SymFun 3.1: Documentation

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Introduction

This is a *Mathematica* package designed to support various manipulations of symmetric functions, with an emphasis on combinatorial applications. It is loosely modeled after John Stembridge's highly successful Maple package ("SF"), but it is somewhat less ambitious: currently it implements only the standard basis conversions and a few other operations such as the Littlewood-Richardson rule.

We assume familiarity with the basic theory of symmetric functions, e.g. Macdonald' s Symmetric Functions and Hall Polynomials or Stanley' s EC2.

Using the Package

Find the package (symfun31.m), open it, and click the button labeled "Run Package". Alternatively, load the package using the command

```
<< symfun31.m
```

In order for the latter method to work, the package must be in a location recognized by *Mathematica*'s $Path variable, for example,

```
"C:\\Program Files\\Wolfram Research\\Mathematica\\7.0\\AddOns\\ExtraPackages"
```

Overview: Conventions, Notation, and Basic Operations

Basic families of symmetric functions (elementary, homogeneous, monomial, power sum, and Schur) may be defined in several ways. The following are expressions that are "atomic", i.e., they are unevaluated by *Mathematica* and hence treated as indeterminates:

```
e[4],h[5],p[3],m[3,2,1],s[3,2,1]
```

The first three combine to give monomials in the obvious way, e.g.,

```
e[3]e[1]^2, h[4]h[2]h[1],p[3]p[2]p[1]
```

These are the standard forms for the elementary, homogenous, and power-sum symmetric function. Expressions of the form

```
e[{3,1,1}], h[{4,2,1}], p[{3,2,1}]
```

are evaluated by *Mathematica*, and give equivalent e, h, and p-functions in the above monomial form.

Appending “x” to the function name and including an “n” parameter yields an evaluation in n variables x[1], x[2],...,x[n], for example,

```
ex[{3, 1, 1}, 3]
```

$x[1]^3 x[2] x[3] + 2 x[1]^2 x[2]^2 x[3] + x[1] x[2]^3 x[3] + 2 x[1]^2 x[2] x[3]^2 + 2 x[1] x[2]^2 x[3]^2 + x[1] x[2] x[3]^3$

```
px[{3, 1, 1}, 3]
```

$x[1]^5 + 2 x[1]^4 x[2] + x[1]^3 x[2]^2 + x[1]^2 x[2]^3 + 2 x[1] x[2]^4 + x[2]^5 + 2 x[1]^4 x[3] + 2 x[1]^3 x[2] x[3] + 2 x[1] x[2]^3 x[3] + 2 x[2]^4 x[3] + x[1]^3 x[3]^2 + x[2]^3 x[3]^2 + x[1]^2 x[3]^3 + 2 x[1] x[2] x[3]^3 + x[2]^2 x[3]^3 + 2 x[1] x[3]^4 + 2 x[2] x[3]^4 + x[3]^5$

The same result can be obtained using "conversion" commands (described fully later in this document), e.g.

```
EToX[e[{3, 1, 1}], 3]
```

$x[1]^3 x[2] x[3] + 2 x[1]^2 x[2]^2 x[3] + x[1] x[2]^3 x[3] + 2 x[1]^2 x[2] x[3]^2 + 2 x[1] x[2]^2 x[3]^2 + x[1] x[2] x[3]^3$

or

```
EToX[e[3] e[1]^2, 3]
```

$x[1]^3 x[2] x[3] + 2 x[1]^2 x[2]^2 x[3] + x[1] x[2]^3 x[3] + 2 x[1]^2 x[2] x[3]^2 + 2 x[1] x[2]^2 x[3]^2 + x[1] x[2] x[3]^3$

Conversions are available between all pairs of families, e.g.,

```
EToS[e[3] e[1]^2]
```

s[2, 2, 1] + s[3, 1, 1] + 2 s[2, 1, 1, 1] + s[1, 1, 1, 1, 1]

```
EToP[e[3] e[1]^2]
```

$$\frac{p[1]^5}{6} - \frac{1}{2} p[1]^3 p[2] + \frac{1}{3} p[1]^2 p[3]$$

Certain other algebraic operations are computed automatically. For example, products of atomic expressions involving the Schur functions are automatically evaluated using the Littlewood-Richardson rule.

```
s[3, 2, 1] s[2, 1]
```

s[3, 3, 3] + 2 s[4, 3, 2] + s[4, 4, 1] + s[5, 2, 2] + s[5, 3, 1] + s[3, 2, 2, 2] + 2 s[3, 3, 2, 1] + 2 s[4, 2, 2, 1] + 2 s[4, 3, 1, 1] + s[5, 2, 1, 1] + s[3, 2, 2, 1, 1] + s[3, 3, 1, 1, 1] + s[4, 2, 1, 1, 1]

Basic Commands: Entering and Manipulating Symmetric Functions

■ H: definitions

h[<λ>]

Expands in terms of h[1],h[2],h[3],...

```
h[{3, 3, 2}]
```

$h[2] h[3]^2$

hx[<lambda>,<n>]

Returns h[λ] expanded in terms of x[1],x[2],...x[n] .

hx[{2}, 3]

$x[1]^2 + x[1]\,x[2] + x[2]^2 + x[1]\,x[3] + x[2]\,x[3] + x[3]^2$

hx[{3, 2}, 2]

$x[1]^5 + 2\,x[1]^4\,x[2] + 3\,x[1]^3\,x[2]^2 + 3\,x[1]^2\,x[2]^3 + 2\,x[1]\,x[2]^4 + x[2]^5$

he[<n>]

Expands h[n] directly into the e's. This syntax works for the "primitive" h, e, and p functions, i.e. they can be converted directly to the others.

he[5]

$e[1]^5 - 4\,e[1]^3\,e[2] + 3\,e[1]\,e[2]^2 + 3\,e[1]^2\,e[3] - 2\,e[2]\,e[3] - 2\,e[1]\,e[4] + e[5]$

hp[5]

$$\frac{p[1]^5}{120} + \frac{1}{12}\,p[1]^3\,p[2] + \frac{1}{8}\,p[1]\,p[2]^2 + \frac{1}{6}\,p[1]^2\,p[3] + \frac{1}{6}\,p[2]\,p[3] + \frac{1}{4}\,p[1]\,p[4] + \frac{p[5]}{5}$$

HList[<n>]

Returns list of all h[λ]'s of degree n, expressed as monomials in h[1],h[2],... (Useful in some technical situations.)

HList[5]

$\{h[5],\ h[1]\,h[4],\ h[2]\,h[3],\ h[1]^2\,h[3],\ h[1]\,h[2]^2,\ h[1]^3\,h[2],\ h[1]^5\}$

HList[<n>, <k>]

Returns list of all h[λ]'s of degree n and Length(λ)≤k, expressed as monomials in h[1],h[2],...

HList[5, 2]

$\{h[5],\ h[1]\,h[4],\ h[2]\,h[3]\}$

HList2[<n>, <k>]

Returns list of all h[λ]'s of degree n and λ[[1]]≤k, expressed as monomials in h[1],h[2],...

HList2[5, 2]

$\{h[1]\,h[2]^2,\ h[1]^3\,h[2],\ h[1]^5\}$

E: definitions

e[<λ>]

Expands in terms of e[1],e[2],e[3],...

e[{3, 2, 2, 1}]

$e[1]\,e[2]^2\,e[3]$

ex[<λ>, <n>]

Returns e[λ] expanded in terms of x[1],x[2],...,x[n] . Similar construction works for e, p, and h.

ex[{3, 2}, 3]

$x[1]^2\,x[2]^2\,x[3] + x[1]^2\,x[2]\,x[3]^2 + x[1]\,x[2]^2\,x[3]^2$

eh[<n>]

Returns e[n] expressed in terms of h[1],h[2],.... . Similar construction works for e, p, and h.

eh[5]

$h[1]^5 - 4\,h[1]^3\,h[2] + 3\,h[1]\,h[2]^2 + 3\,h[1]^2\,h[3] - 2\,h[2]\,h[3] - 2\,h[1]\,h[4] + h[5]$

ep[5]

$$\frac{p[1]^5}{120} - \frac{1}{12}\,p[1]^3\,p[2] + \frac{1}{8}\,p[1]\,p[2]^2 + \frac{1}{6}\,p[1]^2\,p[3] - \frac{1}{6}\,p[2]\,p[3] - \frac{1}{4}\,p[1]\,p[4] + \frac{p[5]}{5}$$

EList[<n>]

Returns list of all e[lambda]'s of degree n, expressed as monomials in e[1],e[2],... (Useful in some technical situations.)

EList[6]

$\{e[6],\ e[1]\,e[5],\ e[2]\,e[4],\ e[1]^2\,e[4],\ e[3]^2,$
$e[1]\,e[2]\,e[3],\ e[1]^3\,e[3],\ e[2]^3,\ e[1]^2\,e[2]^2,\ e[1]^4\,e[2],\ e[1]^6\}$

EList[<n>, <k>]

Returns list of all e[lambda]'s of degree n and lambda[[1]]≤k, expressed as monomials in e[1],e[2],...

EList[6, 2]

$\{e[1]^6,\ e[1]^4\,e[2],\ e[1]^2\,e[2]^2,\ e[2]^3\}$

■ M: definitions

m[<λ>]

Converts m[{λ_1,λ_2,...}] notation into m[λ_1,λ_2,...]. Useful for mapping m onto a list of partitions.

m[{3, 2, 1}]

m[3, 2, 1]

monomial[<exponentlist>]

Returns a single monomial of type <exponentlist>.

monomial[{2, 4, 3, 1}]

$x[1]^2\,x[2]^4\,x[3]^3\,x[4]$

mx[<λ>, <n>]

Returns m[λ] expanded in terms of x[1],x[2],...,x[n].

mx[{3, 2, 1}, 3]

$x[1]^3\,x[2]^2\,x[3] + x[1]^2\,x[2]^3\,x[3] + x[1]^3\,x[2]\,x[3]^2 +$
$x[1]\,x[2]^3\,x[3]^2 + x[1]^2\,x[2]\,x[3]^3 + x[1]\,x[2]^2\,x[3]^3$

MList[<n>]

Returns list of all m[λ]'s of degree n. (Useful in some technical situations.)

MList[4]

{m[4], m[3, 1], m[2, 2], m[2, 1, 1], m[1, 1, 1, 1]}

MList[<n>, <k>]

Returns list of all m[λ]'s of degree n with Length(λ)≤k

```
MList[4, 3]
```

{m[4], m[3, 1], m[2, 2], m[2, 1, 1]}

P: definitions

```
p[<λ>]
```

Expands in terms of p[1],p[2],p[3],...

```
p[{3, 2, 2, 1}]
```

$p[1]\, p[2]^2\, p[3]$

```
px[<λ>,<n>]
```

Returns p[lambda] expanded in terms of x[1],x[2],...,x[n].

```
px[{3, 2, 2, 1}, 2]
```

$$x[1]^8 + x[1]^7\, x[2] + 2\, x[1]^6\, x[2]^2 + 3\, x[1]^5\, x[2]^3 + 2\, x[1]^4\, x[2]^4 + 3\, x[1]^3\, x[2]^5 + 2\, x[1]^2\, x[2]^6 + x[1]\, x[2]^7 + x[2]^8$$

```
px[{4}, 6]
```

$x[1]^4 + x[2]^4 + x[3]^4 + x[4]^4 + x[5]^4 + x[6]^4$

```
pe[<n>]
```

Returns p[n] expanded in terms of e[1],e[2],... Similar construction works for p, e, and h.

```
pe[4]
```

$e[1]^4 - 4\, e[1]^2\, e[2] + 2\, e[2]^2 + 4\, e[1]\, e[3] - 4\, e[4]$

```
ph[4]
```

$-h[1]^4 + 4\, h[1]^2\, h[2] - 2\, h[2]^2 - 4\, h[1]\, h[3] + 4\, h[4]$

```
PList[<n>]
```

Returns list of all p[λ]'s of degree n,expressed as monomials in p[1],p[2],... (Useful in some technical situations.)

```
PList[5]
```

$\{p[5],\ p[1]\, p[4],\ p[2]\, p[3],\ p[1]^2\, p[3],\ p[1]\, p[2]^2,\ p[1]^3\, p[2],\ p[1]^5\}$

```
PList[<n>, <k>]
```

Returns list of all p[λ]'s of degree n, and Length[λ]≤k, expressed as monomials in p[1],p[2],...

```
PList[5, 2]
```

{p[5], p[1] p[4], p[2] p[3]}

S: definitions

```
s[<λ>]
```

Converts s[{λ_1,λ_2,...}] notation into s[λ_1,λ_2,...]. Useful for mapping m onto a list of partitions.

```
s[{4, 2, 1, 1}]
```

s[4, 2, 1, 1]

```
sx[<λ>,<n>]
```

Returns s[λ] expanded in terms of x[1],x[2],...x[n] .

```
sx[{4, 2, 1}, 3]
```

$x[1]^4 x[2]^2 x[3] + x[1]^3 x[2]^3 x[3] + x[1]^2 x[2]^4 x[3] + x[1]^4 x[2] x[3]^2 + 2 x[1]^3 x[2]^2 x[3]^2 + 2 x[1]^2 x[2]^3 x[3]^2 + x[1] x[2]^4 x[3]^2 + x[1]^3 x[2] x[3]^3 + 2 x[1]^2 x[2]^2 x[3]^3 + x[1] x[2]^3 x[3]^3 + x[1]^2 x[2] x[3]^4 + x[1] x[2]^2 x[3]^4$

sh[<λ>]

Returns s[λ] in terms of a linear combination of h[1], h[2], ... Also works if argument is a sequence, or a single integer.

```
sh[{4, 2, 1}]
sh[4, 2, 1]
sh[4]
```

$h[1] h[2] h[4] - h[3] h[4] - h[1]^2 h[5] + h[1] h[6]$

$h[1] h[2] h[4] - h[3] h[4] - h[1]^2 h[5] + h[1] h[6]$

$h[4]$

se[<λ>]

Returns s[λ] in terms of a linear combination of e[1], e[2], ... Also works if argument is a sequence, or a single integer.

```
se[{4, 2, 1}]
se[4, 2, 1]
se[4]
```

$e[1]^2 e[2] e[3] - e[2]^2 e[3] - e[1] e[3]^2 - e[1]^3 e[4] + e[1] e[2] e[4] + e[3] e[4] + e[1]^2 e[5] - e[1] e[6]$

$e[1]^2 e[2] e[3] - e[2]^2 e[3] - e[1] e[3]^2 - e[1]^3 e[4] + e[1] e[2] e[4] + e[3] e[4] + e[1]^2 e[5] - e[1] e[6]$

$e[1]^4 - 3 e[1]^2 e[2] + e[2]^2 + 2 e[1] e[3] - e[4]$

SList[<n>]

Returns list of all s[λ]'s of degree n. (Useful in some technical situations.)

```
SList[7]
```

{s[7], s[6, 1], s[5, 2], s[5, 1, 1], s[4, 3], s[4, 2, 1],
s[4, 1, 1, 1], s[3, 3, 1], s[3, 2, 2], s[3, 2, 1, 1], s[3, 1, 1, 1, 1],
s[2, 2, 2, 1], s[2, 2, 1, 1, 1], s[2, 1, 1, 1, 1, 1], s[1, 1, 1, 1, 1, 1, 1]}

SList[<n>, <k>]

Returns list of all s[λ]'s of degree n with Length(λ)≤k

```
SList[7, 3]
```

{s[7], s[6, 1], s[5, 2], s[5, 1, 1], s[4, 3], s[4, 2, 1], s[3, 3, 1], s[3, 2, 2]}

Basic Commands: Partitions, Degrees, Coefficients, Etc.

Partitions

ConjugateP[<λ>]

Computes the conjugate of a partition λ.

```
ConjugateP[{4, 2, 1, 1, 1, 1, 1, 1}]
```

```
{8, 2, 1, 1}
```

Partitions[<n>,[Rows→ r,Columns→c]]

Returns a list of partitions of n with ≤ r parts and largest part ≤ c. Either, both, or none of the options is possible. (Extension of *Combinatorica*'s Partition command.)

```
Partitions[7, Rows → 3]
```

```
{{7}, {6, 1}, {5, 2}, {5, 1, 1}, {4, 3}, {4, 2, 1}, {3, 3, 1}, {3, 2, 2}}
```

```
Partitions[7, Columns → 3]
```

```
{{3, 3, 1}, {3, 2, 2}, {3, 2, 1, 1}, {3, 1, 1, 1, 1},
 {2, 2, 2, 1}, {2, 2, 1, 1, 1}, {2, 1, 1, 1, 1, 1}, {1, 1, 1, 1, 1, 1, 1}}
```

```
Partitions[7, Rows → 3, Columns → 3]
```

```
{{3, 3, 1}, {3, 2, 2}}
```

Degrees in a polynomial

MonDegrees[<f>,<variable list>]

Returns a sorted list of all of the total degrees of monomials (in the listed variables) appearing in f.

```
f = Product[1 - x[i]^2, {i, 1, 3}] // Expand
MonDegrees[f, {x[1], x[2], x[3]}]
```

$1-x[1]^2-x[2]^2+x[1]^2x[2]^2-x[3]^2+x[1]^2x[3]^2+x[2]^2x[3]^2-x[1]^2x[2]^2x[3]^2$

```
{6, 4, 2, 0}
```

```
MonDegrees[1 + x + y^2 + x y^3 + z^5, {x, y, z}]
MonDegrees[1 + x + y^2 + x y^3 + z^5, {y, z}]
MonDegrees[1 + x + y^2 + x y^3 + z^5, {z}]
```

```
{5, 4, 2, 1, 0}
```

```
{5, 3, 2, 0}
```

```
{5, 0}
```

XDegrees[<f>]

Returns a sorted list of all of the degrees of in x (assumed) appearing in f. Useful when the number of variables is unknown.

```
XDegrees[x[1] x[2]^2 x[3] - 43 x[3424] + 321 x[40]^329 x[47]^32 x[412]]
```

```
{362, 4, 1}
```

```
XDegrees[1 + x[1] + x[1] x[2]]
```

```
{2, 1, 0}
```

SDegrees[<f>]

Returns a sorted list of all of the degrees of s's appearing in f (assuming f is expressed in terms of s's).

```
SDegrees[6 * s[4, 3, 2, 1] + s[3, 1, 1]]
```

```
{10, 5}
```

```
SDegrees[9 + 6 s[4, 3, 2, 1] + s[3, 1, 1]]
SDegrees[6 s[4, 3, 2, 1] + s[3, 1, 1] + x + 9]
```

```
{10, 5, 0}
```

```
{10, 5, 0, x}
```

MDegrees[<f>]

