of the two channels, and the demand of each channel is linear in the channel prices. Delivery drivers are self-scheduling, and the wages offered by the platform determine their labor supply. We investigate four typical contracting schemes in practice: Platform-pricing/no wage-commitment (PN) contract, restaurant-pricing/no wage-commitment (RN) contract, platform-pricing/wage-commitment (PW) contract, and restaurant-pricing/wage-commitment (RW) contract. By deriving the equilibrium market outcomes for these contracts, we first investigate how the self-scheduling nature of drivers affects the contractual relationship between the platform and the restaurant, as well as the resulting market outcomes. We then study the platform and the restaurant's preference over these contracting schemes by comparing the market outcomes under each contract. Finally, we assess how minimum wage regulations, whether based on hourly wages or per delivery wages, impact drivers, the overall food delivery chain, and the welfare of society.

To examine the impact of self-scheduling drivers in the sharing economy on the food delivery market, we first examine a benchmark case in the traditional economy, where the labor supply of drivers is exogenous. In the benchmark case, all the contracting schemes result in identical market outcomes as the online and offline prices and demands are all the same, with only the division of profit differing between firms. In contrast, these contracting schemes yield different market outcomes in the sharing economy. Moreover, we illustrate that the traditional economy demonstrates more intense market competition than that in the sharing economy if the fixed labor supply of the drivers is larger than the endogenously determined labor supply of drivers in the sharing economy. In other words, the sharing economy would soften market competition only if the platform has an ample labor supply than that in the traditional economy.

We also find that the PW contract results in the most fierce competition between the online and offline channels. In particular, under relatively high labor supply costs, the PW contract results in an oversupply of delivery labor in the online channel compared to the centralized scenario. Under this contract, since the platform controls the pricing of the online channel, early commitment to a high supply through high wages for drivers aligns both firms' incentives to set low-profit margins in the online channel, intensifying cross-channel competition. This oversupply of delivery labor is unique to the sharing economy. In a competitive two-sided market, Zhang et al. (2022a) and Hu and Liu (2023) show that wage commitment by competing platforms can intensify market price competition only when competition in the supply market is more intense than that in the demand market. Our findings may seem similar to theirs, but the sequential nature of decisions by the platform and the restaurant rules the results in our setting. Furthermore, while commonly used, we also find that the PN contract leads to the least intense cross-channel competition, ultimately reducing online demand for the platform.

When the platform relegates the online pricing to the restaurant under the RW or RN contract, we show that the wage commitment does not have any impact on the equilibrium outcome. In particular, we show that the RW and RN contracts are equivalent. Moreover, these contracts deliver the best performance for the platform. The reason is that the platform relegates the online channel pricing to the restaurant but sets the commission fee first, allowing the platform to charge high margins for online demand. In contrast, wage commitment can significantly impact the equilibrium outcomes when the platform controls online pricing (under the PN or PW contract). The PN contract allows the platform to adjust online prices and delivery wages simultaneously. Such flexibility for the platform enables the restaurant to soften channel competition by raising its profit margin, which negatively impacts the platform's profitability. For the platform, the PW contract usually performs moderately.

Unlike the platform, the restaurant prefers the PW contract, except when the labor supply costs are extremely high. This contract's advantage for the restaurant lies in its ability to align better the incentives of both the platform and the restaurant to lower their margins and boost online sales. Under this contract, the platform commits to an ample supply of drivers first through high wages. The restaurant would then reduce its margin, knowing that the platform has no incentives to charge a high price for online channel customers as it has already committed to an ample labor supply of drivers. Such alignments in the online channel pricing benefit the restaurant and maximize the whole chain's profit among all these contracting schemes (unless the labor supply cost is high). Given the platform's preference over the RN/RW contract and the PW contract's optimality for the restaurant and the overall food delivery chain, we propose a modified PW contract under which a transfer payment from the restaurant to the platform exists. This modified contract could enhance profits for both the platform and the restaurant and improve the welfare of drivers and customers compared to the platform's most preferred RN/RW contracts.

Our analysis reveals that a minimum wage or a wage rate has different implications for the food delivery market. While commitment to a relatively high minimum wage might help the food delivery chain and all parties involved in the online food delivery market benefit (only the platform loses), commitment to a high wage rate can harm all players in the food delivery market. On the contrary, we show that the platform's commitment to a low enough wage rate can align the restaurant and the platform's pricing decisions, resulting in competitive online channel prices. This intensified cross-channel competition can drive up online orders, benefiting drivers, customers, the restaurant, and the overall food delivery chain.

The rest of this paper is organized as follows. First, we review the relevant literature in Section 2, and we introduce our model in Section 3. The following section elaborates on the analysis, while Section 5 compares the performances of different contracting schemes. Section 6 presents numerical experiments that help us improve our understanding of the problem, and Section 7 incorporates an extension of the model. Finally, we conclude with managerial insights in Section 8.

## 2. Related Literature

This paper contributes to the literature on multi-sided markets. Amid the extensive literature on two-sided markets in economics (e.g., Caillaud and Jullien 2003, Rochet and Tirole 2003, Andrei 2009, Dou and Wu 2021), a growing body of operations management literature has studied the sharing economy. Within this field, some papers focus on the operational aspects of price and wage design. For instance, Banerjee et al. (2016) examine a scenario where the wage is an exogenous proportion of the price to show that static pricing is effective. In contrast, Cachon et al. (2017) study pricing schemes in which both the price and wage are endogenous. They find that surge pricing can achieve nearly optimal profit, and all stakeholders can benefit from surge pricing on a platform with self-scheduling capacity. Hu and Zhou (2020) show that it is optimal for the platform to offer a fixed ratio commission for drivers, which depends on the price and wage sensitivity coefficients of the linear demand and supply functions, while Garg and Nazerzadeh (2022) propose an incentive-compatible pricing mechanism for drivers in response to surge pricing. Taylor (2018) examines how delay sensitivity and agent independence affect a platform's endogenous pricing and waging decisions. Our context differs from the above literature, as it investigates the contractual relationships not only between the platform and drivers but also between the platform and the restaurant.

In two-sided markets, researchers have studied precommitment to wage or price in competitive settings. Hu and Liu (2023) investigate how commitment to price or wages can soften market competition. In particular, they show platforms can benefit from commitment through softened competition if competing platforms commit to wages in the supply market or prices in the demand market, whichever is less competitive. In particular, this study extends the Kreps and Scheinkman equivalency (Kreps and Scheinkman 1983), demonstrating that precommitment to capacity results in reduced price competition. Zhang et al. (2022a) examine three common contracting schemes in two-sided markets, investigating the role of self-scheduling drivers on the platform's profitability. They demonstrate how the relative intensity of competition in the demand vs. supply market affects the platform's choice of contract. Moreover, they reconfirm the findings of Hu and Liu (2023). In contrast to these papers, we consider a three-sided market where market competition is between a platform's online channel and a restaurant's dine-in offline channel, with/without wage commitment.

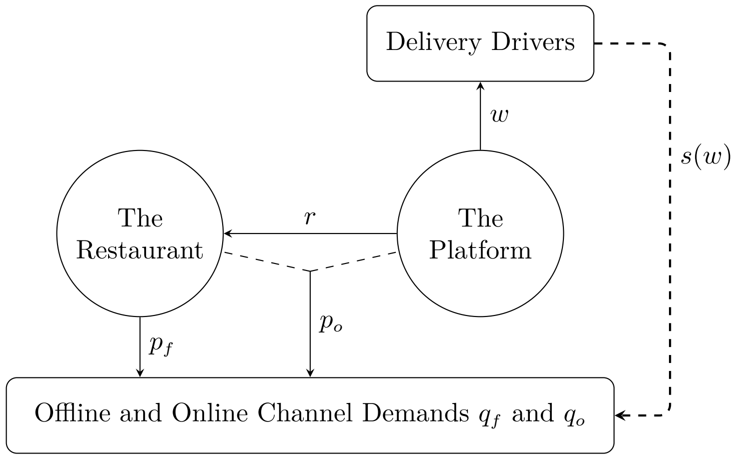
This research mainly contributes to online food delivery services literature. Within this field, one stream of studies investigates factors that affect delivery performance. For example, Mao et al. (2019) empirically show that a driver's individual local area knowledge and prior delivery experience can reduce late deliveries significantly. Based on data from a major Chinese food delivery platform, Zhang et al. (2023) show a high restaurant density reduces the delivery speed. Additionally, Chen and Hu (2024) examine the impact of dedicated versus pooling dispatch strategies on delivery performance, mainly when customers are sensitive to delays. On the importance of delivery performance, Xie et al.

(2024) further examine how delivery performance expectations affect consumer purchase behaviors, and highlight the broader implications of timely deliveries on customer satisfactions and platform revenues. Several studies empirically examine the economic effects of on-demand delivery platforms on restaurants. For example, Li and Wang (2024a) show that, generally, restaurants can benefit from selling through delivery platforms, and the overall positive effect on fast food chains is stronger than that on independent restaurants. Unlike the above findings, Karamshetty et al. (2023) illustrate that the dependence on the platform might reduce the sales revenue from high-margin items.

Another stream of research in this area focuses on the contract design between platforms and restaurants to achieve better performance for the food delivery market members. Specifically, Oh et al. (2023) show that a contract with sharing food revenue and splitting the delivery costs and fees between platforms and restaurants can achieve the first-best profits. Similar to this paper, Feldman et al. (2023) and Chen et al. (2022) also illustrate that simple revenue sharing has inherent drawbacks and fails to coordinate the system. In Feldman et al. (2023), they propose a generalized revenue-sharing contract that can coordinate the system, while Chen et al. (2022) find that a simple revenue-sharing contract with a “price ceiling” on the delivery menu price coordinates the system. Regarding the regulatory issue in the context of on-demand food delivery, Li and Wang (2024b) discusses whether the government should set an upper bound on the commission rates asked by platforms. Zhang et al. (2022b) investigate the government’s policy design to curb traffic incidents brought by delivery drivers. While these studies enhance our understanding of the online food delivery market from various perspectives, they often focus on issues involving only one or two parties, neglecting the market’s three-sided nature. In contrast, our research models the contractual relationships among platforms, restaurants, and drivers to capture the intricate dynamics of this market.

A few recent studies on online food delivery platforms consider the market’s three-sided nature. Bahrami et al. (2023), for instance, characterize the optimal commissions and wages from the perspective of a profit-maximizing or welfare-maximizing platform when customers are time-sensitive. Liu et al. (2023) adopt a state-dependent queuing model to study the platform’s revenue maximization problem, where customers, deliverers, and restaurants make independent participation decisions. Unlike these studies, we consider the sequential moves in contracting in the three-sided market and focus on contractual performance. In this line of research, Sun et al. (2023) are perhaps the closest to our work. Sun et al. (2023) study the three-sidedness of the food delivery market and examine two competing platforms’ optimal choices in a setting where the platforms compete on both prices and service quality. They show that the platforms’ incentive to exploit the market’s three-sided nature is significantly affected by two key factors: whether consumers benefit from service improvement and the intensity of interaction in the buyer-seller market. Our paper differs from this paper in several distinct ways. First, their study considers the competition between platforms, whereas we consider

Figure 1: Schematic View of the Three-Sided Online Food Delivery Market




a single platform in the market and we examine competition between the online and offline channels. Moreover, the emphasis in our paper is on the contractual relationships between a platform, a restaurant, and self-scheduling drivers.

## 3. Model

We consider a stylized three-sided food delivery market in which an online platform connects a restaurant, a group of delivery drivers, and customers seeking catering services. The restaurant contracts with the platform to expand its market base so customers can order food online through the platform. Since delivery drivers are self-scheduling and have alternative working options, the platform must offer competitive wages to incentivize them to fulfill online orders. Besides the online channel, the restaurant also provides a dine-in offline channel, where customers can commute to the restaurant and get dining services. Figure 1 provides a schematic view of the interactions among the players.

### 3.1. Demand Specification

We assume the online and offline channels are differentiated, and customers are sensitive to prices in these channels. We use a linear demand system to model the channel demands as follows.

$$q_o = \tfrac{1}{1+\beta} - \tfrac{1}{1-\beta^2}p_o + \tfrac{\beta}{1-\beta^2}p_f, \tag{1}$$
$$q_f = \tfrac{1}{1+\beta} - \tfrac{1}{1-\beta^2}p_f + \tfrac{\beta}{1-\beta^2}p_o, \tag{2}$$

where $q_o$ is the online demand, i.e., orders placed at the platform, and $q_f$ represents the dine-in offline demand. $p_o$ and $p_f$ represent final prices in these channels; we will provide a detailed explanation of these prices later. The marketing and economics literature has extensively studied such linear demand systems to model differentiated duopolies (Singh and Vives 1984, Jerath and Zhang 2010). In the above model, $\beta$ $(0 \leq \beta \leq 1)$ represents the degree of differentiation between the online and offline channels. When $\beta$ equals zero, these two channels operate independently. As $\beta$ increases, the competition between these two channels becomes more intense. When $\beta$ approaches one, these two channels become fully substitutable, leading to perfect competition between them in the market. Intermediate values of $\beta$ represent varying degrees of differentiation.

We use this demand specification because it has two desirable characteristics: as the differentiation between these two channels increases (i.e., $\beta$ decreases), the price sensitivity $1/(1-\beta^2)$ decreases. This is consistent with the idea that customers are less price-sensitive to more differentiated products. Additionally, the total potential market size in this demand model $2/(1+\beta)$ decreases in $\beta$, which aligns with the intuition that more differentiated products can reach a broader customer base. In the Appendix F, we also explore other popular demand models where the total market size remains unaffected by the degree of product differentiation, and we evaluate how variations in the relative market potentials for online and offline channels impact our findings.

### 3.2. Labor Supply of the Delivery Drivers

The platform must attract enough drivers to provide delivery services for customers ordering at the online channel. We use a linear model to characterize the labor supply of the delivery drivers, which increases with the wage offered by the platform. In particular, we assume that the labor supply of the delivery drivers is given by,

$$s(w) = [-a + bw]^+, \tag{3}$$

where $w$ represents the wage paid by the platform to the delivery drivers, and $s(w)$ denotes the labor supply provided by the delivery drivers. $a$ ($a \geq 0$) represents the attraction of working options other than the online platform for drivers, and $b$ measures the delivery drivers' sensitivity to the platform's wage. A lower value of $a$ indicates that working for the platform is becoming more attractive compared to the outside option (for example, due to the flexibility of the self-scheduling feature of the platform). A lower value of b indicates that the disparity between working for the platform and alternative options is diminishing, thereby increasing the cost for the platform to attract drivers. Specifically, we consider low values of $b$ to represent a costly labor supply of drivers, as the platform must offer higher wages to attract drivers. In addition to the above wage-dependent model, an alternative formulation based on the wage rates is developed in Section 7.

A transaction in the online channel happens only when the platform matches an online order with a delivery driver. Suppose the online demand exceeds the labor supply of the drivers, the platform will randomly assign a limited supply of drivers to the online orders, so the food delivery market will lose the unsatisfied demand. If the labor supply exceeds the online demand, the platform will assign limited online orders to the drivers (randomly), and the extra supply will be wasted. Therefore, the transaction volume of the online channel in our model is given by $\min(s(w), q_o)$. This proportional rationing is a quite common assumption in the literature (e.g., Hu and Liu 2023, Zhang et al. 2022a).

### 3.3. The Platform and the Restaurant

The platform and the restaurant cooperate in the online channel to provide food delivery services to customers; at the same time, the online channel competes with the offline dine-in channel that the restaurant completely controls. Both firms seek to maximize their profits. We can formulate the profit function for the platform as

$$\pi_p^i = \min(s(w), q_o)(m_p^i - w), \tag{4}$$

and the profit function for the restaurant as

$$\pi_r^i = \min(s(w), q_o)m_r^i + p_f q_f, \tag{5}$$

where the superscript $i \in \{PN, RN, PW, RW\}$ represents four different contracting schemes between the platform and the restaurant, as we will introduce in detail in the next subsection, and the subscripts $p$ and $r$ denote the platform and the restaurant, respectively. $m_p^i$ and $m_r^i$ are the online channel profit margins for the platform and the restaurant, respectively; the connections between these margins and online/offline channel prices will be outlined in Section 4.

### 3.4. Contracting Schemes

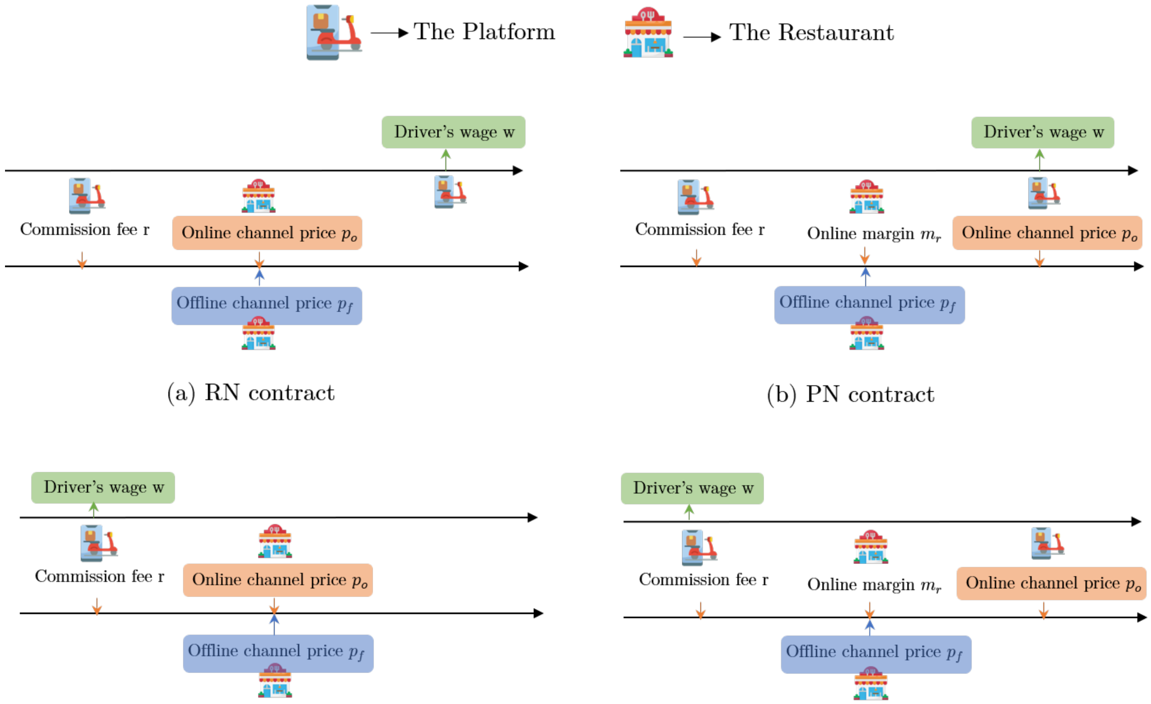
Given that the platform can delegate or control the price setting in the online channel while deciding whether it wants to commit to a wage for the drivers, we study the following four contracting schemes governing the relationships among all the players in this three-sided food delivery market.

*Restaurant-pricing/no wage-commitment (RN) contract* Under this contract, the platform sets a commission fee charged to the restaurant first, allowing the restaurant to set the online channel price alongside the offline channel price. The platform then moves last to set the wage for the delivery drivers. Therefore, the labor supply of the drivers is set after the contracting between the restaurant and platform, i.e., no wage commitment, and the platform has the flexibility of adjusting labor supply after online demand is realized.

*Platform-pricing/no wage-commitment (PN) contract* Under this contract, the platform first asks for a commission fee per delivery from the restaurant. Next, the restaurant sets its profit margin for online orders alongside the dine-in offline prices. Finally, the platform sets the online channel price by posting its delivery fee (or even providing a discount) to online customers. Meanwhile, the platform also sets a wage for the delivery drivers. Under this contract, the restaurant just determines the profit margin per delivery from the online channel but delegates the online pricing decision to the platform. The wage for drivers is also finally set after the contracting (i.e., no wage commitment).

*Restaurant-pricing/wage-commitment (RW) contract* Under this contract, the platform first announces the wage for the delivery drivers alongside the commission fee per delivery it charges to the restaurant. The restaurant then sets the online channel price alongside the offline price for

Figure 2: Sequence of Events under RN, PN, RW, and PW Contracts (Color Online).




(a) RN contract

(b) PN contract

(c) RW contract

(d) PW contract

*Note.* In each figure, the upper timeline denotes the timing of wages for the drivers, which could be set before or after the platform's contracting with the restaurant (wage-commitment or no wage-commitment case, respectively). The lower timeline denotes the timing of contracting between the restaurant and the platform. The restaurant can set the online channel price $p_o$ itself (the restaurant-pricing case) or charge an online margin $m_r$ and delegate the online channel price to the platform (the platform-pricing case).

customers. Different from the above two contracts, the wage for the drivers under the RW contract is set first, so the amount of labor supply is determined before online demands, i.e., there is wage commitment. The restaurant does not delegate the online pricing decision to the platform, but sets the online channel prices itself.

*Platform-pricing/wage-commitment (PW) contract* Under this contract, the platform first announces the wage for the delivery drivers alongside the commission fee per delivery to the restaurant. The restaurant then sets the profit margin and charges the platform for online orders alongside the dine-in offline prices. Finally, the platform announces the delivery fees or discounts to set the online channel price. Like the PN contract, the restaurant asks for a fixed profit margin in the online channel, delegating the online pricing decision to the platform. Note that the wage for drivers is set before the contracting between the firms, so there is wage commitment.

Figure 2 demonstrates the decision sequence of the players in the online food delivery market under these contracts. To rule out trivial cases, we make the following assumption throughout this study.

Assumption 1. *The model parameters satisfy $b \geq \frac{a}{1-\beta}$.*

This assumption implies that the delivery drivers are responsive enough to the wages, so the cost of providing incentives for drivers to make delivery is not excessively high. The assumption ensures that customer demands of the online and offline channels are positive under these contracts. If not, the online channel will be unprofitable to the platform and restaurant. We summarize the notations used in this paper in Table A.1 in the Appendix A, and all proofs are provided thereafter in the Online Supplement.

### 3.5. The Benchmark Model

In this subsection, we study a benchmark model, in which the labor supply of the drivers is exogenously given, i.e., the fixed-labor-supply case. It allows us to understand better the effect of the self-scheduling labor supply in the sharing economy. In what follows, we first characterize the equilibrium decisions of the players in the this benchmark case. Then we investigate the equilibrium outcomes of all contracting schemes in the sharing economy in the next section. We also consider a centralized case in the Appendix C as another benchmark, where a decision maker sets the online and offline channel prices and the drivers' wages to maximize the profit of the entire food delivery market. It allows us to compare the performance of different contracting schemes vs. that of the best achievable one.

In this benchmark case, we study four sub-cases of the contracting schemes introduced in the main model, but following Zhang et al. (2022a), we assume that the platform has a fixed supply of drivers, denoted as $\hat{s}$, and their wage is also constant at $c$. The profit functions of the platform and the restaurant are given by

$$\pi_p^j = \min(\hat{s}, q_o)(m_p^j - c), \tag{6}$$
$$\pi_r^j = \min(\hat{s}, q_o)m_r^j + p_f q_f, \tag{7}$$

where $j \in \{BRN, BRW, BPN, BPW\}$ represents the benchmark sub-cases of four contracting schemes (the RN, RW, PN, and PW contracts) with fixed labor supply of drivers (see Section D in the Appendix for further details). Comparing these benchmark models with the ones with self-scheduling drivers can help us evaluate the impact of the sharing economy. The following lemma establishes the equilibrium outcomes of these benchmark sub-cases.

Lemma 1. *If the labor supply of the drivers is given by $\hat{s}$, we can establish the following:*

*(i) The equilibrium online and offline channel prices of the benchmark sub-cases are the same as follows,*

$$(p_o^{j*}, p_f^{j*}) = \begin{cases} (\frac{2-\beta-2\hat{s}(1-\beta^2)}{2}, \frac{1}{2}) & if \quad \hat{s} \leq \frac{1-\beta-c}{4(1-\beta^2)}, & (8a) \\ (\frac{(3-\beta+c)}{4}, \frac{1}{2}) & if \quad \hat{s} \geq \frac{1-\beta-c}{4(1-\beta^2)}. & (8b) \end{cases}$$

*Here, this holds for all cases of $j \in \{BRN, BRW, BPN, BPW\}$.*

*(ii) The equilibrium online demands of the benchmark sub-cases are also the same as,*

$$q_o^{BRN*} = q_o^{BRW*} = q_o^{BPN*} = q_o^{BPW*} = \begin{cases} \hat{s} & if \quad \hat{s} \leq \frac{1-\beta-c}{4(1-\beta^2)}, & (9a) \\ \frac{1-\beta-c}{4(1-\beta^2)} & if \quad \hat{s} \geq \frac{1-\beta-c}{4(1-\beta^2)}. & (9b) \end{cases}$$

Lemma 1 shows that when the exogenously given labor supply is ample, the platform and the restaurant choose the profit-maximizing prices; otherwise, the online channel price is a function of labor supply, and the fixed size of supply limits the online sales. Additionally, an important observation in the above lemma is that all these contracting schemes result in the same online/offline channel prices and demands when the labor supply of delivery drivers is exogenous. Consequently, the identical market outcomes lead to the same food delivery market's profit across all contracts. Following Lemma 1, we express the contract equivalency for the benchmark sub-cases next.

COROLLARY 1. *If the labor supply of drivers is exogenous, then the RN, RW, PN, and PW contracts result in the same market outcomes.*

This corollary implies that when the supply is exogenously given, the contracting dynamics between the platform and the restaurant do not affect the market outcomes. The change in the decision sequences in these contracts only affects the profit distribution between the firms. In the next section, we demonstrate that the established equivalency in Corollary 1 fails to hold as we incorporate self-scheduling delivery drivers in the sharing economy.

## 4. Analysis

In this section, we first derive the equilibrium outcomes of the online food delivery market in the sharing economy, where the drivers are self-scheduling, responding to changes in the platform's waging under different contracting schemes. Then, we compare the market outcomes of the four contracting schemes with the benchmark sub-cases to show the impact of self-scheduling delivery drivers.

### 4.1. The Restaurant-Pricing/No Wage-Commitment (RN) Contract

Under the RN contract, the platform first announces the commission fee $r$ it charges to the restaurant. Then the restaurant decides the offline price $p_f$ alongside the online price $p_o$ for customers before the platform finally sets up the wage $w$ offered to the delivery drivers. Therefore, the online profit margin for the platform is given by $m_p^{RN} = r$, and it chooses a commission fee $r$ to maximize its profit in the first stage:

$$\max_r \pi_p = \min(s(w), q_o)(r - w). \tag{10}$$

Given the commission fee, the restaurant sets the offline and online prices to maximize the following profit function in the second stage,

$$\max_{p_o, p_f} \pi_r = \min(s(w), q_o)(p_o - r) + q_f p_f. \tag{11}$$

The online profit margin for the restaurant is $m_r^{RN} = p_o - r$ under this contract. Note that the online food price posted by the restaurant can be composed of a combination of a posted food price and a delivery fee fully receivable by the restaurant. Finally, in the last stage of the game, given the commission fee and the online and offline prices, the platform sets the wage for the drivers to provide the labor supply $s(w)$ to maximize the following profit function,

$$\max_{w} \pi_p = \min(s(w), q_o)(r - w). \tag{12}$$

We employ backward induction to derive the equilibrium outcomes under the RN contract, which are characterized in the following lemma.

