Fig. 3: MLP

解题思路：

首先进行正向传播，公式如下：

$$h_1 = x_1 w_{11} + x_2 w_{13} + b_1$$
$$h_2 = x_1 w_{12} + x_2 w_{14} + b_1$$
$$h_{1out} = sigmod(h_1)$$
$$h_{2out} = sigmod(h_2)$$
$$y_1 = h_{1out} w_{21} + h_{2out} w_{23} + b_2$$
$$y_2 = h_{1out} w_{22} + h_{2out} w_{24} + b_2$$

将数据带入公式，计算得到：

$$h1 = 0.3232$$
$$h2 = 0.3232$$
$$h_{1out} = 0.580104$$
$$h_{2out} = 0.580104$$
$$y_1 = 1.21$$
$$y_2 = 1.10$$

接下来，按照MSE的公式计算误差：

$$E = \frac{1}{2}[(y_1 - label1)^2 + (y_2 - label2)^2]$$

代入数据得到：

$$E = \frac{1}{2}[(1.21 - 0.05)^2 + (1.10 - 0.95)] = 0.68405$$

接下来根据结果，进行反向传播，过程如下：

依次求误差E对于各个参数的导数：

首先计算E对于 $w_{21}, w_{23}, w_{22}, w_{24}$ 的导数，利用偏导的链式法则：

$$\frac{\partial E}{\partial w_{21}} = \frac{\partial E}{\partial y_1} \cdot \frac{\partial y_1}{\partial w_{21}} = (y_1 - label_1) * h_{1out}$$

$$\frac{\partial E}{\partial w_{23}} = \frac{\partial E}{\partial y_1} \cdot \frac{\partial y_1}{\partial w_{23}} = (y_1 - label_1) * h_{2out}$$

$$\frac{\partial E}{\partial w_{22}} = \frac{\partial E}{\partial y_1} \cdot \frac{\partial y_1}{\partial w_{22}} = (y_2 - label_1) * h_{1out}$$

$$\frac{\partial E}{\partial w_{24}} = \frac{\partial E}{\partial y_1} \cdot \frac{\partial y_1}{\partial w_{24}} = (y_2 - label_1) * h_{2out}$$

接下来计算E对于b_2的偏导，结果如下：

$$\frac{\partial E}{\partial b_2} = \frac{\partial E}{\partial y_1} \cdot \frac{\partial y_1}{\partial b_2} + \frac{\partial E}{\partial y_2} \cdot \frac{\partial y_2}{\partial b_2} = (y_1 - label_1) + (y_2 - label_2)$$

然后计算E对于$w_{11}, w_{12}, w_{13}, w_{14}$的偏导，利用偏导的链式法则，得到：

$$\frac{\partial E}{\partial w_{11}} = \frac{\partial E}{\partial y_1} \cdot \frac{\partial y_1}{\partial h_{1out}} \cdot \frac{\partial h_{1out}}{\partial h_1} \cdot \frac{\partial h_1}{\partial w_{11}} + \frac{\partial E}{\partial y_2} \cdot \frac{\partial y_2}{\partial h_{1out}} \cdot \frac{\partial h_{1out}}{\partial h_1} \cdot \frac{\partial h_1}{\partial w_{11}}$$
$$= [(y_1 - label_1) * w_{21} + (y_2 - label_2) * w_{22}] * h_{1out}(1 - h_{1out}) * x_1$$

$$\frac{\partial E}{\partial w_{12}} = \frac{\partial E}{\partial y_1} \cdot \frac{\partial y_1}{\partial h_{2out}} \cdot \frac{\partial h_{2out}}{\partial h_2} \cdot \frac{\partial h_2}{\partial w_{12}} + \frac{\partial E}{\partial y_2} \cdot \frac{\partial y_2}{\partial h_{2out}} \cdot \frac{\partial h_{2out}}{\partial h_2} \cdot \frac{\partial h_2}{\partial w_{12}}$$
$$= [(y_1 - label_1) * w_{23} + (y_2 - label_2) * w_{24}] * h_{2out}(1 - h_{2out}) * x_1$$

$$\frac{\partial E}{\partial w_{13}} = \frac{\partial E}{\partial y_1} \cdot \frac{\partial y_1}{\partial h_{1out}} \cdot \frac{\partial h_{1out}}{\partial h_1} \cdot \frac{\partial h_1}{\partial w_{13}} + \frac{\partial E}{\partial y_2} \cdot \frac{\partial y_2}{\partial h_{1out}} \cdot \frac{\partial h_{1out}}{\partial h_1} \cdot \frac{\partial h_1}{\partial w_{13}}$$
$$= [(y_1 - label_1) * w_{21} + (y_2 - label_2) * w_{22}] * h_{1out}(1 - h_{1out}) * x_2$$