Returns a sorted list of all of the degrees of m's appearing in f (assuming f is expressed in terms of m's).

```
MDegrees[5 * m[4, 2] - 4 m[1] - 7 m[2, 2, 2, 2]]
```

```
{8, 6, 1}
```

EDegrees[<f>]

Returns a sorted list of all of the degrees of e's appearing in f (assuming f is expressed in terms of e's).

```
EDegrees[6 e[4] ^4 e[2] e[1] ^3 + e[4]]
```

```
{21, 4}
```

HDegrees[<f>]

Returns a sorted list of all of the degrees of h's appearing in f (assuming f is expressed in terms of h's).

```
HDegrees[h[4] h[2] h[1] ^2 - 4 h[3]]
```

```
{8, 3}
```

PDegrees[<f>]

Returns a sorted list of all of the degrees of p's appearing in f (assuming f is expressed in terms of p's).

```
PDegrees[p[4] p[2] p[1] ^2 - 4 p[3]]
```

```
{8, 3}
```

Coefficients

GetCoefficients[<f>, <variable list>]

Returns a list of the coefficients in f of the variables in <variable list>. The expression f should be a polynomial in subscripted variables, e.g., x[i]'s, h[λ]'s, s[λ],etc. If a monomial in f divides another monomial, only the former coefficient is returned.

```
SList[5]
GetCoefficients[s[3, 2] - 6 s[3, 1, 1] + 93 s[2, 1, 1, 1] - s[5], SList[5]]
```

```
{s[5], s[4, 1], s[3, 2], s[3, 1, 1], s[2, 2, 1], s[2, 1, 1, 1], s[1, 1, 1, 1, 1]}
```

```
{-1, 0, 1, -6, 0, 93, 0}
```

```
g = 7 e[2] ^2 e[1] - e[4] e[2] e[1] - 6 e[3] ^2 e[1] -
   2 e[4] e[1] + 200 e[1] ^5 - 3 e[7] - 81 e[2] e[1] ^5 + 8 e[5];
GetCoefficients[g, EList[5]]
GetCoefficients[g, EList[7]]
GetCoefficients[g, EList[3]]
```

```
{8, -2, 0, 0, 7, 0, 200}
```

```
{-3, 0, 0, 0, 0, -1, 0, -6, 0, 0, 0, 0, 0, -81, 0}
```

```
{0, 0, 0}
```

```
GetCoefficients[g^3 // Expand, EList[21]]
```

Symmetry tests, symmetrization

```
SymmetricQ[<f>,<z>,<n>]
```

Tests whether a polynomial in variables z[1],z[2],...,z[n] is symmetric.

```
SymmetricQ[(x[1] + x[2] + x[3] + x[4] + x[5])^10, x, 5]
```

```
True
```

```
SymmetricQ[(x[1] + x[2] + x[3] + x[4] + x[5])^10, x, 6]
```

```
False
```

```
Symmetrize[<f>,<x>,<n>]
```

Computes $\sum_\sigma f^\sigma$ over all permutations σ of x[1],x[2],...,x[n].

```
f = x[1]^2 + x[2]^2;
Symmetrize[f, x, 5]
SymmetricQ[%, x, 5]
```

$48\,x[1]^2 + 48\,x[2]^2 + 48\,x[3]^2 + 48\,x[4]^2 + 48\,x[5]^2$

```
True
```

Restrict number of variables

```
Restrict[<f>,<n>, <opts>]
```

Takes the symmetric function f and returns the equivalent function when the underlying x-version is restricted to n variables.

This example takes a function in the h - basis, converts it to the s - basis, then restricts the number of variables to 3, eliminating the s - terms with more than 3 parts.

```
f = h[1]^6 - 3 h[3]^2 + h[2]^3 - 5 h[5] h[1]
r1 = HToS[f]
r1 = Restrict[r1, 3]
```

$h[1]^6 + h[2]^3 - 3\,h[3]^2 - 5\,h[1]\,h[5]$

```
-6 s[6] + 3 s[3, 3] + 9 s[4, 2] - s[5, 1] + 6 s[2, 2, 2] + 18 s[3, 2, 1] + 11 s[4, 1, 1] +
 9 s[2, 2, 1, 1] + 10 s[3, 1, 1, 1] + 5 s[2, 1, 1, 1, 1] + s[1, 1, 1, 1, 1, 1]
```

```
-6 s[6] + 3 s[3, 3] + 9 s[4, 2] - s[5, 1] + 6 s[2, 2, 2] + 18 s[3, 2, 1] + 11 s[4, 1, 1]
```

The same result could have been obtained directly using the **HToS** conversion command with the option **Vars -> 3**.

```
r2 = HToS[f, Vars → 3]
r1 == r2
```

```
-6 s[6] + 3 s[3, 3] + 9 s[4, 2] - s[5, 1] + 6 s[2, 2, 2] + 18 s[3, 2, 1] + 11 s[4, 1, 1]
```

```
True
```

Note that the results may be complex in the case of the h - and p - bases. In the other cases, the command operates more simply on individual monomials.

```
f = h[6]
Restrict[f, 4]
```

```
h[6]
```

$-h[1]^6 + 3\,h[1]^4\,h[2] - h[2]^3 - 2\,h[1]^3\,h[3] - 2\,h[1]\,h[2]\,h[3] + h[3]^2 + h[1]^2\,h[4] + 2\,h[2]\,h[4]$

```
f = p[6]
Restrict[f, 4]
```

p[6]

$$-\frac{1}{24}p[1]^6+\frac{3}{8}p[1]^4p[2]-\frac{3}{8}p[1]^2p[2]^2-\frac{p[2]^3}{8}-\frac{2}{3}p[1]^3p[3]+\frac{p[3]^2}{3}+\frac{3}{4}p[1]^2p[4]+\frac{3}{4}p[2]p[4]$$

Input to the **Restrict** command can be a function expressed using more than one basis. The **Bases** option allows the user to specify which bases the variable-restriction gets applied to. (Admittedly, this is not likely to be very useful!).

```
fmixed = e[4] e[2] + p[3]^2 + p[5] + x[1] x[2] x[3] + x[1] x[2] x[4] +
  x[1] x[3] x[4] + x[2] x[3] x[4] + 4 m[2, 2] + s[3, 1, 1, 1, 1, 1, 1, 1, 1] - h[7]
Restrict[fmixed, 3]
Restrict[fmixed, 3, Bases → {E, P, X}]
```

$$e[2]e[4]-h[7]+4m[2,2]+p[3]^2+p[5]+s[3,1,1,1,1,1,1,1,1]+x[1]x[2]x[3]+x[1]x[2]x[4]+x[1]x[3]x[4]+x[2]x[3]x[4]$$

$$-h[1]^3h[2]^2+2h[1]h[2]^3-2h[1]^4h[3]+6h[1]^2h[2]h[3]-3h[2]^2h[3]-3h[1]h[3]^2+4m[2,2]+\frac{p[1]^5}{6}-\frac{5}{6}p[1]^3p[2]+\frac{5}{6}p[1]^2p[3]+\frac{5}{6}p[2]p[3]+p[3]^2+x[1]x[2]x[3]$$

$$-h[7]+4m[2,2]+\frac{p[1]^5}{6}-\frac{5}{6}p[1]^3p[2]+\frac{5}{6}p[1]^2p[3]+\frac{5}{6}p[2]p[3]+p[3]^2+s[3,1,1,1,1,1,1,1,1]+x[1]x[2]x[3]$$

Matrices

■ JTH, JTE : Jacobi-Trudi Matrices

JTH[<λ>]

Gives Jacobi-Trudi Matrix in h-functions, whose determinant equals s[λ].

```
JTH[{3, 2, 1, 1}] // MatrixForm
Det[%] // HToS
```

$$\begin{pmatrix} h[3] & h[4] & h[5] & h[6] \\ h[1] & h[2] & h[3] & h[4] \\ 0 & 1 & h[1] & h[2] \\ 0 & 0 & 1 & h[1] \end{pmatrix}$$

s[3, 2, 1, 1]

JTE[<λ>]

Gives Jacobi-Trudi Matrix in e-functions, whose determinant equals s[λ].

```
JTE[{3, 2, 1, 1}] // MatrixForm
Det[%] // EToS
```

$$\begin{pmatrix} e[4] & e[5] & e[6] \\ e[1] & e[2] & e[3] \\ 0 & 1 & e[1] \end{pmatrix}$$

s[3, 2, 1, 1]

■ K, KInv,KStar,KTr : Kostka matrices

```
K[<n>]
```

Returns the p(n) × p(n) Kostka matrix

```
K[4] // MatrixForm
```

$$\begin{pmatrix} 1 & 1 & 1 & 1 & 1 \\ 0 & 1 & 1 & 2 & 3 \\ 0 & 0 & 1 & 1 & 2 \\ 0 & 0 & 0 & 1 & 3 \\ 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$

```
K[<n>, <k>]
```

Returns the p(n, k) × p(n, k) "restricted" Kostka matrix.

```
K[6, 3] // MatrixForm
```

$$\begin{pmatrix} 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 0 & 1 & 1 & 2 & 1 & 2 & 2 \\ 0 & 0 & 1 & 1 & 1 & 2 & 3 \\ 0 & 0 & 0 & 1 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 & 0 & 1 & 2 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$

```
KInv[<n>]
```

Returns the p(n) × p(n) inverse Kostka matrix

```
KInv[4] // MatrixForm
```

$$\begin{pmatrix} 1 & -1 & 0 & 1 & -1 \\ 0 & 1 & -1 & -1 & 2 \\ 0 & 0 & 1 & -1 & 1 \\ 0 & 0 & 0 & 1 & -3 \\ 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$

```
KInv[<n>, <k>]
```

Returns the p(n, k) × p(n, k) inverse "restricted" Kostka matrix

```
KInv[6, 3] // MatrixForm
```

$$\begin{pmatrix} 1 & -1 & 0 & 1 & 0 & 0 & 0 \\ 0 & 1 & -1 & -1 & 0 & 1 & 0 \\ 0 & 0 & 1 & -1 & -1 & 0 & -1 \\ 0 & 0 & 0 & 1 & 0 & -1 & 1 \\ 0 & 0 & 0 & 0 & 1 & -1 & 1 \\ 0 & 0 & 0 & 0 & 0 & 1 & -2 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$

```
KTr[<n>]
```

Returns the transpose of the p(n) × p(n) Kostka matrix

```
KTr[4] // MatrixForm
```

$$\begin{pmatrix} 1 & 0 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 & 0 \\ 1 & 1 & 1 & 0 & 0 \\ 1 & 2 & 1 & 1 & 0 \\ 1 & 3 & 2 & 3 & 1 \end{pmatrix}$$

```
KTr[<n>, <k>]
```

Returns the transpose of the p(n, k) × p(n, k) "restricted" Kostka matrix

```
KTr[6, 3] // MatrixForm
```

$$\begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 & 0 & 0 & 0 \\ 1 & 1 & 1 & 0 & 0 & 0 & 0 \\ 1 & 2 & 1 & 1 & 0 & 0 & 0 \\ 1 & 1 & 1 & 0 & 1 & 0 & 0 \\ 1 & 2 & 2 & 1 & 1 & 1 & 0 \\ 1 & 2 & 3 & 1 & 1 & 2 & 1 \end{pmatrix}$$

```
KStar[<n>]
```

Returns the inverse transpose of the p(n) × p(n) Kostka matrix

```
KStar[4] // MatrixForm
```

$$\begin{pmatrix} 1 & 0 & 0 & 0 & 0 \\ -1 & 1 & 0 & 0 & 0 \\ 0 & -1 & 1 & 0 & 0 \\ 1 & -1 & -1 & 1 & 0 \\ -1 & 2 & 1 & -3 & 1 \end{pmatrix}$$

```
KStar[<n>, <k>]
```

Returns the inverse transpose of the p(n, k) × p(n, k) "restricted" Kostka matrix

```
KStar[6, 3] // MatrixForm
```

$$\begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ -1 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & -1 & 1 & 0 & 0 & 0 & 0 \\ 1 & -1 & -1 & 1 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & -1 & -1 & 1 & 0 \\ 0 & 0 & -1 & 1 & 1 & -2 & 1 \end{pmatrix}$$

Bigger Examples

`K[8] // MatrixForm`

$$\begin{pmatrix}
1&1&1&1&1&1&1&1&1&1&1&1&1&1&1&1&1&1&1&1&1&1\\
0&1&1&2&1&2&3&1&2&2&3&4&2&3&3&4&5&3&4&5&6&7\\
0&0&1&1&1&2&3&1&2&3&4&6&3&4&5&7&10&6&8&11&15&20\\
0&0&0&1&0&1&3&0&1&1&3&6&1&3&3&6&10&3&6&10&15&21\\
0&0&0&0&1&1&1&1&2&2&3&4&3&4&5&7&10&6&9&13&19&28\\
0&0&0&0&0&1&2&0&1&2&4&8&2&4&6&11&20&8&14&24&40&64\\
0&0&0&0&0&0&1&0&0&0&1&4&0&1&1&4&10&1&4&10&20&35\\
0&0&0&0&0&0&0&1&1&1&1&1&1&2&2&3&4&3&4&6&9&14\\
0&0&0&0&0&0&0&0&1&1&2&3&2&4&5&9&15&7&13&23&40&70\\
0&0&0&0&0&0&0&0&0&1&1&2&1&1&3&5&10&6&9&16&30&56\\
0&0&0&0&0&0&0&0&0&0&1&3&0&1&2&6&15&3&9&21&45&90\\
0&0&0&0&0&0&0&0&0&0&0&1&0&0&0&1&5&0&1&5&15&35\\
0&0&0&0&0&0&0&0&0&0&0&0&1&1&2&3&5&3&6&11&21&42\\
0&0&0&0&0&0&0&0&0&0&0&0&0&1&1&3&6&2&5&12&26&56\\
0&0&0&0&0&0&0&0&0&0&0&0&0&0&1&2&5&3&6&13&30&70\\
0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&1&4&0&2&8&24&64\\
0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&1&0&0&1&6&21\\
0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&1&1&2&5&14\\
0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&1&3&9&28\\
0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&1&5&20\\
0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&1&7\\
0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&1
\end{pmatrix}$$

`KInv[24, 4] // MatrixForm // Timing`

`KTr[12, 5]`

`KStar[6] // MatrixForm`

$$\begin{pmatrix}
1&0&0&0&0&0&0&0&0&0&0\\
-1&1&0&0&0&0&0&0&0&0&0\\
0&-1&1&0&0&0&0&0&0&0&0\\
1&-1&-1&1&0&0&0&0&0&0&0\\
0&0&-1&0&1&0&0&0&0&0&0\\
0&1&0&-1&-1&1&0&0&0&0&0\\
-1&1&1&-1&1&-2&1&0&0&0&0\\
0&0&-1&1&1&-2&0&1&0&0&0\\
0&-1&1&1&0&0&-1&-1&1&0&0\\
1&-1&-2&1&-1&4&-1&1&-3&1&0\\
-1&2&2&-3&1&-6&4&-1&6&-5&1
\end{pmatrix}$$

Chi, ChiInv : Character matrices

`Chi[<n>], Chi2[<n>]`

Returns the p(n) × p(n) character matrix.

`Chi[4] // MatrixForm`

$$\begin{pmatrix}
1&1&1&1&1\\
-1&0&-1&1&3\\
0&-1&2&0&2\\
1&0&-1&-1&3\\
-1&1&1&-1&1
\end{pmatrix}$$

`ChiInv[<n>], ChiInv2[<n>]`

Returns the p(n) × p(n) inverse of the character matrix.

```
ChiInv[4] // MatrixForm
```

$$\begin{pmatrix} \frac{1}{4} & -\frac{1}{4} & 0 & \frac{1}{4} & -\frac{1}{4} \\ \frac{1}{3} & 0 & -\frac{1}{3} & 0 & \frac{1}{3} \\ \frac{1}{8} & -\frac{1}{8} & \frac{1}{4} & -\frac{1}{8} & \frac{1}{8} \\ \frac{1}{4} & \frac{1}{4} & 0 & -\frac{1}{4} & -\frac{1}{4} \\ \frac{1}{24} & \frac{1}{8} & \frac{1}{12} & \frac{1}{8} & \frac{1}{24} \end{pmatrix}$$

- **Bigger Examples**

```
Chi[7] // MatrixForm // Timing
```

$$\left\{0.094\ \text{Second}, \begin{pmatrix} 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ -1 & 0 & -1 & 1 & -1 & 0 & 2 & 0 & -1 & 1 & 3 & 0 & 2 & 4 & 6 \\ 0 & -1 & 1 & -1 & 0 & 0 & 0 & -1 & 2 & 0 & 2 & 2 & 2 & 6 & 14 \\ 1 & 0 & 0 & 0 & 1 & -1 & 1 & 0 & -1 & -1 & 3 & -3 & -1 & 5 & 15 \\ 0 & 0 & -1 & -1 & 1 & 0 & -2 & 2 & -1 & 1 & -1 & 0 & 2 & 4 & 14 \\ 0 & 1 & 0 & 0 & -1 & 1 & -1 & -1 & -1 & -1 & -1 & 1 & -1 & 5 & 35 \\ -1 & 0 & 0 & 0 & 0 & 0 & 0 & 2 & 2 & 0 & 2 & 0 & -4 & 0 & 20 \\ 0 & 0 & 1 & 1 & -1 & -1 & -1 & 0 & 1 & 1 & -3 & -3 & 1 & 1 & 21 \\ 0 & 0 & -1 & 1 & 1 & -1 & 1 & 0 & 1 & -1 & -3 & 3 & 1 & -1 & 21 \\ 0 & -1 & 0 & 0 & 1 & 1 & 1 & -1 & -1 & 1 & -1 & -1 & -1 & -5 & 35 \\ 1 & 0 & 0 & 0 & -1 & -1 & -1 & 0 & -1 & 1 & 3 & 3 & -1 & -5 & 15 \\ 0 & 0 & 1 & -1 & -1 & 0 & 2 & 2 & -1 & -1 & -1 & 0 & 2 & -4 & 14 \\ 0 & 1 & -1 & -1 & 0 & 0 & 0 & -1 & 2 & 0 & 2 & -2 & 2 & -6 & 14 \\ -1 & 0 & 1 & 1 & 1 & 0 & -2 & 0 & -1 & -1 & 3 & 0 & 2 & -4 & 6 \\ 1 & -1 & -1 & 1 & -1 & 1 & -1 & 1 & 1 & -1 & 1 & -1 & 1 & -1 & 1 \end{pmatrix}\right\}$$

```
Chi2[7] // MatrixForm // Timing
```

$$\left\{21.828\ \text{Second}, \begin{pmatrix} 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ -1 & 0 & -1 & 1 & -1 & 0 & 2 & 0 & -1 & 1 & 3 & 0 & 2 & 4 & 6 \\ 0 & -1 & 1 & -1 & 0 & 0 & 0 & -1 & 2 & 0 & 2 & 2 & 2 & 6 & 14 \\ 1 & 0 & 0 & 0 & 1 & -1 & 1 & 0 & -1 & -1 & 3 & -3 & -1 & 5 & 15 \\ 0 & 0 & -1 & -1 & 1 & 0 & -2 & 2 & -1 & 1 & -1 & 0 & 2 & 4 & 14 \\ 0 & 1 & 0 & 0 & -1 & 1 & -1 & -1 & -1 & -1 & -1 & 1 & -1 & 5 & 35 \\ -1 & 0 & 0 & 0 & 0 & 0 & 0 & 2 & 2 & 0 & 2 & 0 & -4 & 0 & 20 \\ 0 & 0 & 1 & 1 & -1 & -1 & -1 & 0 & 1 & 1 & -3 & -3 & 1 & 1 & 21 \\ 0 & 0 & -1 & 1 & 1 & -1 & 1 & 0 & 1 & -1 & -3 & 3 & 1 & -1 & 21 \\ 0 & -1 & 0 & 0 & 1 & 1 & 1 & -1 & -1 & 1 & -1 & -1 & -1 & -5 & 35 \\ 1 & 0 & 0 & 0 & -1 & -1 & -1 & 0 & -1 & 1 & 3 & 3 & -1 & -5 & 15 \\ 0 & 0 & 1 & -1 & -1 & 0 & 2 & 2 & -1 & -1 & -1 & 0 & 2 & -4 & 14 \\ 0 & 1 & -1 & -1 & 0 & 0 & 0 & -1 & 2 & 0 & 2 & -2 & 2 & -6 & 14 \\ -1 & 0 & 1 & 1 & 1 & 0 & -2 & 0 & -1 & -1 & 3 & 0 & 2 & -4 & 6 \\ 1 & -1 & -1 & 1 & -1 & 1 & -1 & 1 & 1 & -1 & 1 & -1 & 1 & -1 & 1 \end{pmatrix}\right\}$$

```
chi10 = Chi[10]
```

```
Det[chi10]
Apply[Times, Flatten[Partitions[10]]]
```

9 263 795 272 057 860 957 392 207 640 004 657 152 000 000 000

9 263 795 272 057 860 957 392 207 640 004 657 152 000 000 000

This last result illustrates the remarkable result that the determinant of the character matrix of S_n equals the product of all of the parts in all of the partitions of n.