LEMMA 2. *Under the RN contract, in equilibrium, the commission fee, the online and offline prices, and the wage for the drivers are*

$$r^{RN*} = \frac{(1-\beta)(1+a(\beta+1)+b(1-\beta^2))}{1+2b(1-\beta^2)}, \tag{13}$$

$$p_o^{RN*} = \frac{1}{2} + \frac{(1-\beta)(1+a(\beta+1)+b(1-\beta^2))}{2+4b(1-\beta^2)}, \tag{14}$$

$$p_f^{RN*} = \frac{1}{2}, \tag{15}$$

$$w^{RN*} = \frac{a+4ab(1-\beta^2)+b(1-\beta)}{4b^2(2-\beta^2)+2b}. \tag{16}$$

Substituting for the equilibrium decisions, we can show that under the RN contract the platform matches supply and demand, i.e., $q_o^{RN*} = s(w^{RN*})$. If the labor supply of the drivers exceeds the online channel demand, the transaction volume matches the online demand. The platform can reduce the wage for the drivers to maintain the same transaction volume but result in a higher profit margin for the online channel. Therefore, in equilibrium, the labor supply cannot exceed the online demand. On the other hand, if the labor supply is smaller than the online channel demand, the transaction volume matches the labor supply. Given the fixed commission fee, the platform's profit depends only on the supply side and is unrelated to the demand side. Therefore, the restaurant can increase the online price to increase its profit margin until the online demand matches the labor supply of drivers.

### 4.2. The Platform-Pricing/No Wage-Commitment (PN) Contract

Under the PN contract, the platform first announces a commission fee $r$ charged to the restaurant. Then, the restaurant sets the dine-in offline price $p_f$ alongside the profit margin $m_r$ it charges the platform for online sales. The platform then sets the online price $p_o$ by posting its delivery fee (or discount) $d$, alongside the delivery drivers' wage $w$. Therefore, the online channel profit margin for the platform is $m_p^{PN} = p_o - m_r = r + d$. The platform chooses a commission fee to maximize its profit at the first stage as,

$$\max_{r} \pi_p = \min(s(w), q_o)(r + d - w). \tag{17}$$

In the second stage, the restaurant decides the offline price alongside the online sales margin to maximize its profit, which is given by

$$\max_{m_r, p_f} \pi_r = \min(s(w), q_o)m_r + q_f p_f. \tag{18}$$

Finally, in the last stage of the game, the platform sets the online food price by posting the delivery fee $d$. Since $p_o = r + d + m_r$, we have,

$$\max_{p_o, w} \pi_p = \min(s(w), q_o)(p_o - m_r - w). \tag{19}$$

Note that if $d \geq 0$, it is the delivery fee that the platform charges to the online customers; otherwise, it is the online price discounts that the platform provides to the customers to better compete with the offline channel controlled by the restaurant. In the rest of this paper, we just nominate $d$ as delivery fee for simplicity. Solving backward, we can characterize the equilibrium outcomes in the following lemma.

LEMMA 3. *Under the PN contract, in equilibrium, the online and offline prices, and the wage for the drivers are given by*

$$p_o^{PN*} = \frac{(1-\beta^2)(a+b(3-\beta))+2(2-\beta)}{4b(1-\beta^2)+4}, \tag{20}$$

$$p_f^{PN*} = \frac{1}{2}, \tag{21}$$

$$w^{PN*} = \frac{a(3+4b(1-\beta^2))+b(1-\beta)}{4b(b(1-\beta^2)+1)}. \tag{22}$$

The first observation in the above lemma is that under the PN contract the labor supply matches the online demand, i.e., $q_o^{PN*} = s(w^{PN*})$. Unlike the RN contract, the platform under the PN contract utilizes both the delivery fee and the wage for drivers to match the labor supply and the demand for the online channel. Intuitively, if the online demand exceeds the labor supply on the platform, the transaction volume will equal the number of drivers available. In this case, if the wage remains constant and the platform slightly increases the delivery fee, it can maintain the same transaction volume while achieving a higher profit margin. Therefore, demand cannot exceed supply in equilibrium. Similarly, supply cannot exceed the online demand in equilibrium because the platform could increase its profit by reducing the wage without lowering the transaction volume.

The next observation indicates that the commission fee set by the platform does not affect the equilibrium outcome. The proof section demonstrates that the restaurant's margin for online sales is independent of $r$, and the online channel price set by the platform (that includes the delivery fee) serves as a perfect substitute for the commission fee (see Equation (G.19) in the Online Supplement G). This means that any increase in the commission fee charged by the platform would lead to a decrease in the online channel price, while a reduction in the commission fee would result in a higher online channel price. The Doordash marketplace falls into this category of contracts. It offers Basic, Plus, and Premier Partnership Plans to restaurants, each with progressively higher commission fees. Customers pay lower service and delivery fees to the platform when the restaurant subscribes to the Plus and Premier Partnership Plans, because these delivery fees decrease in the commission fees paid by restaurants (Doordash 2023)[3].

[3] It is straightforward to establish that the PN contract is equivalent to the common fixed commission contract, in which the platform asks for a commission rate from the restaurant while it has complete control over its delivery fee.

### 4.3. The Platform-Pricing/Wage-Commitment (PW) Contract

Under the PW contract, the platform first commits to the wage $w$ paid to the delivery drivers and asks for a commission fee $r$ from the restaurant. The restaurant then sets the dine-in offline price $p_f$ alongside the profit margin $m_r$ charged to the platform for online orders. Finally, the platform sets the online price $p_o$ by posting its delivery fee $d$. Therefore, the online channel profit margin for the platform is $m_p^{PW} = p_o - m_r = r + d$ under this contract. We can write the platform's problem in the first stage of the game as a function of the wage for drivers and the commission fee,

$$\max_{w,r} \pi_p = \min(s(w), q_o)(r + d - w). \tag{23}$$

In the second stage, the restaurant sets the margin on online orders alongside the dine-in offline channel price to maximize its profit, which is given by

$$\max_{m_r, p_f} \pi_r = \min(s(w), q_o) m_r + q_f p_f. \tag{24}$$

Finally, the platform sets the online channel price by announcing the delivery fee $d$. Since $p_o = r + d + m_r$, the optimal delivery fee can be derived by solving for the optimal online price through,

$$\max_{p_o} \pi_p = \min(s(w), q_o)(p_o - m_r - w). \tag{25}$$

We solve for the equilibrium outcomes presented in the following lemma.

Lemma 4. *Under the PW contract, in equilibrium, the online and offline channel prices, and the wages for the drivers are*

$$p_o^{PW*} = \frac{2a(1-\beta^2) + 2b(3-\beta)(1-\beta^2) + 2 - \beta}{8b(1-\beta^2) + 2}, \tag{26}$$

$$p_f^{PW*} = \frac{1}{2}, \tag{27}$$

$$w^{PW*} = \frac{1 - \beta + 4a(1-\beta^2)}{1 + 4b(1-\beta^2)}. \tag{28}$$

Like the previous two contracting schemes, the online channel demand matches the labor supply in equilibrium under the PW contract. The platform sets the wage for the drivers before competing with the restaurant's offline channel. Once the wage is given, the number of drivers working for the platform (supply capacity) is fixed. If the realized online orders exceed the fixed supply, then the transaction volume is given by the supply of the drivers. Therefore, the restaurant and platform can increase profit margins without changing the transaction volume. Conversely, suppose the labor supply exceeds the online demand. In that case, the platform can decrease delivery drivers' wages in the first stage without changing the transaction volumes and the restaurant's pricing decisions.

### 4.4. The Restaurant-Pricing/Wage-Commitment (RW) Contract

Under the RW contract, the platform first commits to the wage paid to the delivery drivers and asks for a commission fee $r$ from the restaurant. The restaurant then sets the offline and online prices. Therefore, the online channel profit margins for the platform and the restaurant are $m_p^{RW} = r$ and

$m_r^{RW} = p_o - r$, respectively. We can write the platform's problem in the first stage of the game as a function of the wage and the commission fee as,

$$\max_{r,w} \pi_p = \min(s(w), q_o)(r - w). \tag{29}$$

In the second stage of the game, given that the restaurant's profit margin is given by $m_r^{RW} = p_o - r$, it sets the online and offline prices to maximize the following profit function,

$$\max_{p_o, p_f} \pi_r = \min(s(w), q_o)(p_o - r) + q_f p_f. \tag{30}$$

Similar to the RN contract, the online and offline channel demands are realized once the restaurant sets the online and offline prices. Solving backward, we can characterize the equilibrium outcomes in the following lemma, which indicates that the RN and the RW contracts are equivalent.

LEMMA 5. *Under the RW contract, the equilibrium outcomes are the same as those under the RN contract.*

The above lemma demonstrates that when the platform delegates channel pricing to the restaurant, wage commitment no longer impacts the optimal pricing decisions of either firm. In this setup, the platform first sets its commission fee as the Stackelberg leader, aiming to maximize its profit by squeezing the restaurant's margin. Since the restaurant controls online demand through online market pricing, retaining the lever of wage flexibility to influence supply by the platform seems insufficient to affect the equilibrium outcomes. This contrasts with scenarios where the platform manages channel pricing (e.g., in PW and PN contracts). In such cases, wage flexibility functions as a complementary lever to the pricing lever within the platform's operational framework (as detailed later in Section 4.5). In summary, wage commitment only impacts the equilibrium outcome when the platform also controls online channel pricing.

Given the findings in Lemma 5, we only investigate the PN, PW, and RN contracts in the rest of this paper. Next, we present an interesting observation about these contracting schemes in the following corollary.

COROLLARY 2. *The equilibrium dine-in offline price is independent of the contracting schemes between the restaurant, platform, and delivery drivers, and is equal to those of the fixed-labor-supply benchmark case.*

The equilibrium outcomes under all these contracting schemes indicate that the restaurant always prefers to set the dine-in offline channel price equal to $\frac{1}{2}$, independent of the contracting schemes. This observation corroborates that restaurants do not frequently change their dine-in prices, while they might change their online prices more regularly. In particular, online prices on different platforms might be different, under different contract settings between platforms and restaurants. The findings in Corollary 2 also enable us to characterize the competition intensity between the online and offline channels just base on the online channel price, because the dine-in offline price is constant and independent of contracting schemes. Specifically, a higher online channel price indicates softened competition between the online and offline channels, while a lower online price indicates intensified competition between the two channels.

### 4.5. The Impact of Self-Scheduling Drivers

To examine the impact of the self-scheduling drivers, we compare the equilibrium outcomes of all contracts in the fixed-labor-supply case (the benchmark case, see Lemma 1 in §3.5) with those under the sharing economy, where the labor supply of the drivers is self-scheduled. In the fixed-labor-supply case, we assume a fixed number of drivers $\hat{s}$ working for the platform, and the wage for the drivers $c$ is exogenous. In the sharing economy, the drivers are self-scheduling, and the platform can adjust the labor supply of drivers through the wage. To make a "fair" comparison, we assign the value of $c$ in the fixed-labor-supply models to the equilibrium wage value of the three contracting schemes, respectively. The following proposition characterizes our findings.

PROPOSITION 1. *The equilibrium online channel price is lower in the sharing economy case than that of the fixed-labor-supply case if and only if the equilibrium labor supply of drivers in the sharing economy is greater than that of the traditional economy, i.e., the fixed-labor-supply case* $\hat{s}$.

Proposition 1 indicates that the sharing economy with self-scheduling drivers might intensify or soften market competition compared to the fixed-labor-supply case, dependent on the level of the fixed labor supply but regardless of the contracting schemes. In particular, if the fixed supply of the drivers is limited, the platform and the restaurant will jointly set a high online channel price to ensure a large profit margin. In contrast, in the sharing economy, the platform and the restaurant will find it profitable to serve more customers by lowering the price in the online channel and adjusting the delivery drivers' wages. If the fixed labor supply is ample, the dynamics between the platform and the restaurant will lead to fierce competition between the online and offline channels. However, the platform and the restaurant would soften the competition in the sharing economy by adjusting the delivery drivers' wages and online channel prices.

The above finding is similar to the one discussed in Zhang et al. (2022a) in a two-sided market, where two platforms compete for drivers and customers. They show that the effect of the sharing economy on market competition is a function of the exogenous supply of drivers for these platforms. If the fixed supply of drivers for competing platforms exceeds the equilibrium supply in the sharing economy, adopting the sharing economy would soften market competition between these platforms. We extend their findings to a three-sided market, where a platform interacts with drivers and a restaurant to provide food delivery services to online customers, competing with the dine-in offline channel. If the platform has an ample fixed supply of drivers, diverting to a sharing economy model to supply drivers can help the platform soften cross-channel price competition in the market.

In the fixed-labor-supply case, the market outcomes are independent of the contractual structure (see Corollary 1). However, this independence does not hold in the sharing economy, where the platform can leverage the wage to control the labor supply of drivers, and we present the market outcomes in the following proposition.

Proposition 2. *The equilibrium market outcomes in the sharing economy depend on the contract schemes. In particular,*

*(i) The PN contract results in the highest, and the PW contract in the lowest online channel prices, i.e.,* $p_o^{PN*} \geq p_o^{RN*} \geq p_o^{PW*}$.

*(ii) The PW contract generates the highest, and the PN contract the lowest online demands, i.e.,* $q_o^{PW*} \geq q_o^{RN*} \geq q_o^{PN*}$.

*(iii) The PW contract offers the highest, and the PN contract the lowest wages for the drivers, i.e.,* $w^{PW*} \geq w^{RN*} \geq w^{PN*}$.

Part (i) of Proposition 2 shows that the market competition would be softened in the sharing economy if the platform sets the final online channel price, setting the delivery fees, and the delivery driver's wages simultaneously. In the PN contract, the platform has two levers to match drivers' labor supply with online channel demands, i.e., changing the final online channel price by charging different delivery fees or customizing the wages it offers to the delivery drivers. Such flexibility, on the platform side, benefits the restaurant, allowing it to charge a higher margin than the other contracting schemes, in which the platform has only one lever to match labor supply of drivers and online demands, i.e., only the wage or the final online channel price.

Part (i) of Proposition 2 also indicates the most fierce competition between the online and offline channels happens under the PW contract. Under this contract, the platform can set the labor supply of the drivers before getting involved in the competition with the restaurant's dine-in offline channel. We show that the platform should provide a large supply of drivers when competing with the dine-in offline channel under the PW contract. After the platform's commitment to an ample supply of drivers (compared with the other contracting schemes), the restaurant expects low delivery fees in the market as the platform has already committed to an ample supply of drivers. Therefore, the restaurant reduces the margin charged to the platform to benefit from larger online orders. In other words, with a commitment to an ample supply, both firms align their incentives to reduce their margins in the online channel, which increases the online channel's competitiveness. Our findings in Part (i) justify Parts (ii) and (iii), given that Corollary 2 establishes that the offline prices are the same under all contracting schemes. Therefore, a lower online price indicates a higher online demand, which also requires a higher wage to match the online demand and the labor supply of the drivers.

The above findings deviate from the literature on quantity-then-price competition. The literature suggests that when firms initially compete based on quantities and then on channel prices, they tend to limit their capacities to mitigate price competition later in the market (Kreps and Scheinkman 1983). However, this differs from our observations under the PW contract. While the platform's announced wage in the first stage indicates a capacity commitment, it is optimal for the platform to commit to an ample supply (Part (ii) of Proposition 2) to intensify the market competition between the online and offline channels. The PW contract fundamentally differs from the capacity-then-price competition model in the literature. In the PW contract, the platform and the restaurant move sequentially. After the platform's commitment to the labor supply of the drivers, the restaurant moves next to set its margin per unit sold in the online market alongside the dine-in offline prices. Moreover, we assume the restaurant has an unlimited capacity (as it does not need delivery drivers) to serve the dine-in customers. Given these distinct features of the food delivery market, we show that the platform should commit to an ample supply of drivers to induce the restaurant to reduce its margin and make the online channel more competitive. In other words, if the platform commits to a low capacity to curb market competition in the first stage (similar to the capacity-then-price competition), it is the restaurant that would benefit from the softened competition by charging a high margin as the restaurant moves next, which hurts the platform's profitability.

## 5. Comparison of the Contracting Schemes

The online food delivery market features a variety of contracting schemes, prompting a key question for all involved parties: the platform, the restaurant, the customers, and the delivery drivers. Which contracting scheme is the most advantageous from each one's perspective? This question gains significance considering the earlier findings that the equilibrium outcomes of these contracting schemes may display different characteristics. Moreover, this question is relevant in the sharing economy, as all contracting schemes result in the same market outcomes in the traditional economy where the labor supply of the drivers is fixed. In the sharing economy, the platform has to provide the right incentive to the delivery drivers to match their labor supply with the online demand.

### 5.1. The Platform's and the Restaurant's Preference over the Contracting Schemes

The following proposition compares the platform's and the restaurant's equilibrium profits under different contracting schemes. Moreover, we present our findings for the food delivery chain's profit, which is defined as the sum of the platform and the restaurant's profit, i.e., $\pi_{sc}^{i*} = \pi_p^{i*} + \pi_r^{i*}$.

PROPOSITION 3. *We can establish the following:*

*(i) For the platform, if* $b \geq \frac{1}{8(1-\beta^2)}$*, then* $\pi_p^{RN*} \geq \pi_p^{PW*} \geq \pi_p^{PN*}$*; otherwise,* $\pi_p^{RN*} \geq \pi_p^{PN*} \geq \pi_p^{PW*}$*.*

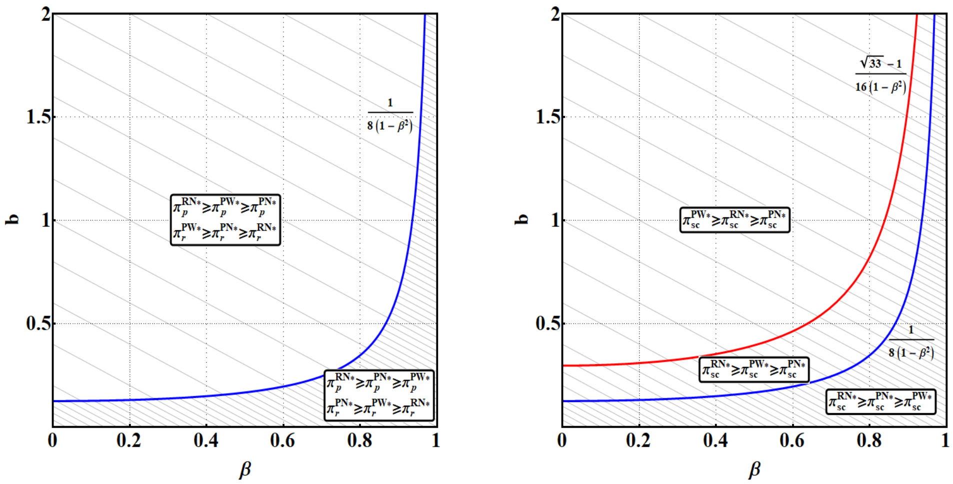
*(ii) For the restaurant, if* $b \geq \frac{1}{8(1-\beta^2)}$, *then* $\pi_r^{PW*} \geq \pi_r^{PN*} \geq \pi_r^{RN*}$; *otherwise,* $\pi_r^{PN*} \geq \pi_r^{PW*} \geq \pi_r^{RN*}$.

*(iii) For the food delivery chain, if* $b \geq \frac{\sqrt{33}-1}{16(1-\beta^2)}$, *then* $\pi_{sc}^{PW*} \geq \pi_{sc}^{RN*} \geq \pi_{sc}^{PN*}$; *if* $\frac{1}{8(1-\beta^2)} \leq b \leq \frac{\sqrt{33}-1}{16(1-\beta^2)}$, *then* $\pi_{sc}^{RN*} \geq \pi_{sc}^{PW*} \geq \pi_{sc}^{PN*}$; *otherwise,* $\pi_{sc}^{RN*} \geq \pi_{sc}^{PN*} \geq \pi_{sc}^{PW*}$.

Part (i) of Proposition 3 indicates that the platform always favors the RN contract, while the worst performance is attributed to the PN/PW contracts. Specifically, when the cost of labor supply is excessively high (i.e., $b \leq \frac{1}{8(1-\beta^2)}$, please refer to Figure 3(a)), the performance of the PN contract for the platform is better than the PW contract; otherwise, the platform prefers the PW contract. It is straightforward to show that the restaurant charges the largest margins under the PN contract, knowing that the platform has two levers to match the drivers' labor supply and online demand. This increases the online price, softening competition between online and offline channels, and benefiting only the restaurant. As a response to such an effect, the platform has two contracting choices: to first commit to the margin and relegate online channel pricing to the restaurant (the RN contract), or first commit to the wage offered to the drivers and still control the price in the online channel (the PW contract). Part (i) demonstrates that the PN contract might have an advantage over the PW contract for the platform only when the labor supply cost is pretty high. Note that the PW contract results in the largest supply of drivers, which requires costly investment in labor supply through high wages. Otherwise, either the RN or PW contract improve the platform's profitability compared to the PN contracts. Furthermore, the platform always prefers RN over PW contracts, because the required wage is high in the PW contracts, while the margin charged needs to be low. This boosts online demand at the cost of profit margin as the platform matches the pre-committed labor supply and online demand.

Part (ii) characterizes the optimal contracting scheme for the restaurant. The RN contract performs the worst for the restaurant. The reason is that the platform sets a high commission fee under the RN contract before the restaurant sets the online channel price. Compared to the PN contract, the restaurant has to reduce its margin to increase the online demand and persuade the platform to offer a higher wage for the delivery drivers, given that the platform has only one lever left (i.e., the delivery wage) to match the labor supply of the drivers and the online channel demand. Altogether, the RN contract results in the worst performance for the restaurant. Part (ii) also indicates that the PW or PN contracts might be the best-performing contract for the restaurant. When the cost of supply is not excessively high (i.e., $b \geq \frac{1}{8(1-\beta^2)}$), the restaurant benefits from the PW contract (see Figure 3(a)), as the high wages committed by the platform ensure an ample supply of drivers, making it profitable for the restaurant to reduce its margin on online orders. As the supply cost significantly increases, providing a large supply becomes particularly challenging for the platform, leading to a substantial decrease in online demand/supply. In this case, the restaurant favors the contract that

Figure 3: Comparison of Player's Equilibrium Profits under Different Contracting Schemes




(a) The platform and the restaurant (b) The food delivery chain

offers a higher margin, namely the PN contract. However, our extensive numerical analysis in Table 2 shows that the performance of the PN contract only barely surpasses that of the PW contract for the restaurant. This is because, as $b$ decreases to extremely low values, both the PN's advantage over the PW contract in margins and the PW's online demand advantage over the PN would diminish. As a result, while the PN's profit for the restaurant can surpass that of the PW contract, the improvement is negligible.

Part (iii) of Proposition 3 shows that from the food delivery chain's perspective, only the PW or the RN contract can arise as the best-performing contract. Like Part (ii), Figure 3(b) shows that the RN contract can arise as the best-performing contract for the food delivery chain only when the labor supply cost is high. Our numerical investigation in Table 2 shows that even though the RN contracts dominate the PW contracts for high labor costs, the performance gap between the PW and RN contracts is quite small. Like the PN contract, the RN contract has an online price advantage over the PW contract (the most fierce cross-channel price competition happens under the PW contract), while the PW contract has an online demand advantage. As the supply becomes quite costly, these advantages diminish, resulting in somewhat similar performances for the food delivery chain. Unsurprisingly, the PN contract arises as the worst-performing contract for the food delivery chain unless the supply cost is excessively high. As discussed earlier, the PN contract results in the minimum online orders among all contracts, as it cannot align the waging and pricing decisions between the platform and the restaurant. Interestingly, when the restaurant prefers the PN contract over the PW contract, i.e., for excessively costly labor supply, the platform and the whole food delivery chain prefer the PN contract over the PW contract. Again, according to Table 2 in Section 6, the potential advantage of the PN contract over the PW contract is quite negligible. Next, we investigate the food delivery chain's online/offline demands and the efficiency of different contracting schemes.

Proposition 4. *We can establish the following:*

*(i) For the food delivery chain's demand, we have* $q_o^{PW*} + q_f^{PW*} \geq q_o^{RN*} + q_f^{RN*} \geq q_o^{PN*} + q_f^{PN*}$.

*(ii) For the demand under the PW contract compared to the centralized case, if* $b \leq \frac{1}{2(1-\beta^2)}$, *then* $q_o^{PW*} \geq q_o^{C*}$ *and* $q_o^{PW*} + q_f^{PW*} \geq q_o^{C*} + q_f^{C*}$; *otherwise,* $q_o^{PW*} \leq q_o^{C*}$ *and* $q_o^{PW*} + q_f^{PW*} \leq q_o^{C*} + q_f^{C*}$.

The PW contract allows the restaurant and the platform to coordinate on an intensified competition between the online and offline channels, leading to low online prices and high online orders and increasing the food delivery chain's total demand, as part (i) of Proposition 4 indicates. Interestingly, this contracting scheme can result in an oversupply of labor force/excessive online demand compared to the centralized case (see the Appendix C) in the online channel. In particular, when the supply cost is relatively high, the demand and supply under the PW contract surpass those in a centralized system. Expensive supply indicates that even in a centralized system, investment in supply is low. The PW contract offers a mechanism that allows the platform to increase online demand by committing to a high supply, as we discussed before. The platform finds it optimal to commit more aggressively to supply when the supply cost is high. In contrast, low supply costs indicate that the centralized model invests heavily in the online channel, and the platform does not need to commit to excessive supply under the PW contract.

In a competitive two-sided market, Zhang et al. (2022a) and Hu and Liu (2023) show that wage commitment can intensify market price competition only when the competition intensity on the supply side is greater than that on the demand side. We uncover intensified market competition under the PW contract for a different reason in online food delivery markets, where the sequential nature of decisions by the platform and the restaurant plays an important role. Unlike the two-sided markets, where two platforms move simultaneously to set their wages and then prices, the platform moves first under the PW contract in the three-sided food delivery market. The restaurant moves next, and then online and offline channels compete. We show that commitment to a high supply of drivers is a lever for the platform to persuade the restaurant that it would charge competitive delivery fees in the market to keep the online channel competitive, given its committed supply of drivers. Such a commitment aligns both firms' incentives so that the restaurant charges low margins to the platform, which makes the online channel more competitive.

### 5.2. The Customers' and the Drivers' Preference over the Contracting Schemes

While we are mainly interested in the food delivery chain's performance, it is also essential to study the effect of these contracting schemes on the other players, i.e., the delivery drivers and the customers.

We can find the equilibrium customer and driver surplus ($CS^{i*}$ and $DS^{i*}$, respectively, for $i \in \{RN, PN, PW\}$), and social welfare ($SW^{i*}$), as defined in Equations (E.2), (E.4) and (E.5) in the Appendix E. The relative performances of the studied contracts are presented in the following proposition.

PROPOSITION 5. *We can establish the following:*
*(i) For the customer surplus in equilibrium, we have* $CS^{PW*} \geq CS^{RN*} \geq CS^{PN*}$.
*(ii) For the driver surplus in equilibrium, we have* $DS^{PW*} \geq DS^{RN*} \geq DS^{PN*}$.
*(iii) For the social welfare in equilibrium, we have* $SW^{PW*} \geq SW^{RN*} \geq SW^{PN*}$.

The customers benefit from the PW contract, as Part (i) of Proposition 5 indicates. Our findings in Proposition 2 help us explain this finding. Given that the offline price is independent of the contracting scheme, we can expect that the total customer surplus decreases as the online price increases. The PW contract results in the lowest online prices, as it aligns both the restaurant and the platform's incentive to charge lower online prices, which benefits the customers. The customers are worst off under the PN contract as it results in the highest online price and the lowest online demand, which hurts the online customers' surplus.

The findings on the delivery drivers' surplus in Part (ii) of Proposition 5 are unsurprising, as the PW contract not only maximizes the wage but also results in the largest online demand. The commitment to an ample labor supply of drivers in the first stage incentivizes both the restaurant and the platform to price the online channel competitively. This, in turn, boosts online demand, necessitates higher wages, and ultimately increases the drivers' surplus.

Part (iii) of Proposition 3 shows that the PW contract arises as the dominant contracting scheme for the food delivery chain. Parts (i) and (ii) of Proposition 5 indicate that for both the drivers and customers, this contracting scheme dominates the others; therefore, it is not surprising that this contract maximizes the social welfare among these contracts. The advantage of this contract lies in its capability to align the restaurant and the platform's online channel pricing, pushing for a larger online demand through lower margins. The only party that loses under this contract is the platform, which prefers the RN contract. Notably, the RN contract performs better than the PN contract for both customers and drivers. The worst performance of the PN contract in providing the right incentive for the restaurant and the platform to coordinate their online pricing hurts both of them, as well as customers and drivers, through high online prices and low delivery wages.

