Enhanced visualization: assignment 4

Camera calibration

I. Theoretical part

A. Intrinsic matrix.

1. Is the intrinsic matrix $\mathbf{M}_{\text{int}}$ invertible? If yes, give its inverse.

B. Direct linear transformation.

1. Show that the cost function $\mathcal{C}(\mathbf{x}) = \|\mathbf{A}\mathbf{x}\|_2^2$ always admits the vector $\mathbf{0}$ as one of its global minima.
2. We now consider the minimization of $\mathcal{C}(\mathbf{x})$ with respect to $\mathbf{x}$ subject to the constraint $\mathbf{x}^{\mathrm{T}}\mathbf{x} = 1$. Is this a convex optimization problem?
3. Give the solution of the constrained problem.

II. Practical part

A. Calibration with a rig. In *moodle* you can find the picture *goal.jpg* from the qualifying match for the European Championship of 2006. See Figure 1. [1]

Figure 1: Image for camera calibration.

[1] Source: `http://www.maths.lth.se/matematiklth/personal/calle/datorseende11/i1.pdf`

1. The soccer field of this match has official dimensions:
   - Goal: 2.44 m × 7.32 m
   - Goal area: 5.5 m × 18.32 m
   - Penalty area: 16.5 m × 40.32 m
   - Soccer pitch: 65 m × 105 m
   - Corner arc radius: 1 m
   - Flag height: 1.5 m

   Download the *notebook* named *calibration_goal.ipynb*. The following steps can be achieved by filling up the code at the points indicated by # ∗ ∗ ∗ ∗ ∗ #.

   Create a matrix of 3D points of the penalty area and the corner (goal vertices, goal area vertices, *etc*). Create also one matrix with the points which are visible in the image in homogeneous coordinates.

   You will assume that the origin is in the middle of the goal line (on the floor). The goal line is the x axis and that it is pointing towards the camera side. The z axis points to outside the goal and the y axis points up.

2. Draw the corresponding cloud of points.

3. For the points which are visible, obtain their pixel coordinates using the command ginput from pylab package.

4. Retrieve the camera parameters using the DLT approach.

5. Test the precision of the camera matrix by projecting the 3D points on the image. You can use the command scatter from matplotlib. You should compare the projections with the hand-picked points.

6. Retrieve the camera position in the world reference frame.

7. Retrieve the pixel position of the ball. Can you say if the ball entered in the goal when the picture was taken?

8. From the image points of the feet of the players can you retrieve their positions in the field? If yes, explain how you can do it.

B. Calibration with planes. In this part we are going to follow the OpenCV tutorial on camera calibration. It is a tutorial that does calibration with planes and it consider a complex model for distortion[2].

Download the Python notebook *calibration.ipynb* and the folder *left.zip* from *moodle*. Extract the images inside *left.zip* inside the same folder where the Python notebook is located.

[2] Source: `http://opencv-python-tutroals.readthedocs.io/en/latest/py_tutorials/py_calib3d/py_calibration/py_calibration.html`

1. Explain what is done in Cell 1 and 2.

2. Read the documentation of the command cv2.calibrateCamera and explain briefly what calibration method is used.

3. What are the output variables mtx, dist, rvecs and tvecs?

4. What is the relation between rvecs and the rotation matrix **R** in the extrinsic matrix?

5. Compare the output images of Cell 3 and the original image. What is the purpose of the commands in Cell 3?

6. What quantity is evaluated in Cell 4? Why is it important?