■ J (transposition matrix), SToEMat, EToSMat, HToH2Mat, H2ToHMat (special matrices for restricted variable situations)

```
J[<n>]
```

Returns the p(n) × p(n) "transposition" matrix that takes every partition to its conjugate

```
J[6] // MatrixForm
```

$$\begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

```
SToEMat[<n>, <k>]
```

Returns the p(n, k) × p(n, k) transition matrix from the S basis to the E basis in a situation where the number of variables is restricted to k

```
SToEMat[8, 3] // MatrixForm
```

$$\begin{pmatrix} 1 & -7 & 15 & 6 & -10 & -20 & 1 & 12 & 6 & -3 \\ 0 & 1 & -5 & -1 & 6 & 8 & -1 & -9 & -3 & 3 \\ 0 & 0 & 1 & -1 & -3 & 2 & 1 & 3 & -2 & -1 \\ 0 & 0 & 0 & 1 & 0 & -4 & 0 & 3 & 3 & -2 \\ 0 & 0 & 0 & 0 & 1 & -2 & -1 & 1 & 2 & -1 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & -2 & -1 & 2 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & -3 & 1 & 2 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & -1 & -1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & -1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$

```
EToSMat[<n>, <k>]
```

Returns the p(n, k) × p(n, k) transition matrix from the E basis to the S basis in a situation where the number of variables is restricted to k

```
EToSMat[8, 3] // MatrixForm
```

$$\begin{pmatrix} 1 & 7 & 20 & 21 & 28 & 64 & 14 & 70 & 56 & 42 \\ 0 & 1 & 5 & 6 & 9 & 24 & 5 & 30 & 26 & 21 \\ 0 & 0 & 1 & 1 & 3 & 8 & 2 & 13 & 12 & 11 \\ 0 & 0 & 0 & 1 & 0 & 4 & 0 & 5 & 6 & 5 \\ 0 & 0 & 0 & 0 & 1 & 2 & 1 & 6 & 5 & 6 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 2 & 3 & 3 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 3 & 2 & 3 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 2 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$

```
HToH2Mat[<n>, <k>]
```

Returns a p(n, k) × p(n, k) matrix that converts from the basis of H-restricted expressed by HList[n,k] to the basis of H-restricted expressed by HList2[n,k]

```
HRestrictionRules[5, 2]
HToH2Mat[5, 2] // MatrixForm
H2ToHMat[5, 2] // MatrixForm
```

$$\{h[5] \to -2\,h[1]^3\,h[2] + 3\,h[1]\,h[2]^2,\ h[1]\,h[4] \to -h[1]^5 + h[1]^3\,h[2] + h[1]\,h[2]^2,\ h[2]\,h[3] \to -h[1]^3\,h[2] + 2\,h[1]\,h[2]^2,\ h[1]^2\,h[3] \to -h[1]^5 + 2\,h[1]^3\,h[2],\ h[1]\,h[2]^2 \to h[1]\,h[2]^2,\ h[1]^3\,h[2] \to h[1]^3\,h[2],\ h[1]^5 \to h[1]^5\}$$

$$\begin{pmatrix} 3 & -2 & 0 \\ 1 & 1 & -1 \\ 2 & -1 & 0 \end{pmatrix}$$

$$\begin{pmatrix} -1 & 0 & 2 \\ -2 & 0 & 3 \\ -3 & -1 & 5 \end{pmatrix}$$

Bigger examples

```
SToEMat[15, 3] // MatrixForm
```

$$\begin{pmatrix}
1 & -14 & 78 & 13 & -220 & -132 & 330 & 495 & 55 & -252 & -840 & -360 & 84 & 630 & 756 & 84 & -8 & -168 \\
0 & 1 & -12 & -1 & 55 & 22 & -120 & -135 & -10 & 126 & 336 & 108 & -56 & -350 & -336 & -28 & 7 & 126 \\
0 & 0 & 1 & -1 & -10 & 9 & 36 & -18 & -9 & -56 & -28 & 48 & 35 & 105 & -42 & -21 & -6 & -69 \\
0 & 0 & 0 & 1 & 0 & -11 & 0 & 45 & 10 & 0 & -84 & -72 & 0 & 70 & 168 & 28 & 0 & -21 \\
0 & 0 & 0 & 0 & 1 & -2 & -8 & 15 & 2 & 21 & -28 & -29 & -20 & -5 & 90 & 13 & 5 & 30 \\
0 & 0 & 0 & 0 & 0 & 1 & 0 & -9 & -1 & 0 & 28 & 16 & 0 & -35 & -63 & -7 & 0 & 15 \\
0 & 0 & 0 & 0 & 0 & 0 & 1 & -3 & 1 & -6 & 17 & -2 & 10 & -20 & -27 & 4 & -4 & -6 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & -1 & 0 & -7 & 6 & 0 & 15 & -3 & -6 & 0 & -10 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & -8 & 0 & 0 & 21 & 7 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & -4 & 3 & -4 & 15 & -6 & -3 & 3 & -6 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & -2 & 0 & -5 & 9 & 2 & 0 & 6 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & -6 & -1 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & -5 & 6 & -1 & -2 & 9 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & -3 & 1 & 0 & -3 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & -1 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & -6 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0
\end{pmatrix}$$

Other matrices: PEMat, PHMat, EPMat, EHMat, HPMat, HEMat

```
PEMat[<n>]
```

Gives n × n matrix whose determinant is p[n] in terms of e[1],...,e[n]

```
PEMat[5] // MatrixForm
Det[%]
EToP[%]
```

$$\begin{pmatrix} e[1] & 1 & 0 & 0 & 0 \\ 2\,e[2] & e[1] & 1 & 0 & 0 \\ 3\,e[3] & e[2] & e[1] & 1 & 0 \\ 4\,e[4] & e[3] & e[2] & e[1] & 1 \\ 5\,e[5] & e[4] & e[3] & e[2] & e[1] \end{pmatrix}$$

$e[1]^5 - 5\,e[1]^3\,e[2] + 5\,e[1]\,e[2]^2 + 5\,e[1]^2\,e[3] - 5\,e[2]\,e[3] - 5\,e[1]\,e[4] + 5\,e[5]$

p[5]

PHMat[<n>]

Gives n × n matrix whose determinant is ±p[n] in terms of h[1],...,h[n] (The ± is implemented in the ph function.)

```
PHMat[5] // MatrixForm
Det[%]
HToP[%]
```

$$\begin{pmatrix} h[1] & 1 & 0 & 0 & 0 \\ 2\,h[2] & h[1] & 1 & 0 & 0 \\ 3\,h[3] & h[2] & h[1] & 1 & 0 \\ 4\,h[4] & h[3] & h[2] & h[1] & 1 \\ 5\,h[5] & h[4] & h[3] & h[2] & h[1] \end{pmatrix}$$

$h[1]^5 - 5\,h[1]^3\,h[2] + 5\,h[1]\,h[2]^2 + 5\,h[1]^2\,h[3] - 5\,h[2]\,h[3] - 5\,h[1]\,h[4] + 5\,h[5]$

p[5]

EPMat[<n>]

Gives n × n matrix whose determinant is n!*e[n] in terms of p[1],...,p[n]

```
EPMat[5] // MatrixForm
Det[%]
PToE[%]
```

$$\begin{pmatrix} p[1] & 1 & 0 & 0 & 0 \\ p[2] & p[1] & 2 & 0 & 0 \\ p[3] & p[2] & p[1] & 3 & 0 \\ p[4] & p[3] & p[2] & p[1] & 4 \\ p[5] & p[4] & p[3] & p[2] & p[1] \end{pmatrix}$$

$p[1]^5 - 10\,p[1]^3\,p[2] + 15\,p[1]\,p[2]^2 + 20\,p[1]^2\,p[3] - 20\,p[2]\,p[3] - 30\,p[1]\,p[4] + 24\,p[5]$

120 e[5]

EHMat[<n>]

Gives n × n matrix whose determinant is e[n] in terms of h[1],...,h[n]

```
EHMat[5] // MatrixForm
Det[%]
HToE[%]
```

$$\begin{pmatrix} h[1] & h[2] & h[3] & h[4] & h[5] \\ 1 & h[1] & h[2] & h[3] & h[4] \\ 0 & 1 & h[1] & h[2] & h[3] \\ 0 & 0 & 1 & h[1] & h[2] \\ 0 & 0 & 0 & 1 & h[1] \end{pmatrix}$$

$h[1]^5 - 4\,h[1]^3\,h[2] + 3\,h[1]\,h[2]^2 + 3\,h[1]^2\,h[3] - 2\,h[2]\,h[3] - 2\,h[1]\,h[4] + h[5]$

e[5]

```
HPMat[<n>]
```

Gives n × n matrix whose determinant is n!*h[n] in terms of p[1],...,p[n]

```
HPMat[5] // MatrixForm
Det[%]
PToH[%]
```

$$\begin{pmatrix} p[1] & -1 & 0 & 0 & 0 \\ p[2] & p[1] & -2 & 0 & 0 \\ p[3] & p[2] & p[1] & -3 & 0 \\ p[4] & p[3] & p[2] & p[1] & -4 \\ p[5] & p[4] & p[3] & p[2] & p[1] \end{pmatrix}$$

$p[1]^5 + 10\,p[1]^3\,p[2] + 15\,p[1]\,p[2]^2 + 20\,p[1]^2\,p[3] + 20\,p[2]\,p[3] + 30\,p[1]\,p[4] + 24\,p[5]$

120 h[5]

```
HEMat[<n>]
```

Gives n × n matrix whose determinant is h[n] in terms of e[1],...,e[n]

```
HEMat[5] // MatrixForm
Det[%]
EToH[%]
```

$$\begin{pmatrix} e[1] & e[2] & e[3] & e[4] & e[5] \\ 1 & e[1] & e[2] & e[3] & e[4] \\ 0 & 1 & e[1] & e[2] & e[3] \\ 0 & 0 & 1 & e[1] & e[2] \\ 0 & 0 & 0 & 1 & e[1] \end{pmatrix}$$

$e[1]^5 - 4\,e[1]^3\,e[2] + 3\,e[1]\,e[2]^2 + 3\,e[1]^2\,e[3] - 2\,e[2]\,e[3] - 2\,e[1]\,e[4] + e[5]$

h[5]

Basis Conversions

- **XTo[HEMPSAll]**
- **XToM**

```
XToM[<f>]
```

If f is a symmetric function expressed in terms of x[1],x[2],..., returns f expressed in terms of the m[lambda]'s. (Assumes that f is symmetric but does not test for this.)

```
XToM[x[1]^3 + 3 x[1] x[2] x[3] + x[3]^3 + x[2]^3]
```

```
m[3] + 3 m[1, 1, 1]
```

NumberOfVars[<f>]

Returns number of x[i] variables in an expression involving x[i]'s.

```
NumberOfVars[x[1] x[2] + x[3]]
```

```
3
```

```
NumberOfVars[x[1]^2 + x[2]^2]
```

```
2
```

```
XToM[x[1]^2 + x[2]^2]
```

```
m[2]
```

```
XToM[(x[1] + x[2]) (x[1] + x[3]) (x[2] + x[3])]
```

```
m[2, 1] + 2 m[1, 1, 1]
```

XTo[Others]

XToB[<f>]

Converts f (expressed in terms of x[i]) into basis B, where B can be H,E,P,S, or "All".

```
f = Product[x[i] + x[j], {i, 1, 3}, {j, i + 1, 3}]
f // XToH
f // XToE
f // XToP
f // XToS
f // XToM
```

```
(x[1] + x[2]) (x[1] + x[3]) (x[2] + x[3])
```

```
h[1] h[2] - h[3]
```

```
e[1] e[2] - e[3]
```

$$\frac{p[1]^3}{3} - \frac{p[3]}{3}$$

```
s[2, 1]
```

```
m[2, 1] + 2 m[1, 1, 1]
```

STo[HEMPXAll]

SToH

SToH[<f>,< opts>]
Options[SToH]={DegreeList→{}, Vars→0}

If f is a symmetric function expressed as a linear combination of s functions, SToH returns the same function expressed as a linear combination of the appropriate h functions. Setting the option DegreeList to a list of the degrees of f saves *Mathematica* some computation, possibly resulting in faster output. Setting the option Vars to some positive integer n restricts the number of variables in the problem to n, resulting in the possible elimation of some terms in the answer.

```
SToH[s[4] + 3 s[2, 2]]
% // HToS
```

$3\,h[2]^2 - 3\,h[1]\,h[3] + h[4]$

s[4] + 3 s[2, 2]

```
SToH[s[4] + 3 s[2, 2], Vars -> 2]
```

$2\,h[1]^4 - 5\,h[1]^2\,h[2] + 4\,h[2]^2$

```
HToX[% - %%, 2]
```

0

- **Bigger Examples**

- **SToE**

```
SToE[<f>, <opts>]
Options[SToE]={DegreeList→{}, Vars→0}
```

If f is a symmetric function expressed as a linear combination of s functions, SToE returns the same function expressed as a linear combination of the appropriate e functions. Setting the option DegreeList to a list of the degrees of f saves *Mathematica* some computation, possibly resulting in faster output. Setting the option Vars to some positive integer n restricts the number of variables in the problem to n, resulting in the possible elimation of some terms in the answer.

```
SToE[4 s[4] - 3 s[3, 1] + 2 s[2, 2] - s[1, 1, 1, 1]]
% // EToS
```

$4\,e[1]^4 - 15\,e[1]^2\,e[2] + 9\,e[2]^2 + 9\,e[1]\,e[3] - 8\,e[4]$

4 s[4] + 2 s[2, 2] - 3 s[3, 1] - s[1, 1, 1, 1]

```
SToE[4 s[4] - 3 s[3, 1] + 2 s[2, 2] - s[1, 1, 1, 1], Vars → 2]
```

$4\,e[1]^4 - 15\,e[1]^2\,e[2] + 9\,e[2]^2$

```
EToX[% - %%, 2]
```

0

- **Bigger examples**

- **SToM**

```
SToM[<f>, <opts>]
Options[SToM]={DegreeList→{}, Vars→0}
```

If f is a symmetric function expressed as a linear combination of s functions, SToM returns the same function expressed as a linear combination of the appropriate m functions. Setting the option DegreeList to a list of the degrees of f saves *Mathematica* some computation, possibly resulting in faster output. Setting the option Vars to some positive integer n restricts the number of variables in the problem to n, resulting in the possible elimation of some terms in the answer.

```
SToM[3 s[5] - 9 s[4, 1] + s[1, 1, 1, 1, 1]]
% // MToS
```

3 m[5] - 6 m[3, 2] - 6 m[4, 1] - 15 m[2, 2, 1] - 15 m[3, 1, 1] - 24 m[2, 1, 1, 1] - 32 m[1, 1, 1, 1, 1]

3 s[5] - 9 s[4, 1] + s[1, 1, 1, 1, 1]

```
SToM[s[3, 2, 1]]
% // MToS
```

$2\,m[2, 2, 2] + m[3, 2, 1] + 4\,m[2, 2, 1, 1] + 2\,m[3, 1, 1, 1] + 8\,m[2, 1, 1, 1, 1] + 16\,m[1, 1, 1, 1, 1, 1]$

s[3, 2, 1]

```
SToM[s[3, 2, 1], Vars → 4]
```

$2\,m[2, 2, 2] + m[3, 2, 1] + 4\,m[2, 2, 1, 1] + 2\,m[3, 1, 1, 1]$

- **Bigger examples**

- **SToP**

```
SToP[<f>,<opts>]
Options[SToP]={DegreeList→{}, Vars→0}
```

If f is a symmetric function expressed as a linear combination of s functions, SToP returns the same function expressed as a linear combination of the appropriate p functions. Setting the option DegreeList to a list of the degrees of f saves *Mathematica* some computation, possibly resulting in faster output. Setting the option Vars to some positive integer n restricts the number of variables in the problem to n, resulting in the possible elimation of some terms in the answer.

```
SToP[2 s[1] + s[3] + 2 s[2, 1] + s[1, 1, 1]]
% // PToS
```