The government's regulatory body is concerned about low payments to delivery drivers, a category of gig workers (Zipperer et al. 2022). To protect gig economy workers, regulators have extensively discussed the employment status of delivery drivers and establishing minimum wages. For example, the New York State Supreme Court began mandating a minimum hourly wage for delivery drivers since 2023 (PYMNTS 2023). We find that when the minimum wage policy was enforced, the wages set by the platforms for drivers were approximately equal to the minimum wage mandated by the government. Specifically, as introduced in PYMNTS (2023), the New York State Supreme Court required a minimum wage of \$17.96 per hour (before tips) in the first quarter of 2024, while the average wage on the delivery platforms in New York was \$16.96[4]. In the second quarter, the government raised the minimum wage requirement to \$19.56 per hour[5], and the average wage on the delivery platforms increased to \$19.88[6].

The reclassification of gig workers as regular employees in several countries (e.g., the USA and China, and expected to be followed by other countries), along with the observation that platforms are often unwilling to pay more than the minimum wage to their delivery drivers, suggests that we should view the minimum wage as a committed wage by the platform. Specifically, platforms must commit to a wage to their employees upon hiring and adhere to wage regulations. As discussed regarding the PW contract, wage commitment can benefit the entire food delivery market, including restaurants, customers, and drivers. However, the platform is disadvantaged compared to its most preferred RN contract. Remember that the RN contract is the least favorable contract for the restaurant. To create a contract that is preferred by all parties (compared to the platform's favored contract RN) in the online food delivery chain, we aim to design a specific contract under which the profits of every player improve. The following corollary establishes the existence of such a contract.

Corollary 3. *When $b \geq \frac{\sqrt{33}-1}{16(1-\beta^2)}$, there exists a modified PW contract with a positive transfer payment $T > 0$ from the restaurant to the platform, under which both the platform's and the restaurant's profits are higher than those under the RN contract. i.e.,*

$$\pi_p^{PW*} + T \geq \pi_p^{RN*}, and\ \pi_r^{PW*} - T \geq \pi_r^{RN*},$$

*so that the modified PW contract is preferred by all players in the food delivery chain.*

Such a modified PW contract exists because, among these contracting schemes, the total food delivery chain's profit is maximized under the PW contract unless the labor supply cost is pretty high (i.e., when $b < \frac{\sqrt{33}-1}{16(1-\beta^2)}$). Even when the PW contract is not the best-performing contract, it performs pretty close to the one that maximizes the food delivery chain's profit (please refer to Table 2 in Section 6, which offers extensive numerical experiments).

When the labor supply cost is not very high (i.e., $b > \frac{\sqrt{33}-1}{16(1-\beta^2)}$), a transfer payment from the restaurant to the platform could make the platform indifferent between the RN and the modified

[4] Pay per hour was less than the minimum wage of \$17.96 because some apps maintained excessive levels of uncompensated on-call time during the quarter. This was allowable and does not indicate a legal violation.

[5] https://www.geekwire.com/2024/new-data-reveals-impact-of-minimum-wage-law-on-food-delivery-drivers-and-orders-in-nyc/

[6] https://www.nyc.gov/assets/dca/downloads/xlsx/Restaurant-Delivery-App-Data-Quarterly.xlsx

PW contract, maximizing the platform's profit among all contracting schemes. At the same time, the restaurant is willing to pay such a transfer payment as it strictly prefers PW over the RN or PN contracts. Such a fixed transfer payment does not affect the equilibrium market outcomes since channel prices and drivers' wages are the same as the PW contracts. Therefore, customer and driver surplus remains the same as those under the original PW contract. In the case of a high labor supply cost (i.e., $b < \frac{\sqrt{33}-1}{16(1-\beta^2)}$), a transfer payment can make the platform indifferent between the RN and PW contracts. Such a transfer payment makes the restaurant worse off, but as our numerical experiments indicate since the delivery chain's profit loss under the PW contract compared to the RN contract is pretty small, the loss for the restaurant under the modified contract would be negligible. In practice, some platforms like Sesame have started to charge restaurants fixed monthly fees (Joe 2021), which can be treated as a form of transfer payment. Moreover, this modified PW contract interests platforms trying to increase their online market share. It allows the food delivery chain to increase online sales while benefiting both platforms and restaurants.

As discussed earlier, the minimum wage can be viewed as the platform's committed wage, particularly as the regulators continue to reclassify gig workers as employees. Therefore, given that the highest wages are paid under the PW (and also, the modified PW ) contracts, a relatively high minimum wage, as long as it is less than $w^{PW*}$, might benefit not only the drivers but also society and the whole chain. In Section 7, we show that commitment to a relatively high minimum wage rate has different implications from the minimum wage commitment for the food delivery chain. This reveals an interesting observation for regulators designing new minimum wage or wage rate regulations. Next, the following section uses extensive numerical experiments to improve our understanding of the contracting schemes studied and their relative performances.

## 6. Numerical Experiments

In this section, we use extensive numerical experiments to evaluate the relative performance of the studied contracting schemes for the platform, the restaurant, and the food delivery chain. To this end, we consider the following parameter ranges: $\beta \in [0.01, 1]$, $a \in [0.01, 1]$, and $b \in [0.1, 1.9]$. We divide the ranges for $\beta$ and $a$ ($b$) into 100 (10) equally placed intervals and use a combination of these values to solve for the optimal contracting terms for all the contracting schemes and the centralized case. We also assess the feasibility of these contracting terms, ensuring that both the online and offline channels remain active in equilibrium. In total, we analyze 44,458 different feasible scenarios.

We denote the loss of profit for firm $f$ in scheme $t$ compared to scheme $k$ as $L_f^{t,k} = \frac{\pi_f^k - \pi_f^t}{\pi_f^k}$, with $\pi_f^t < \pi_f^k$, $f \in \{r, p, sc\}$ and $t, k \in \{RN, PN, PW, C\}$. Tables 2 summarizes our findings. It demonstrates that the PW contract performs well for the food delivery chain compared to the centralized solution, as the loss of profit stands at 0.54% on average, with a maximum of 4.12% among all tested scenarios.

Table 2: The Performance of Different Contracting Schemes

| Loss of profit | Average | 10 percentile | 90 percentile | Maximum |
|---|---|---|---|---|
| $L_{sc}^{PW,C}$ | 0.0054 | 0.0000 | 0.0165 | 0.0412 |
| $L_{sc}^{RN,C}$ | 0.0105 | 0.0001 | 0.0287 | 0.0609 |
| $L_{sc}^{PN,C}$ | 0.0209 | 0.0005 | 0.0533 | 0.0971 |
| $L_{sc}^{PW,RN}$ | 0.0011 | 0.0000 | 0.0026 | 0.0215 |
| $L_{sc}^{RN,PW}$ | 0.0055 | 0.0002 | 0.0135 | 0.0234 |
| $L_{sc}^{PW,PN}$ | 0.0008 | 0.0000 | 0.0028 | 0.0052 |
| $L_{r}^{PW,PN}$ | 0.0006 | 0.0000 | 0.0019 | 0.0035 |
| $L_{r}^{PN,PW}$ | 0.0112 | 0.0003 | 0.0279 | 0.0514 |

Table 3: Demand Gap for Different Contracting Schemes

| Demand Gap | Average | Minimum | 10 percentile | 90 percentile | Maximum |
|---|---|---|---|---|---|
| $K_{o}^{PW,C}$ | 0.2124 | 0 | 0.0661 | 0.3077 | 0.8899 |
| $K_{o}^{RN,C}$ | 0.3205 | 0.0190 | 0.2294 | 0.3853 | 0.3958 |
| $K_{o}^{PN,C}$ | 0.5 | 0.5 | 0.5 | 0.5 | 0.5 |
| $K_{sc}^{PW,C}$ | 0.0284 | 0 | 0.0011 | 0.0714 | 0.1269 |
| $K_{sc}^{RN,C}$ | 0.0416 | 0 | 0.0032 | 0.0939 | 0.1543 |
| $K_{sc}^{PN,C}$ | 0.0596 | 0 | 0.0067 | 0.1268 | 0.1949 |

The performance of the next best contract, i.e., the RN contract, is also good at 1.05% loss of profit on average. The loss of profit for the PN contract is more significant, averaging at 2.09%.

We also use Table 2 to illustrate that while the PW contract may not always be the optimal choice for the food delivery chain or for the restaurant, as shown in Proposition 3, the profit loss is relatively minor compared to the best-performing contracts. If the supply chain profit under the PW contract is less than that under the RN contract, the average profit loss is 0.1%. Additionally, the profit loss for both the supply chain and the restaurant under the PW contract compared to the PN contract is quite negligible (0.08% and 0.06%, respectively). Therefore, we claim that the PW contract arises as the preferred contract for both the food delivery chain and the restaurant.

In addition to the profit loss, we also study the demand gap between each contract and the centralized case, focusing on both the online demand and total demand. We define the demand gap in contract scheme $t$ compared to the centralized case C as $K_g^{t,C} = \frac{|Q_g^C - Q_g^t|}{Q_g^C}$, where $g \in \{o, sc\}$ and $t \in \{RN, PN, PW\}$, and $Q_g$ denotes the demand for the online channel $o$ or the total supply chain $sc$. Table 3 illustrates the results. Despite its prevalence in practice, it is noteworthy that the PN contract induces 50% less online demand than the centralized online demand. This also justifies the poor performance of this contract compared to the centralized case. The online demand gap with the centralized case reduces significantly as the platform adopts the RN or PW contracts. It is also notable that even the PW contract results in about 21.24% loss/excess in online demand. For the total demand, the change in offline demand moderates the changes in total demand under the PW contract, as the total demand loss/excess stands at only 2.84%. Such a small gap in total demand justifies our earlier finding that the PW contract can achieve more than 99.5% of the total profit for the food delivery chain (on average). The lowest demand loss/excess under the PW contract indicates better coordination between the restaurant and the platform to serve online customers compared to the centralized case.

## 7. Extension: Labor Supply Model with Wage Rate

The labor supply model of delivery drivers used in the main body of this study has a limitation in that it only considers the supply as a function of the wage offered by the platform. In other words, it assumes that the platform pays the delivery drivers a certain wage per unit of time. In practice, it is also common for drivers to get paid based on the number of deliveries they make. Therefore, these drivers' motivation to participate in food delivery is also a function of the online channel demand rate (i.e., how busy they are with deliveries). While the supply model of the main body captures the main characteristics of the labor supply of the drivers, it does not explicitly account for the impact of the online demand rate on the supply of the delivery drivers. In the rest of this subsection, we present a framework to address this issue and demonstrate the robustness of our findings in the main model. Moreover, we reveal the implications of introducing the wage rate into the labor supply model for the regulators designing mechanisms to protect the gig economy workers (i.e., the drivers in our model).

Gig workers are in high demand, with platforms competing to hire them. They can choose when and where to work based on the wages offered (Zhang et al. 2022a). As a result, many gig drivers work for multiple platforms simultaneously and can switch between them in real-time. To model this phenomenon, we follow the literature on differentiated duopolies (Abhishek et al. 2016, Sun et al. 2023) and use a utility framework similar to Equation (E.3) (in the Appendix E) to model the drivers' choice. Specifically, a representative driver can work for the platform or an outside option. If he chooses to work for the outside option, he will receive a fixed amount of income per unit of time $y$; If he works for the platform, his expected wage is given by $wq_o$, i.e., the wage rate paid by the platform times the online channel demand. Assume that the representative driver maximizes the following utility function,

$$\arg\max_{l_p, l_o} U(wq_o, y) = l_p wq_o + l_o y - \tfrac{1}{2}l_p^2 - \tfrac{1}{2}l_o^2 - \phi l_p l_o, \tag{31}$$

where $l_p$ and $l_o$ denote the amount of labor that a representative driver allocates to work for the platform and the outside option, respectively, and $\phi$ represents the substitutability of working for the platform vs. the outside option. Assuming that the thickness of the drivers' supply market is $N$, it is straightforward to show that the supply of drivers for the platform is given by

$$s = \tfrac{wq_o - \phi y}{1-\phi^2} N. \tag{32}$$

The above formulation of drivers' labor supply captures the indirect network effect of online demand on the supply side. In particular, increasing online demand also implies a greater interest from drivers to work for the platform. This feature is mainly overlooked in the main supply model, given by Equation (3). Substituting the supply equation, we solve for the equilibrium market outcomes under the three contracting schemes represented in the following proposition, and all other results and proofs of this extension section are provided in the Online Supplement H and J.

Proposition 6. *If the labor supply of the delivery drivers is given by* (32)*, we have,*

*(i) The online channel price satisfies* $p_o^{PN*} \geq p_o^{RN*} \geq p_o^{PW*}$.

*(ii) The wage rate for the delivery drivers satisfies* $w^{PW*} \leq w^{RN*} \leq w^{PN*}$*, but the wage paid to delivery drivers satisfies* $q_o^{PW*}w^{PW*} \geq q_o^{RN*}w^{RN*} \geq q_o^{PN*}w^{PN*}$.

*(iii) The online channel demand satisfies* $q_o^{PW*} \geq q_o^{RN*} \geq q_o^{PN*}$.

Proposition 6 confirms that introducing wage rates does not impact our main findings. In particular, while the wage rate under the PW contract is the minimum among all schemes, the total wage paid to delivery drivers that incorporate demand rates (i.e., $wq_o$) is still the maximum under the PW contract. This finding also aligns with Parts (i) and (iii), confirming that the wage rate-induced labor supply does not change the conclusions of the main model. In particular, the PN contract has the highest online channel price, while the PW contract results in the most fierce market competition. The intuition behind these findings is quite the same as that of the main model.

The mechanism in which the PW contract delivers its good performance somehow differs between the main and the wage rate models. When supply is only a function of the wage (i.e., the main model), the platform has to commit to a high wage in the first stage to induce enough participation by the drivers. However, when driver supply is a function of the wage and the demand rate together, the platform should commit to a low enough wage rate, inducing the restaurant to reduce the margin to provide the right incentive for the platform to post low delivery fees (knowing that the platform pays a low wage rate); this aligns the platform and the restaurant's incentive to make the online channel competitive against the dine-in offline channel, by setting low online prices.

Next, we investigate how the introduction of the wage rate affects the players' profitability in the food delivery chain under different contracting schemes.

Proposition 7. *If the supply of the delivery drivers is given by* (32)*, we have,*

*(i) If* $0 < N \leq \frac{1-\phi^2}{8-8\beta^2}$*, then* $\pi_p^{RN*} \geq \pi_p^{PN*} \geq \pi_p^{PW*}$*; otherwise,* $\pi_p^{RN*} \geq \pi_p^{PW*} \geq \pi_p^{PN*}$.

*(ii) If* $0 < N \leq \frac{1-\phi^2}{8-8\beta^2}$*, then* $\pi_r^{PN*} \geq \pi_r^{PW*} \geq \pi_r^{RN*}$*; otherwise,* $\pi_r^{PW*} \geq \pi_r^{PN*} \geq \pi_r^{RN*}$.

*(iii) If* $0 < N \leq \frac{1-\phi^2}{8-8\beta^2}$*, then* $\pi_{sc}^{RN*} \geq \pi_{sc}^{PN*} \geq \pi_{sc}^{PW*}$*; if* $\frac{1-\phi^2}{8-8\beta^2} < N \leq \frac{(\sqrt{33}-1)(\phi^2-1)}{16(\beta^2-1)}$*, then* $\pi_{sc}^{RN*} \geq \pi_{sc}^{PW*} \geq \pi_{sc}^{PN*}$*; otherwise,* $\pi_{sc}^{PW*} \geq \pi_{sc}^{RN*} \geq \pi_{sc}^{PN*}$.

*(iv)For the customer's surplus in equilibrium:* $CS^{PW*} \geq CS^{RN*} \geq CS^{PN*}$.

*(v)For the driver's surplus in equilibrium:* $DS^{PW*} \geq DS^{RN*} \geq DS^{PN*}$.

While the utility formulation of the drivers' supply is quite different from the supply formulation in (3), Proposition 7 shows that our findings in the main model are pretty robust. In particular, Proposition 7 indicates that when the supply market is pretty thin (i.e., $N \leq \frac{1-\phi^2}{8-8\beta^2}$), which indicates raising supply is excessively costly, the platform prefers the RN contract while the restaurant prefers the PN contract. A thicker market, as it implies a cheaper labor supply, makes the restaurant prefer the PW contract. Proposition 7 also characterizes how different contracting schemes affect the drivers and customers. In particular, we can show that the PW contract might benefit not only the restaurant but also the drivers and customers, which aligns with our previous findings.

From a government perspective, regulations have been implemented setting a minimum delivery fee per delivery (minimum wage rate). These regulations can also be viewed as a form of wage commitment imposed on platforms. For example, the State Court of New York has ruled that the food delivery platforms should pay the delivery drivers 50 cents per minute of delivery before tips (Lindeque 2024). In contrast to the primary base model, we find that when the government sets a minimum wage rate, it is not optimal for the wage rate to be set too high because the optimal wage rate under the PW contract is the small wage rate among all contracts. This commitment to a relatively low wage rate allows both the restaurant and the platform to align with charging low margins to keep the online channel competitive vs. the offline channel in the food market. Competitive online pricing increases online orders, maximizing the wage drivers receive under the PW with committed wage rates compared to the other contracts. To conclude, while a relatively high minimum wage per hour can benefit the drivers and the food delivery chain, a relatively high minimum wage rate can hurt the drivers and the whole food delivery chain.

## 8. Discussions and Conclusions

The emergence of the sharing economy has prompted the evolution of digital platforms, which have yet to be extensively studied in the literature. This paper addresses this gap by examining online food delivery platforms' pricing and waging decisions within a three-sided market. We provide an analytical framework focusing on the key trade-offs a food delivery platform encounters as it contracts with restaurants and gig economy drivers to provide food delivery services to customers.

In such a three-sided online food delivery market, the match of online customers' demands and labor supply of drivers requires careful management of the platform's relationship with the self-scheduling drivers and restaurants, as they are providing the food offered on the platform. Without self-scheduling drivers, we show that all the introduced contracting schemes have the same market outcome for the food delivery chain. This observation falls apart as we incorporate the self-scheduling nature of the delivery drivers' labor supply. These contracts might result in different market outcomes, highlighting the importance of modeling the self-scheduling of the drivers in a three-sided market.

Compared to the traditional economy with a fixed labor supply, resorting to self-scheduling service providers could soften market competition if the traditional economy suffers from insufficient capacity.

Under the sharing economy, the platform always prefers RN/RW contracts. In contrast, the restaurant and the whole food delivery chain prefer the PW contract (unless the labor supply is excessively costly). The PW contract maximizes the online demand by aligning the platform and the restaurant's incentives so that the restaurant charges low online profit margins to the platform. Specifically, while the platform and the restaurant move sequentially to set their margins for online sales, the platform's commitment to a high wage for drivers aligns both the platform's and the restaurant's incentives to offer competitive online prices. Given the inferior performance of the PW contract for the platform, we propose a modified PW contract under which the platform is compensated with a transfer payment from the restaurant if the PW contract is implemented so that it would benefit not only the food delivery chain but also customers and drivers.

Finally, we investigate different implementations of the minimum wage requirement contemplated by regulators to protect drivers and increase social welfare. Given the optimality of the PW contract for the food delivery chain, which maximizes the drivers' and customers' surplus and social welfare, we show that a relatively high minimum wage can still benefit all of them. In contrast, if the regulator aims to set a minimum wage rate, then a relatively low rate can protect the drivers while benefiting the food delivery chain, as it aligns the platform and the restaurant's incentives to increase the competitiveness of the online channel.

This paper focuses solely on modeling the competition between a single restaurant's online food delivery and offline dine-in channels. Future research could enhance the robustness of our findings by examining competition among multiple restaurants and platforms, introducing new driving forces that could reshape equilibrium outcomes. Additionally, another avenue for future research could explore how waiting times affect online customers' behavior and the subsequent impact on the operations strategy of food delivery platforms, which is a common issue faced by these on-demand service providers.

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# Appendix to “Pricing and Waging in Three-Sided Food Delivery Markets”

## Appendix A: Notations

Table A.1: Table of Notations

| Symbols | Description |
|---|---|
| $\beta$ | the degree of differentiation between online and offline channels |
| $a$ | attraction of outside work options for drivers |
| $b$ | delivery drivers' sensitivity to the platform's wage |
| $p_f$ | offline (full) channel price |
| $p_o$ | online (full) channel price |
| $m_p$ | online profit margin of the platform |
| $m_r$ | online profit margin of the restaurant |
| $d$ | delivery fee |
| $r$ | commission fee asked by the platform |
| $w$ | driver's wage |
| $q_o$ | online demand |
| $q_f$ | offline (dine-in) demand |
| $s$ | supply of delivery drivers |
| $\pi_f$ | firm $f$'s profit, $f \in \{p, r, sc\}$ |
| $L_f^{t,k}$ | loss of profit for firm $f$ in scheme $t$ compared to scheme $k$, where $f \in \{p, r, sc\}$, and $t, k \in \{RN, PN, PW, C\}$ |
| $K_g^{t,C}$ | demand gap in scheme $t$ compared to the centralized case, where $g \in \{o, sc\}$, and $t \in \{RN, PN, PW\}$ |
| $l_o$ | the amount of labor that the driver allocates to work for the outside option |
| $l_p$ | the amount of labor that the driver allocates to work for the platform |
| $\phi$ | the substitutability of working for the platform vs. the outside option |
| $y$ | fixed amount of income per unit of time for outside option |
| $N$ | The thickness of drivers' supply |
| $\alpha$ | relative potential market size of the dine-in channel vs. online channel |

## Appendix B: Features of Well-Known Food Delivery Platforms: Table B.1

## Appendix C: The Centralized Case

In this benchmark case, a centralized decision maker sets the online and offline channel prices, $p_o$ and $p_f$, and the wages for the delivery drivers $w$ to maximize the profit of the food delivery supply chain:

$$\pi_{sc}^C = \max_{w, p_o, p_f} \min(s(w), q_o)(p_o - w) + p_f q_f, \tag{C.1}$$

where $\pi_{sc}^C$ is the sum of the sales profit from the online and offline channels. This case serves as a benchmark to evaluate the performance of different contracting schemes for the entire food delivery market. The following lemma characterizes the equilibrium outcomes (denoted by superscript “C”) and the proof of Lemma is provided in Section J in the Online Supplement.

Table B.1: Features of Well-Known Food Delivery Platforms

| Name | Region | Channel price set by | Wage commitment | Contract |
|---|---|---|---|---|
| Ele.me | East Asia | Platform | Yes | PW[a] |
| Ele.me | East Asia | Restaurant | No | RN[b] |
| Meituan | East Asia | Platform | Yes | PW[c] |
| Meituan | East Asia | Restaurant | Yes | RW[d] |
| Uber Eats | North America | Platform | No | PN[e] |
| Doordash | North America | Platform | No | PN[f] |
| Grubhub | North America | Platform | No | PN[g] |
| Uber Eats | North America | Platform | Yes | PW[h] |
| Doordash | North America | Platform | Yes | PW[i] |
| Grubhub | North America | Platform | Yes | PW[j] |
| FoodPanda | Southeast Asia | Platform | No | PN[k] |
| Zomato | South Asia | Restaurant[l] | No[m] | RN |
| Deliveroo | Europe | Restaurant[n] | No[o] | RN |

[a] When the contract between the restaurant and the platform falls under the "Employed Driver Program" in Ele.me (Eleme 2024), the platform provides drivers with stable wages and fixed earnings, with drivers deciding whether to join the platform based on the wages promised by the platform (Cheng 2021). Furthermore, under the "Employed Driver Program", the platform retains control over the final online channel price by determining the delivery fee on top of the food price set by the restaurant (Eleme 2024)

[b] When the contract between the restaurant and the platform falls under the "Spark Program" in Ele.me (Eleme 2024), the online channel price paid by the customer for online orders is collected by the restaurant and the drivers are crowdsourced.

[c] Similar to Ele.me, Meituan also provides the "Employed Driver Program" (https://peisong.meituan.com/download).

[d] Meituan allows the restaurant to set the delivery fee when the restaurant joins the "Restaurant Delivery Service Program". This program allows restaurants to set delivery fees while utilizing the platform's delivery drivers for order fulfillment, providing restaurants with greater autonomy in managing their operations. Figure B.1 below illustrates the interface on Meituan where restaurants set their delivery fees.

[e] https://merchants.ubereats.com/us/en/pricing/

[f] https://merchants.doordash.com/en-ca/products/marketplace

[g] https://get.grubhub.com/products/marketplace/

[h] Some government agencies have introduced minimum wage policies to ensure the welfare of delivery riders, such as those implemented in New York and Seattle. These minimum wage policies can be viewed as instances where platforms effectively commit to delivery wages, as the wages offered by platforms closely align with the government-mandated minimum wage levels (https://www.nyc.gov/assets/dca/downloads/xlsx/Restaurant-Delivery-App-Data-Quarterly.xlsx).

[i] Similar to the PW contract in Uber Eats

[j] Similar to the PW contract in Uber Eats

[k] https://www.supliful.com/blog/foodpanda-business-model-canvas-explained

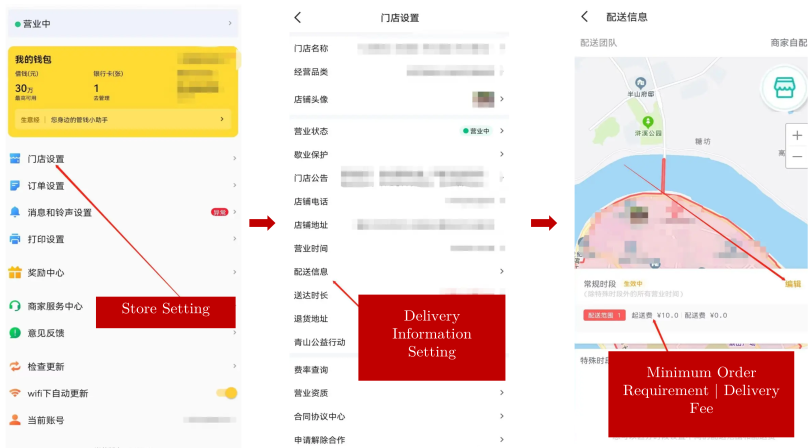
[l] https://blog.menuviel.com/zomato-fees-and-commissions-for-restaurants/

[m] https://www.zomato.com/deliver-food/

[n] https://www.deliverect.com/en-gb/blog/online-food-delivery/deliveroo-101-the-essential-guide-for-restaurants

[o] https://deliveroo.co.uk/

Figure B.1: Illustration of the Restaurants Interfaces on Meituan




Lemma C1. *In the centralized model, the online and offline channel prices and the wages for the drivers are,*

$$p_o^{C*} = \frac{(1-\beta^2)(a+b)+2-\beta}{2b(1-\beta^2)+2}, \tag{C.2}$$
$$p_f^{C*} = \frac{1}{2}, \tag{C.3}$$
$$w^{C*} = \frac{a+b(1-\beta)+2ab(1-\beta^2)}{2b(1+b(1-\beta^2))}. \tag{C.4}$$

## Appendix D: More Details for the Fixed-Labor-Supply Case

In this section, we detail the firm's profit under each contract for the benchmark sub-cases with fixed labor supplies. The resulting equilibrium outcomes of these four contracts are provided in the Online Supplement G.1.

**Restaurant-pricing/no wage-commitment contract (BRN).** Under this contract, the platform first announces its commission fee of $r$ to the restaurant. Then, the restaurant sets the offline channel price $p_f$ and the online channel price $p_o$. In this sub-case, the profits of both the platform and the restaurant are given by:

$$\pi_p^{BRN}(r) = \min(\hat{s}, q_o)(r-c), \tag{D.1}$$
$$\pi_r^{BRN}(p_o, p_f) = \min(\hat{s}, q_o)(p_o - r) + p_f q_f, \tag{D.2}$$

where $\min(\hat{s}, q_o)$ denotes the realized online demand, with $\hat{s}$ being exogenously given. In this case, the platform's margin is $m_p^{BRN} = r$, and the restaurant's online margin is $m_p^{BRN} = p_o - r$.