$$\frac{\partial E}{\partial w_{14}} = \frac{\partial E}{\partial y_1} \cdot \frac{\partial y_1}{\partial h_{2out}} \cdot \frac{\partial h_{2out}}{\partial h_2} \cdot \frac{\partial h_2}{\partial w_{14}} + \frac{\partial E}{\partial y_2} \cdot \frac{\partial y_2}{\partial h_{2out}} \cdot \frac{\partial h_{2out}}{\partial h_2} \cdot \frac{\partial h_2}{\partial w_{14}}$$
$$= [(y_1 - label_1) * w_{23} + (y_2 - label_2) * w_{24}] * h_{2out}(1 - h_{2out}) * x_2$$

再计算E对于b_1的偏导，利用偏导的链式法则，得到：

$$\frac{\partial E}{\partial b_1} = \frac{\partial E}{\partial y_1} \cdot \frac{\partial y_1}{\partial h_{1out}} \cdot \frac{\partial h_{1out}}{\partial h_1} \cdot \frac{\partial h_1}{\partial b_1} + \frac{\partial E}{\partial y_2} \cdot \frac{\partial y_2}{\partial h_{1out}} \cdot \frac{\partial h_{1out}}{\partial h_1} \cdot \frac{\partial h_1}{\partial b_1}$$
$$+ \frac{\partial E}{\partial y_1} \cdot \frac{\partial y_1}{\partial h_{2out}} \cdot \frac{\partial h_{2out}}{\partial h_2} \cdot \frac{\partial h_2}{\partial b_1} + \frac{\partial E}{\partial y_2} \cdot \frac{\partial y_2}{\partial h_{2out}} \cdot \frac{\partial h_{2out}}{\partial h_2} \cdot \frac{\partial h_2}{\partial b_1}$$
$$= (y_1 - label_1) * w_{21} * h_{1out}(1 - h_{1out}) + (y_2 - label_2) * w_{22} * h_{1out}(1 - h_{1out})$$
$$+ (y_1 - label_1) * w_{23} * h_{2out}(1 - h_{2out}) + (y_2 - label_2) * w_{24} * h_{2out}(1 - h_{2out})$$

将数据带入公式得：

$$\frac{\partial E}{\partial w_{21}} = 0.67292064$$

$$\frac{\partial E}{\partial w_{22}} = 0.0870156$$

$$\frac{\partial E}{\partial w_{23}} = 0.67292064$$

$$\frac{\partial E}{\partial w_{24}} = 0.0870156$$

$$\frac{\partial E}{\partial b_2} = 1.31$$

$$\frac{\partial E}{\partial w_{11}} = 0.0126176174877312$$

$$\frac{\partial E}{\partial w_{12}} = 0.01360169421843456$$

$$\frac{\partial E}{\partial w_{13}} = 0.0189264262315968$$

$$\frac{\partial E}{\partial w_{14}} = 0.02040254132765184$$

$$\frac{\partial E}{\partial b_1} = 0.359650815070176$$

对所有参数，更新公式为：

$$w_{new} = w - \alpha \frac{\partial E}{\partial w}$$

按照更新公式（计算结果保留4位小数）：

$$w'_{11} = 0.24 - 0.5 * 0.0126176174877312 \approx 0.2337$$
$$w'_{12} = 0.3 - 0.5 * 0.01360169421843456 \approx 0.2932$$
$$w'_{13} = 0.2 - 0.5 * 0.0189264262315968 \approx 0.1905$$
$$w'_{14} = 0.16 - 0.5 * 0.02040254132765184 \approx 0.1498$$
$$b'_1 = 0.28 - 0.5 * 0.359650815070176 \approx 0.1002$$
$$w'_{21} = 0.5 - 0.5 * 0.67292064 \approx 0.1635$$
$$w'_{22} = 0.45 - 0.5 * 0.0870156 \approx 0.4065$$
$$w'_{23} = 0.55 - 0.5 * 0.67292064 \approx 0.2135$$
$$w'_{24} = 0.4 - 0.5 * 0.0870156 \approx 0.3565$$
$$b'_2 = 0.6 - 0.5 * 1.31 = -0.055$$