$2\,p[1] + p[1]^3$

$2\,s[1] + s[3] + 2\,s[2, 1] + s[1, 1, 1]$

```
SToP[s[4, 3, 2, 1]]
% // PToS
```

$$\frac{p[1]^{10}}{4725} - \frac{1}{315}\,p[1]^7\,p[3] - \frac{1}{27}\,p[1]\,p[3]^3 + \frac{1}{75}\,p[1]^5\,p[5] + \frac{1}{15}\,p[1]^2\,p[3]\,p[5] - \frac{p[5]^2}{25} - \frac{1}{21}\,p[1]^3\,p[7] + \frac{1}{21}\,p[3]\,p[7]$$

s[4, 3, 2, 1]

```
SToP[s[4, 3, 2, 1], Vars → 4]
```

$$\frac{p[1]^{10}}{1728} - \frac{7}{288}\,p[1]^6\,p[2]^2 + \frac{1}{24}\,p[1]^4\,p[2]^3 - \frac{1}{64}\,p[1]^2\,p[2]^4 - \frac{1}{144}\,p[1]^7\,p[3] + \frac{5}{48}\,p[1]^5\,p[2]\,p[3] - \frac{17}{144}\,p[1]^3\,p[2]^2\,p[3] + \frac{1}{48}\,p[1]\,p[2]^3\,p[3] - \frac{7}{72}\,p[1]^4\,p[3]^2 + \frac{1}{12}\,p[1]^2\,p[2]\,p[3]^2 - \frac{1}{72}\,p[2]^2\,p[3]^2 - \frac{1}{27}\,p[1]\,p[3]^3 + \frac{1}{144}\,p[1]^6\,p[4] - \frac{1}{12}\,p[1]^4\,p[2]\,p[4] + \frac{1}{16}\,p[1]^2\,p[2]^2\,p[4] + \frac{11}{72}\,p[1]^3\,p[3]\,p[4] - \frac{1}{24}\,p[1]\,p[2]\,p[3]\,p[4] + \frac{1}{36}\,p[3]^2\,p[4] - \frac{1}{16}\,p[1]^2\,p[4]^2$$

```
PToS[%]
PToS[%%, Vars → 4]
```

```
s[4, 3, 2, 1] - s[2, 2, 2, 2, 2] - s[3, 2, 2, 2, 1] + s[3, 3, 2, 1, 1] +
 s[4, 2, 2, 1, 1] + s[4, 3, 1, 1, 1] - 2 s[2, 2, 2, 2, 1, 1] + s[3, 3, 1, 1, 1, 1] +
 s[4, 2, 1, 1, 1, 1] - 2 s[2, 2, 2, 1, 1, 1, 1] - 2 s[2, 2, 1, 1, 1, 1, 1, 1] -
 s[3, 1, 1, 1, 1, 1, 1, 1] - 2 s[2, 1, 1, 1, 1, 1, 1, 1, 1] - s[1, 1, 1, 1, 1, 1, 1, 1, 1, 1]
```

```
s[4, 3, 2, 1]
```

STo X

```
SToX[<f>,<vars>]
```

If f is a symmetric function expressed as a linear combination of s functions, SToX returns the same function expressed in variables x[1], ..., x[n].

```
SToX[2 s[1] + s[3] + 2 s[2, 1] + s[1, 1, 1], 4]
XToS[%]
```

```
2 x[1] + x[1]^3 + 2 x[2] + 3 x[1]^2 x[2] + 3 x[1] x[2]^2 + x[2]^3 + 2 x[3] +
 3 x[1]^2 x[3] + 6 x[1] x[2] x[3] + 3 x[2]^2 x[3] + 3 x[1] x[3]^2 + 3 x[2] x[3]^2 +
 x[3]^3 + 2 x[4] + 3 x[1]^2 x[4] + 6 x[1] x[2] x[4] + 3 x[2]^2 x[4] + 6 x[1] x[3] x[4] +
 6 x[2] x[3] x[4] + 3 x[3]^2 x[4] + 3 x[1] x[4]^2 + 3 x[2] x[4]^2 + 3 x[3] x[4]^2 + x[4]^3
```

```
2 s[1] + s[3] + 2 s[2, 1] + s[1, 1, 1]
```

```
SToX[s[3, 2, 1], 3]
```

```
x[1]^3 x[2]^2 x[3] + x[1]^2 x[2]^3 x[3] + x[1]^3 x[2] x[3]^2 +
 2 x[1]^2 x[2]^2 x[3]^2 + x[1] x[2]^3 x[3]^2 + x[1]^2 x[2] x[3]^3 + x[1] x[2]^2 x[3]^3
```

```
SToX[s[4, 3, 2, 1], 4]
XToS[%]
```

```
x[1]^4 x[2]^3 x[3]^2 x[4] + x[1]^3 x[2]^4 x[3]^2 x[4] +
 x[1]^4 x[2]^2 x[3]^3 x[4] + 2 x[1]^3 x[2]^3 x[3]^3 x[4] + x[1]^2 x[2]^4 x[3]^3 x[4] +
 x[1]^3 x[2]^2 x[3]^4 x[4] + x[1]^2 x[2]^3 x[3]^4 x[4] + x[1]^4 x[2]^3 x[3] x[4]^2 +
 x[1]^3 x[2]^4 x[3] x[4]^2 + 2 x[1]^4 x[2]^2 x[3]^2 x[4]^2 + 4 x[1]^3 x[2]^3 x[3]^2 x[4]^2 +
 2 x[1]^2 x[2]^4 x[3]^2 x[4]^2 + x[1]^4 x[2] x[3]^3 x[4]^2 + 4 x[1]^3 x[2]^2 x[3]^3 x[4]^2 +
 4 x[1]^2 x[2]^3 x[3]^3 x[4]^2 + x[1] x[2]^4 x[3]^3 x[4]^2 + x[1]^3 x[2] x[3]^4 x[4]^2 +
 2 x[1]^2 x[2]^2 x[3]^4 x[4]^2 + x[1] x[2]^3 x[3]^4 x[4]^2 + x[1]^4 x[2]^2 x[3] x[4]^3 +
 2 x[1]^3 x[2]^3 x[3] x[4]^3 + x[1]^2 x[2]^4 x[3] x[4]^3 + x[1]^4 x[2] x[3]^2 x[4]^3 +
 4 x[1]^3 x[2]^2 x[3]^2 x[4]^3 + 4 x[1]^2 x[2]^3 x[3]^2 x[4]^3 + x[1] x[2]^4 x[3]^2 x[4]^3 +
 2 x[1]^3 x[2] x[3]^3 x[4]^3 + 4 x[1]^2 x[2]^2 x[3]^3 x[4]^3 + 2 x[1] x[2]^3 x[3]^3 x[4]^3 +
 x[1]^2 x[2] x[3]^4 x[4]^3 + x[1] x[2]^2 x[3]^4 x[4]^3 + x[1]^3 x[2]^2 x[3] x[4]^4 +
 x[1]^2 x[2]^3 x[3] x[4]^4 + x[1]^3 x[2] x[3]^2 x[4]^4 + 2 x[1]^2 x[2]^2 x[3]^2 x[4]^4 +
 x[1] x[2]^3 x[3]^2 x[4]^4 + x[1]^2 x[2] x[3]^3 x[4]^4 + x[1] x[2]^2 x[3]^3 x[4]^4
```

```
s[4, 3, 2, 1]
```

SToAll

```
SToAll[<f>, <opts>]
Options[SToAll]={Targets→{E,H,P,M, X}, Vars→0}
```

If f is a symmetric function expressed as a linear combination of s functions, SToAll returns the same function expressed as linear combinations of the appropriate functions as specified in Targets. Setting the option Vars to some positive integer n restricts the number of variables in the problem to n, resulting in the possible elimation of some terms in the answer. Note that the X's will only be displayed if Vars is specified.

```
SToAll[s[3, 2, 1]] // ColumnForm
```

s[3, 2, 1]
e[1] e[2] e[3] – e[3]^2 – e[1]^2 e[4] + e[1] e[5]
h[1] h[2] h[3] – h[3]^2 – h[1]^2 h[4] + h[1] h[5]
$\frac{p[1]^6}{45} - \frac{1}{9} p[1]^3 p[3] - \frac{p[3]^2}{9} + \frac{1}{5} p[1] p[5]$
2 m[2, 2, 2] + m[3, 2, 1] + 4 m[2, 2, 1, 1] + 2 m[3, 1, 1, 1] + 8 m[2, 1, 1, 1, 1] + 16 m[1, 1, 1, 1, 1,

```
SToAll[s[3, 2, 1] – 2 s[2, 2, 2], Vars → 3, Targets → {E, M, X}]
```

{–2 s[2, 2, 2] + s[3, 2, 1], e[1] e[2] e[3] – 3 e[3]^2, m[3, 2, 1], x[1]^3 x[2]^2 x[3] + x[1]^2 x[2]^3 x[3] + x[1]^3 x[2] x[3]^2 + x[1] x[2]^3 x[3]^2 + x[1]^2 x[2] x[3]^3 + x[1] x[2]^2 x[3]^3}

- **PTo[HEMSXAll]**

- **PRestrictionRules**

```
PRestrictionRules[<deg>, <var>]
```

PRestrictionRules[deg, var] creates a list of rules that express p functions of degree deg in terms of other p funtions of degree deg when the number of variables is restricted to var. The basis used for this is p[lambda] functions where lambda[[1]]≤var (note that this is NOT the same basis generated by PList[deg, var]).

```
PRestrictionRules[4, 2]
PList[4, 2]
```

$\left\{p[4] \to -\frac{1}{2} p[1]^4 + p[1]^2 p[2] + \frac{p[2]^2}{2},\ p[1]\, p[3] \to -\frac{1}{2} p[1]^4 + \frac{3}{2} p[1]^2 p[2],\ p[2]^2 \to p[2]^2,\ p[1]^2 p[2] \to p[1]^2 p[2],\ p[1]^4 \to p[1]^4\right\}$

$\{p[4],\ p[1]\, p[3],\ p[2]^2\}$

- **PToH**

```
PToH[<f>,< opts>]
Options[PToH]={DegreeList→{}, Vars→0}
```

If f is a symmetric function expressed as a linear combination of p functions, PToH returns the same function expressed as a linear combination of the appropriate h functions. Setting the option DegreeList to a list of the degrees of f saves *Mathematica* some computation, possibly resulting in faster output. Setting the option Vars to some positive integer n restricts the number of variables in the problem to n, resulting in the possible elimation of some terms in the answer.

```
PToH[p[1]^2 p[4]]
% // HToP
```

$-h[1]^6 + 4 h[1]^4 h[2] - 2 h[1]^2 h[2]^2 - 4 h[1]^3 h[3] + 4 h[1]^2 h[4]$

$p[1]^2 p[4]$

```
Restrict[%, 3]
PToH[p[1]^2 p[4], Vars → 3]
```

$3 h[1]^6 - 8 h[1]^4 h[2] + 2 h[1]^2 h[2]^2 + 4 h[1]^3 h[3]$

$3 h[1]^6 - 8 h[1]^4 h[2] + 2 h[1]^2 h[2]^2 + 4 h[1]^3 h[3]$

```
Restrict[p[1]^2 p[4], 3]
PToH[%]
```

$$\frac{p[1]^6}{6} - p[1]^4\, p[2] + \frac{1}{2}\, p[1]^2\, p[2]^2 + \frac{4}{3}\, p[1]^3\, p[3]$$

$3\, h[1]^6 - 8\, h[1]^4\, h[2] + 2\, h[1]^2\, h[2]^2 + 4\, h[1]^3\, h[3]$

▪ PToE

```
PToE[<f>,< opts>]
Options[PToE]={DegreeList→{}, Vars→0}
```

If f is a symmetric function expressed as a linear combination of p functions, PToE returns the same function expressed as a linear combination of the appropriate e functions. Setting the option DegreeList to a list of the degrees of f saves *Mathematica* some computation, possibly resulting in faster output. Setting the option Vars to some positive integer n restricts the number of variables in the problem to n, resulting in the possible elimation of some terms in the answer.

```
PToE[p[1]^2 p[4]]
% // EToP
```

$e[1]^6 - 4\, e[1]^4\, e[2] + 2\, e[1]^2\, e[2]^2 + 4\, e[1]^3\, e[3] - 4\, e[1]^2\, e[4]$

$p[1]^2\, p[4]$

```
Restrict[%, 3]
PToE[p[1]^2 p[4], Vars → 3]
```

$e[1]^6 - 4\, e[1]^4\, e[2] + 2\, e[1]^2\, e[2]^2 + 4\, e[1]^3\, e[3]$

$e[1]^6 - 4\, e[1]^4\, e[2] + 2\, e[1]^2\, e[2]^2 + 4\, e[1]^3\, e[3]$

▪ PToM

```
PToM[<f>,< opts>]
Options[PToM]={DegreeList→{}, Vars→0}
```

If f is a symmetric function expressed as a linear combination of p functions, PToM returns the same function expressed as a linear combination of the appropriate m functions. Setting the option DegreeList to a list of the degrees of f saves *Mathematica* some computation, possibly resulting in faster output. Setting the option Vars to some positive integer n restricts the number of variables in the problem to n, resulting in the possible elimation of some terms in the answer.

```
PToM[p[3] - 5 p[1]^3]
% // MToP
```

$-4\, m[3] - 15\, m[2, 1] - 30\, m[1, 1, 1]$

$-5\, p[1]^3 + p[3]$

```
Restrict[%, 2]
PToM[p[3] - 5 p[1]^3, Vars → 2]
```

$$-\frac{11}{2}\, p[1]^3 + \frac{3}{2}\, p[1]\, p[2]$$

$-4\, m[3] - 15\, m[2, 1]$

```
PToM[p[4] - 3 p[3] p[1] + 2 p[1]^4]
Restrict[%, 3]
PToM[p[4] - 3 p[3] p[1] + 2 p[1]^4, Vars → 3]
```

```
12 m[2, 2] + 5 m[3, 1] + 24 m[2, 1, 1] + 48 m[1, 1, 1, 1]
```

```
12 m[2, 2] + 5 m[3, 1] + 24 m[2, 1, 1]
```

```
12 m[2, 2] + 5 m[3, 1] + 24 m[2, 1, 1]
```

PToS

```
PToS[<f>,<options>]
Options[PToS]={DegreeList→{}, Vars→0}
```

If f is a symmetric function expressed as a linear combination of p functions, PToS returns the same function expressed as a linear combination of the appropriate s functions. Setting the option DegreeList to a list of the degrees of f saves *Mathematica* some computation, possibly resulting in faster output. Setting the option Vars to some positive integer n restricts the number of variables in the problem to n, resulting in the possible elimation of some terms in the answer.

```
PToS[p[1]^3 + 2 p[3]]
% // SToP
```

```
3 s[3] + 3 s[1, 1, 1]
```

$p[1]^3 + 2\,p[3]$

```
PToS[p[1]^3 + 2 p[3], Vars → 2]
```

```
3 s[3]
```

PToX

```
PToX[<f>,<n>]
```

If f is a symmetric function expressed as a linear combination of p functions, PToX returns the same function expressed in variables x[1], ..., x[n].

```
PToX[p[2] p[1]^2, 3]
% // XToP
```

$$x[1]^4 + 2\,x[1]^3\,x[2] + 2\,x[1]^2\,x[2]^2 + 2\,x[1]\,x[2]^3 + x[2]^4 + 2\,x[1]^3\,x[3] + 2\,x[1]^2\,x[2]\,x[3] + 2\,x[1]\,x[2]^2\,x[3] + 2\,x[2]^3\,x[3] + 2\,x[1]^2\,x[3]^2 + 2\,x[1]\,x[2]\,x[3]^2 + 2\,x[2]^2\,x[3]^2 + 2\,x[1]\,x[3]^3 + 2\,x[2]\,x[3]^3 + x[3]^4$$

$p[1]^2\,p[2]$

PToAll

```
PToAll[<f>, <opts>]
Options[PToAll]={Targets→{E,H,S,M, X}, Vars→0}
```

If f is a symmetric function expressed as a linear combination of p functions, PToAll returns the same function expressed as linear combinations of the appropriate functions as specified in Targets. Setting the option Vars to some positive integer n restricts the number of variables in the problem to n, resulting in the possible elimation of some terms in the answer. Note that the X's will only be displayed if Vars is specified.

```
Sum[p[i] p[j], {i, 3}, {j, 3}];
PToAll[%]
```

```
{p[1]^2 + 2 p[1] p[2] + p[2]^2 + 2 p[1] p[3] + 2 p[2] p[3] + p[3]^2,
 e[1]^2 + 2 e[1]^3 + 3 e[1]^4 + 2 e[1]^5 + e[1]^6 - 4 e[1] e[2] - 10 e[1]^2 e[2] -
  10 e[1]^3 e[2] - 6 e[1]^4 e[2] + 4 e[2]^2 + 12 e[1] e[2]^2 + 9 e[1]^2 e[2]^2 + 6 e[1] e[3] +
  6 e[1]^2 e[3] + 6 e[1]^3 e[3] - 12 e[2] e[3] - 18 e[1] e[2] e[3] + 9 e[3]^2,
 h[1]^2 - 2 h[1]^3 + 3 h[1]^4 - 2 h[1]^5 + h[1]^6 + 4 h[1] h[2] - 10 h[1]^2 h[2] +
  10 h[1]^3 h[2] - 6 h[1]^4 h[2] + 4 h[2]^2 - 12 h[1] h[2]^2 + 9 h[1]^2 h[2]^2 + 6 h[1] h[3] -
  6 h[1]^2 h[3] + 6 h[1]^3 h[3] + 12 h[2] h[3] - 18 h[1] h[2] h[3] + 9 h[3]^2,
 s[2] + 2 s[3] + 3 s[4] + 2 s[5] + s[6] + s[1, 1] - s[3, 1] + 2 s[3, 2] + 2 s[3, 3] -
  2 s[4, 1] - s[5, 1] - 2 s[1, 1, 1] - s[2, 1, 1] - 2 s[2, 2, 1] + 2 s[2, 2, 2] -
  2 s[3, 2, 1] + s[4, 1, 1] + 3 s[1, 1, 1, 1] + 2 s[2, 1, 1, 1] + s[3, 1, 1, 1] -
  2 s[1, 1, 1, 1, 1] - s[2, 1, 1, 1, 1] + s[1, 1, 1, 1, 1, 1], m[2] + 2 m[3] + 3 m[4] +
  2 m[5] + m[6] + 2 m[1, 1] + 2 m[2, 1] + 2 m[2, 2] + 2 m[3, 1] + 2 m[3, 2] + 2 m[3, 3]}
```

```
Sum[p[i] p[j], {i, 3}, {j, 3}];
PToAll[%, Vars → 2, Targets → {E, M, X}]
```

```
{p[1]^2 + 2 p[1] p[2] + p[2]^2 + 2 p[1] p[3] + 2 p[2] p[3] + p[3]^2,
 e[1]^2 + 2 e[1]^3 + 3 e[1]^4 + 2 e[1]^5 + e[1]^6 - 4 e[1] e[2] - 10 e[1]^2 e[2] -
  10 e[1]^3 e[2] - 6 e[1]^4 e[2] + 4 e[2]^2 + 12 e[1] e[2]^2 + 9 e[1]^2 e[2]^2,
 m[2] + 2 m[3] + 3 m[4] + 2 m[5] + m[6] + 2 m[1, 1] + 2 m[2, 1] + 2 m[2, 2] + 2 m[3, 1] +
  2 m[3, 2] + 2 m[3, 3], x[1]^2 + 2 x[1]^3 + 3 x[1]^4 + 2 x[1]^5 + x[1]^6 + 2 x[1] x[2] +
  2 x[1]^2 x[2] + 2 x[1]^3 x[2] + x[2]^2 + 2 x[1] x[2]^2 + 2 x[1]^2 x[2]^2 + 2 x[1]^3 x[2]^2 +
  2 x[2]^3 + 2 x[1] x[2]^3 + 2 x[1]^2 x[2]^3 + 2 x[1]^3 x[2]^3 + 3 x[2]^4 + 2 x[2]^5 + x[2]^6}
```