**Platform-pricing/no wage-commitment contract (BPN).** Under this contract, the platform first announces its commission fee $r$ to the restaurant. Then, the restaurant sets its offline channel price alongside the online sales margin, $m_r$ for each online order. Finally, the platform determines the online channel price $p_o$ by setting the delivery fee $d$ for online orders. The platform and restaurant's profit are given by

$$\pi_p^{BPN}(r, d) = \min(\hat{s}, q_o)(p_o - m_r - c), \tag{D.3}$$
$$\pi_r^{BPN}(m_r, p_f) = \min(\hat{s}, q_o)m_r + p_f q_f. \tag{D.4}$$

Here, the platform's margin $m_p^{BPN} = p_o - m_r = r + d$.

**Platform-pricing/wage-commitment (BPW).** Since the wage is exogenously fixed at a constant $c$, the BPW contract follows the same decision sequence as the BPN contract. Specifically, the platform first announces the commission fee, followed by the restaurant setting its margin for online and offline channel prices. Finally, the platform determines the online channel price by setting the delivery fee. As a result, the profits for the platform and the restaurant are given by Equations (D.3) and (D.4).

**Restaurant-pricing/wage-commitment (BRW).** Similarly, the BRW contract follows the same decision sequence as the BRN contract because the wage is exogenously fixed at a constant $c$. As a result, the profits for the platform and the restaurant are given by Equations (D.1) and (D.2).

## Appendix E: Drivers/Customer's Surplus

To derive the delivery drivers' surplus while working for the platform, we apply the classical theory of monopoly markets (e.g., Tirole 1988). The surplus is given by

$$DS^i = \int_0^{-a+bw^i} (w^i - \tfrac{s+a}{b})\, ds = \tfrac{(-a+bw^i)^2}{2b}, \tag{E.1}$$

as a function of the supply parameters $(a, b)$ and the equilibrium wages under the contracting scheme $i \in \{RN, PN, PW\}$. Substituting optimal wage $w^{i*}$ into Equation (E.1), we can get

$$\begin{aligned} DS^{RN*} &= \tfrac{(a+b(\beta-1))^2}{8b(1-2b(\beta^2-1))^2}, \\ DS^{PN*} &= \tfrac{(b(1-\beta)-a)^2}{32b(-b\beta^2+b+1)^2}, \\ DS^{PW*} &= \tfrac{(a+b(\beta-1))^2}{2b(1-4b(\beta^2-1))^2}. \end{aligned} \tag{E.2}$$

The market demand functions (1) and (2) follow from the consumption quadratic utility of a representative customer:

$$U(q_o, q_f, p_o, p_f) = q_o + q_f - \tfrac{1}{2}q_o^2 - \tfrac{1}{2}q_f^2 - \beta q_o q_f - p_o q_o - p_f q_f. \tag{E.3}$$

Hence, the customer's surplus under each contract is equivalent to the customer's utility, that is, $CS^i = U^i(q_o^i, q_f^i, p_o^i, p_f^i)$, $i \in \{RN, PN, PW\}$. Substituting optimal channel prices and demands $p_o^{i*}$, $p_f^{i*}$, $q_o^{i*}$, $q_f^{i*}$ into Equation (E.3), we can get

$$\begin{aligned} CS^{RN*} &= \tfrac{a^2(1-\beta^2)-2ab(\beta-1)^2(\beta+1)+b(\beta-1)(\beta+1)(b(\beta-1)(3\beta+5)-4)+1}{8(1-2b(\beta^2-1))^2}, \\ CS^{PN*} &= \tfrac{a^2(1-\beta^2)-2ab(\beta-1)^2(\beta+1)+b(\beta-1)(\beta+1)(b(\beta-1)(3\beta+5)-8)+4}{32(-b\beta^2+b+1)^2}, \\ CS^{PW*} &= \tfrac{4a^2(1-\beta^2)-8ab(1-\beta)^2(\beta+1)+4b(1-\beta^2)(b(1-\beta)(3\beta+5)+2)+1}{8(1-4b(\beta^2-1))^2}. \end{aligned} \tag{E.4}$$

In addition, we define the total social welfare as follows ($i \in \{RN, PN, PW\}$):

$$SW^i = \pi_p^i + \pi_r^i + DS^i + CS^i. \tag{E.5}$$

## Appendix F: An Alternative Demand Model

As mentioned earlier, one of the features of the online and offline channel demands in our main model is that the total market size increases as the two channels become more differentiated. However, this feature might only sometimes hold. In this subsection, we study a demand model that assumes the total potential market size is independent of the degree of differentiation between the channels. In particular, we adopt the model of Raju et al. (1995) and assume the online and offline channel demands as

$$q_o = 1 - p_o + \beta(p_f - p_o), \tag{F.1}$$

$$q_f = \alpha - p_f + \beta(p_o - p_f), \tag{F.2}$$

where $\alpha$ represents the relative potential size of the dine-in offline vs. the online channel. It is essential to investigate the robustness of our findings as $\alpha$ changes. In particular, when $\alpha \leq 1$, the online channel potential can be larger than that of the dine-in offline channel. In contrast, $\alpha > 1$ implies a larger potential market size for the dine-in channel that customers prefer to get served at the restaurant.

Following the analysis in Section 4, we can derive the equilibrium solutions for each type of contract, which we omit here, and please refer to the Online Supplement I for detailed information. We then investigate how the introduction of the alternative demand model affects the player's profitability in the food delivery chain under different contracting schemes. Results are shown in the following proposition.

Proposition F1. *We can establish the following when the demand function is given by* (F.1) *and* (F.2).

*(i) The online channel price satisfies* $p_o^{PN*} \geq p_o^{RN*} \geq p_o^{PW*}$.

*(ii) The delivery drivers wage satisfies* $w^{PW*} \geq w^{RN*} \geq w^{PN*}$.

*(iii) The online channel demand satisfies* $q_o^{PW*} \geq q_o^{RN*} \geq q_o^{PN*}$.

*(iv) For the equilibrium profit of the platform, if* $b \geq \frac{1+\beta}{8}$, *then* $\pi_p^{RN*} \geq \pi_p^{PW*} \geq \pi_p^{PN*}$; *otherwise,* $\pi_p^{RN*} \geq \pi_p^{PN*} \geq \pi_p^{PW*}$.

*(v) For the equilibrium profit of the restaurant, if* $b \geq \frac{1+\beta}{8}$, *then* $\pi_r^{PW*} \geq \pi_r^{PN*} \geq \pi_r^{RN*}$; *otherwise,* $\pi_r^{PN*} \geq \pi_r^{PW*} \geq \pi_r^{RN*}$.

*(vi) For the equilibrium profit of the supply chain, if* $b \geq \frac{(\sqrt{33}-1)(\beta+1)}{16}$, $\pi_{sc}^{PW*} \geq \pi_{sc}^{RN*} \geq \pi_{sc}^{PN*}$; *if* $\frac{1+\beta}{8} \leq b < \frac{(\sqrt{33}-1)(\beta+1)}{16}$, *then* $\pi_{sc}^{RN*} \geq \pi_{sc}^{PW*} \geq \pi_{sc}^{PN*}$; *otherwise,* $\pi_{sc}^{RN*} \geq \pi_{sc}^{PN*} \geq \pi_{sc}^{PW*}$.

Proposition F1 shows that our findings in the main model still hold as we incorporate different market potentials for the online and offline channels. In particular, the most fierce channel competition happens under the PW contract, while the PN contract softens channel competition. Similar to our findings in Proposition 7, as long as raising the supply of drivers is not excessively expensive (i.e., when $b$ is not very low), the platform prefers the RN contract. In contrast, the restaurant prefers the PW contract. Moreover, if working for the platform is attractive enough (i.e., $b$ is large enough), the food delivery chain would benefit from the PW contract through larger online orders. Altogether, our findings in this extension subsection demonstrate that the relative market size parameter $\alpha$ does not play a significant role in the relative performance of these contracts.

# Online Supplement to “Pricing and Waging in Three-Sided Food Delivery Markets”

## Appendix G: Proofs of Main Results

### G.1. Proof of Lemma 1

In this proof, we solve for the equilibrium decisions in the BRN (BRW) and BPN (BPW) contracts, respectively. Since the profit functions and decision sequences for both the platform and the restaurant in BRW and BPW are identical to those in the BRN and BPN contracts (please refer to the Section D), the equilibrium results are the same as those.

**Restaurant-pricing/no wage-commitment contract (BRN).** Using backward induction, we first solve for the restaurant's optimization problem:

$$\pi_r^{BRN}(p_o,p_f)=\min(\hat{s},q_o)(p_o-r)+p_f q_f,$$

In this benchmark, the exogenous supply ($\hat{s}$) may either exceed or fall short of demand, leading us to consider the following two cases.

(i) When supply exceeds demand ($\hat{s}\geq q_o$), the restaurant's optimization problem becomes

$$\begin{aligned}\pi_r^{BRN}=&\max_{p_o,p_f} q_o(p_o-r)+p_f q_f,\\ s.t.\quad &\hat{s}\geq q_o.\end{aligned}\tag{G.1}$$

The restaurant's profit is jointly concave in $p_o$ and $p_f$ because the Hessian matrix is negative definite. The Hessian matrix is given by

$$H=\begin{bmatrix}\frac{\partial^2\pi_r^{BRN}}{\partial p_o^2} & \frac{\partial^2\pi_r^{BRN}}{\partial p_o\partial p_f}\\ \frac{\partial^2\pi_r^{BRN}}{\partial p_f\partial p_o} & \frac{\partial^2\pi_r^{BRN}}{\partial p_f^2}\end{bmatrix}=\begin{bmatrix}\frac{2}{\beta^2-1} & \frac{2\beta}{1-\beta^2}\\ \frac{2\beta}{1-\beta^2} & \frac{2}{\beta^2-1}\end{bmatrix}.$$

By applying KKT(Karush-Kuhn-Tucker) conditions, we can rewrite problem (G.1) as follows.

$$\begin{aligned}\mathcal{L}(p_o,p_f,\lambda)=&\,q_o(p_o-r)+p_f q_f+\lambda(\hat{s}-q_o),\\ s.t.\quad &\frac{\partial\mathcal{L}}{\partial p_o}=\frac{(1-\beta^2)\lambda-\beta-2p_o+2\beta p_f+r+1}{1-\beta^2}=0,\\ &\frac{\partial\mathcal{L}}{\partial p_f}=\frac{\beta^3\lambda-\beta(\lambda-2p_o+r+1)-2p_f+1}{1-\beta^2}=0,\\ &\hat{s}-q_o\geq 0,\\ &\lambda\geq 0,\\ &\lambda(\hat{s}-q_o)=0.\end{aligned}$$

By solving the above problem, the restaurant's optimal online and offline channel prices are determined as follows:

$$p_o^*,p_f^*=\begin{cases}1-\frac{\beta}{2}+(\beta^2-1)\hat{s},\,\frac{1}{2} & if\quad r\leq\hat{r}, \quad\text{(G.2a)}\\ \frac{r+1}{2},\frac{1}{2} & if\quad r\geq\hat{r}, \quad\text{(G.2b)}\end{cases}$$

where $\hat{r}=(1-\beta)(1-2\beta\hat{s}-2\hat{s})$. Substituting $p_o^*$ and $p_f^*$ into the restaurant's profit, we have the restaurant's optimal profit as follows:

$$\pi_r^{BRN*}=\begin{cases}\frac{1}{4}-\hat{s}(\beta+r-1)+(\beta^2-1)\hat{s}^2 & if\quad r\leq\hat{r},\\ \frac{-2\beta+r(2\beta+r-2)+2}{4-4\beta^2} & if\quad r\geq\hat{r}.\end{cases}$$

(ii) When the supply is smaller than the demand ($\hat{s}\leq q_o$), the restaurant's optimization problem becomes

$$\begin{aligned}\pi_r^{BRN}=&\max_{p_o,p_f}\hat{s}(p_o-r)+p_f q_f,\\ s.t.\quad &\hat{s}\leq q_o.\end{aligned}$$

In this case, the restaurant's profit increases with $p_o$, as $\frac{\partial \pi_r^{BRN}}{\partial p_o} = \hat{s} + \frac{\beta p_f}{1-\beta^2} > 0$. Therefore, the restaurant will continue to raise $p_o$ until the demand matches the fixed supply $\hat{s}$, at which point the optimal online price is $p_o^* = 1 - \frac{\beta}{2} + (\beta^2 - 1)\hat{s}$. Substituting $p_o^*$ into the restaurant's profit, we obtain the optimal offline channel price $p_f^* = \frac{1}{2}$. Consequently, we have the restaurant's optimal profit $\pi_r^{BRN*} = \frac{1}{4} - \hat{s}(\beta + r - 1) + (\beta^2 - 1)\hat{s}^2$.

Combining the Case (i) and Case (ii), we can show that Case (ii) is dominated by Case (i) since

$$\pi_r^{BRN*}|_{\hat{s}\geq q_o} - \pi_r^{BRN*}|_{\hat{s}\leq q_o} = \begin{cases} 0 & if \quad r \leq \hat{r}, \\ \frac{(\beta+r-2(\beta^2-1)\hat{s}-1)^2}{4(1-\beta^2)} > 0 & if \quad r \geq \hat{r}. \end{cases}$$

Hence, the restaurant's best responses are listed in Equation (G.2). Next, we consider the platform's optimization problem in the following two cases.

(a) Anticipating $p_o^* = 1 - \frac{\beta}{2} + (\beta^2 - 1)\hat{s}$, $p_f^* = \frac{1}{2}$, then the platform's profit in Equation (D.1) becomes

$$\begin{aligned} \pi_p^{BRN} &= \hat{s}(r - c), \\ s.t. \quad & r \leq \hat{r}, \end{aligned}$$

which increases with $r$. Hence, the platform increases the commission until $r^* = \hat{r}$, resulting $\pi_p^{BRN*} = \hat{s}((\beta - 1)(2(\beta+1)\hat{s} - 1) - c)$.

(b) Anticipating $p_o^* = \frac{r+1}{2}$, $p_f^* = \frac{1}{2}$, then the platform's profit in Equation (D.1) becomes

$$\begin{aligned} \pi_p^{BRN} &= \frac{(c-r)(\beta+r-1)}{2(1-\beta^2)}, \\ s.t. \quad & r \geq \hat{r}, \end{aligned}$$

which is concave in $r$ since $\frac{\partial^2 \pi_p^{BRN}}{\partial r^2} = -\frac{1}{1-\beta^2} < 0$. Hence, the platform profit is maximized at $\tilde{r} = \frac{1}{2}(1 - \beta + c)$, at which $\frac{\partial \pi_p^{BRN}}{\partial r} = 0$. Additionally, the platform has the constraint $r \geq \hat{r}$. Comparing $\hat{r}$ and $\tilde{r}$, we have the following two subcases:

(b-1) If $\hat{s} \geq \frac{1-\beta-c}{4(1-\beta^2)}$, then $\tilde{r} \geq \hat{r}$. Hence, the optimal $r$ for the platform is $r^* = \tilde{r}$, resulting $\pi_p^{BRN*} = \frac{(1-\beta-c)^2}{8(1-\beta^2)}$.

(b-2) If $\hat{s} \leq \frac{1-\beta-c}{4(1-\beta^2)}$, then $\hat{r} \geq \tilde{r}$. Hence the optimal $r$ for the platform is $r^* = \hat{r}$, resulting $\pi_p^{BRN*} = \hat{s}((\beta - 1)(2(\beta+1)\hat{s} - 1) - c)$.

Combining Case (a) and Case (b), we can show that the platform's profit in Case (a) is dominated by that in Case (b), i.e.,

$$\pi_p^{BRN*}|_{r\geq\hat{r}} - \pi_p^{BRN*}|_{r\leq\hat{r}} = \begin{cases} 0 & if \quad \hat{s} \leq \frac{1-\beta-c}{4(1-\beta^2)}, \\ \frac{(\beta+c-4(\beta^2-1)\hat{s}-1)^2}{8(1-\beta^2)} > 0 & if \quad \hat{s} \geq \frac{1-\beta-c}{4(1-\beta^2)}. \end{cases}$$

Therefore, the equilibrium commission $r^*$ in the BRN contract lies in Case (b). Specifically, we have

$$r^* = \begin{cases} \hat{r} & if \quad \hat{s} \leq \frac{1-\beta-c}{4(1-\beta^2)}, \quad \text{(G.3a)} \\ \tilde{r} & if \quad \hat{s} \geq \frac{1-\beta-c}{4(1-\beta^2)}. \quad \text{(G.3b)} \end{cases}$$

Consequently, we have the following equilibrium results:

If $\hat{s} \leq \frac{1-\beta-c}{4(1-\beta^2)}$,

$$\begin{aligned} p_o^{BRN*} &= \frac{2-\beta-2(1-\beta^2)\hat{s}}{2}, & p_f^{BRN*} &= \frac{1}{2}, \\ q_o^{BRN*} &= \hat{s}, & q_f^{BRN*} &= \frac{1}{2} - \beta\hat{s}, \\ \pi_p^{BRN*} &= \hat{s}(1 - \beta - c + 2\hat{s}(\beta^2 - 1)), & \pi_r^{BRN*} &= \frac{4(1-\beta^2)\hat{s}^2+1}{4}, \\ \pi_{sc}^{BRN*} &= \frac{4\hat{s}(-\beta-c+1)+4(\beta^2-1)\hat{s}^2+1}{4}. \end{aligned} \quad \text{(G.4)}$$

If $\hat{s} > \frac{1-\beta-c}{4(1-\beta^2)}$,

$$
\begin{aligned}
&p_o^{BRN*} = \frac{3-\beta+c}{4}, && p_f^{BRN*} = \frac{1}{2},\\
&q_o^{BRN*} = \frac{-\beta-c+1}{4-4\beta^2}, && q_f^{BRN*} = \frac{\beta(-1-\beta+c)+2}{4(1-\beta^2)},\\
&\pi_p^{BRN*} = \frac{(-\beta-c+1)^2}{8(1-\beta^2)}, && \pi_r^{BRN*} = \frac{-\beta(3\beta+2)+c^2-2(1-\beta)c+5}{16(1-\beta^2)},\\
&\pi_{sc}^{BRN*} = \frac{\beta^2+6\beta(1-c)-3c(c-2)-7}{16(\beta^2-1)}.
\end{aligned} \tag{G.5}
$$

**Platform-pricing/no wage-commitment contract (BPN)**. Using backward induction, we first solve the platform's optimization problem. As before, the supply may either exceed demand or fall short of it.

(i) When supply exceeds demand ($\hat{s} \geq q_o$), we have the following optimization problem for the platform:

$$
\begin{aligned}
\pi_p^{BPN} &= \max_{p_o} q_o(p_o - m_r - c),\\
s.t. \quad & \hat{s} \geq q_o = \frac{1}{1+\beta} - \frac{1}{1-\beta^2}p_o + \frac{\beta}{1-\beta^2}p_f.
\end{aligned}
$$

The platform's profit is concave in $p_o$ because $\frac{\partial^2 \pi_p^{BPN}}{\partial p_o^2} = \frac{2}{\beta^2-1} < 0$. By applying KKT conditions, we can rewrite the above problem as follows:

$$
\begin{aligned}
\mathcal{L}(p_o, \lambda) &= q_o(p_o - m_r - c) + \lambda(\hat{s} - q_o),\\
s.t. \quad & \frac{\partial \mathcal{L}}{\partial p_o} = \frac{c+\lambda+m_r-2p_o+\beta(-\beta\lambda+p_t-1)+1}{1-\beta^2} = 0,\\
& \hat{s} - q_o \geq 0,\\
& \lambda \geq 0,\\
& \lambda(\hat{s} - q_o) = 0.
\end{aligned}
$$

By solving this, the platform's optimal delivery fee can be listed as follows:

$$
p_o^* = \begin{cases} \beta(p_f - 1) + \left(\beta^2 - 1\right)\hat{s} + 1 & if \quad m_r \leq \hat{m}_r, \quad \text{(G.6a)}\\ \frac{1+c+m_r+\beta(p_f-1)}{2} & if \quad m_r \geq \hat{m}_r. \quad \text{(G.6b)} \end{cases}
$$

Note that $\hat{m}_r = 1 - c + \beta(p_f - 1) + 2(\beta^2 - 1)\hat{s}$. Substituting $p_o^*$ into the platform's profit, we have the platform's optimal profit as follows:

$$
\pi_p^{BPN*} = \begin{cases} -\hat{s}(c + m_r + \hat{s} - 1) + \beta(p_f - 1)\hat{s} + \beta^2\hat{s}^2 & if \quad m_r \leq \hat{m}_r,\\ \frac{(\beta+c+m_r-\beta p_f-1)^2}{4(1-\beta^2)} & if \quad m_r \geq \hat{m}_r. \end{cases}
$$

(ii) When the supply is less than the demand ($\hat{s} \leq q_o$), the platform's optimization problem becomes

$$
\begin{aligned}
\pi_p^{BPN} &= \max_{p_o} \hat{s}(p_o - m_r - c),\\
s.t. \quad & \hat{s} \leq q_o = \frac{1}{1+\beta} - \frac{1}{1-\beta^2}p_o + \frac{\beta}{1-\beta^2}p_f.
\end{aligned}
$$

In this case, the platform's profit increases with $p_o$ since $\frac{\partial \pi_p^{BPN}}{\partial p_o} = \hat{s} > 0$. Hence, the platform always increases $p_o$ until demand decreases to $\hat{s}$, at which point $p_o^* = \beta(p_f - 1) + (\beta^2 - 1)\hat{s} + 1$, leading to $\pi_p^{BPN*} = -\hat{s}(c + m_r + \hat{s} - 1) + \beta(p_f - 1)\hat{s} + \beta^2\hat{s}^2$.

Combining Case (i) and (ii) ($\hat{s} \geq q_o$ and $\hat{s} \leq q_o$), we show that Case (ii) is dominated by Case (i) since

$$
\pi_p^{BPN*}|_{\hat{s} \geq q_o} - \pi_p^{BPN*}|_{\hat{s} \leq q_o} = \begin{cases} 0 & if \quad m_r \leq \hat{m}_r,\\ \frac{(\beta+c+m_r-\beta p_f-2\beta^2\hat{s}+2\hat{s}-1)^2}{4(1-\beta^2)} & if \quad m_r \geq \hat{m}_r. \end{cases}
$$

Therefore, the platform's best response is listed in Equation (G.6). Next, we consider the restaurant's optimization problem in the following two cases.

(a) Anticipating the platform's optimal delivery fee $p_o^* = \beta(p_f - 1) + (\beta^2 - 1)\hat{s} + 1$, the restaurant's optimization problem in Equation (D.4) becomes

$$
\begin{aligned}
\pi_r^{BPN} &= \max_{m_r, p_f} q_o m_r + p_f q_f = m_r\hat{s} - p_f(p_f + \beta\hat{s} - 1),\\
s.t. \quad & m_r \leq \hat{m}_r.
\end{aligned}
$$

The restaurant's profit increases with $m_r$ given any $p_f$. Hence, the restaurant increases the online margin to $m_r^* = \hat{m}_r$. Then, substituting $m_r^* = \hat{m}_r$ into the restaurant's profit, we can get the optimal offline channel price $p_f^* = \frac{1}{2}$ satisfying $\frac{\partial \pi_r^{BPN}}{\partial p_f} = 0$. Substituting $m_r^*$ and $p_f^*$ into the restaurant's profit, we can get $\pi_r^{BPN*} = \frac{-4\hat{s}(\beta+c-1)+8(\beta^2-1)\hat{s}^2+1}{4}$.

(b) Anticipating the platform's delivery fee $p_o^* = \frac{1+c+m_r+\beta(p_f-1)}{2}$, then the restaurant's optimization problem becomes

$$\begin{aligned} &\pi_r^{BPN} = \max_{m_r,p_f} q_o m_r + p_f q_f = \tfrac{m_r(\beta+c-2\beta p_f-1)+\beta p_f(\beta-c-\beta p_f+1)+m_r^2+2(p_f-1)p_f}{2(\beta^2-1)}, \\ &s.t. \quad m_r \geq \hat{m}_r. \end{aligned}$$

The restaurant's profit is joint concave in $m_r$ and $p_f$ because the Hessian matrix is negative definite. Specifically, the Hessian matrix is given by

$$H = \begin{bmatrix} \frac{\partial^2 \pi_r^{BPN}}{\partial m_r^2} & \frac{\partial^2 \pi_r^{BPN}}{\partial m_r \partial p_f} \\ \frac{\partial^2 \pi_r^{BPN}}{\partial p_f \partial m_r} & \frac{\partial^2 \pi_r^{BPN}}{\partial p_f^2} \end{bmatrix} = \begin{bmatrix} \frac{1}{\beta^2-1} & -\frac{\beta}{\beta^2-1} \\ -\frac{\beta}{\beta^2-1} & \frac{2-\beta^2}{\beta^2-1} \end{bmatrix}.$$

By applying KKT conditions, we can get the optimal prices

$$m_r^*, p_f^* = \begin{cases} \frac{-\beta+2(1-c)+4(\beta^2-1)\hat{s}}{2}, \frac{1}{2} & if \quad \hat{s} \leq \frac{1-\beta-c}{4(1-\beta^2)}, \qquad \text{(G.7a)} \\ \frac{1-c}{2}, \frac{1}{2} & if \quad \hat{s} \geq \frac{1-\beta-c}{4(1-\beta^2)}. \qquad \text{(G.7b)} \end{cases}$$

Consequently, the restaurant's optimal profit is $\pi_r^{BPN*} = \frac{-4\hat{s}(\beta+c-1)+8(\beta^2-1)\hat{s}^2+1}{4}$ if $\hat{s} \leq \frac{1-\beta-c}{4(1-\beta^2)}$; otherwise, the the restaurant's optimal profit is $\pi_r^{BPN*} = \frac{(1-\beta)(\beta+3)+c^2+2(\beta-1)c}{8-8\beta^2}$.

Combining Case (a) and Case (b), we can show that the restaurant's profit in Case (a) is dominated by that in Case (b), i.e.,

$$\pi_r^{BPN*}|_{m_r \geq \hat{m}_r} - \pi_r^{BPN*}|_{m_r \leq \hat{m}_r} = \begin{cases} 0 & if \quad \hat{s} \leq \frac{1-\beta-c}{4(1-\beta^2)}, \\ \frac{(\beta+c-4(\beta^2-1)s-1)^2}{8(1-\beta^2)} > 0 & if \quad \hat{s} \geq \frac{1-\beta-c}{4(1-\beta^2)}. \end{cases}$$

Therefore, the equilibrium prices for the restaurant lie in Equations (G.7).

