Augmented reality Test I MAM4 - February 2017

Name: _____ Signature: _____

- Calculator and documents are allowed.
- Test duration : 30 minutes.
- You have to answer the questions directly in this sheet.
- All communication between students is forbidden.
- You should turn off your mobile phone and put it in your backpack/handbag.

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1. A shear transformation in 2D is a function that takes a point $\begin{bmatrix} x \\ y \end{bmatrix}$ to a point $\begin{bmatrix} x+ay \\ y+bx \end{bmatrix}$ for $a \neq 0$ or $b \neq 0$.
   (a) Is this transformation nonlinear? Affine? And/or linear?

   (b) If the transformation is linear, give its matrix representation for Cartesian coordinates. If it is nonlinear explain why.

   (c) Draw the points $\begin{bmatrix} 0 \\ 0 \end{bmatrix}$, $\begin{bmatrix} 0 \\ 1 \end{bmatrix}$, $\begin{bmatrix} 1 \\ 0 \end{bmatrix}$, $\begin{bmatrix} 1 \\ 1 \end{bmatrix}$ and the resulting points after a shear transformation with $a = 1$ and $b = 0$.

   (d) Based on the previous graphic, is this a rigid body transformation? Why?

   (e) Justify your answer mathematically.

   (f) What is the inverse of the shear transformation in homogeneous coordinates?

2. Given a transformation T which is a composition of N invertible transformations $T_1, \cdots, T_N$, that is in matrix representation $\mathbf{T} = \mathbf{T}_N \mathbf{T}_{N-1} \cdots \mathbf{T}_1$.

   (a) What is the expression of $\mathbf{T}^{-1}$ as a function of $\mathbf{T}_1^{-1}, \cdots, \mathbf{T}_N^{-1}$?

   (b) If the i-th transformation in Cartesian coordinates is represented by $\mathbf{T}_i = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$, what is $\mathbf{T}_i^{-1}$ in homogeneous coordinates?