- **MTo[HEPSXAll]**

- **MToH**

```
MToH[<f>, <opts>]
Options[MToH]={DegreeList→{}, Vars→0}
```

If f is a symmetric function expressed as a linear combination of m functions, MToH returns the same function expressed as a linear combination of the appropriate h functions. Setting the option DegreeList to a list of the degrees of f saves *Mathematica* some computation, possibly resulting in faster output. Setting the option Vars to some positive integer n restricts the number of variables in the problem to n, resulting in the possible elimation of some terms in the answer.

```
MToH[m[1, 1, 1, 1] + 2 m[2, 2] - 3 m[3, 1] + 4 m[4]]
% // HToM
```

```
-7 h[1]^4 + 26 h[1]^2 h[2] - 7 h[2]^2 - 31 h[1] h[3] + 23 h[4]
```

```
4 m[4] + 2 m[2, 2] - 3 m[3, 1] + m[1, 1, 1, 1]
```

```
MToH[m[1, 1, 1, 1] + 2 m[2, 2] - 3 m[3, 1] + 4 m[4], Vars → 3]
% // HToM
```

```
16 h[1]^4 - 43 h[1]^2 h[2] + 16 h[2]^2 + 15 h[1] h[3]
```

```
4 m[4] + 2 m[2, 2] - 3 m[3, 1] + 24 m[1, 1, 1, 1]
```

- **Bigger Examples**

```
MToH[m[9, 1] + m[4, 3, 3] - 10 m[6, 3] + 14 m[13, 2]] // Timing
```

```
MToH[m[9, 1] + m[4, 3, 3] - 10 m[6, 3] + 14 m[13, 2], Vars → 5] // Timing
```

```
{36.037, 20 h[1]^9 - h[1]^10 - 28 h[1]^15 - 180 h[1]^7 h[2] + 11 h[1]^8 h[2] + 238 h[1]^13 h[2] +
  540 h[1]^5 h[2]^2 - 43 h[1]^6 h[2]^2 - 532 h[1]^11 h[2]^2 - 590 h[1]^3 h[2]^3 + 68 h[1]^4 h[2]^3 -
  700 h[1]^9 h[2]^3 + 150 h[1] h[2]^4 - 34 h[1]^2 h[2]^4 + 4620 h[1]^7 h[2]^4 + 2 h[2]^5 - 6902 h[1]^5 h[2]^5 +
  3864 h[1]^3 h[2]^6 - 518 h[1] h[2]^7 + 180 h[1]^6 h[3] - 11 h[1]^7 h[3] - 210 h[1]^12 h[3] -
  930 h[1]^4 h[2] h[3] + 81 h[1]^5 h[2] h[3] + 1008 h[1]^10 h[2] h[3] + 1170 h[1]^2 h[2]^2 h[3] -
  166 h[1]^3 h[2]^2 h[3] + 420 h[1]^8 h[2]^2 h[3] - 150 h[2]^3 h[3] + 67 h[1] h[2]^3 h[3] -
  8638 h[1]^6 h[2]^3 h[3] + 14546 h[1]^4 h[2]^4 h[3] - 7672 h[1]^2 h[2]^5 h[3] + 518 h[2]^6 h[3] +
  390 h[1]^3 h[3]^2 - 40 h[1]^4 h[3]^2 - 476 h[1]^9 h[3]^2 - 720 h[1] h[2] h[3]^2 + 141 h[1]^2 h[2] h[3]^2 +
  462 h[1]^7 h[2] h[3]^2 - 33 h[2]^2 h[3]^2 + 5068 h[1]^5 h[2]^2 h[3]^2 - 9912 h[1]^3 h[2]^3 h[3]^2 +
  4732 h[1] h[2]^4 h[3]^2 + 120 h[3]^3 - 42 h[1] h[3]^3 - 182 h[1]^6 h[3]^3 - 1442 h[1]^4 h[2] h[3]^3 +
  2464 h[1]^2 h[2]^2 h[3]^3 - 924 h[2]^3 h[3]^3 + 392 h[1]^3 h[3]^4 - 154 h[1] h[2] h[3]^4 -
  42 h[3]^5 - 150 h[1]^5 h[4] + 5 h[1]^6 h[4] + 600 h[1]^3 h[2] h[4] - 38 h[1]^4 h[2] h[4] +
  728 h[1]^9 h[2] h[4] - 450 h[1] h[2]^2 h[4] + 69 h[1]^2 h[2]^2 h[4] - 4914 h[1]^7 h[2]^2 h[4] -
  10 h[2]^3 h[4] + 10920 h[1]^5 h[2]^3 h[4] - 8918 h[1]^3 h[2]^4 h[4] + 1820 h[1] h[2]^5 h[4] -
  450 h[1]^2 h[3] h[4] + 37 h[1]^3 h[3] h[4] - 364 h[1]^8 h[3] h[4] + 360 h[2] h[3] h[4] -
  117 h[1] h[2] h[3] h[4] + 5572 h[1]^6 h[2] h[3] h[4] - 16282 h[1]^4 h[2]^2 h[3] h[4] +
  13076 h[1]^2 h[2]^3 h[3] h[4] - 1456 h[2]^4 h[3] h[4] + 45 h[3]^2 h[4] - 1540 h[1]^5 h[3]^2 h[4] +
  7896 h[1]^3 h[2] h[3]^2 h[4] - 5642 h[1] h[2]^2 h[3]^2 h[4] - 1652 h[1]^2 h[3]^3 h[4] +
  784 h[2] h[3]^3 h[4] + 90 h[1] h[4]^2 - 3 h[1]^2 h[4]^2 + 420 h[1]^7 h[4]^2 + 12 h[2] h[4]^2 -
  3038 h[1]^5 h[2] h[4]^2 + 5180 h[1]^3 h[2]^2 h[4]^2 - 1778 h[1] h[2]^3 h[4]^2 + 1890 h[1]^4 h[3] h[4]^2 -
  4802 h[1]^2 h[2] h[3] h[4]^2 + 994 h[2]^2 h[3] h[4]^2 + 1260 h[1] h[3]^2 h[4]^2 - 588 h[1]^3 h[4]^3 +
  476 h[1] h[2] h[4]^3 - 210 h[3] h[4]^3 + 150 h[1]^4 h[5] - h[1]^5 h[5] - 450 h[1]^2 h[2] h[5] +
  13 h[1]^3 h[2] h[5] - 364 h[1]^8 h[2] h[5] + 90 h[2]^2 h[5] - 18 h[1] h[2]^2 h[5] +
  2366 h[1]^6 h[2]^2 h[5] - 4914 h[1]^4 h[2]^3 h[5] + 3458 h[1]^2 h[2]^4 h[5] - 364 h[2]^5 h[5] +
  360 h[1] h[3] h[5] - 15 h[1]^2 h[3] h[5] + 21 h[2] h[3] h[5] - 1512 h[1]^5 h[2] h[3] h[5] +
  4956 h[1]^3 h[2]^2 h[3] h[5] - 3444 h[1] h[2]^3 h[3] h[5] + 28 h[1]^4 h[3]^2 h[5] -
  1344 h[1]^2 h[2] h[3]^2 h[5] + 686 h[2]^2 h[3]^2 h[5] + 420 h[1] h[3]^3 h[5] - 90 h[4] h[5] +
  3 h[1] h[4] h[5] - 420 h[1]^6 h[4] h[5] + 2828 h[1]^4 h[2] h[4] h[5] - 4158 h[1]^2 h[2]^2 h[4] h[5] +
  784 h[2]^3 h[4] h[5] - 1316 h[1]^3 h[3] h[4] h[5] + 2632 h[1] h[2] h[3] h[4] h[5] -
  630 h[3]^2 h[4] h[5] + 868 h[1]^2 h[4]^2 h[5] - 266 h[2] h[4]^2 h[5] - 182 h[1]^3 h[2] h[5]^2 +
  364 h[1] h[2]^2 h[5]^2 - 182 h[1]^2 h[3] h[5]^2 + 154 h[2] h[3] h[5]^2 - 210 h[1] h[4] h[5]^2 -
  70 h[5]^3 - 150 h[1]^3 h[6] + h[1]^4 h[6] + 360 h[1] h[2] h[6] - 15 h[1]^2 h[2] h[6] + 6 h[2]^2 h[6] -
  270 h[3] h[6] + 24 h[1] h[3] h[6] - 12 h[4] h[6] + 90 h[1]^2 h[7] + 2 h[1]^3 h[7] - 90 h[2] h[7] +
  12 h[1] h[2] h[7] - 21 h[3] h[7] - 90 h[1] h[8] - 9 h[1]^2 h[8] + 90 h[9] + 9 h[1] h[9]}
```

MToE

```
MToE[<f>, <opts>]
Options[MToE]={DegreeList→{}, Vars→0}
```

If f is a symmetric function expressed as a linear combination of m functions, MToE returns the same function expressed as a linear combination of the appropriate e functions. Setting the option DegreeList to a list of the degrees of f saves *Mathematica* some computation, possibly resulting in faster output. Setting the option Vars to some positive integer n restricts the number of variables in the problem to n, resulting in the possible elimation of some terms in the answer.

```
MToE[m[5] - 5 m[4, 1] + m[3, 2] - 2 m[3, 1, 1] + m[2, 2, 1]]
% // EToM
```

```
e[1]^5 - 10 e[1]^3 e[2] + 21 e[1] e[2]^2 + 6 e[1]^2 e[3] - 26 e[2] e[3] - 6 e[1] e[4] + 20 e[5]
```

```
m[5] + m[3, 2] - 5 m[4, 1] + m[2, 2, 1] - 2 m[3, 1, 1]
```

```
MToE[m[5] - 5 m[4, 1] + m[3, 2] - 2 m[3, 1, 1] + m[2, 2, 1], Vars → 3]
```

$$e[1]^{5}-10\,e[1]^{3}\,e[2]+21\,e[1]\,e[2]^{2}+6\,e[1]^{2}\,e[3]-26\,e[2]\,e[3]$$