Next, anticipating the restaurant's optimal decisions, the platform determines the commission fee $r$ in the first stage. However, we find that the platform's profit is independent of the commission fee $r$, leading to the following equilibrium results:

If $\hat{s} \leq \frac{1-\beta-c}{4(1-\beta^2)}$,

$$\begin{aligned} &m_r^{BPN*} = \tfrac{-\beta+2(1-c)+4(\beta^2-1)\hat{s}}{2}, && d^{BPN*} = c - r + \hat{s}(1-\beta^2), \\ &p_o^{BPN*} = \tfrac{2-\beta-2(1-\beta^2)\hat{s}}{2}, && p_f^{BPN*} = \tfrac{1}{2}, \\ &q_o^{BPN*} = \hat{s}, && q_f^{BPN*} = \tfrac{1}{2} - \beta\hat{s}, \\ &\pi_p^{BPN*} = \hat{s}(1-\beta^2), && \pi_r^{BPN*} = \tfrac{-4\hat{s}(\beta+c-1)+8(\beta^2-1)\hat{s}^2+1}{4}, \\ &\pi_{sc}^{BPN*} = \tfrac{4\hat{s}(-\beta-c+1)+4(\beta^2-1)\hat{s}^2+1}{4}. \end{aligned} \tag{G.8}$$

If $\hat{s} > \frac{1-\beta-c}{4(1-\beta^2)}$,

$$\begin{aligned} &m_r^{BPN*} = \tfrac{2-\beta-2(1-\beta^2)s}{2}, && d^{BPN*} = \tfrac{-\beta+3c-4r+1}{4}, \\ &p_o^{BPN*} = \tfrac{3-\beta+c}{4}, && p_f^{BPN*} = \tfrac{1}{2}, \\ &q_o^{BPN*} = \tfrac{-\beta-c+1}{4-4\beta^2}, && q_f^{BPN*} = \tfrac{\beta(-1-\beta+c)+2}{4(1-\beta^2)}, \\ &\pi_p^{BPN*} = \tfrac{(\beta+c-1)^2}{16(1-\beta^2)}, && \pi_r^{BPN*} = \tfrac{(1-\beta)(\beta+3)+c^2+2(\beta-1)c}{8-8\beta^2}, \\ &\pi_{sc}^{BPN*} = \tfrac{\beta^2+6\beta(1-c)-3c(c-2)-7}{16(\beta^2-1)}. \end{aligned} \tag{G.9}$$

In summary, comparing equilibrium decisions in four types of contracts (see Equations (G.4), (G.5), (G.8), and (G.9)), we can get our results in Lemma 1. □

### G.2. Proof of Lemma 2

**Stage 3: platform determines the wage $w$, or equivalently, the supply $s(w)$.** The total amount of supply is defined as $s=-a+bw$ in Equation (3). This can be rearranged to express $w$ as $w=\frac{a+s}{b}$. To simplify our analysis, we consider the scenario where the platform's decision is the supply $s$. In this context, the platform has no incentive to choose $s$ such that $s>q_o$ because it could always increase its profit by reducing $s$ (the platform's profit decreases as $s$ increases). Thus, in equilibrium, $s\le q_o$. Therefore, the platform's optimization problem can be formulated as follows:

$$\begin{aligned} &\pi_p^{RN}=\max_s s(r-\tfrac{a+s}{b}),\\ &s.t.\quad s\le q_o=\tfrac{1}{1+\beta}-\tfrac{1}{1-\beta^2}p_o+\tfrac{\beta}{1-\beta^2}p_f. \end{aligned} \tag{G.10}$$

The platform's profit is concave in $s$. Next, by applying KKT conditions, we can rewrite the above problem as follows.

$$\begin{aligned} \mathcal{L}(s,\lambda)&=s(r-\tfrac{a+s}{b})+\lambda(q_o-s),\\ s.t.\quad &\tfrac{\partial\mathcal{L}}{\partial s}=-\tfrac{a+b\lambda-br+2s}{b}=0,\\ &q_o-s\ge 0,\\ &\lambda\ge 0,\\ &\lambda(q_o-s)=0. \end{aligned}$$

By solving this, we can get the platform's optimal supply:

$$s^*=\begin{cases}\frac{br-a}{2} & if\quad p_o\le\bar{p}_o, \quad (G.11a)\\ q_o=\frac{1-\beta+\beta p_f-p_o}{1-\beta^2} & if\quad p_o\ge\bar{p}_o. \quad (G.11b)\end{cases}$$

Note that $\bar{p}_o=\frac{-a\beta^2+a+b\left(\beta^2-1\right)r+2\beta(p_f-1)+2}{2}$.

**Stage 2: restaurant determines online channel price $p_o$ and the offline channel price $p_f$.** Anticipating the varying optimal supply levels chosen by the platform, the restaurant's pricing decisions will differ accordingly.

(i) Anticipating the platform's optimal supply $s^*=\frac{br-a}{2}$, then the restaurant's optimization problem becomes

$$\begin{aligned} &\pi_r^{RN}=\max_{p_o,p_f} s(p_o-r)+q_fp_f=\tfrac{br-a}{2}(p_o-r)+q_fp_f,\\ &s.t.\quad p_o\le\bar{p}_o=\tfrac{-a\beta^2+a+b\left(\beta^2-1\right)r+2\beta(p_f-1)+2}{2}. \end{aligned} \tag{G.12}$$

Taking the derivative of the restaurant's profit with respect to $p_o$, we obtain $\frac{\partial\pi_r^{RN}}{\partial p_o}=\frac{(1-\beta^2)(br-a)+2\beta p_f}{2(1-\beta^2)}>0$. Hence, given any $p_f$, the restaurant increases $p_o$ until $p_o^*=\bar{p}_o(p_f)$. Substituting $p_o^*=\bar{p}_o(p_f)$ into restaurant's profit, we have $\frac{\partial^2\pi_r^{RN}}{\partial p_f^2}=\frac{2(1-\beta^2)}{\beta^2-1}<0$. Thus, the restaurant's optimal $p_f$ satisfying $\frac{\partial\pi_r^{RN}}{\partial p_f}=0$, which is $p_f^*=\frac{1}{2}$. Hence, the restaurant's optimal prices are $p_o^*=\bar{p}_o(p_f^*)$ and $p_f^*=\frac{1}{2}$, respectively, and the restaurant's optimal profit is $\pi_r^{RN*}=\frac{(a-br)(a(\beta^2-1)+r(-b\beta^2+b+2)-2(1-\beta))+1}{4}$.

(ii) Anticipating the platform's optimal supply $s^*=\frac{1-\beta+\beta p_f-p_o}{1-\beta^2}$, then the restaurant's optimization problem becomes

$$\begin{aligned} &\pi_r^{RN}=\max_{p_o,p_f} s(p_o-r)+q_fp_f=\tfrac{1-\beta+\beta p_f-p_o}{1-\beta^2}(p_o-r)+q_fp_f,\\ &s.t.\quad p_o\ge\bar{p}_o=\tfrac{-a\beta^2+a+b\left(\beta^2-1\right)r+2\beta(p_f-1)+2}{2}. \end{aligned} \tag{G.13}$$

The restaurant's profit is jointly concave in $p_o$ and $p_f$ as the Hessian matrix is negative definite, and the Hessian matrix is given by

$$H=\begin{bmatrix}\frac{\partial^2\pi_r^{RN}}{\partial p_o^2} & \frac{\partial^2\pi_r^{RN}}{\partial p_o\partial p_f}\\ \frac{\partial^2\pi_r^{RN}}{\partial p_f\partial p_o} & \frac{\partial^2\pi_r^{RN}}{\partial p_f^2}\end{bmatrix}=\begin{bmatrix}\frac{2}{\beta^2-1} & \frac{2\beta}{1-\beta^2}\\ \frac{2\beta}{1-\beta^2} & \frac{2}{\beta^2-1}\end{bmatrix}.$$

By applying KKT conditions, the restaurant's optimal prices can be listed as follows:

$$p_o^*, p_f^* = \begin{cases} \frac{1+r}{2}, \quad \frac{1}{2} & if \quad r \geq \bar{r}, \quad \text{(G.14a)} \\ \frac{-a\beta^2+a+b\left(\beta^2-1\right)r-\beta+2}{2}, \quad \frac{1}{2} & if \quad r \leq \bar{r}. \quad \text{(G.14b)} \end{cases}$$

where $\bar{r} = \frac{(\beta-1)(a\beta+a+1)}{b(\beta^2-1)-1}$.

Substituting optimal prices into the restaurant's profit, we can get the corresponding profit of the restaurant is $\pi_r^{RN*} = \frac{r^2-2r(1-\beta)+2(1-\beta)}{4-4\beta^2}$ if $r \geq \bar{r}$; otherwise, $\pi_r^{RN*} = \frac{(a-br)(a(\beta^2-1)+r(-b\beta^2+b+2)+2(\beta-1))+1}{4}$.

Combining Case (i) and Case (ii), we can show that the restaurant's profit in Case (i) is dominated by that in Case (ii) since

$$\pi_r^{RN*}|_{p_o \geq \bar{p}_o} - \pi_r^{RN*}|_{p_o \leq \bar{p}_o} = \begin{cases} \frac{(\beta^2(a-br)-a+br+\beta+r-1)^2}{4(1-\beta^2)} > 0 & if \quad r \geq \bar{r}, \\ 0 & if \quad r \leq \bar{r}. \end{cases}$$

Therefore, the optimal prices for the restaurant in this stage are given in Equation (G.14).

**Stage 1: platform determines the commission fee $r$.** We consider the following two cases.

(i) Anticipating the restaurant's optimal channel prices $p_o^* = \frac{-a\beta^2+a+b\left(\beta^2-1\right)r-\beta+2}{2}$, $p_f^* = \frac{1}{2}$, the optimal supply in the system becomes $s^* = \frac{1-\beta+\beta p_f^*-p_o^*}{1-\beta^2} = \frac{-a+br}{2}$, so the platform's optimization problem becomes

$$\begin{aligned} \pi_p^{RN} &= \max_r s(r-w) = \frac{(a-br)^2}{4b}, \\ s.t. \quad & r \leq \bar{r}. \end{aligned} \tag{G.15}$$

The platform's profit is convex in $r$ since $\frac{\partial^2 \pi_p^{RN}}{\partial r^2} = \frac{b}{2} > 0$, and the platform's profit gets the minimum value when $r = \frac{a}{b}$. $\bar{r} > \frac{a}{b}$ since $\bar{r} - \frac{a}{b} = \frac{a-b(1-\beta)}{b(b(\beta^2-1)-1)} > 0$. Additionally, the online demand in this stage becomes $s = \frac{-a+br}{2}$. Hence, the commission fee $r$ needs to satisfy $r \geq \frac{a}{b}$ to ensure $s \geq 0$. Therefore, the platform's profit increases in $r$ when $\frac{a}{b} \leq r \leq \bar{r}$, and its optimal commission fee is $r^* = \bar{r}$. Substituting $r^* = \bar{r}$ into the platform's profit, we have $\pi_p^{RN*} = \frac{(a+b(\beta-1))^2}{4b(-b\beta^2+b+1)^2}$.

(ii) Anticipating the restaurant's prices $p_o^* = \frac{1+r}{2}$, $p_f^* = \frac{1}{2}$, the optimal supply in the system becomes $s^* = \frac{\beta+r-1}{2(\beta^2-1)}$, so the platform's optimization problem becomes

$$\begin{aligned} \pi_p^{RN} &= \max_r s(r-w) = \frac{(-\beta-r+1)(2a(\beta^2-1)-2b(\beta^2-1)r+\beta+r-1)}{4b(1-\beta^2)^2}, \\ s.t. \quad & r \geq \bar{r}. \end{aligned} \tag{G.16}$$

The platform's profit is concave in $r$ since $\frac{\partial^2 \pi_p^{RN}}{\partial r^2} = \frac{1}{\beta^2-1} - \frac{1}{2b(\beta^2-1)^2} < 0$. Hence, there exists a

$$r_m = \frac{(1-\beta)(a(-\beta-1)+b(\beta^2-1)-1)}{2b(\beta^2-1)-1}$$

satisfying $\frac{\partial \pi_p^{RN}}{\partial r} = 0$ such that the platform's profit is maximized. $r_m > \bar{r}$ because $r_m - \bar{r} = \frac{b(\beta^2-1)^2(-a+b(1-\beta))}{(b(\beta^2-1)-1)(2b(\beta^2-1)-1)} > 0$. Hence, in this case, the optimal $r^* = r_m$. Substituting $r^*$ into the platform's profit, we have $\pi_p^{RN*} = \frac{(a+b(\beta-1))^2}{4b(2b(1-\beta^2)+1)}$.

Combining Case (i) and Case (ii), the platform's profit in Case (i) is dominated by that in Case (ii) since

$$\pi_p^{RN*}|_{r \geq \bar{r}} - \pi_p^{RN*}|_{r \leq \bar{r}} = \frac{b(\beta^2-1)^2(a+b(\beta-1))^2}{4(-b\beta^2+b+1)^2(2b(1-\beta^2)+1)} > 0.$$

Therefore, the equilibrium commission fee is $r^* = r_m$. Substituting $r^*$ into the optimal decisions in subsequent stages yields the equilibrium results presented in Lemma 2. Furthermore, we have the following equilibrium demands and profits:

$$\begin{aligned}
&q_o^{RN*} = s^{RN*} = \tfrac{a-b(1-\beta)}{4b(\beta^2-1)-2}, \quad q_f^{RN*} = \tfrac{-a\beta+b(-\beta-2)(1-\beta)-1}{4b(\beta^2-1)-2},\\
&\pi_p^{RN*} = -\tfrac{(-a-b\beta+b)^2}{4b(2b(\beta^2-1)-1)},\\
&\pi_r^{RN*} = \tfrac{a^2(-(\beta^2-1))+2ab(-\beta-1)(1-\beta)^2+b(-\beta-1)(1-\beta)(b(-3\beta-5)(1-\beta)-4)+1}{4(1-2b(\beta^2-1))^2},\\
&\pi_{sc}^{RN*} = \tfrac{a^2(1-3b(\beta^2-1))+2ab(1-\beta)(3b(\beta^2-1)-1)+b^2(1-\beta)(b(-\beta-7)(-\beta-1)(1-\beta)+3\beta+5)+b}{4b(1-2b(\beta^2-1))^2}.
\end{aligned} \tag{G.17}$$

□

### G.3. Proof of Lemma 3

**Stage 3: platform determines the wage $w$ and the online channel price $p_o$.** We first show that $q_o = s$ is the platform's optimal choice in this stage because neither $q_o > s$ nor $q_o < s$ can be optimal. If $q_o > s$, the platform can slightly increase the delivery fee $d$ (which slightly increases the channel price $p_o$ and reduces the demand) such that $\min(q_o, s)$ remains unaltered, thereby increasing the platform's profit. If $q_o < s$, the platform can slightly decrease the wage $w$ (which slightly reduces the supply) such that $\min(q_o, s)$ remains unaltered, thus increasing the platform's profit. Given $q_o = s$, the platform's optimization problem becomes

$$\begin{aligned}
&\pi_p^{PN} = \max_{w,p_o} s(p_o - m_r - w),\\
&s.t. \quad s = q_o = \tfrac{1}{1+\beta} - \tfrac{1}{1-\beta^2}p_o + \tfrac{\beta}{1-\beta^2}p_f.
\end{aligned} \tag{G.18}$$

By applying the Lagrange multiplier method, we have

$$\begin{aligned}
&\mathcal{L}(w, p_o, \lambda) = s(p_o - m_r - w) + \lambda(q_o - s),\\
&s.t. \quad \tfrac{\partial \mathcal{L}}{\partial w} = 0, \quad \tfrac{\partial \mathcal{L}}{\partial p_o} = 0, \quad \tfrac{\partial \mathcal{L}}{\partial \lambda} = 0.
\end{aligned}$$

By solving the above problem, the platform's optimal wage and delivery fee are listed as follows:

$$w^* = \tfrac{a(2b(\beta^2-1)-1)+b(m_r+\beta(1-p_f)-1)}{2b(-b(1-\beta^2)-1)}, \quad p_o^* = \tfrac{a\left(\beta^2-1\right)+b\left(\beta^2-1\right)(m_r+\beta(p_f-1)+1)+2\beta-2\beta p_f-2}{2b(\beta^2-1)-2}.$$

**Stage 2: the restaurant sets the online price margin $m_r$ and the offline channel price $p_f$.** Anticipating the optimal $w^*$ and $p_o^*$, the restaurant maximizes the following profit

$$\pi_r^{PN} = m_r s + q_f p_f = \tfrac{a(m_r-\beta p_f)+b(m_r^2+m_r(\beta-2\beta p_f-1)+p_f(\beta^2+\beta-\beta^2 p_f+2p_f-2))+2(p_f-1)p_f}{2b(\beta^2-1)-2}$$

by setting $m_r$ and $p_f$. We find that the restaurant's profit is jointly concave in $m_r$ and $p_f$ as the Hessian matrix is negative definite. The Hessian matrix is given by

$$H = \begin{bmatrix} \frac{\partial^2 \pi_r^{PN}}{\partial m_r^2} & \frac{\partial^2 \pi_r^{PN}}{\partial m_r \partial p_f} \\ \frac{\partial^2 \pi_r^{PN}}{\partial p_f \partial m_r} & \frac{\partial^2 \pi_r^{PN}}{\partial p_f^2} \end{bmatrix} = \begin{bmatrix} \frac{2b}{2b(\beta^2-1)-2} & -\frac{2b\beta}{2b(\beta^2-1)-2} \\ -\frac{2b\beta}{2b(\beta^2-1)-2} & \frac{b(4-2\beta^2)+4}{2b(\beta^2-1)-2} \end{bmatrix}.$$

Hence, the optimal online margin $m_r$ and the offline channel price $p_f$ satisfy $\frac{\partial \pi_r^{PN}}{\partial m_r} = 0$, $\frac{\partial \pi_r^{PN}}{\partial p_f} = 0$, simultaneously, which are characterized by $m_r^* = \frac{b-a}{2b}$ and $p_f^* = \frac{1}{2}$.

**Stage 1: the platform determines the commission fee $r$.** Anticipating the restaurant's optimal $m_r^*$ and $p_f^*$, the platform's profit becomes independent of $r$. Hence, substituting optimal $m_r^*$ and $p_f^*$ back into $p_o^*$ and $w^*$, we can get the equilibrium results in Lemma 3. Furthermore, we have the following equilibrium results:

$$\begin{aligned}
&d^{PN*} = \tfrac{a(3b(1-\beta^2)+2)+b(1-\beta)(b(1-\beta^2)+2)}{4b(b(1-\beta^2)+1)} - r,\\
&q_o^{PN*} = \tfrac{a-b(1-\beta)}{4b(\beta^2-1)-4}, \quad q_f^{PN*} = \tfrac{-a\beta+b(-\beta-2)(1-\beta)-2}{4b(\beta^2-1)-4},\\
&\pi_p^{PN*} = \tfrac{(-a-b\beta+b)^2}{16b(b(1-\beta^2)+1)}, \quad \pi_r^{PN*} = \tfrac{-a^2+2ab(1-\beta)+b(b(-\beta-3)(1-\beta)-2)}{8b(b(\beta^2-1)-1)},\\
&\pi_{sc}^{PN*} = \tfrac{-3a^2+6ab(1-\beta)+b(b(-\beta-7)(1-\beta)-4)}{16b(b(\beta^2-1)-1)}.
\end{aligned} \tag{G.19}$$

□

### G.4. Proof of Lemma 4

**Stage 3: platform determines the online channel price $p_o$.** Similar to the proof of Lemma 2, we equivalently consider the scenario where the platform's decision variable is the supply $s$ in the first stage. Given the fixed $s$, the platform has no incentive to set the online channel price $p_o$ such that $s < q_o$ because it can increase $p_o$ slightly to raise its profit while keeping $\min(s, q_o)$ unchanged. In equilibrium, this implies that $q_o \le s$. Consequently, the platform's maximization problem can be formulated as follows

$$\begin{aligned} \pi_p^{PW} &= \max_{p_o} q_o(p_o - m_r - \tfrac{a+s}{b}), \\ s.t. \quad & q_o = \tfrac{1}{1+\beta} - \tfrac{1}{1-\beta^2} p_o + \tfrac{\beta}{1-\beta^2} p_f \le s. \end{aligned} \tag{G.20}$$

The platform's profit is concave in $p_o$ since $\frac{\partial^2 \pi_p^{PW}}{\partial p_o^2} = \frac{2}{\beta^2 - 1} < 0$. By applying KKT conditions, we can rewrite the above problem as follows:

$$\begin{aligned} \mathcal{L}(p_o, \lambda) &= q_o(p_o - m_r - \tfrac{a+s}{b}) + \lambda(s - q_o), \\ s.t. \quad & \tfrac{\partial \mathcal{L}}{\partial p_o} = -\tfrac{a+b(\lambda+m_r-2p_o+\beta(p_f-1)+1)+s}{b(\beta^2-1)} = 0, \\ & s - q_o \ge 0, \\ & \lambda \ge 0, \\ & \lambda(s - q_o) = 0. \end{aligned}$$

By solving this, the platform's optimal online channel price can be listed as follows:

$$p_o^* = \begin{cases} \beta(p_f - 1) + \left(\beta^2 - 1\right) s + 1 & if \quad m_r \le \tilde{m}_r, \quad \text{(G.21a)} \\ \frac{a+b(m_r+\beta(p_f-1)+1)+s}{2b} & if \quad m_r \ge \tilde{m}_r. \quad \text{(G.21b)} \end{cases}$$

Note that $\tilde{m}_r = \frac{-a+b\beta(p_f-1)+2b(\beta^2-1)s+b-s}{b}$.

**Stage 2: the restaurant sets the online price margin $m_r$ and the offline channel price $p_f$.**

(i) Anticipating the platform's online channel price $p_o^* = \beta(p_f - 1) + (\beta^2 - 1)\, s + 1$, then the restaurant's optimization problem becomes

$$\begin{aligned} \pi_r^{PW} &= \max_{m_r, p_f} q_o m_r + q_f p_f = m_r s - p_f(p_f + \beta s - 1), \\ s.t. \quad & m_r \le \tilde{m}_r. \end{aligned} \tag{G.22}$$

Taking the derivative of the restaurant's profit with respect to $m_r$, we obtain $\frac{\partial \pi_r^{PW}}{\partial m_r} = s > 0$. Hence, given any $p_f$, the restaurant increases the margin until $m_r = \tilde{m}_r$. Substituting $m_r = \tilde{m}_r$ into restaurant's profit, we have $\frac{\partial^2 \pi_r^{PW}}{\partial p_f^2} = -2 < 0$. Thus, the restaurant's optimal $p_f$ satisfying $\frac{\partial \pi_r^{PW}}{\partial p_f} = 0$, which is $p_f = \frac{1}{2}$. Therefore, the restaurant's optimal prices are $m_r^* = \frac{-2a+b(-\beta+4(\beta^2-1)s+2)-2s}{2b}$ and $p_f^* = \frac{1}{2}$. In this case, the restaurant's profit is $\pi_r^{PW*} = \frac{-4s(a+s)+4(1-\beta)bs(1-2(\beta+1)s)+b}{4b}$.

(ii) Anticipating the platform's online channel price $p_o^* = \frac{a+b(m_r+\beta(p_f-1)+1)+s}{2b}$, then the restaurant's optimization problem becomes

$$\begin{aligned} \pi_r^{PW} &= \max_{m_r, p_f} q_o m_r + q_f p_f = \frac{a(m_r-\beta p_f)+b(m_r^2+m_r(\beta-2\beta p_f-1)+\beta p_f(\beta-\beta p_f+1)+2(p_f-1)p_f)+s(m_r-\beta p_f)}{2b(\beta^2-1)}, \\ s.t. \quad & m_r \ge \tilde{m}_r. \end{aligned} \tag{G.23}$$

We find that the restaurant's profit is jointly concave in $m_r$ and $p_f$ as the Hessian matrix is negative definite, and the Hessian matrix is given by

$$H = \begin{bmatrix} \frac{\partial^2 \pi_r^{PW}}{\partial m_r^2} & \frac{\partial^2 \pi_r^{PW}}{\partial m_r \partial p_f} \\ \frac{\partial^2 \pi_r^{PW}}{\partial p_f \partial m_r} & \frac{\partial^2 \pi_r^{PW}}{\partial p_f^2} \end{bmatrix} = \begin{bmatrix} \frac{1}{\beta^2-1} & \frac{\beta}{1-\beta^2} \\ \frac{\beta}{1-\beta^2} & \frac{2-\beta^2}{\beta^2-1} \end{bmatrix}.$$

By applying KKT conditions, the restaurant's optimal prices can be listed as follows:

$$m_r^*, p_f^* = \begin{cases} \frac{-a+b-s}{2b}, \quad \frac{1}{2} & if \quad s \geq \bar{s}, \quad \text{(G.24a)} \\ \frac{-2a+b(-\beta+4(\beta^2-1)s+2)-2s}{2b}, \quad \frac{1}{2} & if \quad s \leq \bar{s}, \quad \text{(G.24b)} \end{cases}$$

where $\bar{s} = \frac{a+b(\beta-1)}{4b(\beta^2-1)-1}$.

Substituting optimal prices into the restaurant's profit, we can get the corresponding profit of the restaurant is $\pi_r^{PW*} = -\frac{a^2+2a(b(\beta-1)+s)-b^2(\beta-1)(\beta+3)+2b(\beta-1)s+s^2}{8b^2(\beta^2-1)}$ if $s \geq \bar{s}$; otherwise, $\pi_r^{PW*} = \frac{-4s(a+s)+4(\beta-1)bs(2(\beta+1)s-1)+b}{4b}$.

Combining Case (i) and Case (ii), we can show that the restaurant's profit in Case (i) is dominated by that in Case (ii) since

$$\pi_r^{PW*}|_{m \geq \tilde{m}} - \pi_r^{PW*}|_{m \leq \tilde{m}} = \begin{cases} \frac{(a+b(\beta-4\beta^2 s+4s-1)+s)^2}{8b^2(1-\beta^2)} > 0 & if \quad s \geq \bar{s}, \\ 0 & if \quad s \leq \bar{s}. \end{cases}$$

Hence, the restaurant's optimal prices at this stage are given in Equation (G.24).

**Stage 1: platform determines the supply $s$.** We consider the following two cases.

(i) Anticipating the restaurant's optimal prices $m_r^* = \frac{-a+b-s}{2b}$, $p_f^* = \frac{1}{2}$, the platform's optimization problem becomes

$$\begin{aligned} \pi_p^{PW} &= \max_s q_o(p_o^* - m_r^* - \tfrac{a+s}{b}) = \frac{(a+b(\beta-1)+s)^2}{16b^2(1-\beta^2)}, \\ s.t. \quad & s \geq \bar{s}. \end{aligned} \tag{G.25}$$

The platform's profit is convex in $s$ since $\frac{\partial^2 \pi_p^{PW}}{\partial s^2} = \frac{1}{8b^2(1-\beta^2)} > 0$, and the platform's profit is minimized when $s = -a + b(1-\beta)$. We can show that $-a + b(1-\beta) - \bar{s} = \frac{4b(\beta^2-1)(a+b(\beta-1))}{4b(1-\beta^2)+1} > 0$. Additionally, the online demand in this stage becomes $q_o = \frac{a+b(\beta-1)+s}{4b(\beta^2-1)}$, and it decreases with $s$ since $\frac{\partial q_o}{\partial s} = \frac{1}{4b(\beta^2-1)} < 0$, and $q_o = 0$ when $s = -a+b(1-\beta)$. Hence, $s$ needs to satisfy $s \leq -a+b(1-\beta)$ to ensure the positive demand. Therefore, the platform's profit decreases in $s$ when $\bar{s} \leq s \leq -a+b(1-\beta)$, and its optimal supply is $s^* = \bar{s}$.

(ii) Anticipating the restaurant's prices $m_r^* = \frac{-2a+b(-\beta+4(\beta^2-1)s+2)-2s}{2b}$, $p_f^* = \frac{1}{2}$, then the platform's optimization problem becomes

$$\begin{aligned} \pi_p^{PW} &= \max_s q_o(p_o^* - m_r^* - \tfrac{a+s}{b}) = s^2(1-\beta^2), \\ s.t. \quad & s \leq \bar{s}. \end{aligned} \tag{G.26}$$

The platform's profit is convex increasing in $s$, so the platform's optimal supply is $s^* = \bar{s}$.

