Problem 1.11. a) Since $a_j = (2q+1)^{-1}$, $-q \le j \le q$, we have

$$\begin{aligned}
\sum_{j=-q}^{q} a_j m_{t-j} &= \frac{1}{2q+1} \sum_{j=-q}^{q} \left(c_0 + c_1 \left(t-j\right)\right) \\
&= \frac{1}{2q+1} \left(c_0 \left(2q+1\right) + c_1 \sum_{j=-q}^{q} \left(t-j\right) \right) = c_0 + \frac{c_1}{2q+1} \left(t\left(2q+1\right) - \sum_{j=-q}^{q} j \right) \\
&= c_0 + c_1 t - \frac{c_1}{2q+1} \left(\sum_{j=1}^{q} j + \sum_{j=1}^{q} -j \right) \\
&= c_0 + c_1 t = m_t
\end{aligned}$$

b) We have

$$\mathbb{E}\left[A_t\right] = \mathbb{E}\left[\sum_{j=-q}^{q} a_j Z_{t-j} \right] = \sum_{j=-q}^{q} a_j \mathbb{E}\left[Z_{t-j}\right] = 0 \quad \text{and}$$

$$\operatorname{Var}\left(A_t\right) = \operatorname{Var}\left(\sum_{j=-q}^{q} a_j Z_{t-j} \right) = \sum_{j=-q}^{q} a_j^2 \operatorname{Var}\left(Z_{t-j}\right) = \frac{1}{\left(2q+1\right)^2} \sum_{j=-q}^{q} \sigma^2 = \frac{\sigma^2}{2q+1}$$

We see that the variance $\operatorname{Var}(A_t)$ is small for large q. Hence, the process A_t will be close to its mean (which is zero) for large q.

Problem 1.15. a) Put

$$\begin{aligned}
Z_t &= \nabla\nabla_{12} X_t = (1-B)(1-B^{12})X_t = (1-B)(X_t - X_{t-12}) \\
&= X_t - X_{t-12} - X_{t-1} + X_{t-13} \\
&= a + bt + s_t + Y_t - a - b(t-12) - s_{t-12} - Y_{t-12} - a - b(t-1) - s_{t-1} - Y_{t-1} \\
&\quad + a + b(t-13) + s_{t-13} + Y_{t-13} \\
&= Y_t - Y_{t-1} - Y_{t-12} + Y_{t-13}.
\end{aligned}$$

We have $\mu_Z(t) = \mathbb{E}[Z_t] = 0$ and

$$\begin{aligned}
\gamma_Z(t+h,t) &= \operatorname{Cov}\left(Z_{t+h}, Z_t\right) \\
&= \operatorname{Cov}\left(Y_{t+h} - Y_{t+h-1} - Y_{t+h-12} + Y_{t+h-13}, Y_t - Y_{t-1} - Y_{t-12} + Y_{t-13}\right) \\
&= \gamma_Y(h) - \gamma_Y(h+1) - \gamma_Y(h+12) + \gamma_Y(h+13) - \gamma_Y(h-1) + \gamma_Y(h) \\
&\quad + \gamma_Y(h+11) - \gamma_Y(h+12) - \gamma_Y(h-12) + \gamma_Y(h-11) \\
&\quad + \gamma_Y(h) - \gamma_Y(h+1) + \gamma_Y(h-13) - \gamma_Y(h-12) - \gamma_Y(h-1) + \gamma_Y(h) \\
&= 4\gamma_Y(h) - 2\gamma_Y(h+1) - 2\gamma_Y(h-1) + \gamma_Y(h+11) + \gamma_Y(h-11) \\
&\quad - 2\gamma_Y(h+12) - 2\gamma_Y(h-12) + \gamma_Y(h+13) + \gamma_Y(h-13).
\end{aligned}$$

Since $\mu_Z(t)$ and $\gamma_Z(t+h,t)$ do not depend on t, $\{Z_t : t \in \mathbb{Z}\}$ is (weakly) stationary.
b) We have $X_t = (a+bt)s_t + Y_t$. Hence,

$$\begin{aligned}
Z_t &= \nabla_{12}^2 X_t = (1-B^{12})(1-B^{12})X_t = (1-B^{12})(X_t - X_{t-12}) \\
&= X_t - X_{t-12} - X_{t-12} + X_{t-24} = X_t - 2X_{t-12} + X_{t-24} \\
&= (a+bt)s_t + Y_t - 2(a + b(t-12)s_{t-12} + Y_{t-12}) + (a + b(t-24))s_{t-24} + Y_{t-24} \\
&= a(s_t - 2s_{t-12} + s_{t-24}) + b(ts_t - 2(t-12)s_{t-12} + (t-24)s_{t-24}) \\
&\quad + Y_t - 2Y_{t-12} + Y_{t-24} \\
&= Y_t - 2Y_{t-12} + Y_{t-24}.
\end{aligned}$$

Now we have $\mu_Z(t) = \mathbb{E}[Z_t] = 0$ and

$$\begin{aligned}
\gamma_Z(t+h,t) &= \operatorname{Cov}\left(Z_{t+h}, Z_t\right) \\
&= \operatorname{Cov}\left(Y_{t+h} - 2Y_{t+h-12} + Y_{t+h-24}, Y_t - 2Y_{t-12} + Y_{t-24}\right) \\
&= \gamma_Y(h) - 2\gamma_Y(h+12) + \gamma_Y(h+24) - 2\gamma_Y(h-12) + 4\gamma_Y(h) \\
&\quad - 2\gamma_Y(h+12) + \gamma_Y(h-24) - 2\gamma_Y(h-12) + \gamma_Y(h) \\
&= 6\gamma_Y(h) - 4\gamma_Y(h+12) - 4\gamma_Y(h-12) + \gamma_Y(h+24) + \gamma_Y(h-24).
\end{aligned}$$

Since $\mu_Z(t)$ and $\gamma_Z(t+h,t)$ do not depend on t, $\{Z_t : t \in \mathbb{Z}\}$ is (weakly) stationary.