■ Bigger Examples

```
MToE[m[15] - 3 m[6, 4, 2, 2, 1]] // Timing
```

$$\{61.641\,\text{Second},\ e[1]^{15}-15\,e[1]^{13}\,e[2]+90\,e[1]^{11}\,e[2]^{2}-275\,e[1]^{9}\,e[2]^{3}+450\,e[1]^{7}\,e[2]^{4}-$$
$$378\,e[1]^{5}\,e[2]^{5}+140\,e[1]^{3}\,e[2]^{6}-15\,e[1]\,e[2]^{7}+15\,e[1]^{12}\,e[3]-165\,e[1]^{10}\,e[2]\,e[3]+$$
$$675\,e[1]^{8}\,e[2]^{2}\,e[3]-1260\,e[1]^{6}\,e[2]^{3}\,e[3]+1050\,e[1]^{4}\,e[2]^{4}\,e[3]-315\,e[1]^{2}\,e[2]^{5}\,e[3]+$$
$$15\,e[2]^{6}\,e[3]+75\,e[1]^{9}\,e[3]^{2}-540\,e[1]^{7}\,e[2]\,e[3]^{2}+1260\,e[1]^{5}\,e[2]^{2}\,e[3]^{2}-$$
$$1050\,e[1]^{3}\,e[2]^{3}\,e[3]^{2}+225\,e[1]\,e[2]^{4}\,e[3]^{2}+140\,e[1]^{6}\,e[3]^{3}-525\,e[1]^{4}\,e[2]\,e[3]^{3}+$$
$$450\,e[1]^{2}\,e[2]^{2}\,e[3]^{3}-50\,e[2]^{3}\,e[3]^{3}+75\,e[1]^{3}\,e[3]^{4}-75\,e[1]\,e[2]\,e[3]^{4}+3\,e[3]^{5}-$$
$$15\,e[1]^{11}\,e[4]+150\,e[1]^{9}\,e[2]\,e[4]-540\,e[1]^{7}\,e[2]^{2}\,e[4]+840\,e[1]^{5}\,e[2]^{3}\,e[4]-$$
$$525\,e[1]^{3}\,e[2]^{4}\,e[4]+90\,e[1]\,e[2]^{5}\,e[4]-135\,e[1]^{8}\,e[3]\,e[4]+840\,e[1]^{6}\,e[2]\,e[3]\,e[4]-$$
$$1575\,e[1]^{4}\,e[2]^{2}\,e[3]\,e[4]+900\,e[1]^{2}\,e[2]^{3}\,e[3]\,e[4]-75\,e[2]^{4}\,e[3]\,e[4]-$$
$$315\,e[1]^{5}\,e[3]^{2}\,e[4]+900\,e[1]^{3}\,e[2]\,e[3]^{2}\,e[4]-450\,e[1]\,e[2]^{2}\,e[3]^{2}\,e[4]-$$
$$150\,e[1]^{2}\,e[3]^{3}\,e[4]+60\,e[2]\,e[3]^{3}\,e[4]+60\,e[1]^{7}\,e[4]^{2}-315\,e[1]^{5}\,e[2]\,e[4]^{2}+$$
$$450\,e[1]^{3}\,e[2]^{2}\,e[4]^{2}-150\,e[1]\,e[2]^{3}\,e[4]^{2}+225\,e[1]^{4}\,e[3]\,e[4]^{2}-450\,e[1]^{2}\,e[2]\,e[3]\,e[4]^{2}+$$
$$90\,e[2]^{2}\,e[3]\,e[4]^{2}+90\,e[1]\,e[3]^{2}\,e[4]^{2}-50\,e[1]^{3}\,e[4]^{3}+60\,e[1]\,e[2]\,e[4]^{3}-$$
$$15\,e[3]\,e[4]^{3}+15\,e[1]^{10}\,e[5]-135\,e[1]^{8}\,e[2]\,e[5]+420\,e[1]^{6}\,e[2]^{2}\,e[5]-$$
$$525\,e[1]^{4}\,e[2]^{3}\,e[5]+225\,e[1]^{2}\,e[2]^{4}\,e[5]-15\,e[2]^{5}\,e[5]+120\,e[1]^{7}\,e[3]\,e[5]-$$
$$630\,e[1]^{5}\,e[2]\,e[3]\,e[5]+900\,e[1]^{3}\,e[2]^{2}\,e[3]\,e[5]-300\,e[1]\,e[2]^{3}\,e[3]\,e[5]+$$
$$225\,e[1]^{4}\,e[3]^{2}\,e[5]-450\,e[1]^{2}\,e[2]\,e[3]^{2}\,e[5]+90\,e[2]^{2}\,e[3]^{2}\,e[5]+60\,e[1]\,e[3]^{3}\,e[5]-$$
$$105\,e[1]^{6}\,e[4]\,e[5]+450\,e[1]^{4}\,e[2]\,e[4]\,e[5]-453\,e[1]^{2}\,e[2]^{2}\,e[4]\,e[5]+$$
$$66\,e[2]^{3}\,e[4]\,e[5]-294\,e[1]^{3}\,e[3]\,e[4]\,e[5]+348\,e[1]\,e[2]\,e[3]\,e[4]\,e[5]-$$
$$36\,e[3]^{2}\,e[4]\,e[5]+84\,e[1]^{2}\,e[4]^{2}\,e[5]-51\,e[2]\,e[4]^{2}\,e[5]+45\,e[1]^{5}\,e[5]^{2}-$$
$$147\,e[1]^{3}\,e[2]\,e[5]^{2}+84\,e[1]\,e[2]^{2}\,e[5]^{2}+81\,e[1]^{2}\,e[3]\,e[5]^{2}-33\,e[2]\,e[3]\,e[5]^{2}-$$
$$9\,e[1]\,e[4]\,e[5]^{2}-25\,e[5]^{3}-15\,e[1]^{9}\,e[6]+120\,e[1]^{7}\,e[2]\,e[6]-315\,e[1]^{5}\,e[2]^{2}\,e[6]+$$
$$300\,e[1]^{3}\,e[2]^{3}\,e[6]-75\,e[1]\,e[2]^{4}\,e[6]-105\,e[1]^{6}\,e[3]\,e[6]+450\,e[1]^{4}\,e[2]\,e[3]\,e[6]-$$
$$441\,e[1]^{2}\,e[2]^{2}\,e[3]\,e[6]+42\,e[2]^{3}\,e[3]\,e[6]-168\,e[1]^{3}\,e[3]^{2}\,e[6]+216\,e[1]\,e[2]\,e[3]^{2}\,e[6]-$$
$$42\,e[3]^{3}\,e[6]+90\,e[1]^{5}\,e[4]\,e[6]-306\,e[1]^{3}\,e[2]\,e[4]\,e[6]+192\,e[1]\,e[2]^{2}\,e[4]\,e[6]+$$
$$216\,e[1]^{2}\,e[3]\,e[4]\,e[6]-96\,e[2]\,e[3]\,e[4]\,e[6]-81\,e[1]\,e[4]^{2}\,e[6]-66\,e[1]^{4}\,e[5]\,e[6]+$$
$$144\,e[1]^{2}\,e[2]\,e[5]\,e[6]-21\,e[2]^{2}\,e[5]\,e[6]-120\,e[1]\,e[3]\,e[5]\,e[6]+48\,e[4]\,e[5]\,e[6]+$$
$$21\,e[1]^{3}\,e[6]^{2}-27\,e[1]\,e[2]\,e[6]^{2}+69\,e[3]\,e[6]^{2}+15\,e[1]^{8}\,e[7]-105\,e[1]^{6}\,e[2]\,e[7]+$$
$$225\,e[1]^{4}\,e[2]^{2}\,e[7]-165\,e[1]^{2}\,e[2]^{3}\,e[7]+45\,e[2]^{4}\,e[7]+90\,e[1]^{5}\,e[3]\,e[7]-$$
$$264\,e[1]^{3}\,e[2]\,e[3]\,e[7]+108\,e[1]\,e[2]^{2}\,e[3]\,e[7]+72\,e[1]^{2}\,e[3]^{2}\,e[7]+$$
$$24\,e[2]\,e[3]^{2}\,e[7]-102\,e[1]^{4}\,e[4]\,e[7]+258\,e[1]^{2}\,e[2]\,e[4]\,e[7]-147\,e[2]^{2}\,e[4]\,e[7]-$$
$$126\,e[1]\,e[3]\,e[4]\,e[7]+87\,e[4]^{2}\,e[7]+78\,e[1]^{3}\,e[5]\,e[7]-36\,e[1]\,e[2]\,e[5]\,e[7]-$$
$$30\,e[3]\,e[5]\,e[7]-63\,e[1]^{2}\,e[6]\,e[7]-24\,e[2]\,e[6]\,e[7]+42\,e[1]\,e[7]^{2}-15\,e[1]^{7}\,e[8]+$$
$$90\,e[1]^{5}\,e[2]\,e[8]-135\,e[1]^{3}\,e[2]^{2}\,e[8]+30\,e[1]\,e[2]^{3}\,e[8]-72\,e[1]^{4}\,e[3]\,e[8]+$$
$$102\,e[1]^{2}\,e[2]\,e[3]\,e[8]+51\,e[2]^{2}\,e[3]\,e[8]+12\,e[1]\,e[3]^{2}\,e[8]+84\,e[1]^{3}\,e[4]\,e[8]-$$
$$48\,e[1]\,e[2]\,e[4]\,e[8]-90\,e[3]\,e[4]\,e[8]-117\,e[1]^{2}\,e[5]\,e[8]-18\,e[2]\,e[5]\,e[8]+$$
$$102\,e[1]\,e[6]\,e[8]+21\,e[7]\,e[8]+15\,e[1]^{6}\,e[9]-132\,e[1]^{4}\,e[2]\,e[9]+303\,e[1]^{2}\,e[2]^{2}\,e[9]-$$
$$129\,e[2]^{3}\,e[9]+114\,e[1]^{3}\,e[3]\,e[9]-378\,e[1]\,e[2]\,e[3]\,e[9]+96\,e[3]^{2}\,e[9]-$$
$$153\,e[1]^{2}\,e[4]\,e[9]+324\,e[2]\,e[4]\,e[9]+186\,e[1]\,e[5]\,e[9]-285\,e[6]\,e[9]+66\,e[1]^{5}\,e[10]-$$
$$288\,e[1]^{3}\,e[2]\,e[10]+219\,e[1]\,e[2]^{2}\,e[10]+315\,e[1]^{2}\,e[3]\,e[10]-246\,e[2]\,e[3]\,e[10]-$$
$$276\,e[1]\,e[4]\,e[10]+225\,e[5]\,e[10]-66\,e[1]^{4}\,e[11]+153\,e[1]^{2}\,e[2]\,e[11]+$$
$$75\,e[2]^{2}\,e[11]-180\,e[1]\,e[3]\,e[11]-3\,e[4]\,e[11]+165\,e[1]^{3}\,e[12]-384\,e[1]\,e[2]\,e[12]+$$
$$309\,e[3]\,e[12]-165\,e[1]^{2}\,e[13]+57\,e[2]\,e[13]+399\,e[1]\,e[14]-525\,e[15]\}$$

```
MToE[m[15] - 3 m[6, 4, 2, 2, 1], Vars → 6] // Timing
```

```
{26.094 Second, e[1]^15 - 15 e[1]^13 e[2] + 90 e[1]^11 e[2]^2 - 275 e[1]^9 e[2]^3 + 450 e[1]^7 e[2]^4 -
  378 e[1]^5 e[2]^5 + 140 e[1]^3 e[2]^6 - 15 e[1] e[2]^7 + 15 e[1]^12 e[3] - 165 e[1]^10 e[2] e[3] +
  675 e[1]^8 e[2]^2 e[3] - 1260 e[1]^6 e[2]^3 e[3] + 1050 e[1]^4 e[2]^4 e[3] - 315 e[1]^2 e[2]^5 e[3] +
  15 e[2]^6 e[3] + 75 e[1]^9 e[3]^2 - 540 e[1]^7 e[2] e[3]^2 + 1260 e[1]^5 e[2]^2 e[3]^2 -
  1050 e[1]^3 e[2]^3 e[3]^2 + 225 e[1] e[2]^4 e[3]^2 + 140 e[1]^6 e[3]^3 - 525 e[1]^4 e[2] e[3]^3 +
  450 e[1]^2 e[2]^2 e[3]^3 - 50 e[2]^3 e[3]^3 + 75 e[1]^3 e[3]^4 - 75 e[1] e[2] e[3]^4 + 3 e[3]^5 -
  15 e[1]^11 e[4] + 150 e[1]^9 e[2] e[4] - 540 e[1]^7 e[2]^2 e[4] + 840 e[1]^5 e[2]^3 e[4] -
  525 e[1]^3 e[2]^4 e[4] + 90 e[1] e[2]^5 e[4] - 135 e[1]^8 e[3] e[4] + 840 e[1]^6 e[2] e[3] e[4] -
  1575 e[1]^4 e[2]^2 e[3] e[4] + 900 e[1]^2 e[2]^3 e[3] e[4] - 75 e[2]^4 e[3] e[4] -
  315 e[1]^5 e[3]^2 e[4] + 900 e[1]^3 e[2] e[3]^2 e[4] - 450 e[1] e[2]^2 e[3]^2 e[4] -
  150 e[1]^2 e[3]^3 e[4] + 60 e[2] e[3]^3 e[4] + 60 e[1]^7 e[4]^2 - 315 e[1]^5 e[2] e[4]^2 +
  450 e[1]^3 e[2]^2 e[4]^2 - 150 e[1] e[2]^3 e[4]^2 + 225 e[1]^4 e[3] e[4]^2 - 450 e[1]^2 e[2] e[3] e[4]^2 +
  90 e[2]^2 e[3] e[4]^2 + 90 e[1] e[3]^2 e[4]^2 - 50 e[1]^3 e[4]^3 + 60 e[1] e[2] e[4]^3 -
  15 e[3] e[4]^3 + 15 e[1]^10 e[5] - 135 e[1]^8 e[2] e[5] + 420 e[1]^6 e[2]^2 e[5] -
  525 e[1]^4 e[2]^3 e[5] + 225 e[1]^2 e[2]^4 e[5] - 15 e[2]^5 e[5] + 120 e[1]^7 e[3] e[5] -
  630 e[1]^5 e[2] e[3] e[5] + 900 e[1]^3 e[2]^2 e[3] e[5] - 300 e[1] e[2]^3 e[3] e[5] +
  225 e[1]^4 e[3]^2 e[5] - 450 e[1]^2 e[2] e[3]^2 e[5] + 90 e[2]^2 e[3]^2 e[5] + 60 e[1] e[3]^3 e[5] -
  105 e[1]^6 e[4] e[5] + 450 e[1]^4 e[2] e[4] e[5] - 453 e[1]^2 e[2]^2 e[4] e[5] +
  66 e[2]^3 e[4] e[5] - 294 e[1]^3 e[3] e[4] e[5] + 348 e[1] e[2] e[3] e[4] e[5] -
  36 e[3]^2 e[4] e[5] + 84 e[1]^2 e[4]^2 e[5] - 51 e[2] e[4]^2 e[5] + 45 e[1]^5 e[5]^2 -
  147 e[1]^3 e[2] e[5]^2 + 84 e[1] e[2]^2 e[5]^2 + 81 e[1]^2 e[3] e[5]^2 - 33 e[2] e[3] e[5]^2 -
  9 e[1] e[4] e[5]^2 - 25 e[5]^3 - 15 e[1]^9 e[6] + 120 e[1]^7 e[2] e[6] - 315 e[1]^5 e[2]^2 e[6] +
  300 e[1]^3 e[2]^3 e[6] - 75 e[1] e[2]^4 e[6] - 105 e[1]^6 e[3] e[6] + 450 e[1]^4 e[2] e[3] e[6] -
  441 e[1]^2 e[2]^2 e[3] e[6] + 42 e[2]^3 e[3] e[6] - 168 e[1]^3 e[3]^2 e[6] +
  216 e[1] e[2] e[3]^2 e[6] - 42 e[3]^3 e[6] + 90 e[1]^5 e[4] e[6] - 306 e[1]^3 e[2] e[4] e[6] +
  192 e[1] e[2]^2 e[4] e[6] + 216 e[1]^2 e[3] e[4] e[6] - 96 e[2] e[3] e[4] e[6] -
  81 e[1] e[4]^2 e[6] - 66 e[1]^4 e[5] e[6] + 144 e[1]^2 e[2] e[5] e[6] - 21 e[2]^2 e[5] e[6] -
  120 e[1] e[3] e[5] e[6] + 48 e[4] e[5] e[6] + 21 e[1]^3 e[6]^2 - 27 e[1] e[2] e[6]^2 + 69 e[3] e[6]^2}
```

MToP

```
MToP[<f>, <opts>]
Options[MToP]={DegreeList→{}, Vars→0}
```

If f is a symmetric function expressed as a linear combination of m functions, MToP returns the same function expressed as a linear combination of the appropriate p functions. Setting the option DegreeList to a list of the degrees of f saves *Mathematica* some computation, possibly resulting in faster output. Setting the option Vars to some positive integer n restricts the number of variables in the problem to n, resulting in the possible elimation of some terms in the answer.

```
MToP[m[4, 2, 1] - 3 m[5, 2] + 2 m[3, 2, 2]]
% // PToM
```

```
p[2]^2 p[3] + p[1] p[2] p[4] - 2 p[3] p[4] - 6 p[2] p[5] - p[1] p[6] + 7 p[7]
```

```
-3 m[5, 2] + 2 m[3, 2, 2] + m[4, 2, 1]
```

```
MToP[m[4, 2, 1] - 3 m[5, 2] + 2 m[3, 2, 2], Vars → 3]
```

$$\frac{p[1]^7}{9} - \frac{7}{12} p[1]^5 p[2] + \frac{2}{3} p[1]^3 p[2]^2 + \frac{1}{4} p[1] p[2]^3 +$$
$$\frac{25}{36} p[1]^4 p[3] - \frac{8}{3} p[1]^2 p[2] p[3] - \frac{11}{12} p[2]^2 p[3] + \frac{22}{9} p[1] p[3]^2$$

■ MToS

```
MToS[<f>, <opts>]
Options[MToS]={DegreeList→{}, Vars→0}
```

If f is a symmetric function expressed as a linear combination of m functions, MToS returns the same function expressed as a linear combination of the appropriate s functions. Setting the option DegreeList to a list of the degrees of f saves *Mathematica* some computation, possibly resulting in faster output. Setting the option Vars to some positive integer n restricts the number of variables in the problem to n, resulting in the possible elimation of some terms in the answer.

```
MToS[m[5, 2] - 6 m[3, 1, 1] + 5 m[4, 2, 1, 1] - 9 m[7]]
% // SToM
```

```
-9 s[7] - s[4, 3] + s[5, 2] + 9 s[6, 1] + 6 s[2, 2, 1] - 6 s[3, 1, 1] - s[3, 2, 2] +
 s[3, 3, 1] - 10 s[5, 1, 1] + 6 s[2, 1, 1, 1] + s[2, 2, 2, 1] + 10 s[2, 2, 2, 2] -
 5 s[3, 2, 2, 1] - 5 s[3, 3, 1, 1] + 10 s[4, 1, 1, 1] + 5 s[4, 2, 1, 1] - 18 s[1, 1, 1, 1, 1] -
 s[2, 2, 1, 1, 1] - 5 s[2, 2, 2, 1, 1] - 10 s[3, 1, 1, 1, 1] + 10 s[3, 2, 1, 1, 1] -
 15 s[4, 1, 1, 1, 1] + 11 s[2, 1, 1, 1, 1, 1] - 5 s[2, 2, 1, 1, 1, 1] + 15 s[3, 1, 1, 1, 1, 1] -
 11 s[1, 1, 1, 1, 1, 1, 1] - 30 s[2, 1, 1, 1, 1, 1, 1] + 60 s[1, 1, 1, 1, 1, 1, 1, 1]
```

```
-9 m[7] + m[5, 2] - 6 m[3, 1, 1] + 5 m[4, 2, 1, 1]
```

```
MToS[m[5, 2] - 6 m[3, 1, 1] + 5 m[4, 2, 1, 1] - 9 m[7], Vars → 3]
```

```
-9 s[7] - s[4, 3] + s[5, 2] + 9 s[6, 1] +
 6 s[2, 2, 1] - 6 s[3, 1, 1] - s[3, 2, 2] + s[3, 3, 1] - 10 s[5, 1, 1]
```

■ MToX

```
MToX[<f>, <n>]
```

If f is a symmetric function expressed as a linear combination of m functions, MToX returns the same function expressed in the variables x[1], ..., x[n].

```
MToX[m[5, 2] + m[4], 3]
% // XToM
```

$$x[1]^4 + x[1]^5 x[2]^2 + x[2]^4 + x[1]^2 x[2]^5 + x[1]^5 x[3]^2 + x[2]^5 x[3]^2 + x[3]^4 + x[1]^2 x[3]^5 + x[2]^2 x[3]^5$$

```
m[4] + m[5, 2]
```

■ MToAll

```
MToAll[<f>, <opts>]
Options[MToAll]={Targets→{E,P,S,H, X}, Vars→0}
```

If f is a symmetric function expressed as a linear combination of m functions, MToAll returns the same function expressed as linear combinations of the appropriate functions as specified in Targets. Setting the option Vars to some positive integer n restricts the number of variables in the problem to n, resulting in the possible elimation of some terms in the answer. Note that the X's will only be displayed if Vars is specified.

```
MToAll[m[3, 2, 1]]
```

$$\{m[3, 2, 1], e[1]\, e[2]\, e[3] - 3 e[3]^2 - 3 e[1]^2 e[4] + 4 e[2]\, e[4] + 7 e[1]\, e[5] - 12 e[6],$$
$$p[1]\, p[2]\, p[3] - p[3]^2 - p[2]\, p[4] - p[1]\, p[5] + 2 p[6],$$
$$-2 s[2, 2, 2] + s[3, 2, 1] - 2 s[3, 1, 1, 1] + 4 s[2, 1, 1, 1, 1] - 6 s[1, 1, 1, 1, 1, 1],$$
$$-6 h[1]^6 + 34 h[1]^4 h[2] - 48 h[1]^2 h[2]^2 + 8 h[2]^3 - 30 h[1]^3 h[3] + 61 h[1]\, h[2]\, h[3] - 15 h[3]^2 + 21 h[1]^2 h[4] - 20 h[2]\, h[4] - 17 h[1]\, h[5] + 12 h[6]\}$$

```
MToAll[m[4, 1, 1] - 2 m[3, 2, 1], Vars → 3, Targets → {E, S, X}]
```

```
{-2 m[3, 2, 1] + m[4, 1, 1],
 e[1]^3 e[3] - 5 e[1] e[2] e[3] + 9 e[3]^2, 5 s[2, 2, 2] - 3 s[3, 2, 1] + s[4, 1, 1],
 x[1]^4 x[2] x[3] - 2 x[1]^3 x[2]^2 x[3] - 2 x[1]^2 x[2]^3 x[3] + x[1] x[2]^4 x[3] - 2 x[1]^3 x[2] x[3]^2 -
  2 x[1] x[2]^3 x[3]^2 - 2 x[1]^2 x[2] x[3]^3 - 2 x[1] x[2]^2 x[3]^3 + x[1] x[2] x[3]^4}
```

- **ETo[HMPSXAll]**

- **EToH**

```
EToH[<f>, <opts>]
Options[EToH]={DegreeList→{}, Vars→0}
```

If f is a symmetric function expressed as a linear combination of e functions, EToH returns the same function expressed as a linear combination of the appropriate h functions. Setting the option DegreeList to a list of the degrees of f saves *Mathematica* some computation, possibly resulting in faster output. Setting the option Vars to some positive integer n restricts the number of variables in the problem to n, resulting in the possible elimation of some terms in the answer.

```
EToH[e[3] e[2] e[1] + e[4] e[2]]
% // HToE
```

```
2 h[1]^6 - 7 h[1]^4 h[2] + 6 h[1]^2 h[2]^2 - h[2]^3 +
 3 h[1]^3 h[3] - 3 h[1] h[2] h[3] - h[1]^2 h[4] + h[2] h[4]
```

```
e[1] e[2] e[3] + e[2] e[4]
```

```
EToH[e[3] e[2] e[1] + e[4] e[2], Vars → 4]
```

```
2 h[1]^6 - 7 h[1]^4 h[2] + 6 h[1]^2 h[2]^2 - h[2]^3 +
 3 h[1]^3 h[3] - 3 h[1] h[2] h[3] - h[1]^2 h[4] + h[2] h[4]
```

```
EToH[e[3] e[2] e[1] + e[4] e[2], Vars → 3]
```

```
h[1]^6 - 3 h[1]^4 h[2] + 2 h[1]^2 h[2]^2 + h[1]^3 h[3] - h[1] h[2] h[3]
```

```
Product[1 + e[i], {i, 1, 3}] // Expand
EToH[%]
% // HToE
```

```
1 + e[1] + e[2] + e[1] e[2] + e[3] + e[1] e[3] + e[2] e[3] + e[1] e[2] e[3]
```

```
1 + h[1] + h[1]^2 + 2 h[1]^3 + h[1]^4 + h[1]^5 + h[1]^6 - h[2] - 3 h[1] h[2] -
 2 h[1]^2 h[2] - 3 h[1]^3 h[2] - 3 h[1]^4 h[2] + 2 h[1] h[2]^2 + 2 h[1]^2 h[2]^2 +
 h[3] + h[1] h[3] + h[1]^2 h[3] + h[1]^3 h[3] - h[2] h[3] - h[1] h[2] h[3]
```

```
1 + e[1] + e[2] + e[1] e[2] + e[3] + e[1] e[3] + e[2] e[3] + e[1] e[2] e[3]
```