Combining these two cases, we can get the platform's optimal $s^* = \bar{s}$. Then, substituting $s^*$ into the $m_r^*$, $p_f^*$, and $d^*$, we can get the equilibrium results in Lemma 4. Furthermore, we have the following equilibrium results:

$$\begin{aligned} m_r^{PW*} &= \frac{4\beta^2(a-b)-4a+\beta+4b}{2-8b(\beta^2-1)}, \quad d^{PW*} = \frac{(1-\beta)(3a(\beta+1)-b\beta^2+b+1)}{4b(1-\beta^2)+1} - r, \\ q_o^{PW*} &= \frac{a-b(1-\beta)}{4b(\beta^2-1)-1}, \quad q_f^{PW*} = \frac{-2a\beta+2b(-\beta-2)(1-\beta)-1}{8b(\beta^2-1)-2}, \\ \pi_p^{PW*} &= -\frac{(\beta^2-1)(-a-b\beta+b)^2}{(1-4b(\beta^2-1))^2}, \\ \pi_r^{PW*} &= \frac{-8a^2(\beta^2-1)+16ab(-\beta-1)(1-\beta)^2+8b(\beta^2-1)(b(-\beta-3)(1-\beta)-1)+1}{4(1-4b(\beta^2-1))^2}, \\ \pi_{sc}^{PW*} &= \frac{-12a^2(\beta^2-1)+24ab(-\beta-1)(1-\beta)^2+4b(\beta^2-1)(b(-\beta-7)(1-\beta)-2)+1}{4(1-4b(\beta^2-1))^2}. \end{aligned} \tag{G.27}$$

□

### G.5. Proof of Lemma 5

**Stage 2: restaurant determines the online channel price $p_o$ and the offline channel price $p_f$.** We consider the restaurant's price decisions with constraint $q_o \leq s$. If $q_o > s$, the restaurant will increase the online price such that the demand ($s$) remains unaltered. Additionally, given the offline price fixed, the increase in online price increases the offline demand, leading to larger restaurant's profit. Thus, in equilibrium, $q_o \leq s$. Therefore, the restaurant's optimization problem can be formulated as follows:

$$\begin{aligned} \pi_r^{RW} &= \max_{p_o, p_f} q_o(p_o - r) + p_f q_f, \\ s.t. \quad & q_o \leq s. \end{aligned} \tag{G.28}$$

The restaurant's profit is jointly concave in $p_o$ and $p_f$ as the Hessian matrix is negative definite, and the Hessian matrix is given by

$$H = \begin{bmatrix} \frac{\partial^2 \pi_r^{RN}}{\partial p_o^2} & \frac{\partial^2 \pi_r^{RN}}{\partial p_o \partial p_f} \\ \frac{\partial^2 \pi_r^{RN}}{\partial p_f \partial p_o} & \frac{\partial^2 \pi_r^{RN}}{\partial p_f^2} \end{bmatrix} = \begin{bmatrix} \frac{2}{\beta^2-1} & \frac{2\beta}{1-\beta^2} \\ \frac{2\beta}{1-\beta^2} & \frac{2}{\beta^2-1} \end{bmatrix}.$$

By applying KKT conditions, the restaurant's optimal prices can be listed as follows:

$$p_o^*, p_f^* = \begin{cases} \frac{1+r}{2}, \quad \frac{1}{2} & if \quad r \geq \bar{r}, \quad \text{(G.29a)} \\ \frac{-\beta+2\left(\beta^2-1\right)s+2}{2}, \quad \frac{1}{2} & if \quad r \leq \bar{r}. \quad \text{(G.29b)} \end{cases}$$

where $\bar{r} = (\beta - 1)(2(\beta + 1)s - 1)$.

**Stage 1: platform determines the commission fee $r$ and the supply $s$.** We consider the following two cases.

(i) Anticipating the restaurant's optimal channel prices $p_o^* = \frac{-\beta+2\left(\beta^2-1\right)s+2}{2}$, $p_f^* = \frac{1}{2}$, the online demand in the system becomes $q_o^* = s$, so the platform's optimization problem becomes

$$\begin{aligned} \pi_p^{RW} &= \max_{r,s} s\left(r - \tfrac{a+s}{b}\right), \\ s.t. \quad & r \leq \bar{r}. \end{aligned} \tag{G.30}$$

The platform's profit increases in $r$ since $\frac{\partial \pi_p^{RW}}{\partial r} > 0$. Hence, the platform's optimal commission fee is $r^* = \bar{r}$. Substituting $r^* = \bar{r}$ into the platform's profit, we have

$$\pi_p^{RW*} = \frac{s(-a+b(\beta-1)(2(\beta+1)s-1)-s)}{b}.$$

Its concave in $s$ since $\frac{\partial^2 \pi_p^{RW}}{\partial s^2} = \frac{2\left(2b\beta^2-2b-1\right)}{b} < 0$. Hence, the optimal $s^* = \frac{a+b(\beta-1)}{4b(\beta^2-1)-2}$ satisfying $\frac{\partial \pi_p^{RW}}{\partial s} = 0$. Substituting $s^*$ back into the platform's profit, we have $\pi_p^{RW*} = -\frac{(a+b(\beta-1))^2}{4b(2b(\beta^2-1)-1)}$.

(ii) Anticipating the restaurant's prices $p_o^* = \frac{1+r}{2}$, $p_f^* = \frac{1}{2}$, the online demand in the system becomes $q_o = \frac{\beta+r-1}{2(\beta^2-1)}$, so the platform's optimization problem becomes

$$\begin{aligned} \pi_p^{RW} &= \max_{r,s} q_o\left(r \tfrac{a+s}{b}\right) = -\frac{(\beta+r-1)(a-br+s)}{2b(\beta^2-1)}, \\ s.t. \quad & r \geq \bar{r}. \end{aligned} \tag{G.31}$$

By applying KKT, we can get the optimal $r^* = \frac{(\beta-1)\left(a\beta+a-b\beta^2+b+1\right)}{2b(\beta^2-1)-1}$, and $s^* = \frac{a+b(\beta-1)}{4b(\beta^2-1)-2}$, leading to $\pi_p^{RW*} = -\frac{(a+b(\beta-1))^2}{4b(2b(\beta^2-1)-1)}$.

Combining Case (i) and Case (ii), we find these two cases lead to the same optimal decisions and profits. In summary, we have the following equilibrium prices, demands and profits:

$$
\begin{aligned}
p_o^{RW*} &= \tfrac{1}{2} + \tfrac{(1-\beta)(1+a(\beta+1)+b(1-\beta^2))}{2+4b(1-\beta^2)}, \quad p_f^{RW*} = \tfrac{1}{2},\\
w^{RW*} &= \tfrac{a+4ab(1-\beta^2)+b(1-\beta)}{4b^2(2-\beta^2)+2b},\\
q_o^{RW*} &= s^{RW*} = \tfrac{a-b(1-\beta)}{4b(\beta^2-1)-2}, \quad q_f^{RW*} = \tfrac{-a\beta+b(-\beta-2)(1-\beta)-1}{4b(\beta^2-1)-2},\\
\pi_p^{RW*} &= -\tfrac{(-a-b\beta+b)^2}{4b(2b(\beta^2-1)-1)},\\
\pi_r^{RW*} &= \tfrac{a^2(-(\beta^2-1))+2ab(-\beta-1)(1-\beta)^2+b(-\beta-1)(1-\beta)(b(-3\beta-5)(1-\beta)-4)+1}{4(1-2b(\beta^2-1))^2},\\
\pi_{sc}^{RW*} &= \tfrac{a^2(1-3b(\beta^2-1))+2ab(1-\beta)(3b(\beta^2-1)-1)+b^2(1-\beta)(b(-\beta-7)(-\beta-1)(1-\beta)+3\beta+5)+b}{4b(1-2b(\beta^2-1))^2}.
\end{aligned}
\tag{G.32}
$$

Comparing the above equilibrium results with those in the RN contract, we find that they are the same (see Equation (G.17)). $\square$

## G.6. Proof of Proposition 1

First, we compare the online channel price under the RW contract with its benchmark (BRW). To make a fair comparison, we substitute $c = w^{RW*}$ (see Equation (16)) into the equilibrium online channel price in the BRW case (see Equation (8)), we can get

$$
p_o^{BRW*} = \begin{cases} \frac{2-\beta-2\hat{s}(1-\beta^2)}{2} & if \quad \hat{s} \leq \frac{(4b(1-\beta^2)+1)(b(1-\beta)-a)}{8b(1-\beta^2)(2b(1-\beta^2)+1)},\\ \frac{a(4b(\beta^2-1)-1)+b(-4b(\beta-3)(\beta-1)(\beta+1)+3\beta-7)}{8b(2b(\beta^2-1)-1)} & if \quad \hat{s} \geq \frac{(4b(1-\beta^2)+1)(b(1-\beta)-a)}{8b(1-\beta^2)(2b(1-\beta^2)+1)}. \end{cases}
$$

Then we compare $p_o^{RN*}$ with $p_o^{BRN*}$. If $\hat{s} \leq \frac{(4b(1-\beta^2)+1)(b(1-\beta)-a)}{8b(1-\beta^2)(2b(1-\beta^2)+1)}$,

$$
p_o^{RN*} - p_o^{BRN*} = \tfrac{1}{2} + \tfrac{(1-\beta)(1+a(\beta+1)+b(1-\beta^2))}{2+4b(1-\beta^2)} - \tfrac{2-\beta-2\hat{s}(1-\beta^2)}{2} = \tfrac{(1-\beta^2)(a+b(\beta-4\beta^2\hat{s}+4\hat{s}-1)+2\hat{s})}{4b(1-\beta^2)+2},
$$

which is smaller than 0 when $\hat{s} \leq \frac{a+b(\beta-1)}{4b(\beta^2-1)-2} = q_o^{RN*}$ (see Equation (G.17)); otherwise, $p_o^{RN*} - p_o^{BRN*} \geq 0$ when $\hat{s} \geq q_o^{RN*}$. If $\hat{s} \geq \frac{(4b(1-\beta^2)+1)(b(1-\beta)-a)}{8b(1-\beta^2)(2b(1-\beta^2)+1)}$,

$$
p_o^{RN*} - p_o^{BRN*} = \tfrac{1}{2} + \tfrac{(1-\beta)(1+a(\beta+1)+b(1-\beta^2))}{2+4b(1-\beta^2)} - \tfrac{a(4b(\beta^2-1)-1)+b(-4b(\beta-3)(\beta-1)(\beta+1)+3\beta-7)}{8b(2b(\beta^2-1)-1)} = \tfrac{a+b(\beta-1)}{8b(2b(\beta^2-1)-1)} > 0.
$$

Since we can show that $\frac{(4b(1-\beta^2)+1)(b(1-\beta)-a)}{8b(1-\beta^2)(2b(1-\beta^2)+1)} - q_o^{RN*} = \frac{b(1-\beta)-a}{8b(1-\beta^2)(2b(1-\beta^2)+1)} > 0$, we have $p_o^{RN*} < p_o^{BRN*}$ if $\hat{s} < q_o^{RN*}$.

Second, we compare the online channel price under the PN contract with its benchmark (BPN). Similarly, we substitute $c = w^{PN*}$ (see Equation (22)) into the equilibrium online channel price in the BPN case (see Equation (8)), we can get

$$
p_o^{BPN*} = \begin{cases} \frac{2-\beta-2\hat{s}(1-\beta^2)}{2} & if \quad \hat{s} \leq \frac{(4b(1-\beta^2)+3)(b(1-\beta)-a)}{16b(1-\beta^2)(b(1-\beta^2)+1)},\\ \frac{1}{4}(3-\beta+\frac{a+b(\beta-1)}{4b(b(\beta^2-1)-1)}+\frac{a}{b}) & if \quad \hat{s} \geq \frac{(4b(1-\beta^2)+3)(b(1-\beta)-a)}{16b(1-\beta^2)(b(1-\beta^2)+1)}. \end{cases}
$$

Then we compare $p_o^{PN*}$ with $p_o^{BPN*}$. If $\hat{s} \leq \frac{(4b(1-\beta^2)+3)(b(1-\beta)-a)}{16b(1-\beta^2)(b(1-\beta^2)+1)}$,

$$
p_o^{PN*} - p_o^{BPN*} = \tfrac{(1-\beta^2)(a+b(3-\beta))+2(2-\beta)}{4b(1-\beta^2)+4} - \tfrac{2-\beta-2\hat{s}(1-\beta^2)}{2} = \tfrac{(1-\beta^2)(a+b(\beta-4\beta^2\hat{s}+4\hat{s}-1)+4\hat{s})}{4+4b(1-\beta^2)},
$$

which is smaller than 0 when $\hat{s} < \frac{a+b\beta-b}{4(b\beta^2-b-1)} = q_o^{PN*}$ (see Equation (G.19)); otherwise, $p_o^{PN*} - p_o^{BPN*} \geq 0$ when $\hat{s} \geq q_o^{PN*}$. If $\hat{s} \geq \frac{(4b(1-\beta^2)+3)(b(1-\beta)-a)}{16b(1-\beta^2)(b(1-\beta^2)+1)}$,

$$
p_o^{PN*} - p_o^{BPN*} = \tfrac{(1-\beta^2)(a+b(3-\beta))+2(2-\beta)}{4b(1-\beta^2)+4} - \tfrac{1}{4}(3-\beta+\tfrac{a+b(\beta-1)}{4b(b(\beta^2-1)-1)}+\tfrac{a}{b}) = \tfrac{3(a+b(\beta-1))}{16b(b(\beta^2-1)-1)} > 0.
$$

Since we can show that $\frac{(4b(1-\beta^2)+3)(b(1-\beta)-a)}{16b(1-\beta^2)(b(1-\beta^2)+1)} - q_o^{PN*} = -\frac{3(a+b(\beta-1))}{16b(\beta^2-1)(b(\beta^2-1)-1)} > 0$, we have $p_o^{PN*} < p_o^{BPN*}$ if $\hat{s} < q_o^{PN*}$.

Third, we compare the online channel price under the PW contract with its benchmark (BPW). Similarly, we substitute $c = w^{PW*}$ (see Equation (28)) into the equilibrium online channel price in the BPW case (see Equation (8)), we can get

$$p_o^{BPW*} = \begin{cases} \frac{2-\beta-2\hat{s}(1-\beta^2)}{2} & if \quad \hat{s} \leq \frac{a+b(\beta-1)}{4b(\beta^2-1)-1}, \\ \frac{1}{4}(3-\beta+\frac{(\beta-1)(4a(\beta+1)+1)}{4b(\beta^2-1)-1}) & if \quad \hat{s} \geq \frac{a+b(\beta-1)}{4b(\beta^2-1)-1}. \end{cases}$$

Then we compare $p_o^{PW*}$ with $p_o^{BPW*}$. If $\hat{s} \leq \frac{a+b(\beta-1)}{4b(\beta^2-1)-1}$,

$$p_o^{PW*} - p_o^{BPW*} = \frac{2a(1-\beta^2)+2b(3-\beta)(1-\beta^2)+2-\beta}{8b(1-\beta^2)+2} - \frac{2-\beta-2\hat{s}(1-\beta^2)}{2} = \frac{(1-\beta^2)(a+b(\beta-4\beta^2\hat{s}+4\hat{s}-1)+\hat{s})}{4b(1-\beta^2)+1},$$

which is smaller than 0 when $\hat{s} < \frac{a+b(\beta-1)}{4b(\beta^2-1)-1} = q_o^{PW*}$ (see Equation (G.27)); otherwise, $p_o^{PW*} - p_o^{BPW*} \geq 0$ when $\hat{s} \geq q_o^{PW*}$. If $\hat{s} \geq \frac{a+b(\beta-1)}{4b(\beta^2-1)-1}$,

$$p_o^{PW*} - p_o^{BPW*} = 0.$$

Since $\frac{a+b(\beta-1)}{4b(\beta^2-1)-1} - q_o^{PW*} = 0$, we have $p_o^{PW*} < p_o^{BPW*}$ if $\hat{s} < q_o^{PW*}$.

Fourth, we compare the online channel price under the RW contract with its benchmark (BRW). Because the equilibrium online channel price and wage in the RW(BRW) contract is the same as that in the RN(BRN) contract, so we can get the same results in this scenario. Combining all four cases, we have our results in Proposition 1. □

### G.7. Proof of Proposition 2

(i) Based on Equations (14), (20), and (26), we can derive

$$p_o^{PN*} - p_o^{RN*} = \frac{(1-\beta^2)(b(1-\beta)-a)}{4(b(1-\beta^2)+1)(2b(1-\beta^2)+1)} > 0; \quad p_o^{RN*} - p_o^{PW*} = \frac{(1-\beta^2)(b(1-\beta)-a)}{2(2b(1-\beta^2)+1)(4b(1-\beta^2)+1)} > 0.$$

(ii) Based on Equations (G.17), (G.19), and (G.27), we can derive

$$q_o^{PW*} - q_o^{RN*} = \frac{-a+b(1-\beta)}{2(2b(1-\beta^2)+1)(4b(1-\beta^2)+1)} > 0; \quad q_o^{RN*} - q_o^{PN*} = \frac{-a+b(1-\beta)}{4(b(1-\beta^2)+1)(2b(1-\beta^2)+1)} > 0.$$

(iii) Based on Equations (16), (22), and (28), we can derive

$$w^{PW*} - w^{RN*} = \frac{b(1-\beta)-a}{2b(2b(1-\beta^2)+1)(4b(1-\beta^2)+1)} > 0; \quad w^{RN*} - w^{PN*} = \frac{b(1-\beta)-a}{4b(b(1-\beta^2)+1)(2b(1-\beta^2)+1)} > 0.$$

□

### G.8. Proof of Proposition 3

Based on Equations (G.17), (G.19), and (G.27), we can derive

(i) Platform's profit:

$$\pi_p^{PW*} - \pi_p^{PN*} = -\frac{(1-8b(1-\beta^2))(a+b(\beta-1))^2}{16b(1-4b(\beta^2-1))^2(1+b(1-\beta^2))}.$$

Hence, $\pi_p^{PW*} - \pi_p^{PN*} > 0$ if $b > \frac{1}{8(1-\beta^2)}$; otherwise, $\pi_p^{PW*} - \pi_p^{PN*} \leq 0$. Additionally, we have

$$\pi_p^{RN*} - \pi_p^{PW*} = \frac{(1+4b(1-\beta^2)(1+2b(1-\beta^2)))(a+b(\beta-1))^2}{4b(1-4b(\beta^2-1))^2(2b(1-\beta^2)+1)} > 0;$$
$$\pi_p^{RN*} - \pi_p^{PN*} = \frac{(2b(1-\beta^2)+3)(a+b(\beta-1))^2}{16b(b(1-\beta^2)+1)(2b(1-\beta^2)+1)} > 0.$$

(ii) Restaurant's profit:

$$\pi_r^{PW*} - \pi_r^{PN*} = -\frac{(1-8b(1-\beta^2))(a+b(\beta-1))^2}{8b(1-4b(\beta^2-1))^2(1+b(1-\beta^2))}.$$

Hence, $\pi_r^{PW*} - \pi_r^{PN*} > 0$ if $b > \frac{1}{8(1-\beta^2)}$; otherwise, $\pi_r^{PW*} - \pi_r^{PN*} \leq 0$. Additionally, we have

$$\pi_r^{PN*} - \pi_r^{RN*} = \frac{(2b(1-\beta^2+b(\beta^2-1)^2)+1)(a+b(\beta-1))^2}{8b(1-2b(\beta^2-1))^2(b(1-\beta^2)+1)} > 0;$$
$$\pi_r^{PW*} - \pi_r^{RN*} = \frac{(1-\beta^2)(8b(1-\beta^2)(2b(1-\beta^2)+3)+7)(a+b(\beta-1))^2}{4(4b(1-\beta^2)+1)^2(2b(1-\beta^2)+1)^2} > 0.$$

(iii) Supply chain profit:

$$\pi_{sc}^{PW*} - \pi_{sc}^{RN*} = \frac{(8b^2(1-\beta^2)^2 - b\beta^2 + b - 1)(a+b(\beta-1))^2}{4b(4b(1-\beta^2)+1)^2(2b(1-\beta^2)+1)^2}.$$

Hence, $\pi_{sc}^{PW*} - \pi_{sc}^{RN*} > 0$ when $8b^2(1-\beta^2)^2 - b\beta^2 + b - 1 > 0$, that is, $b > \frac{\sqrt{33}-1}{16(1-\beta^2))}$; otherwise, $\pi_{sc}^{PW*} - \pi_{sc}^{RN*} \leq 0$. Additionally, we have

$$\pi_{sc}^{PW*} - \pi_{sc}^{PN*} = \frac{3(8b(\beta^2-1)+1)(a+b(\beta-1))^2}{16b(1-4b(\beta^2-1))^2(b(\beta^2-1)-1)}.$$

Hence, $\pi_{sc}^{PW*} - \pi_{sc}^{PN*} > 0$ if $b > \frac{1}{8(1-\beta^2)}$; otherwise, $\pi_{sc}^{PW*} - \pi_{sc}^{PN*} \leq 0$. Combining

$$\pi_{sc}^{RN*} - \pi_{sc}^{PN*} = \frac{(4b(1-\beta^2)+1)(a+b(\beta-1))^2}{16b(1-2b(\beta^2-1))^2(b(1-\beta^2)+1)} > 0,$$

we have our results in Proposition 3 (iii). $\square$

### G.9. Proof of Proposition 4

(i) Based on Equations (J.1) , (G.17), (G.19), and (G.27), we can derive

$$q_o^{PW*} + q_f^{PW*} - (q_o^{RN*} + q_f^{RN*}) = \frac{(1-\beta)(b(1-\beta)-a)}{2(2b(1-\beta^2)+1)(4b(1-\beta^2)+1)} > 0;$$
$$q_o^{RN*} + q_f^{RN*} - (q_o^{PN*} + q_f^{PN*}) = \frac{(1-\beta)(b(1-\beta)-a)}{4(b(1-\beta^2)+1)(2b(1-\beta^2)+1)} > 0.$$

(ii) Similarly, we can derive

$$q_o^{PW*} - q_o^{C*} = \frac{(2b(\beta^2-1)+1)(b(1-\beta)-a)}{2(b(1-\beta^2)+1)(4b(1-\beta^2)+1)};$$
$$q_o^{PW*} + q_f^{PW*} - (q_o^{C*} + q_f^{C*}) = \frac{(1-\beta)(2b(\beta^2-1)+1)(b(1-\beta)-a)}{2(b(1-\beta^2)+1)(4b(1-\beta^2)+1)}.$$

Hence, $q_o^{PW*} > q_o^{C*}$ and $q_o^{PW*} + q_f^{PW*} > q_o^{C*} + q_f^{C*}$ when $1 - 2b(1-\beta^2) > 0$. $\square$

### G.10. Proof of Proposition 5

Based on Equations (E.2) and (E.4), we can derive

$$CS^{PW*} - CS^{RN*} = \frac{(1-\beta^2)(8b(1-\beta^2)+3)(a+b(\beta-1))^2}{8(1-4b(\beta^2-1))^2(1-2b(\beta^2-1))^2} > 0;$$
$$CS^{RN*} - CS^{PN*} = \frac{(1-\beta^2)(4b(1-\beta^2)+3)(a+b(\beta-1))^2}{32(1-2b(\beta^2-1))^2(b(\beta^2-1)-1)^2} > 0;$$
$$DS^{PW*} - DS^{RN*} = \frac{(8b(1-\beta^2)+3)(a+b(\beta-1))^2}{8b(1-4b(\beta^2-1))^2(1-2b(\beta^2-1))^2} > 0;$$
$$DS^{RN*} - DS^{PN*} = \frac{(4b(1-\beta^2)+3)(a+b(\beta-1))^2}{32b(1-2b(\beta^2-1))^2(b(\beta^2-1)-1)^2} > 0.$$

Additionally, based on Equations (E.5), (E.2), (E.4), (G.17), (G.19), and (G.27), we can derive

$$SW^{PW*} - SW^{RN*} = \frac{(b(1-\beta)(\beta+1)(24b(1-\beta^2)+13)+1)(a+b(\beta-1))^2}{8b(1-4b(\beta^2-1))^2(1-2b(\beta^2-1))^2} > 0;$$
$$SW^{RN*} - SW^{PN*} = \frac{(12b(\beta^2-1)-5)(a+b(\beta-1))^2}{32b(1-2b(\beta^2-1))^2(b(\beta^2-1)-1)} > 0.$$

$\square$

## Appendix H: Analytical Results of Labor Supply Model with Wage Rate

ASSUMPTION H.1. *When the platform determines the wage rate, the model parameters satisfy* $1-\beta-y\phi \geq 0$.

This assumption ensures that online and dine-in demands are positive regardless of the contracting schemes. If not, the platform may close the online channel. In the following, we will briefly list the platform and restaurant's optimization problem in each type of contract and then characterize the equilibrium solutions respectively.

**The Restaurant-Pricing/No Wage-Commitment (RN) Contract** In the first stage, the platform chooses the commission fee $r$ to maximize the profit:

$$\max_{r} \pi_p = \min(s(w), q_o)(r - wq_o), \tag{H.1}$$

where $w$ is the wage rate offered by the platform, and $wq_o$ is the driver's wage.

In the second stage, given the commission fee $r$, the restaurant sets the online and the offline channel prices to maximize the following profit function,

$$\max_{p_o, p_f} \pi_r = \min(s(w), q_o)(p_o - r) + q_f p_f. \tag{H.2}$$

Note that the online channel margin is given by $m_r = p_o - r$.

Finally, in the last stage of the game, given the commission fee $r$ and channel prices in two channels ($p_o$, $p_f$), the platform maximizes its profit by solving for optimal wage rate,

$$\max_{w} \pi_p = \min(s(w), q_o)(r - wq_o). \tag{H.3}$$

Solving backward, we characterize the equilibrium outcomes in the next lemma.

LEMMA H1. *When the platform determines the wage rate, then under the RN contract, the equilibrium commission fee, the equilibrium online and offline channel prices, and the equilibrium wage rate are*

$$r^{RN*} = \frac{(1-\beta)((\beta+1)N(1-\beta+y\phi)+1-\phi^2)}{2(1-\beta^2)N-\phi^2+1}, \tag{H.4}$$
$$p_o^{RN*} = \frac{(2-\beta)(1-\phi^2)+(1-\beta^2)N(-\beta+y\phi+3)}{2(2(1-\beta^2)N-\phi^2+1)}, \tag{H.5}$$
$$p_f^{RN*} = \frac{1}{2}, \tag{H.6}$$
$$w^{RN*} = \frac{1-(1-\beta)\phi^2-\beta+y\phi(4(1-\beta^2)N+1)-y\phi^3}{N(1-\beta-y\phi)}. \tag{H.7}$$

**The Platform-Pricing/No Wage-Commitment (PN) Contract** In the first stage, the platform chooses the commission fee $r$ to maximize its profit:

$$\max_{r} \pi_p = \min(s(w), q_o)(r + d - wq_o). \tag{H.8}$$

Note that the online channel profit margin for the platform is $m_p^{PN} = p_o - m_r = r + d$. In the second stage, the restaurant decides the offline channel price alongside the online sales margin to maximize its profit, given by

$$\max_{m_r, p_f} \pi_r = \min(s(w), q_o)m_r + q_f p_f. \tag{H.9}$$

Finally, in the last stage of the game, the platform sets the online channel price $p_o$ by posting the delivery fee, $d$, and the wage rate, $w$, to maximize its profit:

$$\max_{w,p_o} \pi_p = \min(s(w), q_o)(p_o - m_r - wq_o). \tag{H.10}$$

Note that the online channel price is given by $p_o = r + m_r + d$. We employ backward induction to solve for the equilibrium outcomes, as outlined in Lemma H2.

Lemma H2. *When the platform determines the wage rate, then under the PN contract, the equilibrium online and offline channel prices and the equilibrium wage rate are*

$$p_o^{PN*} = \frac{2(2-\beta)(1-\phi^2)+(1-\beta^2)N(-\beta+y\phi+3)}{4((1-\beta^2)N-\phi^2+1)}, \tag{H.11}$$
$$p_f^{PN*} = \frac{1}{2}, \tag{H.12}$$
$$w^{PN*} = \frac{-(1-\beta)\phi^2-\beta+y\phi(4(1-\beta^2)N+3)-3y\phi^3+1}{N(-\beta-y\phi+1)}. \tag{H.13}$$

**The Platform-Pricing/Wage-Commitment (PW) Contract** Under the PW contract, the platform first commits to the wage rate paid to the delivery drivers and asks for a commission from the restaurant. We can write the platform's problem in the first stage of the game as

$$\max_{w,r} \pi_p = \min(s(w), q_o)(p_o - m_r - wq_o). \tag{H.14}$$

Note that the online channel profit margin for the platform is $m_p^{PW} = p_o - m_r = r + d$. In the second stage, the restaurant sets its margin on online orders alongside the dine-in channel prices to maximize its profit, given by

$$\max_{m_r,p_f} \pi_r = \min(s(w), q_o)m_r + q_f p_f. \tag{H.15}$$

Finally, the platform sets the online channel price $p_o = r + d + m_r$ by announcing its delivery fee $d$ in the third stage to maximize its profit

$$\max_{p_o} \pi_p = \min(s(w), q_o)(p_o - m_r - wq_o). \tag{H.16}$$

The following lemma characterizes the equilibrium outcome.