- **EToM**

```
EToM[<f>, <opts>]
Options[EToM]={DegreeList→{}, Vars→0}
```

If f is a symmetric function expressed as a linear combination of e functions, EToM returns the same function expressed as a linear combination of the appropriate m functions. Setting the option DegreeList to a list of the degrees of f saves *Mathematica* some computation, possibly resulting in faster output. Setting the option Vars to some positive integer n restricts the number of variables in the problem to n, resulting in the possible elimation of some terms in the answer.

```
Product[1 + e[i], {i, 3}] // Expand
EToM[%]
% // MToE
```

1 + e[1] + e[2] + e[1] e[2] + e[3] + e[1] e[3] + e[2] e[3] + e[1] e[2] e[3]

1 + m[1] + m[1, 1] + m[2, 1] + 4 m[1, 1, 1] + m[2, 1, 1] + m[2, 2, 1] +
3 m[2, 2, 2] + m[3, 2, 1] + 4 m[1, 1, 1, 1] + 3 m[2, 1, 1, 1] + 8 m[2, 2, 1, 1] +
3 m[3, 1, 1, 1] + 10 m[1, 1, 1, 1, 1] + 22 m[2, 1, 1, 1, 1] + 60 m[1, 1, 1, 1, 1, 1]

1 + e[1] + e[2] + e[1] e[2] + e[3] + e[1] e[3] + e[2] e[3] + e[1] e[2] e[3]

```
Product[1 + e[i], {i, 3}] // Expand
EToM[%, Vars → 3]
```

1 + e[1] + e[2] + e[1] e[2] + e[3] + e[1] e[3] + e[2] e[3] + e[1] e[2] e[3]

1 + m[1] + m[1, 1] + m[2, 1] + 4 m[1, 1, 1] + m[2, 1, 1] + m[2, 2, 1] + 3 m[2, 2, 2] + m[3, 2, 1]

▪ EToP

```
EToP[<f>, <opts>]
Options[EToP]={DegreeList→{}, Vars→0}
```

If f is a symmetric function expressed as a linear combination of e functions, EToP returns the same function expressed as a linear combination of the appropriate p functions. Setting the option DegreeList to a list of the degrees of f saves *Mathematica* some computation, possibly resulting in faster output. Setting the option Vars to some positive integer n restricts the number of variables in the problem to n, resulting in the possible elimation of some terms in the answer.

```
EToP[e[3] e[2] e[1] - 2 e[2]^2 e[1]^2]
% // PToE
```

$$-\frac{5}{12} p[1]^6 + \frac{2}{3} p[1]^4 p[2] - \frac{1}{4} p[1]^2 p[2]^2 + \frac{1}{6} p[1]^3 p[3] - \frac{1}{6} p[1] p[2] p[3]$$

$$-2 e[1]^2 e[2]^2 + e[1] e[2] e[3]$$

```
EToP[e[3] e[2] e[1] - 2 e[2]^2 e[1]^2, Vars → 3]
```

$$-\frac{5}{12} p[1]^6 + \frac{2}{3} p[1]^4 p[2] - \frac{1}{4} p[1]^2 p[2]^2 + \frac{1}{6} p[1]^3 p[3] - \frac{1}{6} p[1] p[2] p[3]$$

```
EToP[e[3] e[2] e[1] - 2 e[2]^2 e[1]^2, Vars → 2]
```

$$-\frac{1}{2} p[1]^6 + p[1]^4 p[2] - \frac{1}{2} p[1]^2 p[2]^2$$

▪ EToS

```
EToS[<f>, <opts>]
Options[EToS]={DegreeList→{}, Vars→0}
```

If f is a symmetric function expressed as a linear combination of e functions, EToS returns the same function expressed as a linear combination of the appropriate s functions. Setting the option DegreeList to a list of the degrees of f saves *Mathematica* some computation, possibly resulting in faster output. Setting the option Vars to some positive integer n restricts the number of variables in the problem to n, resulting in the possible elimation of some terms in the answer.

```
Sum[e[i] e[j], {i, 1, 3}, {j, 1, 3}]
EToS[%]
% // SToE
```

$e[1]^2 + 2\,e[1]\,e[2] + e[2]^2 + 2\,e[1]\,e[3] + 2\,e[2]\,e[3] + e[3]^2$

s[2] + s[1, 1] + 2 s[2, 1] + s[2, 2] + 2 s[1, 1, 1] + 3 s[2, 1, 1] + 2 s[2, 2, 1] + s[2, 2, 2] + 3 s[1, 1, 1, 1] + 2 s[2, 1, 1, 1] + s[2, 2, 1, 1] + 2 s[1, 1, 1, 1, 1] + s[2, 1, 1, 1, 1] + s[1, 1, 1, 1, 1, 1]

$e[1]^2 + 2\,e[1]\,e[2] + e[2]^2 + 2\,e[1]\,e[3] + 2\,e[2]\,e[3] + e[3]^2$

```
Sum[e[i] e[j], {i, 1, 3}, {j, 1, 3}]
EToS[%, Vars → 3]
```

$e[1]^2 + 2\,e[1]\,e[2] + e[2]^2 + 2\,e[1]\,e[3] + 2\,e[2]\,e[3] + e[3]^2$

s[2] + s[1, 1] + 2 s[2, 1] + s[2, 2] + 2 s[1, 1, 1] + 3 s[2, 1, 1] + 2 s[2, 2, 1] + s[2, 2, 2]

■ EToX

```
EToX[<f>, <n>]
```

If f is a symmetric function expressed as a linear combination of e functions, EToX returns the same function expressed in the variables x[1], ..., x[n].

```
EToX[e[1]^2 e[2], 4]
% // XToE
```

$x[1]^3\,x[2] + 2\,x[1]^2\,x[2]^2 + x[1]\,x[2]^3 + x[1]^3\,x[3] + 5\,x[1]^2\,x[2]\,x[3] + 5\,x[1]\,x[2]^2\,x[3] + x[2]^3\,x[3] + 2\,x[1]^2\,x[3]^2 + 5\,x[1]\,x[2]\,x[3]^2 + 2\,x[2]^2\,x[3]^2 + x[1]\,x[3]^3 + x[2]\,x[3]^3 + x[1]^3\,x[4] + 5\,x[1]^2\,x[2]\,x[4] + 5\,x[1]\,x[2]^2\,x[4] + x[2]^3\,x[4] + 5\,x[1]^2\,x[3]\,x[4] + 12\,x[1]\,x[2]\,x[3]\,x[4] + 5\,x[2]^2\,x[3]\,x[4] + 5\,x[1]\,x[3]^2\,x[4] + 5\,x[2]\,x[3]^2\,x[4] + x[3]^3\,x[4] + 2\,x[1]^2\,x[4]^2 + 5\,x[1]\,x[2]\,x[4]^2 + 2\,x[2]^2\,x[4]^2 + 5\,x[1]\,x[3]\,x[4]^2 + 5\,x[2]\,x[3]\,x[4]^2 + 2\,x[3]^2\,x[4]^2 + x[1]\,x[4]^3 + x[2]\,x[4]^3 + x[3]\,x[4]^3$

$e[1]^2\,e[2]$

■ EToAll

```
EToAll[<f>, <opts>]
Options[EToAll]={Targets→{H,P,S,M, X}, Vars→0}
```

If f is a symmetric function expressed as a linear combination of e functions, EToAll returns the same function expressed as linear combinations of the appropriate functions as specified in Targets. Setting the option Vars to some positive integer n restricts the number of variables in the problem to n, resulting in the possible elimation of some terms in the answer. Note that the X's will only be displayed if Vars is specified.

```
Sum[e[i] e[j], {i, 3}, {j, 3}]
EToAll[%]
```

$$e[1]^2 + 2\,e[1]\,e[2] + e[2]^2 + 2\,e[1]\,e[3] + 2\,e[2]\,e[3] + e[3]^2$$

$$\Big\{e[1]^2 + 2\,e[1]\,e[2] + e[2]^2 + 2\,e[1]\,e[3] + 2\,e[2]\,e[3] + e[3]^2,$$
$$h[1]^2 + 2\,h[1]^3 + 3\,h[1]^4 + 2\,h[1]^5 + h[1]^6 - 2\,h[1]\,h[2] - 6\,h[1]^2\,h[2] - 6\,h[1]^3\,h[2] - 4\,h[1]^4\,h[2] + h[2]^2 + 4\,h[1]\,h[2]^2 + 4\,h[1]^2\,h[2]^2 + 2\,h[1]\,h[3] + 2\,h[1]^2\,h[3] + 2\,h[1]^3\,h[3] - 2\,h[2]\,h[3] - 4\,h[1]\,h[2]\,h[3] + h[3]^2,$$
$$p[1]^2 + p[1]^3 + \frac{7\,p[1]^4}{12} + \frac{p[1]^5}{6} + \frac{p[1]^6}{36} - p[1]\,p[2] - \frac{3}{2}\,p[1]^2\,p[2] - \frac{2}{3}\,p[1]^3\,p[2] - \frac{1}{6}\,p[1]^4\,p[2] + \frac{p[2]^2}{4} + \frac{1}{2}\,p[1]\,p[2]^2 + \frac{1}{4}\,p[1]^2\,p[2]^2 + \frac{2}{3}\,p[1]\,p[3] + \frac{1}{3}\,p[1]^2\,p[3] + \frac{1}{9}\,p[1]^3\,p[3] - \frac{1}{3}\,p[2]\,p[3] - \frac{1}{3}\,p[1]\,p[2]\,p[3] + \frac{p[3]^2}{9},$$
$$s[2] + s[1,1] + 2\,s[2,1] + s[2,2] + 2\,s[1,1,1] + 3\,s[2,1,1] + 2\,s[2,2,1] + s[2,2,2] + 3\,s[1,1,1,1] + 2\,s[2,1,1,1] + s[2,2,1,1] + 2\,s[1,1,1,1,1] + s[2,1,1,1,1] + s[1,1,1,1,1,1],$$
$$m[2] + 2\,m[1,1] + 2\,m[2,1] + m[2,2] + 6\,m[1,1,1] + 4\,m[2,1,1] + 2\,m[2,2,1] + m[2,2,2] + 14\,m[1,1,1,1] + 6\,m[2,1,1,1] + 2\,m[2,2,1,1] + 20\,m[1,1,1,1,1] + 6\,m[2,1,1,1,1] + 20\,m[1,1,1,1,1,1]\Big\}$$

```
Sum[e[i] e[j], {i, 3}, {j, 3}]
EToAll[%, Vars → 2, Targets → {P, M, X}]
```

$$e[1]^2 + 2\,e[1]\,e[2] + e[2]^2 + 2\,e[1]\,e[3] + 2\,e[2]\,e[3] + e[3]^2$$

$$\Big\{e[1]^2 + 2\,e[1]\,e[2] + e[2]^2 + 2\,e[1]\,e[3] + 2\,e[2]\,e[3] + e[3]^2,$$
$$p[1]^2 + p[1]^3 + \frac{p[1]^4}{4} - p[1]\,p[2] - \frac{1}{2}\,p[1]^2\,p[2] + \frac{p[2]^2}{4},\ m[2] + 2\,m[1,1] + 2\,m[2,1] + m[2,2],$$
$$x[1]^2 + 2\,x[1]\,x[2] + 2\,x[1]^2\,x[2] + x[2]^2 + 2\,x[1]\,x[2]^2 + x[1]^2\,x[2]^2\Big\}$$

- **HTo[EMPSXAll]**
- **HRestrictionRules**

HRestrictionRules[<**deg**>, <**var**>]

HRestrictionRules[deg, var] creates a list of rules that express h functions of degree deg in terms of other h funtions of degree deg when the number of variables is restricted to var. The basis used for this is h[lambda] functions where lambda[[1]]≤var (note that this is NOT the same basis generated by HList[deg, var]).

```
HRestrictionRules[4, 2]
HList[4, 2]
```

$$\{h[4] \to -h[1]^4 + h[1]^2\,h[2] + h[2]^2,\ h[1]\,h[3] \to -h[1]^4 + 2\,h[1]^2\,h[2],\ h[2]^2 \to h[2]^2,\ h[1]^2\,h[2] \to h[1]^2\,h[2],\ h[1]^4 \to h[1]^4\}$$

$$\{h[4],\ h[1]\,h[3],\ h[2]^2\}$$

```
HRestrictionRules[6, 1]
HList[6, 1]
```

```
{h[6] → h[1]^6, h[1] h[5] → h[1]^6, h[2] h[4] → h[1]^6,
 h[1]^2 h[4] → h[1]^6, h[3]^2 → h[1]^6, h[1] h[2] h[3] → h[1]^6, h[1]^3 h[3] → h[1]^6,
 h[2]^3 → h[1]^6, h[1]^2 h[2]^2 → h[1]^6, h[1]^4 h[2] → h[1]^6, h[1]^6 → h[1]^6}
```

```
{h[6]}
```

HToE

```
HToE[<f>, <opts>]
Options[HToE]={DegreeList→{}, Vars→0}
```

If f is a symmetric function expressed as a linear combination of h functions, HToE returns the same function expressed as a linear combination of the appropriate e functions. Setting the option DegreeList to a list of the degrees of f saves *Mathematica* some computation, possibly resulting in faster output. Setting the option Vars to some positive integer n restricts the number of variables in the problem to n, resulting in the possible elimation of some terms in the answer.

```
1 + Sum[h[i], {i, 1, 6}]
HToE[%]
% // EToH
```

```
1 + h[1] + h[2] + h[3] + h[4] + h[5] + h[6]
```

```
1 + e[1] + e[1]^2 + e[1]^3 + e[1]^4 + e[1]^5 + e[1]^6 - e[2] - 2 e[1] e[2] - 3 e[1]^2 e[2] -
 4 e[1]^3 e[2] - 5 e[1]^4 e[2] + e[2]^2 + 3 e[1] e[2]^2 + 6 e[1]^2 e[2]^2 - e[2]^3 + e[3] +
 2 e[1] e[3] + 3 e[1]^2 e[3] + 4 e[1]^3 e[3] - 2 e[2] e[3] - 6 e[1] e[2] e[3] +
 e[3]^2 - e[4] - 2 e[1] e[4] - 3 e[1]^2 e[4] + 2 e[2] e[4] + e[5] + 2 e[1] e[5] - e[6]
```

```
1 + h[1] + h[2] + h[3] + h[4] + h[5] + h[6]
```

```
1 + Sum[h[i], {i, 1, 6}]
HToE[%, Vars → 3]
```

```
1 + h[1] + h[2] + h[3] + h[4] + h[5] + h[6]
```

```
1 + e[1] + e[1]^2 + e[1]^3 + e[1]^4 + e[1]^5 + e[1]^6 - e[2] - 2 e[1] e[2] - 3 e[1]^2 e[2] -
 4 e[1]^3 e[2] - 5 e[1]^4 e[2] + e[2]^2 + 3 e[1] e[2]^2 + 6 e[1]^2 e[2]^2 - e[2]^3 + e[3] +
 2 e[1] e[3] + 3 e[1]^2 e[3] + 4 e[1]^3 e[3] - 2 e[2] e[3] - 6 e[1] e[2] e[3] + e[3]^2
```

HToM

```
HToM[<f>, <opts>]
Options[HToM]={DegreeList→{},Vars→0}
```

If f is a symmetric function expressed as a linear combination of h functions, HToM returns the same function expressed as a linear combination of the appropriate m functions. Setting the option DegreeList to a list of the degrees of f saves *Mathematica* some computation, possibly resulting in faster output. Setting the option Vars to some positive integer n restricts the number of variables in the problem to n, resulting in the possible elimation of some terms in the answer.

```
1 + Sum[h[i], {i, 1, 6}]
HToM[%]
% // MToH
```

1 + h[1] + h[2] + h[3] + h[4] + h[5] + h[6]

1 + m[1] + m[2] + m[3] + m[4] + m[5] + m[6] + m[1, 1] + m[2, 1] + m[2, 2] + m[3, 1] + m[3, 2] + m[3, 3] + m[4, 1] + m[4, 2] + m[5, 1] + m[1, 1, 1] + m[2, 1, 1] + m[2, 2, 1] + m[2, 2, 2] + m[3, 1, 1] + m[3, 2, 1] + m[4, 1, 1] + m[1, 1, 1, 1] + m[2, 1, 1, 1] + m[2, 2, 1, 1] + m[3, 1, 1, 1] + m[1, 1, 1, 1, 1] + m[2, 1, 1, 1, 1] + m[1, 1, 1, 1, 1, 1]

1 + h[1] + h[2] + h[3] + h[4] + h[5] + h[6]

```
1 + Sum[h[i], {i, 1, 6}]
HToM[%, Vars → 3]
```

1 + h[1] + h[2] + h[3] + h[4] + h[5] + h[6]

1 + m[1] + m[2] + m[3] + m[4] + m[5] + m[6] + m[1, 1] + m[2, 1] + m[2, 2] + m[3, 1] + m[3, 2] + m[3, 3] + m[4, 1] + m[4, 2] + m[5, 1] + m[1, 1, 1] + m[2, 1, 1] + m[2, 2, 1] + m[2, 2, 2] + m[3, 1, 1] + m[3, 2, 1] + m[4, 1, 1]

HToP

```
HToP[<f>, <opts>]
Options[HToP]={DegreeList→{}, Vars→0}
```

If f is a symmetric function expressed as a linear combination of h functions, HToP returns the same function expressed as a linear combination of the appropriate p functions. Setting the option DegreeList to a list of the degrees of f saves *Mathematica* some computation, possibly resulting in faster output. Setting the option Vars to some positive integer n restricts the number of variables in the problem to n, resulting in the possible elimation of some terms in the answer.

```
HToP[h[3] h[2] h[1] - 2 h[2]^2 h[1]^2]
% // PToH
```

$$-\frac{5}{12} p[1]^6 - \frac{2}{3} p[1]^4 p[2] - \frac{1}{4} p[1]^2 p[2]^2 + \frac{1}{6} p[1]^3 p[3] + \frac{1}{6} p[1] p[2] p[3]$$

$$-2 h[1]^2 h[2]^2 + h[1] h[2] h[3]$$

```
HToP[h[3] h[2] h[1] - 2 h[2]^2 h[1]^2, Vars → 3]
```

$$-\frac{5}{12} p[1]^6 - \frac{2}{3} p[1]^4 p[2] - \frac{1}{4} p[1]^2 p[2]^2 + \frac{1}{6} p[1]^3 p[3] + \frac{1}{6} p[1] p[2] p[3]$$

```
HToP[h[3] h[2] h[1] - 2 h[2]^2 h[1]^2, Vars → 2]
```

$$-\frac{1}{2} p[1]^6 - \frac{1}{2} p[1]^4 p[2]$$

HToS

```
HToS[<f>, <opts>]
Options[HToS]={DegreeList→{}, Vars→0}
```

If f is a symmetric function expressed as a linear combination of h functions, HToS returns the same function expressed as a linear combination of the appropriate s functions. Setting the option DegreeList to a list of the degrees of f saves *Mathematica* some computation, possibly resulting in faster output. Setting the option Vars to some positive integer n restricts the number of variables in the problem to n, resulting in the possible elimation of some terms in the answer.

```
1 + Sum[h[i] h[j], {i, 3}, {j, 3}]
HToS[%]
% // SToH
```

$1 + h[1]^2 + 2\, h[1]\, h[2] + h[2]^2 + 2\, h[1]\, h[3] + 2\, h[2]\, h[3] + h[3]^2$

$1 + s[2] + 2\, s[3] + 3\, s[4] + 2\, s[5] + s[6] + s[1, 1] + 2\, s[2, 1] + s[2, 2] + 3\, s[3, 1] + 2\, s[3, 2] + s[3, 3] + 2\, s[4, 1] + s[4, 2] + s[5, 1]$

$1 + h[1]^2 + 2\, h[1]\, h[2] + h[2]^2 + 2\, h[1]\, h[3] + 2\, h[2]\, h[3] + h[3]^2$

```
1 + Sum[h[i] h[j], {i, 3}, {j, 3}]
HToS[%, Vars → 1]
```

$1 + h[1]^2 + 2\, h[1]\, h[2] + h[2]^2 + 2\, h[1]\, h[3] + 2\, h[2]\, h[3] + h[3]^2$

$1 + s[2] + 2\, s[3] + 3\, s[4] + 2\, s[5] + s[6]$

HToX

```
HToX[<f>, <n>]
```

If f is a symmetric function expressed as a linear combination of h functions, HToX returns the same function expressed in the variables x[1], ..., x[n].

```
HToX[h[1] h[2], 3]
```

$x[1]^3 + 2\, x[1]^2\, x[2] + 2\, x[1]\, x[2]^2 + x[2]^3 + 2\, x[1]^2\, x[3] + 3\, x[1]\, x[2]\, x[3] + 2\, x[2]^2\, x[3] + 2\, x[1]\, x[3]^2 + 2\, x[2]\, x[3]^2 + x[3]^3$

HToAll

```
HToAll[<f>, <opts>]
Options[HToAll]={Targets→{E,P,S,M, X}, Vars→0}
```

If f is a symmetric function expressed as a linear combination of h functions, HToAll returns the same function expressed as linear combinations of the appropriate functions as specified in Targets. Setting the option Vars to some positive integer n restricts the number of variables in the problem to n, resulting in the possible elimation of some terms in the answer. Note that the X's will only be displayed if Vars is specified.

```
1 + Sum[h[i] h[j], {i, 3}, {j, 3}]
HToAll[%]
```

$$1+h[1]^2+2\,h[1]\,h[2]+h[2]^2+2\,h[1]\,h[3]+2\,h[2]\,h[3]+h[3]^2$$

$$\begin{aligned}
\{&1+h[1]^2+2\,h[1]\,h[2]+h[2]^2+2\,h[1]\,h[3]+2\,h[2]\,h[3]+h[3]^2,\\
&1+e[1]^2+2\,e[1]^3+3\,e[1]^4+2\,e[1]^5+e[1]^6-2\,e[1]\,e[2]-6\,e[1]^2\,e[2]-\\
&\quad 6\,e[1]^3\,e[2]-4\,e[1]^4\,e[2]+e[2]^2+4\,e[1]\,e[2]^2+4\,e[1]^2\,e[2]^2+2\,e[1]\,e[3]+\\
&\quad 2\,e[1]^2\,e[3]+2\,e[1]^3\,e[3]-2\,e[2]\,e[3]-4\,e[1]\,e[2]\,e[3]+e[3]^2,\\
&1+p[1]^2+p[1]^3+\frac{7\,p[1]^4}{12}+\frac{p[1]^5}{6}+\frac{p[1]^6}{36}+p[1]\,p[2]+\frac{3}{2}\,p[1]^2\,p[2]+\frac{2}{3}\,p[1]^3\,p[2]+\\
&\quad \frac{1}{6}\,p[1]^4\,p[2]+\frac{p[2]^2}{4}+\frac{1}{2}\,p[1]\,p[2]^2+\frac{1}{4}\,p[1]^2\,p[2]^2+\frac{2}{3}\,p[1]\,p[3]+\\
&\quad \frac{1}{3}\,p[1]^2\,p[3]+\frac{1}{9}\,p[1]^3\,p[3]+\frac{1}{3}\,p[2]\,p[3]+\frac{1}{3}\,p[1]\,p[2]\,p[3]+\frac{p[3]^2}{9},\\
&1+s[2]+2\,s[3]+3\,s[4]+2\,s[5]+s[6]+s[1,1]+2\,s[2,1]+s[2,2]+\\
&\quad 3\,s[3,1]+2\,s[3,2]+s[3,3]+2\,s[4,1]+s[4,2]+s[5,1],\\
&1+m[2]+2\,m[3]+3\,m[4]+2\,m[5]+m[6]+2\,m[1,1]+4\,m[2,1]+7\,m[2,2]+\\
&\quad 6\,m[3,1]+6\,m[3,2]+4\,m[3,3]+4\,m[4,1]+3\,m[4,2]+2\,m[5,1]+6\,m[1,1,1]+\\
&\quad 10\,m[2,1,1]+10\,m[2,2,1]+7\,m[2,2,2]+8\,m[3,1,1]+6\,m[3,2,1]+\\
&\quad 4\,m[4,1,1]+14\,m[1,1,1,1]+14\,m[2,1,1,1]+10\,m[2,2,1,1]+\\
&\quad 8\,m[3,1,1,1]+20\,m[1,1,1,1,1]+14\,m[2,1,1,1,1]+20\,m[1,1,1,1,1,1]\}
\end{aligned}$$

```
1 + Sum[h[i] h[j], {i, 3}, {j, 3}]
HToAll[%, Vars → 2, Targets → {E, M, X}]
```

$$1+h[1]^2+2\,h[1]\,h[2]+h[2]^2+2\,h[1]\,h[3]+2\,h[2]\,h[3]+h[3]^2$$

$$\begin{aligned}
\{&1+h[1]^2+2\,h[1]\,h[2]+h[2]^2+2\,h[1]\,h[3]+2\,h[2]\,h[3]+h[3]^2,\\
&1+e[1]^2+2\,e[1]^3+3\,e[1]^4+2\,e[1]^5+e[1]^6-2\,e[1]\,e[2]-6\,e[1]^2\,e[2]-\\
&\quad 6\,e[1]^3\,e[2]-4\,e[1]^4\,e[2]+e[2]^2+4\,e[1]\,e[2]^2+4\,e[1]^2\,e[2]^2,\\
&1+m[2]+2\,m[3]+3\,m[4]+2\,m[5]+m[6]+2\,m[1,1]+4\,m[2,1]+7\,m[2,2]+\\
&\quad 6\,m[3,1]+6\,m[3,2]+4\,m[3,3]+4\,m[4,1]+3\,m[4,2]+2\,m[5,1],\\
&1+x[1]^2+2\,x[1]^3+3\,x[1]^4+2\,x[1]^5+x[1]^6+2\,x[1]\,x[2]+4\,x[1]^2\,x[2]+\\
&\quad 6\,x[1]^3\,x[2]+4\,x[1]^4\,x[2]+2\,x[1]^5\,x[2]+x[2]^2+4\,x[1]\,x[2]^2+7\,x[1]^2\,x[2]^2+\\
&\quad 6\,x[1]^3\,x[2]^2+3\,x[1]^4\,x[2]^2+2\,x[2]^3+6\,x[1]\,x[2]^3+6\,x[1]^2\,x[2]^3+4\,x[1]^3\,x[2]^3+\\
&\quad 3\,x[2]^4+4\,x[1]\,x[2]^4+3\,x[1]^2\,x[2]^4+2\,x[2]^5+2\,x[1]\,x[2]^5+x[2]^6\}
\end{aligned}$$

Miscellaneous

■ LR

LRProduct[<μ>,<ν>] or s[<μ>]s[<ν>]

Computes the expansion of s[μ]s[ν], expressed as a linear combination of s[λ]'s.

$s[<\mu>]^{<n>}$

Computes the expression s[μ]^n

s[<μ>]s[<ν>]s[<ψ>]... //Expand

Computes the product s[μ] s[ν] s[φ]... Products of more than two terms need to be followed by "//Expand" in order to expand completely. There is no need to type //Expand for $s[<\mu>]^n$

```
LRProduct[{2, 1}, {2, 1}]
```

```
s[3, 3] + s[4, 2] + s[2, 2, 2] + 2 s[3, 2, 1] + s[4, 1, 1] + s[2, 2, 1, 1] + s[3, 1, 1, 1]
```

```
s[2, 1] s[2, 1]
```

```
s[3, 3] + s[4, 2] + s[2, 2, 2] + 2 s[3, 2, 1] + s[4, 1, 1] + s[2, 2, 1, 1] + s[3, 1, 1, 1]
```

```
s[2, 1] s[2, 1] s[2, 1]
s[2, 1] s[2, 1] s[2, 1] // Expand
```

```
s[2, 1] (s[3, 3] + s[4, 2] + s[2, 2, 2] + 2 s[3, 2, 1] + s[4, 1, 1] + s[2, 2, 1, 1] + s[3, 1, 1, 1])
```

```
2 s[5, 4] + s[6, 3] + 2 s[3, 3, 3] + 8 s[4, 3, 2] + 4 s[4, 4, 1] +
 4 s[5, 2, 2] + 6 s[5, 3, 1] + 2 s[6, 2, 1] + 4 s[3, 2, 2, 2] + 8 s[3, 3, 2, 1] +
 9 s[4, 2, 2, 1] + 9 s[4, 3, 1, 1] + 6 s[5, 2, 1, 1] + s[6, 1, 1, 1] +
 2 s[2, 2, 2, 2, 1] + 6 s[3, 2, 2, 1, 1] + 4 s[3, 3, 1, 1, 1] + 6 s[4, 2, 1, 1, 1] +
 2 s[5, 1, 1, 1, 1] + s[2, 2, 2, 1, 1, 1] + 2 s[3, 2, 1, 1, 1, 1] + s[4, 1, 1, 1, 1, 1]
```

```
s[1] s[1] s[1] s[1] s[1] s[1] s[1] s[1] s[1] // Expand // Timing
```

```
{31.45, s[9] + 42 s[5, 4] + 48 s[6, 3] + 27 s[7, 2] + 8 s[8, 1] + 42 s[3, 3, 3] + 168 s[4, 3, 2] +
  84 s[4, 4, 1] + 120 s[5, 2, 2] + 162 s[5, 3, 1] + 105 s[6, 2, 1] + 28 s[7, 1, 1] +
  84 s[3, 2, 2, 2] + 168 s[3, 3, 2, 1] + 216 s[4, 2, 2, 1] + 216 s[4, 3, 1, 1] +
  189 s[5, 2, 1, 1] + 56 s[6, 1, 1, 1] + 42 s[2, 2, 2, 2, 1] + 162 s[3, 2, 2, 1, 1] +
  120 s[3, 3, 1, 1, 1] + 189 s[4, 2, 1, 1, 1] + 70 s[5, 1, 1, 1, 1] + 48 s[2, 2, 2, 1, 1, 1] +
  105 s[3, 2, 1, 1, 1, 1] + 56 s[4, 1, 1, 1, 1, 1] + 27 s[2, 2, 1, 1, 1, 1, 1] +
  28 s[3, 1, 1, 1, 1, 1, 1] + 8 s[2, 1, 1, 1, 1, 1, 1, 1] + s[1, 1, 1, 1, 1, 1, 1, 1, 1]}
```

```
s[1]^9 // Timing
```

```
{31.372, s[9] + 42 s[5, 4] + 48 s[6, 3] + 27 s[7, 2] + 8 s[8, 1] + 42 s[3, 3, 3] + 168 s[4, 3, 2] +
  84 s[4, 4, 1] + 120 s[5, 2, 2] + 162 s[5, 3, 1] + 105 s[6, 2, 1] + 28 s[7, 1, 1] +
  84 s[3, 2, 2, 2] + 168 s[3, 3, 2, 1] + 216 s[4, 2, 2, 1] + 216 s[4, 3, 1, 1] +
  189 s[5, 2, 1, 1] + 56 s[6, 1, 1, 1] + 42 s[2, 2, 2, 2, 1] + 162 s[3, 2, 2, 1, 1] +
  120 s[3, 3, 1, 1, 1] + 189 s[4, 2, 1, 1, 1] + 70 s[5, 1, 1, 1, 1] + 48 s[2, 2, 2, 1, 1, 1] +
  105 s[3, 2, 1, 1, 1, 1] + 56 s[4, 1, 1, 1, 1, 1] + 27 s[2, 2, 1, 1, 1, 1, 1] +
  28 s[3, 1, 1, 1, 1, 1, 1] + 8 s[2, 1, 1, 1, 1, 1, 1, 1] + s[1, 1, 1, 1, 1, 1, 1, 1, 1]}
```

Finally, a "check" that the calculation is correct:

```
s[2, 2] s[3, 1] s[1] // Expand
aaa = SToX[%, 7];
bbb = (SToX[s[2, 2], 7]) (SToX[s[3, 1], 7]) (SToX[s[1], 7]) // Expand;
aaa == bbb
```

```
s[5, 4] + s[6, 3] + s[3, 3, 3] + 3 s[4, 3, 2] + s[4, 4, 1] + 2 s[5, 2, 2] +
 3 s[5, 3, 1] + s[6, 2, 1] + s[3, 2, 2, 2] + 2 s[3, 3, 2, 1] + 3 s[4, 2, 2, 1] +
 2 s[4, 3, 1, 1] + 2 s[5, 2, 1, 1] + s[3, 2, 2, 1, 1] + s[4, 2, 1, 1, 1]
```

```
True
```

Bigger examples

```
LRProduct[{4, 3, 2, 1}, {3, 2, 1}]
```

```
s[4, 4, 4, 4] + 3 s[5, 4, 4, 3] + 2 s[5, 5, 3, 3] + 3 s[5, 5, 4, 2] + s[5, 5, 5, 1] +
 3 s[6, 4, 3, 3] + 3 s[6, 4, 4, 2] + 4 s[6, 5, 3, 2] + 2 s[6, 5, 4, 1] + s[6, 6, 2, 2] +
 s[6, 6, 3, 1] + s[7, 3, 3, 3] + 2 s[7, 4, 3, 2] + s[7, 4, 4, 1] + s[7, 5, 2, 2] +
 s[7, 5, 3, 1] + s[4, 3, 3, 3, 3] + 3 s[4, 4, 3, 3, 2] + 2 s[4, 4, 4, 2, 2] + 3 s[4, 4, 4, 3, 1] +
 3 s[5, 3, 3, 3, 2] + 6 s[5, 4, 3, 2, 2] + 6 s[5, 4, 3, 3, 1] + 6 s[5, 4, 4, 2, 1] +
 2 s[5, 5, 2, 2, 2] + 6 s[5, 5, 3, 2, 1] + 3 s[5, 5, 4, 1, 1] + 3 s[6, 3, 3, 2, 2] +
 3 s[6, 3, 3, 3, 1] + 3 s[6, 4, 2, 2, 2] + 8 s[6, 4, 3, 2, 1] + 3 s[6, 4, 4, 1, 1] +
 4 s[6, 5, 2, 2, 1] + 4 s[6, 5, 3, 1, 1] + s[6, 6, 2, 1, 1] + s[7, 3, 2, 2, 2] + 2 s[7, 3, 3, 2, 1] +
 2 s[7, 4, 2, 2, 1] + 2 s[7, 4, 3, 1, 1] + s[7, 5, 2, 1, 1] + s[4, 3, 3, 2, 2, 2] +
 2 s[4, 3, 3, 3, 2, 1] + s[4, 4, 2, 2, 2, 2] + 4 s[4, 4, 3, 2, 2, 1] + 3 s[4, 4, 3, 3, 1, 1] +
 3 s[4, 4, 4, 2, 1, 1] + s[5, 3, 2, 2, 2, 2] + 4 s[5, 3, 3, 2, 2, 1] + 3 s[5, 3, 3, 3, 1, 1] +
 4 s[5, 4, 2, 2, 2, 1] + 8 s[5, 4, 3, 2, 1, 1] + 3 s[5, 4, 4, 1, 1, 1] + 3 s[5, 5, 2, 2, 1, 1] +
 3 s[5, 5, 3, 1, 1, 1] + 2 s[6, 3, 2, 2, 2, 1] + 4 s[6, 3, 3, 2, 1, 1] + 4 s[6, 4, 2, 2, 1, 1] +
 4 s[6, 4, 3, 1, 1, 1] + 2 s[6, 5, 2, 1, 1, 1] + s[7, 3, 2, 2, 1, 1] + s[7, 3, 3, 1, 1, 1] +
 s[7, 4, 2, 1, 1, 1] + s[4, 3, 3, 2, 2, 1, 1] + s[4, 3, 3, 3, 1, 1, 1] + s[4, 4, 2, 2, 2, 1, 1] +
 2 s[4, 4, 3, 2, 1, 1, 1] + s[4, 4, 4, 1, 1, 1, 1] + s[5, 3, 2, 2, 2, 1, 1] +
 2 s[5, 3, 3, 2, 1, 1, 1] + 2 s[5, 4, 2, 2, 1, 1, 1] + 2 s[5, 4, 3, 1, 1, 1, 1] +
 s[5, 5, 2, 1, 1, 1, 1] + s[6, 3, 2, 2, 1, 1, 1] + s[6, 3, 3, 1, 1, 1, 1] + s[6, 4, 2, 1, 1, 1, 1]
```

Pieri

Pieri[<λ>,<k>]

Pieri[λ, k] computes the list of partitions that are "k-Pieri" over lambda (the partitions that come about by adding k boxes to the Ferrer's diagram of λ with no more than one box added per column). Corresponds to the special case of the LR rule where one multiplies by s[n].

```
Pieri[{3, 3, 3}, 6]
```

```
{{9, 3, 3}, {6, 3, 3, 3}, {7, 3, 3, 2}, {8, 3, 3, 1}}
```

```
LRProduct[{3, 3, 3}, {6}]
```

```
s[9, 3, 3] + s[6, 3, 3, 3] + s[7, 3, 3, 2] + s[8, 3, 3, 1]
```

```
LRProduct[{3, 3, 3}, {1}]
```

```
s[4, 3, 3] + s[3, 3, 3, 1]
```

```
Pieri[{2, 1}, 1]
```

```
{{2, 2}, {3, 1}, {2, 1, 1}}
```

```
SMult[{2, 1}, 1]
```

```
s[2, 2] + s[3, 1] + s[2, 1, 1]
```