Lemma H3. *When the platform determines the wage rate, then under the PW contract, the equilibrium online and offline channel prices and the equilibrium wage rate are*

$$p_o^{PW*} = -\frac{(\beta-2)(\phi^2-1)+2(\beta^2-1)N(\beta-y\phi-3)}{2(4(\beta^2-1)N+\phi^2-1)}, \tag{H.17}$$
$$p_f^{PW*} = \frac{1}{2}, \tag{H.18}$$
$$w^{PW*} = \frac{(\beta-1)(4(\beta+1)Ny\phi-\phi^2+1)}{N(\beta+y\phi-1)}. \tag{H.19}$$

**The Restaurant-Pricing/Wage-Commitment (RW) Contract** Under the RW contract, the platform first commits to the wage paid to the delivery drivers and asks for a commission fee $r$ from the restaurant. The restaurant then sets the offline and online prices. Therefore, the online channel profit margins for the platform and the restaurant are $m_p^{RW} = r$ and $m_r^{RW} = p_o - r$, respectively. We can write the platform's problem in the first stage of the game as a function of the wage and the commission fee as,

$$\max_{r,w} \pi_p = \min(s(w), q_o)(r - wq_o). \tag{H.20}$$

In the second stage of the game, given that the restaurant's profit margin is given by $m_r^{RW} = p_o - r$, it sets the online and offline prices to maximize the following profit function,

$$\max_{p_o, p_f} \pi_r = \min(s(w), q_o)(p_o - r) + q_f p_f. \tag{H.21}$$

Solving backward, we can characterize the equilibrium outcomes in the following lemma, which indicates that the RN and the RW contracts are equivalent.

Lemma H4. *Under the RW contract, the equilibrium outcome is the same as that under the RN contract.*

Next, we will derive the optimal driver's surplus. Recall that the driver's utility is defined as

$$U = \max_{l_p, l_o} l_p w q_o + l_o y - \tfrac{1}{2} l_p^2 - \tfrac{1}{2} l_o^2 - \phi l_p l_o.$$

Solving the driver's optimization problem, we have

$$l_p^* = \tfrac{w q_o - \phi y}{1 - \phi^2}; \quad l_o^* = \tfrac{y - w q_o \phi}{1 - \phi^2}.$$

Substituting the optimal amount of labor $l_p^*$ and $l_o^*$ into the driver's utility, we can get

$$U = \tfrac{q_o^2 w^2 - 2 q_o y w \phi + y^2}{2 - 2\phi^2}, \tag{H.22}$$

which demonstrates one driver's optimal utility. Given the system contains $N$ amount of drivers, the driver's surplus in this scenario is equivalent to the driver's utility multiplied by the number of drivers in the system, i.e., $DS^i = U^i N$, $i \in \{RN, PN, PW\}$. Substituting optimal wage rate $w^{i*}$ into the driver's surplus, we can get

$$\begin{aligned} DS^{RN*} &= \tfrac{N(-(\beta-1)^2(\phi^2-1) + y^2(\phi^2(16(\beta^2-1)N-7) + 4(1-2(\beta^2-1)N)^2 + 3\phi^4) - 2(\beta-1)y\phi(\phi^2-1))}{8(2(\beta^2-1)N+\phi^2-1)^2}, \\ DS^{PN*} &= \tfrac{N(-(\beta-1)^2(\phi^2-1) + y^2(\phi^2(32(\beta^2-1)N-31) + 16(-\beta^2 N+N+1)^2 + 15\phi^4) - 2(\beta-1)y\phi(\phi^2-1))}{32((\beta^2-1)N+\phi^2-1)^2}, \\ DS^{PW*} &= \tfrac{N(-(\beta-1)^2(\phi^2-1) + y^2(\phi^2(8(\beta^2-1)N-1) + (1-4(\beta^2-1)N)^2) - 2(\beta-1)y\phi(\phi^2-1))}{2(4(\beta^2-1)N+\phi^2-1)^2}. \end{aligned} \tag{H.23}$$

For customers, since we adopt the same demand functions as the main model, the customer's surplus is the same as Equation (E.3). Substituting optimal channel prices and demands $p_o^{i*}$, $p_f^{i*}$, $q_o^{i*}$, $q_f^{i*}$ into Equation (E.3), we can get

$$\begin{aligned} CS^{RN*} &= \tfrac{(\beta^2-1)N^2(3\beta^2+\beta(2-2y\phi)+y\phi(2-y\phi)-5) + 4(\beta^2-1)N(\phi^2-1) + \phi^4 - 2\phi^2 + 1}{8(2(\beta^2-1)N+\phi^2-1)^2}, \\ CS^{PN*} &= \tfrac{(\beta^2-1)N^2(3\beta^2+\beta(2-2y\phi)+y\phi(2-y\phi)-5) + 8(\beta^2-1)N(\phi^2-1) + 4(\phi^2-1)^2}{32((\beta^2-1)N+\phi^2-1)^2}, \\ CS^{PW*} &= \tfrac{4(\beta^2-1)N^2(3\beta^2+\beta(2-2y\phi)+y\phi(2-y\phi)-5) + 8(\beta^2-1)N(\phi^2-1) + \phi^4 - 2\phi^2 + 1}{8(4(\beta^2-1)N+\phi^2-1)^2}. \end{aligned} \tag{H.24}$$

## Appendix I: Analytical Results of the Alternative Demand Model

Following the same analysis in Section 4, we can get the equilibrium solutions in the following Lemmas for each type of contract. The solving process is similar to the main part and the wage rate model, so we omit it here. Similar to the previous two cases, we make the following assumption to ensure the positive online demand.

Assumption I.1. *Consider the demand functions in* (F.1) *and* (F.2), *the model parameters satisfy* $-a(1+\beta) + b \geq 0$.

Lemma I1. *Consider the demand functions in* (F.1) *and* (F.2)*, then under the RN contract, the equilibrium channel prices, driver's wage, demands, and profits are*

$$\begin{aligned}
p_o^{RN*} &= \tfrac{1}{4}\left(\tfrac{2a+1}{2b+\beta+1} + \tfrac{1-\alpha}{2\beta+1} + \alpha + \tfrac{1}{\beta+1} + 1\right),\\
p_f^{RN*} &= \tfrac{\alpha\beta+\alpha+\beta}{4\beta+2},\\
w^{RN*} &= \tfrac{4ab+a\beta+a+b}{4b^2+2\beta b+2b},\\
q_o^{RN*} &= \tfrac{b-a(\beta+1)}{2(2b+\beta+1)},\\
q_f^{RN*} &= \tfrac{1}{2}\left(\tfrac{\beta(a\beta+a+b+\beta+1)}{(\beta+1)(2b+\beta+1)} + \alpha\right),\\
\pi_p^{RN*} &= \tfrac{(a\beta+a-b)^2}{4b(\beta+1)(2b+\beta+1)},\\
\pi_r^{RN*} &= \tfrac{1}{16}\left(\tfrac{4a^2-1}{2b+\beta+1} - \tfrac{2(2a+1)^2 b}{(2b+\beta+1)^2} + \tfrac{2(\alpha-1)^2}{2\beta+1} + 2(\alpha+1)^2 - \tfrac{3}{\beta+1}\right),\\
\pi_{sc}^{RN*} &= \tfrac{1}{16}\left(\tfrac{4a^2(\beta+1)(3b+\beta+1)-8ab(3b+\beta+1)-b(8b+3\beta+3)}{b(2b+\beta+1)^2} + \tfrac{4(\alpha\beta+\alpha+\beta)^2+6\beta+3}{(\beta+1)(2\beta+1)}\right).
\end{aligned} \tag{I.1}$$

Lemma I2. *Consider the demand functions in* (F.1) *and* (F.2)*, then under the PN contract, the equilibrium channel prices, driver's wage, demands, and profits are*

$$\begin{aligned}
p_o^{PN*} &= \tfrac{1}{4}\left(\tfrac{a+1}{b+\beta+1} + \tfrac{1-\alpha}{2\beta+1} + \alpha + \tfrac{1}{\beta+1} + 1\right),\\
p_f^{PN*} &= \tfrac{\alpha\beta+\alpha+\beta}{4\beta+2},\\
w^{PN*} &= \tfrac{1}{4}\left(\tfrac{a+1}{b+\beta+1} + \tfrac{3a}{b}\right),\\
q_o^{PN*} &= \tfrac{b-a(\beta+1)}{4(b+\beta+1)},\\
q_f^{PN*} &= \tfrac{1}{4}\left(\tfrac{\beta((a+2)\beta+a+b+2)}{(\beta+1)(b+\beta+1)} + 2\alpha\right),\\
\pi_p^{PN*} &= \tfrac{(a\beta+a-b)^2}{16b(\beta+1)(b+\beta+1)},\\
\pi_r^{PN*} &= \tfrac{1}{8}\left(\tfrac{a^2}{b} - \tfrac{(a+1)^2}{b+\beta+1} + \tfrac{(\alpha-1)^2}{2\beta+1} + (\alpha+1)^2 - \tfrac{1}{\beta+1}\right),\\
\pi_{sc}^{PN*} &= \tfrac{1}{16}\left(\tfrac{3a^2}{b} - \tfrac{3(a+1)^2}{b+\beta+1} + \tfrac{2(\alpha-1)^2}{2\beta+1} + 2(\alpha+1)^2 - \tfrac{1}{\beta+1}\right).
\end{aligned} \tag{I.2}$$

Lemma I3. *Consider the demand functions in* (F.1) *and* (F.2)*, then under the PW contract, the equilibrium channel prices, driver's wage, demands, and profits are*

$$\begin{aligned}
p_o^{PW*} &= \tfrac{1}{4}\left(\tfrac{4a+1}{4b+\beta+1} + \tfrac{1-\alpha}{2\beta+1} + \alpha + \tfrac{1}{\beta+1} + 1\right),\\
p_f^{PW*} &= \tfrac{\alpha\beta+\alpha+\beta}{4\beta+2},\\
w^{PW*} &= \tfrac{4a+1}{4b+\beta+1},\\
q_o^{PW*} &= \tfrac{b-a(\beta+1)}{4b+\beta+1},\\
q_f^{PW*} &= \tfrac{1}{2}\left(\tfrac{\beta(2a(\beta+1)+2b+\beta+1)}{(\beta+1)(4b+\beta+1)} + \alpha\right),\\
\pi_p^{PW*} &= \tfrac{(a\beta+a-b)^2}{(\beta+1)(4b+\beta+1)^2},\\
\pi_r^{PW*} &= \tfrac{1}{8}\left(\tfrac{16a^2-1}{4b+\beta+1} - \tfrac{4(4a+1)^2 b}{(4b+\beta+1)^2} + \tfrac{(\alpha-1)^2}{2\beta+1} + (\alpha+1)^2 - \tfrac{1}{\beta+1}\right),\\
\pi_{sc}^{PW*} &= \tfrac{1}{16}\left(\tfrac{3(4a+1)(4a(\beta+1)-8b-\beta-1)}{(4b+\beta+1)^2} + \tfrac{4\alpha((\alpha+2)\beta+\alpha)}{2\beta+1} - \tfrac{1}{\beta+1} + \tfrac{2}{2\beta+1} + 2\right).
\end{aligned} \tag{I.3}$$

## Appendix J: Proofs of Results in the Appendix and the Supplement

### J.1. Proof of Lemma C1

The total amount of supply is defined as $s = -a + bw$ in Equation (3). Equivalently, this can be expressed as $w = \frac{a+s}{b}$. To simplify our analysis, we consider the scenario where the centralized decision maker determines the supply $s$. We first show $s = q_o$ in centralized decision maker's optimal choice because neither $s \geq q_o$ nor $s \leq q_o$ can be optimal. If $s \geq q_o$, the decision maker can slightly decrease $s$ (which slightly reduces the wage cost) such that $min(s, q_o)$remains unaltered, which increases its profit by Equation (C.1). If $s \leq q_o$, the decision maker can slightly increase $p_o$ (which slightly reduces the demand) such that $min(s, q_o)$ remains unaltered, which increases its profit by Equation (C.1).

Next, we derive the equilibrium decisions. Because $s = q_o$ under the optimal choice, the optimization problem in Equation (C.1) can be converted to

$$\begin{aligned} \pi_{sc}^{C} &= \max_{s,p_o,p_f} s(p_o - \tfrac{a+s}{b}) + p_f q_f, \\ s.t. \quad & s = q_o. \end{aligned}$$

By applying KKT(Karush-Kuhn-Tucker) conditions, we can rewrite this problem as follows.

$$\mathcal{L}(p_o, p_f, s, \lambda) = s(p_o - \tfrac{a+s}{b}) + p_f q_f + \lambda(s - q_o),$$

By the first-order conditions, the optimal choice is given by the following system of equations:

$$\begin{aligned} \tfrac{\partial \mathcal{L}}{\partial p_o} &= \tfrac{a+b(-\beta+\lambda-2p_o+2\beta p_f+1)+s}{b-b\beta^2} = 0, \\ \tfrac{\partial \mathcal{L}}{\partial p_f} &= \tfrac{\beta(a+s)+b(\beta\lambda+\beta-2\beta p_o+2p_f-1)}{b(\beta^2-1)} = 0, \\ \tfrac{\partial \mathcal{L}}{\partial s} &= 0, \\ \tfrac{\partial \mathcal{L}}{\partial \lambda} &= \tfrac{-p_o+\beta(p_f-1)+\left(\beta^2-1\right)s+1}{\beta^2-1} = 0. \end{aligned}$$

Consequently, we have the following equilibrium prices, quantities and profits:

$$\begin{aligned} p_o^{C*} &= \tfrac{(1-\beta^2)(a+b)+2-\beta}{2b(1-\beta^2)+2}; \quad p_f^{C*} = \tfrac{1}{2}; \\ w^{C*} &= \tfrac{a+b(1-\beta)+2ab(1-\beta^2)}{2b(1+b(1-\beta^2))}; \\ q_o^{C*} &= s^{C*} = \tfrac{b(1-\beta)-a}{2b(1-\beta^2)+2}; \quad q_f^{C*} = \tfrac{a\beta+b(1-\beta)+1}{2b(1-\beta^2)+2}; \\ \pi_{sc}^{C*} &= \tfrac{a^2-2ab(1-\beta)+2b^2(1-\beta)+b}{4b(b(1-\beta^2)+1)}. \end{aligned} \tag{J.1}$$

□

### J.2. Proof of Lemma H1

**Stage 3: platform determines the wage rate $w$, or equivalently, the supply $s(w)$.** The total amount of supply is defined as $s = \frac{wq_o-\phi y}{1-\phi^2}N$ in Equation (32). Equivalently, we can get $w = \frac{Ny\phi+s(1-\phi^2)}{q_oN}$. Next, we consider the scenario where the platform's decision is the supply $s$ for tractability. Similar to the RN contract in the base model, the platform chooses a $s$ such that in equilibrium, $s \le q_o$. Hence, the platform's maximization problem can be written as

$$\begin{aligned} \pi_p^{RN} &= \max_s s(r - \tfrac{Ny\phi+s(1-\phi^2)}{N}), \\ s.t. \quad & s \le q_o = \tfrac{1}{1+\beta} - \tfrac{1}{1-\beta^2}p_o + \tfrac{\beta}{1-\beta^2}p_f. \end{aligned} \tag{J.2}$$

It is obvious that the platform's profit is concave in $s$. By applying KKT conditions, the platform's optimal supply can be listed as follows:

$$s^* = \begin{cases} \tfrac{N(r-y\phi)}{2(1-\phi^2)} & if \quad p_o \le \bar{p}_o, \quad \text{(J.3a)} \\ q_o = \tfrac{1-\beta+\beta p_f-p_o}{1-\beta^2} & if \quad p_o \ge \bar{p}_o. \quad \text{(J.3b)} \end{cases}$$

Note that $\bar{p}_o = \frac{\left(\beta^2-1\right)N(r-W\phi)+2\left(1-\phi^2\right)(\beta(p_f-1)+1)}{2(1-\phi^2)}$.

**Stage 2: restaurant determines online channel price $p_o$ and the offline channel price $p_f$.**

(i) Anticipating the platform's optimal supply $s^* = \frac{N(r-y\phi)}{2(1-\phi^2)}$, then the restaurant's optimization problem becomes

$$\begin{aligned} \pi_r^{RN} &= \max_{p_o,p_f} sp_o + q_f p_f = \tfrac{N(r-y\phi)}{2(1-\phi^2)}p_o + q_f p_f, \\ s.t. \quad & p_o \le \bar{p}_o. \end{aligned} \tag{J.4}$$

Taking the derivative of the restaurant's profit with respect to $p_o$, we obtain

$$\frac{\partial \pi_r^{RN}}{\partial p_o} = \frac{N(r-y\phi)}{2-2\phi^2} + \frac{\beta p_f}{1-\beta^2} > 0$$

as long as $s > 0$ ($r > y\phi$). Thus, given any $p_f$, the restaurant increases $p_o$ until $p_o = \bar{p}_o$. Substituting $p_o = \bar{p}_o$ into restaurant's profit, we have $\frac{\partial^2 \pi_r^{RN}}{\partial p_f^2} = -2 < 0$. Hence, the restaurant's optimal $p_f$ satisfying $\frac{\partial \pi_r^{RN}}{\partial p_f} = 0$, which is $p_f^* = \frac{1}{2}$. Therefore, the restaurant's optimal prices are

$$p_o^* = \bar{p}_o(p_f^*), \quad p_f^* = \tfrac{1}{2},$$

respectively, and the restaurant's optimal profit is

$$\pi_r^{RN*} = \tfrac{1}{4}\left(\tfrac{(\beta^2-1)N^2(r-y\phi)^2}{(\phi^2-1)^2} + \tfrac{2N(-\beta-r+1)(r-y\phi)}{1-\phi^2} + 1\right).$$

(ii) Anticipating the platform's optimal supply $s^* = \frac{1-\beta+\beta p_f - p_o}{1-\beta^2}$, then the restaurant's optimization problem becomes

$$\begin{aligned} \pi_r^{RN} &= \max_{p_o, p_f} s(p_o - r) + q_f p_f = \tfrac{1-\beta+\beta p_f - p_o}{1-\beta^2}(p_o - r) + q_f p_f, \\ s.t. \quad & p_o \geq \bar{p}_o. \end{aligned} \tag{J.5}$$

The restaurant's profit is jointly concave in $p_o$ and $p_f$ as the Hessian matrix is negative definite. Specifically, the Hessian matrix is given by

$$H = \begin{bmatrix} \frac{\partial^2 \pi_r^{RN}}{\partial p_o^2} & \frac{\partial^2 \pi_r^{RN}}{\partial p_o \partial p_f} \\ \frac{\partial^2 \pi_r^{RN}}{\partial p_f \partial p_o} & \frac{\partial^2 \pi_r^{RN}}{\partial p_f^2} \end{bmatrix} = \begin{bmatrix} \frac{2}{\beta^2-1} & \frac{2\beta}{1-\beta^2} \\ \frac{2\beta}{1-\beta^2} & \frac{2}{\beta^2-1} \end{bmatrix}.$$

By applying KKT conditions, the restaurant's optimal prices can be listed as follows:

$$p_o^*, p_f^* = \begin{cases} \frac{1+r}{2}, \quad \frac{1}{2} & if \quad r \geq \bar{r}, \\ \frac{1}{2}\left(2 - \beta - \frac{(\beta^2-1)N(r-y\phi)}{\phi^2-1}\right), \quad \frac{1}{2} & if \quad r \leq \bar{r}, \end{cases}$$

where $\bar{r} = \frac{(\beta-1)((\beta+1)Ny\phi - \phi^2+1)}{(\beta^2-1)N+\phi^2-1}$. Substituting optimal prices into the restaurant's profit, we can get the corresponding profit of the restaurant is $\pi_r^{RN*} = \frac{-2\beta+r(2\beta+r-2)+2}{4-4\beta^2}$ if $r \geq \bar{r}$; otherwise, $\pi_r^{RN*} = \frac{1}{4}\left(\frac{(\beta^2-1)N^2(r-y\phi)^2}{(\phi^2-1)^2} + \frac{2N(-\beta-r+1)(r-y\phi)}{1-\phi^2} + 1\right)$.

Combining Case (i) and Case (ii), we can show that the restaurant's profit in Case (i) is dominated by that in Case (ii) since

$$\pi_r^{RN*}|_{p_o \geq \bar{p}_o} - \pi_r^{RN*}|_{p_o \leq \bar{p}_o} = \begin{cases} -\frac{(\beta-\beta^2 Nr + Nr + (\beta^2-1)Ny\phi - \phi^2(\beta+r-1)+r-1)^2}{4(\beta^2-1)(\phi^2-1)^2} > 0 & if \quad r \geq \bar{r}, \\ 0 & if \quad r \leq \bar{r}. \end{cases}$$

Thus the optimal prices for the restaurant in this stage are:

$$p_o^*, p_f^* = \begin{cases} \frac{1+r}{2}, \quad \frac{1}{2} & if \quad r \geq \bar{r}, \\ \frac{1}{2}\left(2 - \beta - \frac{(\beta^2-1)N(r-y\phi)}{\phi^2-1}\right), \quad \frac{1}{2} & if \quad r \leq \bar{r}, \end{cases}$$

where $\bar{r} = \frac{(\beta-1)((\beta+1)Ny\phi - \phi^2+1)}{(\beta^2-1)N+\phi^2-1}$.

**Stage 1: platform determines the commission fee $r$.** We consider the following two cases.

(i) Anticipating the restaurant's optimal channel prices $p_o^* = \frac{1}{2}\left(2 - \beta - \frac{(\beta^2-1)N(r-y\phi)}{\phi^2-1}\right)$, $\quad p_f^* = \frac{1}{2}$, the platform's optimization problem becomes

$$\begin{aligned} \pi_p^{RN} &= \max_r s(r - wq_o) = \tfrac{N(r-y\phi)^2}{4(1-\phi^2)}, \\ s.t. \quad & r \leq \bar{r}. \end{aligned} \tag{J.6}$$

The platform's profit is convex in $r$ since $\frac{\partial^2 \pi_p^{RN}}{\partial r^2} = \frac{N}{2(1-\phi^2)} > 0$, and the platform's profit gets the minimum value when $r = y\phi$. $\bar{r} > y\phi$ since $\bar{r} - y\phi = \frac{(1-\phi^2)(1-\beta-y\phi)}{(1-\beta^2)N+1-\phi^2} > 0$. Additionally, the online demand in this stage becomes $s = \frac{N(r-y\phi)}{2-2\phi^2}$. Hence, the commission fee $r$ needs to satisfy $r \geq y\phi$ to ensure $s \geq 0$. Therefore, the platform's profit increases in $r$ when $y\phi \le r \le \bar{r}$, and its optimal commission fee is $r^* = \bar{r}$. Substituting $r^*$ into the platform's profit, we have $\pi_p^{RN*} = \frac{N(1-\phi^2)(1-\beta-y\phi)^2}{4((1-\beta^2)N+1-\phi^2)^2}$.

(ii) Anticipating the restaurant's prices $p_o^* = \frac{1+r}{2}$, $p_f^* = \frac{1}{2}$, then the platform's optimization problem becomes

$$\begin{aligned} &\pi_p^{RN} = \max_r s(r - wq_o) = \tfrac{(1-\beta-r)(r(2(1-\beta^2)N-\phi^2+1)+(\beta-1)(2(\beta+1)Ny\phi-\phi^2+1))}{4(1-\beta^2)^2N}, \\ &s.t. \quad r \ge \bar{r}. \end{aligned} \tag{J.7}$$

The platform's profit is concave in $r$ since $\frac{\partial^2 \pi_p^{RN}}{\partial r^2} = -\frac{2(1-\beta^2)N+(1-\phi^2)}{2(\beta^2-1)^2N} < 0$. Hence, there exists a

$$r_m = \tfrac{(1-\beta)((\beta+1)N(-\beta+y\phi+1)-\phi^2+1)}{2(1-\beta^2)N-\phi^2+1}$$

satisfying $\frac{\partial \pi_p^{RN}}{\partial r} = 0$ such that the platform's profit is maximized. $r_m > \bar{r}$ because $r_m - \bar{r} = \frac{(\beta^2-1)^2N^2(-\beta-y\phi+1)}{((1-\beta^2)N-\phi^2+1)(2(1-\beta^2)N-\phi^2+1)} > 0$. Hence, in this case, the optimal $r^* = r_m$. Substituting $r^*$ into the platform's profit, we have $\pi_p^{RN*} = \frac{N(\beta+y\phi-1)^2}{4(2(1-\beta^2)N-\phi^2+1)}$.

Combining Case (i) and Case (ii), the platform's profit in Case (i) is dominated by that in Case (ii) since

$$\pi_p^{RN*}|_{r\ge\bar{r}} - \pi_p^{RN*}|_{r\le\bar{r}} = \tfrac{(\beta^2-1)^2N^3(\beta+y\phi-1)^2}{4((\beta^2-1)N+\phi^2-1)^2(2(1-\beta^2)N+1-\phi^2)} > 0.$$

Therefore, the equilibrium commission fee is $r^* = r_m$. Substituting $r^*$ into the optimal decisions in later stages, we can get the equilibrium results in Lemma H1. Furthermore, we have the following equilibrium results:

$$\begin{aligned} &q_o^{RN*} = s^{RN*} = \tfrac{N(-\beta-y\phi+1)}{2(2(1-\beta^2)N-\phi^2+1)}, \\ &q_f^{RN*} = \tfrac{N(-\beta^2-\beta+\beta y\phi+2)-\phi^2+1}{2(2(1-\beta^2)N-\phi^2+1)}, \\ &\pi_p^{RN*} = \tfrac{N(\beta+y\phi-1)^2}{4(2(1-\beta^2)N-\phi^2+1)}, \\ &\pi_r^{RN*} = \tfrac{(\beta^2-1)N^2(3\beta^2+\beta(2-2y\phi)+y\phi(2-y\phi)-5)+4(\beta^2-1)N(\phi^2-1)+\phi^4-2\phi^2+1}{4(2(\beta^2-1)N+\phi^2-1)^2}, \\ &\pi_{sc}^{RN*} = \tfrac{(\beta^2-1)N^2(\beta^2+\beta(6-6y\phi)-3y\phi(y\phi-2)-7)+N(\phi^2-1)(3\beta^2+\beta(2-2y\phi)+y\phi(2-y\phi)-5)+(\phi^2-1)^2}{4(2(\beta^2-1)N+\phi^2-1)^2}. \end{aligned} \tag{J.8}$$

□

### J.3. Proof of Lemma H2

**Stage 3: platform determines the wage rate $w$ and the online channel price $p_o$.** Similarly to the proof of Lemma 3, we can show that in equilibrium, $q_o = s$. Given $q_o = s$, the platform's optimization problem becomes

$$\begin{aligned} &\pi_p^{PN} = \max_{w,p_o} s(p_o - m_r - wq_o), \\ &s.t. \quad s = q_o = \tfrac{1}{1+\beta} - \tfrac{1}{1-\beta^2}p_o + \tfrac{\beta}{1-\beta^2}p_f. \end{aligned} \tag{J.9}$$

By applying the Lagrange multiplier method, we have

$$\begin{aligned} &\mathcal{L}(w,p_o,\lambda) = s(p_o - m_r - wq_o) + \lambda(q_o - s), \\ &s.t. \quad \tfrac{\partial \mathcal{L}}{\partial w} = 0, \quad \tfrac{\partial \mathcal{L}}{\partial p_o} = 0, \quad \tfrac{\partial \mathcal{L}}{\partial \lambda} = 0. \end{aligned} \tag{J.10}$$

By solving the above problem, the platform's optimal wage and delivery fee are listed as follows:

$$\begin{aligned} &w^* = \tfrac{\beta-\phi^2(\beta+m_r-\beta p_f-1)+m_r+y\phi(2(\beta^2-1)N-1)-\beta p_f+y\phi^3-1}{N(\beta+m_r-\beta p_f+y\phi-1)}, \\ &p_o^* = \tfrac{\left(\beta^2-1\right)N(m_r+\beta(p_f-1)+y\phi+1)+2\left(\phi^2-1\right)(\beta(p_f-1)+1)}{2((\beta^2-1)N+\phi^2-1)}. \end{aligned}$$

**Stage 2: the restaurant sets the online price margin $m_r$ and the offline channel price $p_f$.** Anticipating the optimal $w^*$ and $d^*$, the restaurant maximizes the following profit

$$\pi_r^{PN} = m_r s + q_f p_f = \tfrac{m_r^2N+m_rN(\beta-2\beta p_f+y\phi-1)+Np_f(\beta^2+\beta-(\beta^2-2)p_f-\beta y\phi-2)-2(p_f-1)p_f(\phi^2-1)}{2((\beta^2-1)N+\phi^2-1)}$$

by setting $m_r$ and $p_f$. We find that the restaurant's profit is jointly concave in $m_r$ and $p_f$ as the Hessian matrix is negative definite, with the Hessian matrix given by

$$H=\begin{bmatrix}\frac{\partial^2\pi_r^{PN}}{\partial m_r^2} & \frac{\partial^2\pi_r^{PN}}{\partial m_r\partial p_f}\\ \frac{\partial^2\pi_r^{PN}}{\partial p_f\partial m_r} & \frac{\partial^2\pi_r^{PN}}{\partial p_f^2}\end{bmatrix}=\begin{bmatrix}\frac{N}{(\beta^2-1)N+\phi^2-1} & -\frac{\beta N}{(\beta^2-1)N+\phi^2-1}\\ -\frac{\beta N}{(\beta^2-1)N+\phi^2-1} & \frac{2(2-\beta^2)N+4(1-\phi^2)}{2((\beta^2-1)N+\phi^2-1)}\end{bmatrix}.$$

Hence, the optimal online margin $m_r$ and the offline channel price $p_f$ satisfy $\frac{\partial\pi_r^{PN}}{\partial m_r}=0$, $\frac{\partial\pi_r^{PN}}{\partial p_f}=0$, simultaneously, which is characterized in the following two equations:

$$m_r^*=\tfrac{1-y\phi}{2};\quad p_f^*=\tfrac{1}{2}.$$

**Stage 1: the platform sets the commission fee $r$.** Anticipating the restaurant's optimal $m_r^*$ and $p_f^*$, the platform's profit becomes independent of $r$. Hence, substituting optimal $m^*$ and $p_f^*$ back into $d^*$ and $w^*$, we can get the equilibrium results in Lemma H2. Furthermore, we have the following equilibrium results:

$$\begin{aligned}
d^{PN*}&=-\tfrac{(\beta^2-1)N(\beta-3y\phi-1)+2(\phi^2-1)(\beta-y\phi-1)}{4((\beta^2-1)N+\phi^2-1)}-r,\\
q_o^{PN*}&=\tfrac{N(\beta+y\phi-1)}{4((\beta^2-1)N+\phi^2-1)},\\
q_f^{PN*}&=\tfrac{N(\beta^2+\beta-\beta y\phi-2)+2\phi^2-2}{4((\beta^2-1)N+\phi^2-1)},\\
\pi_p^{PN*}&=-\tfrac{N(\beta+y\phi-1)^2}{16((\beta^2-1)N+\phi^2-1)},\\
\pi_r^{PN*}&=\tfrac{N(\beta^2+\beta(2-2y\phi)+y\phi(2-y\phi)-3)+2\phi^2-2}{8((\beta^2-1)N+\phi^2-1)},\\
\pi_{sc}^{PN*}&=\tfrac{(\beta-1)(\beta+7)N+\phi^2(4-3Ny^2)-6(\beta-1)Ny\phi-4}{16((\beta^2-1)N+\phi^2-1)}.
\end{aligned}\tag{J.11}$$

□

### J.4. Proof of Lemma H3

**Stage 3: platform determines the online channel price $p_o$.** The total amount of supply is defined as $s=\frac{wq_o-\phi y}{1-\phi^2}N$ in Equation (32). Equivalently, we can get $w=\frac{Ny\phi+s(1-\phi^2)}{q_oN}$. To simplify our analysis, we consider the scenario where the platform's decision is the supply $s$ in the first stage. Given $s$, the platform has no incentive to choose delivery fee $d$ such that $s<q_o$ because otherwise, the platform can increase $d$ slightly to increase its profit such that $\min(s,q_o)$ remains unaltered. In other words, in equilibrium, $q_o\le s$. Hence, the platform's maximization problem can be written as

$$\begin{aligned}
\pi_p^{PW}&=\max_{p_o} q_o(p_o-m_r-\tfrac{Ny\phi+s(1-\phi^2)}{N}),\\
s.t.\quad q_o&=\tfrac{1}{1+\beta}-\tfrac{1}{1-\beta^2}p_o+\tfrac{\beta}{1-\beta^2}p_f\le s.
\end{aligned}\tag{J.12}$$

The platform's profit is concave in the delivery fee since $\frac{\partial\pi_p^{PW}}{\partial p_o}=\frac{2}{\beta^2-1}<0$. By applying KKT conditions, the platform's optimal online channel price can be listed as follows:

$$p_o^*=\begin{cases}\beta(p_f-1)+\left(\beta^2-1\right)s+1 & if\quad m_r\le\tilde m_r, \quad\text{(J.13a)}\\ \frac{N(m_r+\beta(p_f-1)+y\phi+1)-s\phi^2+s}{2N} & if\quad m_r\ge\tilde m_r. \quad\text{(J.13b)}\end{cases}$$

Note that $\tilde m_r=\frac{s(2(\beta^2-1)N+\phi^2-1)}{N}+\beta(p_f-1)-y\phi+1$.

**Stage 2: the restaurant sets the online price margin $m_r$ and the offline channel price $p_f$.**

(i) Anticipating the platform's delivery fee $p_o^*=\beta(p_f-1)+(\beta^2-1)s+1$, then the restaurant's optimization problem becomes

$$\begin{aligned}
\pi_r^{PW}&=\max_{m_r,p_f} q_om_r+q_fp_f=m_rs-p_f(p_f+\beta s-1),\\
s.t.\quad & m_r\le\tilde m_r.
\end{aligned}\tag{J.14}$$

Taking the derivative of the restaurant's profit with respect to $m_r$, we obtain $\frac{\partial \pi_r^{PW}}{\partial m_r} = s > 0$. Hence, given any $p_f$, the restaurant increases the margin until $m_r = \tilde{m}_r$. Substituting $m_r = \tilde{m}_r$ into restaurant's profit, we have $\frac{\partial^2 \pi_r^{PW}}{\partial p_f^2} = -2 < 0$. Thus, the restaurant's optimal $p_f$ satisfying $\frac{\partial \pi_r^{PW}}{\partial p_f} = 0$, which is $p_f = \frac{1}{2}$. Hence, the restaurant's optimal prices are:

$$m_r^* = 1 - \frac{\beta}{2} + \frac{s(2(\beta^2-1)N+\phi^2-1)}{N} - y\phi, \quad p_f^* = \frac{1}{2}.$$

In this case, the restaurant's profit is $\pi_r^{PW*} = \frac{s^2(2(\beta^2-1)N+\phi^2-1)}{N} + s(1-\beta-y\phi) + \frac{1}{4}$.

(ii) Anticipating the platform's delivery fee $p_o^* = \frac{N(m_r+\beta(p_f-1)+y\phi+1)-s\phi^2+s}{2N}$, then the restaurant's optimization problem becomes

$$\begin{aligned} \pi_r^{PW} &= \max_{m_r,p_f} q_o m_r + q_f p_f = \frac{m_r^2 N + m_r(N(\beta-2\beta p_f+y\phi-1)-s\phi^2+s)+p_f(N(\beta^2(1-p_t)+\beta+2p_f-\beta y\phi-2)+\beta s(\phi^2-1))}{2(\beta^2-1)N}, \\ s.t. &\quad m_r \geq \tilde{m}_r. \end{aligned}$$

We find that the restaurant's profit is jointly concave in $m_r$ and $p_f$ as the Hessian matrix is negative definite, with the Hessian matrix given by

$$H = \begin{bmatrix} \frac{\partial^2 \pi_r^{PW}}{\partial m_r^2} & \frac{\partial^2 \pi_r^{PW}}{\partial m_r \partial p_f} \\ \frac{\partial^2 \pi_r^{PW}}{\partial p_f \partial m_r} & \frac{\partial^2 \pi_r^{PW}}{\partial p_f^2} \end{bmatrix} = \begin{bmatrix} \frac{1}{\beta^2-1} & \frac{\beta}{1-\beta^2} \\ \frac{\beta}{1-\beta^2} & \frac{2-\beta^2}{\beta^2-1} \end{bmatrix}.$$

By applying KKT conditions, the restaurant's optimal prices can be listed as follows:

$$m_r^*, p_f^* = \begin{cases} \frac{N(1-y\phi)+s(\phi^2-1)}{2N}, \quad \frac{1}{2} & if \quad s \geq \bar{s}, \\ 1 - y\phi - \frac{\beta}{2} + \frac{s(2(\beta^2-1)N+\phi^2-1)}{N}, \quad \frac{1}{2} & if \quad s \leq \bar{s}, \end{cases}$$

where $\bar{s} = \frac{N(1-\beta-y\phi)}{4(1-\beta^2)N+1-\phi^2}$.

Substituting optimal prices into the restaurant's profit, we can get the corresponding profit of the restaurant is $\pi_r^{PW*} = \frac{N^2(\beta^2+\beta(2-2y\phi)+y\phi(2-y\phi)-3)+2Ns(\phi^2-1)(\beta+y\phi-1)-s^2(\phi^2-1)^2}{8(\beta^2-1)N^2}$ if $s \geq \bar{s}$; otherwise, $\pi_r^{PW*} = \frac{s^2(2(\beta^2-1)N+\phi^2-1)}{N} + s(1-\beta-y\phi) + \frac{1}{4}$.

Combining Case (i) and Case (ii), we can show that the restaurant's profit in Case (i) is dominated by that in Case (ii) since

$$\pi_r^{PW*}|_{m_r \geq \tilde{m}_r} - \pi_r^{PW*}|_{m_r \leq \tilde{m}_r} = \begin{cases} -\frac{(N(\beta-4(\beta^2-1)s+y\phi-1)-s\phi^2+s)^2}{8(\beta^2-1)N^2} > 0 & if \quad s \geq \bar{s}, \\ 0 & if \quad s \leq \bar{s}. \end{cases}$$

Hence, the restaurant's optimal prices at this stage are:

$$m_r^*, p_f^* = \begin{cases} \frac{N(1-y\phi)+s(\phi^2-1)}{2N}, \quad \frac{1}{2} & if \quad s \geq \bar{s}, \quad \text{(J.16a)} \\ 1 - y\phi - \frac{\beta}{2} + \frac{s(2(\beta^2-1)N+\phi^2-1)}{N}, \quad \frac{1}{2} & if \quad s \leq \bar{s}, \quad \text{(J.16b)} \end{cases}$$

where $\bar{s} = \frac{N(1-\beta-y\phi)}{4(1-\beta^2)N+1-\phi^2}$.

**Stage 1: platform determines the supply** $s$ We consider the following two cases.

(i) Anticipating the restaurant's optimal prices $m_r^* = \frac{N(1-y\phi)+s(\phi^2-1)}{2N}$, $p_f^* = \frac{1}{2}$, the platform's optimization problem becomes

$$\begin{aligned} \pi_p^{PW} &= \max_s q_o(d + r - \frac{Ny\phi - s\phi^2 + s}{N}) = \frac{(N(\beta+y\phi-1)-s\phi^2+s)^2}{16(1-\beta^2)N^2}, \\ s.t. &\quad s \geq \bar{s}. \end{aligned} \tag{J.17}$$

The platform's profit is convex in $s$ since $\frac{\partial^2 \pi_p^{PW}}{\partial s^2} = \frac{(\phi^2-1)^2}{8(1-\beta^2)N^2} > 0$, and the platform's profit is minimized when $s = \frac{N(\beta+y\phi-1)}{\phi^2-1}$. We can show that $\frac{N(\beta+y\phi-1)}{\phi^2-1} - \bar{s} = \frac{4(\beta^2-1)N^2(\beta+y\phi-1)}{(\phi^2-1)(4(\beta^2-1)N+\phi^2-1)} > 0$. Additionally, the online demand in this stage becomes $q_o = \frac{N(\beta+y\phi-1)-s\phi^2+s}{4(\beta^2-1)N}$, and it decreases with $s$ since $\frac{\partial q_o}{\partial s} = -\frac{\phi^2-1}{4(\beta^2-1)N} < 0$, and $q_o = 0$ when $s = \frac{N(\beta+y\phi-1)}{\phi^2-1}$. Hence, $s$ needs to satisfy $s \leq \frac{N(\beta+y\phi-1)}{\phi^2-1}$ to ensure the positive demand. Therefore, the platform's profit decreases in $s$ when $\bar{s} \leq s \leq \frac{N(\beta+y\phi-1)}{\phi^2-1}$, and its optimal supply is $s^* = \bar{s}$.

(ii) Anticipating the restaurant's prices $m_r^* = 1 - y\phi - \frac{\beta}{2} + \frac{s(2(\beta^2-1)N+\phi^2-1)}{N}$, $p_f^* = \frac{1}{2}$, then the platform's optimization problem becomes

$$\begin{aligned} \pi_p^{PW} &= \max_s q_o(d + r - \tfrac{Ny\phi - s\phi^2 + s}{N}) = s^2(1-\beta^2), \\ s.t. \quad & s \leq \bar{s}. \end{aligned} \tag{J.18}$$

The platform's profit is convex increasing in $s$, so the platform's optimal supply is $s^* = \bar{s}$.

Combining these two cases, we can get the platform's optimal $s^* = \bar{s}$. Then, substituting $s^*$ into the $m_r^*$, $p_f^*$, and $p_o^*$, we can get the equilibrium results in Lemma H3. Furthermore, we have the following equilibrium results:

$$\begin{aligned} m_r^{PW*} &= \frac{\beta(\phi^2-1)-4(\beta^2-1)N(y\phi-1)}{2(4(\beta^2-1)N+\phi^2-1)}, \\ d^{PW*} &= \frac{(\beta-1)((\beta+1)N(\beta-3y\phi-1)+\phi^2-1)}{4(\beta^2-1)N+\phi^2-1} - r, \\ q_o^{PW*} &= \frac{N(\beta+y\phi-1)}{4(\beta^2-1)N+\phi^2-1}, \\ q_f^{PW*} &= \frac{1}{2} - \frac{\beta N(\beta+y\phi-1)}{4(\beta^2-1)N+\phi^2-1}, \\ \pi_p^{PW*} &= -\frac{(\beta^2-1)N^2(\beta+y\phi-1)^2}{(4(\beta^2-1)N+\phi^2-1)^2}, \\ \pi_r^{PW*} &= \frac{8(\beta^2-1)N^2(\beta^2+\beta(2-2y\phi)+y\phi(2-y\phi)-3)+8(\beta^2-1)N(\phi^2-1)+\phi^4-2\phi^2+1}{4(4(\beta^2-1)N+\phi^2-1)^2}, \\ \pi_{sc}^{PW*} &= \frac{4(\beta^2-1)N^2(\beta^2+\beta(6-6y\phi)-3y\phi(y\phi-2)-7)+8(\beta^2-1)N(\phi^2-1)+(\phi^2-1)^2}{4(4(\beta^2-1)N+\phi^2-1)^2}. \end{aligned} \tag{J.19}$$

□

### J.5. Proof of Proposition 6

(i) Based on Equations (H.5), (H.11), and (H.17), we can derive

$$p_o^{PN*} - p_o^{RN*} = \frac{(1-\beta^2)N(\phi^2-1)(\beta+y\phi-1)}{4((\beta^2-1)N+\phi^2-1)(2(\beta^2-1)N+\phi^2-1)} > 0; \quad p_o^{RN*} - p_o^{PW*} = \frac{(1-\beta^2)N(\phi^2-1)(\beta+y\phi-1)}{2(2(\beta^2-1)N+\phi^2-1)(4(\beta^2-1)N+\phi^2-1)} > 0.$$

(ii) Based on Equations (H.7), (H.13), (H.19), (J.8), (J.11), and (J.19), we can derive

$$w^{PN*} - w^{RN*} = \frac{2y\phi(\phi^2-1)}{N(\beta+y\phi-1)} > 0; \quad w^{RN*} - w^{PW*} = \frac{y\phi(\phi^2-1)}{N(\beta+y\phi-1)} > 0;$$

$$q_o^{PW*}w^{PW*} - q_o^{RN*}w^{RN*} = -\frac{(\phi^2-1)^2(\beta+y\phi-1)}{2(2(\beta^2-1)N+\phi^2-1)(4(\beta^2-1)N+\phi^2-1)} > 0;$$

$$q_o^{RN*}w^{RN*} - q_o^{PN*}w^{PN*} = -\frac{(\phi^2-1)^2(\beta+y\phi-1)}{4((\beta^2-1)N+\phi^2-1)(2(\beta^2-1)N+\phi^2-1)} > 0.$$

(iii) Based on Equations (J.8), (J.11), and (J.19), we can derive

$$q_o^{PW*} - q_o^{RN*} = \frac{N(\phi^2-1)(\beta+y\phi-1)}{2(2(\beta^2-1)N+\phi^2-1)(4(\beta^2-1)N+\phi^2-1)} > 0; \quad q_o^{RN*} - q_o^{PN*} = \frac{N(\phi^2-1)(\beta+y\phi-1)}{4((\beta^2-1)N+\phi^2-1)(2(\beta^2-1)N+\phi^2-1)} > 0.$$

□

### J.6. Proof of Proposition 7

Based on Equations (J.8), (J.11), and (J.19), we can derive

(i) Platform's profit:

$$\pi_p^{PW*} - \pi_p^{PN*} = -\frac{N(\phi^2-1)(8(\beta^2-1)N-\phi^2+1)(\beta+y\phi-1)^2}{16((\beta^2-1)N+\phi^2-1)(4(\beta^2-1)N+\phi^2-1)^2}.$$

Hence, $\pi_p^{PW*} - \pi_p^{PN*} > 0$ if $N > \frac{1-\phi^2}{8-8\beta^2}$; otherwise, $\pi_p^{PW*} - \pi_p^{PN*} \leq 0$. Additionally, we have

$$\pi_p^{RN*} - \pi_p^{PW*} = -\frac{N(8(\beta^2-1)^2N^2+4(\beta^2-1)N(\phi^2-1)+(\phi^2-1)^2)(\beta+y\phi-1)^2}{4(2(\beta^2-1)N+\phi^2-1)(4(\beta^2-1)N+\phi^2-1)^2} > 0;$$
$$\pi_p^{RN*} - \pi_p^{PN*} = -\frac{N(2(\beta^2-1)N+3(\phi^2-1))(\beta+y\phi-1)^2}{16((\beta^2-1)N+\phi^2-1)(2(\beta^2-1)N+\phi^2-1)} > 0.$$

(ii) Restaurant's profit:

$$\pi_r^{PW*} - \pi_r^{PN*} = -\frac{N(\phi^2-1)(8(\beta^2-1)N-\phi^2+1)(\beta+y\phi-1)^2}{8((\beta^2-1)N+\phi^2-1)(4(\beta^2-1)N+\phi^2-1)^2}.$$

Hence, $\pi_r^{PW*} - \pi_r^{PN*} > 0$ if $N > \frac{1-\phi^2}{8-8\beta^2}$; otherwise, $\pi_r^{PW*} - \pi_r^{PN*} \leq 0$. Additionally, we have

$$\pi_r^{PN*} - \pi_r^{RN*} = \frac{N(2(\beta^2-1)^2N^2+2(\beta^2-1)N(\phi^2-1)+(\phi^2-1)^2)(\beta+y\phi-1)^2}{8((\beta^2-1)N+\phi^2-1)(2(\beta^2-1)N+\phi^2-1)^2} > 0;$$
$$\pi_r^{PW*} - \pi_r^{RN*} = -\frac{(\beta^2-1)N^2(16(\beta^2-1)^2N^2+24(\beta^2-1)N(\phi^2-1)+7(\phi^2-1)^2)(\beta+y\phi-1)^2}{4(2(\beta^2-1)N+\phi^2-1)^2(4(\beta^2-1)N+\phi^2-1)^2} > 0.$$

(iii) Supply chain profit:

$$\pi_{sc}^{PW*} - \pi_{sc}^{RN*} = -\frac{N(\phi^2-1)(8(\beta^2-1)^2N^2+(\beta^2-1)N(\phi^2-1)-(\phi^2-1)^2)(\beta+y\phi-1)^2}{4(2(\beta^2-1)N+\phi^2-1)^2(4(\beta^2-1)N+\phi^2-1)^2}.$$

Hence, $\pi_{sc}^{PW*} - \pi_{sc}^{RN*} > 0$ when $8(\beta^2-1)^2N^2 + (\beta^2-1)N(\phi^2-1) - (\phi^2-1)^2 > 0$, that is, $N > \frac{\sqrt{33}-1}{16(1-\beta^2))}$; otherwise, $\pi_{sc}^{PW*} - \pi_{sc}^{RN*} \leq 0$. Additionally, we have

$$\pi_{sc}^{PW*} - \pi_{sc}^{PN*} = -\frac{3N(\phi^2-1)(8(\beta^2-1)N-\phi^2+1)(\beta+y\phi-1)^2}{16((\beta^2-1)N+\phi^2-1)(4(\beta^2-1)N+\phi^2-1)^2}.$$

Hence, $\pi_{sc}^{PW*} - \pi_{sc}^{PN*} > 0$ if $N > \frac{1-\phi^2}{8-8\beta^2}$; otherwise, $\pi_{sc}^{PW*} - \pi_{sc}^{PN*} \leq 0$. Combining

$$\pi_{sc}^{RN*} - \pi_{sc}^{PN*} = -\frac{N(\phi^2-1)(4(\beta^2-1)N+\phi^2-1)(\beta+y\phi-1)^2}{16((\beta^2-1)N+\phi^2-1)(2(\beta^2-1)N+\phi^2-1)^2} > 0,$$

we have our results in Proposition 7 (iii).

(iv) Customer surplus:

$$CS^{PW*} - CS^{RN*} = -\frac{(\beta^2-1)N^2(\phi^2-1)(8(\beta^2-1)N+3(\phi^2-1))(\beta+y\phi-1)^2}{8(2(\beta^2-1)N+\phi^2-1)^2(4(\beta^2-1)N+\phi^2-1)^2} > 0;$$
$$CS^{RN*} - CS^{PN*} = -\frac{(\beta^2-1)N^2(\phi^2-1)(4(\beta^2-1)N+3(\phi^2-1))(\beta+y\phi-1)^2}{32((\beta^2-1)N+\phi^2-1)^2(2(\beta^2-1)N+\phi^2-1)^2} > 0;$$

(v) Driver surplus:

$$DS^{PW*} - DS^{RN*} = -\frac{N(\phi^2-1)^2(8(\beta^2-1)N+3(\phi^2-1))(\beta+y\phi-1)^2}{8(2(\beta^2-1)N+\phi^2-1)^2(4(\beta^2-1)N+\phi^2-1)^2} > 0;$$
$$DS^{RN*} - DS^{PN*} = -\frac{N(\phi^2-1)^2(4(\beta^2-1)N+3(\phi^2-1))(\beta+y\phi-1)^2}{32((\beta^2-1)N+\phi^2-1)^2(2(\beta^2-1)N+\phi^2-1)^2} > 0.$$

□

### J.7. Proof of Proposition F1

Based on Equations (I.1), (I.2), and (I.3), we can derive

(i) Online channel price:

$$p_o^{PN*} - p_o^{RN*} = \frac{b-a(\beta+1)}{4(b+\beta+1)(2b+\beta+1)} > 0; \quad p_o^{RN*} - p_o^{PW*} = \frac{b-a(\beta+1)}{2(2b+\beta+1)(4b+\beta+1)} > 0.$$

(ii) Driver's wage:

$$w^{PW*} - w^{RN*} = -\frac{(\beta+1)(a\beta+a-b)}{2b(2b+\beta+1)(4b+\beta+1)} > 0; \quad w^{RN*} - w^{PN*} = -\frac{(\beta+1)(a\beta+a-b)}{4b(b+\beta+1)(2b+\beta+1)} > 0;$$

(iii) Online demand:

$$q_o^{PW*} - q_o^{RN*} = -\frac{(\beta+1)(a\beta+a-b)}{2(2b+\beta+1)(4b+\beta+1)} > 0;$$

$$q_o^{RN*} - q_o^{PN*} = -\frac{(\beta+1)(a\beta+a-b)}{4(b+\beta+1)(2b+\beta+1)} > 0.$$

(iv) Platform's profit:

$$\pi_p^{PW*} - \pi_p^{PN*} = \frac{(8b-\beta-1)(b-a(\beta+1))^2}{16b(b+\beta+1)(4b+\beta+1)^2}.$$

Hence, $\pi_p^{PW*} - \pi_p^{PN*} > 0$ if $b > \frac{1+\beta}{8}$; otherwise, $\pi_p^{PW*} - \pi_p^{PN*} \leq 0$. Additionally, we have

$$\pi_p^{RN*} - \pi_p^{PW*} = \frac{(8b^2+4(\beta+1)b+(\beta+1)^2)(a\beta+a-b)^2}{4b(\beta+1)(2b+\beta+1)(4b+\beta+1)^2} > 0;$$
$$\pi_p^{RN*} - \pi_p^{PN*} = \frac{(2b+3\beta+3)(b-a(\beta+1))^2}{16b(\beta+1)(b+\beta+1)(2b+\beta+1)} > 0.$$

(v) Restaurant's profit:

$$\pi_r^{PW*} - \pi_r^{PN*} = \frac{(8b-\beta-1)(a\beta+a-b)^2}{8b(b+\beta+1)(4b+\beta+1)^2}.$$

Hence, $\pi_r^{PW*} - \pi_r^{PN*} > 0$ if $b > \frac{1+\beta}{8}$; otherwise, $\pi_r^{PW*} - \pi_r^{PN*} \leq 0$. Additionally, we have

$$\pi_r^{PN*} - \pi_r^{RN*} = \frac{(2b^2+2(\beta+1)b+(\beta+1)^2)(a\beta+a-b)^2}{8b(\beta+1)(b+\beta+1)(2b+\beta+1)^2} > 0;$$
$$\pi_r^{PW*} - \pi_r^{RN*} = \frac{(16b^2+24(\beta+1)b+7(\beta+1)^2)(a\beta+a-b)^2}{4(\beta+1)(2b+\beta+1)^2(4b+\beta+1)^2} > 0.$$

(vi) Supply chain profit:

$$\pi_{sc}^{PW*} - \pi_{sc}^{RN*} = \frac{(8b^2+(\beta+1)b-(\beta+1)^2)(a\beta+a-b)^2}{4b(2b+\beta+1)^2(4b+\beta+1)^2}.$$

Hence, $\pi_{sc}^{PW*} - \pi_{sc}^{RN*} > 0$ when $8b^2 + (\beta+1)b - (\beta+1)^2 > 0$, that is, $b > \frac{\sqrt{33}-1}{16(1-\beta^2))}$; otherwise, $\pi_{sc}^{PW*} - \pi_{sc}^{RN*} \leq 0$. Additionally, we have

$$\pi_{sc}^{PW*} - \pi_{sc}^{PN*} = \frac{3(8b-\beta-1)(a\beta+a-b)^2}{16b(b+\beta+1)(4b+\beta+1)^2}.$$

Hence, $\pi_{sc}^{PW*} - \pi_{sc}^{PN*} > 0$ if $b > \frac{1+\beta}{8}$; otherwise, $\pi_{sc}^{PW*} - \pi_{sc}^{PN*} \leq 0$. Combining

$$\pi_{sc}^{RN*} - \pi_{sc}^{PN*} = \frac{(4b+\beta+1)(a\beta+a-b)^2}{16b(b+\beta+1)(2b+\beta+1)^2} > 0,$$

we have our results in Proposition F1 (vi). $\square$