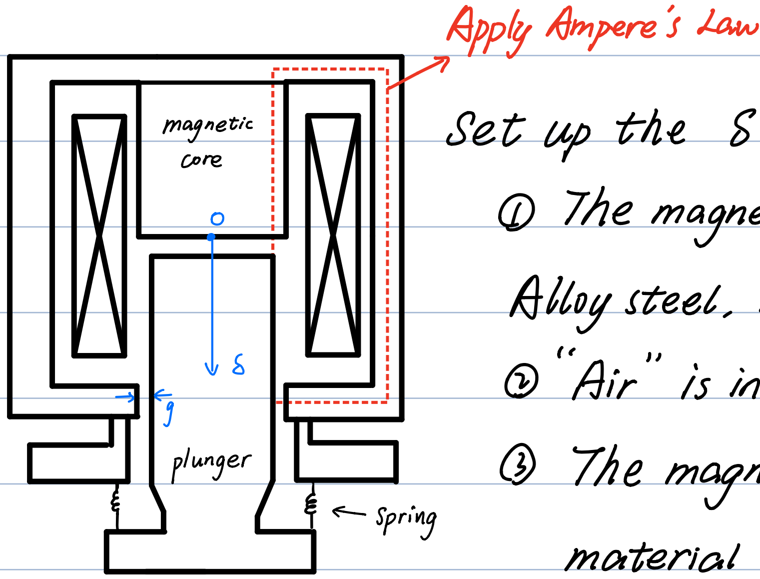
MEC 108

Coursework 1

Student ID: 1931254	
Full name: Zhongnan Li	
Email address: Zhongnan.li19@student.xjtlu.edu.cn	

Solution:

(a) The simplified schematic diagram is shown below.



Set up the δ - coordinate system, downwards.

① The magnetic core is shown in the figure. Alloy steel, AISI 440C or 42CrMo are used.

② "Air" is in the "air gap, g".

③ The magnetic iron, DT4 is the plunger material whose nature is high magnetic permeability, high saturation magnetic induction (B_s), high flux, stable magnetism. ④ As shown above.

(b) Apply Ampere's Law following the loop shown in the figure.

Since $H = \frac{\phi}{\mu A}$ and $\mu \to \infty$, ϕ is finite,

$$\oint_{LM} \vec{H} \cdot d\vec{l} = H_s l_s + H_g l_g = \frac{B_\delta}{\mu_0} \cdot \delta + \frac{H_g}{\mu_0} \cdot g = Ni \quad \text{—— (1)}$$

Assuming leakage flux and fringing effects are neglected, thus magnetic fluxes ϕ through air gaps δ and g are the same.

$$\phi = A_\delta \cdot B_\delta = A_g \cdot B_g$$

$$\Rightarrow \pi R^2 \cdot B_\delta = 2\pi R h \cdot B_g$$

$$\Rightarrow B_g = \frac{R}{2h} B_\delta \quad \text{—— (2)}$$

Combine (1) and (2), we obtain

$$B_\delta = \frac{\mu_0 N i}{\delta + g(\frac{R}{2h})}$$

Since we consider μ is constant, $L = \frac{\lambda}{i} = \frac{N\phi}{i}$

$$\phi = B_\delta A_\delta = \frac{\mu_0 N i}{\delta + g(\frac{R}{2h})} \cdot \pi R^2$$

$$L = \frac{N\phi}{i} = \frac{N}{i} \cdot \frac{\mu_0 N i}{\delta + g(\frac{R}{2h})} \cdot \pi R^2 = \frac{\pi \mu_0 N^2 R^2}{\delta + g(\frac{R}{2h})} = \frac{4.89 \times 10^{-5}}{\delta + 4\times10^{-3}} \ m.$$

(C) The magnetic force imposed on the plunger

$$f_{fld} = -\left.\frac{\partial W_{fld}}{\partial \delta}\right|_{\lambda} = \frac{\lambda^2}{2L^2}\frac{dL}{d\delta} = \frac{i^2}{2}\frac{dL}{d\delta}$$

① For f_{fld} as a function of λ

$$\frac{dL}{d\delta} = -\frac{\mu_0 \pi R^2 N^2}{[\delta + g(\frac{R}{2h})]^2} \cdot \frac{1}{\mu_0 \pi R^2 N^2}$$

$$= -\frac{L^2}{\mu_0 \pi R^2 N^2}$$

$$f_{fld} = \frac{\lambda^2}{2L^2} \cdot \left(-\frac{L^2}{\mu_0 \pi R^2 N^2}\right)$$

$$= -\frac{\lambda^2}{2\mu_0 \pi R^2 N^2} = -\frac{\lambda^2}{9.78\times10^{-5}} \ (N)$$

② For f_{fld} as a function of i

$$f_{fld} = -\frac{\mu_0 \pi R^2 N^2}{2[\delta + g(\frac{R}{2h})]^2} \cdot i^2 = -\frac{2.45\times10^{-5}}{\delta + 4\times10^{-3}} \cdot i^2 \ (N)$$

The direction of f_{fld} is to increase δ,

as indicated by the minus sign.

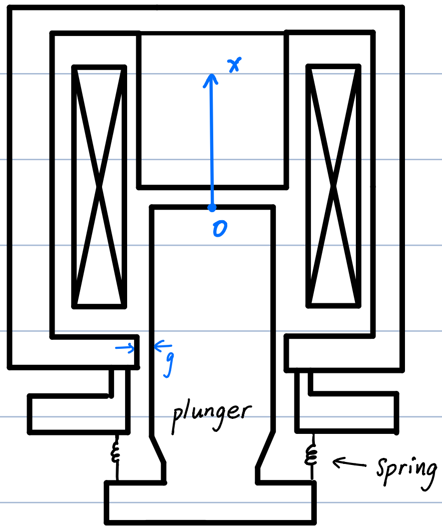
(d) The Electromechanical - Energy - Conversion System is considered as " Lossless ", since,

① Neglect Leakage flux

② Neglect core-loss

③ Mechanical losses are seperated from the system and expressed as f_D.

And external force f_t is not considered. $f_t = 0$.



Set up the x-coordinate system with

$$x = \delta_0 - \delta$$

when $t=0$, $\delta = \delta_0$, $x=0$

$\delta_{min} = 0.5\ mm \Rightarrow x_{max} = (\delta_0 - 0.5)\ mm$

With $\delta_0 = 1.5\ mm$, $x \in [0, 1]\ mm$

The voltage equation

$$V_{in} = iR_c + e$$

$$e = \frac{d\lambda}{dt} = \frac{d}{dt}(Li) = L\frac{di}{dt} + i\frac{dL}{dt}$$

Since $L(\delta) = \dfrac{\mu_0 \pi^2 R^2 N^2}{\delta + g\left(\frac{R}{2h}\right)}$, $x = \delta_0 - \delta$

Thus $L(x) = \dfrac{\mu_0 \pi^2 R^2 N^2}{(\delta_0 - x) + g\left(\frac{R}{2h}\right)}$ in x-coordinate system

Therefore $V_{in} = iR_c + e$

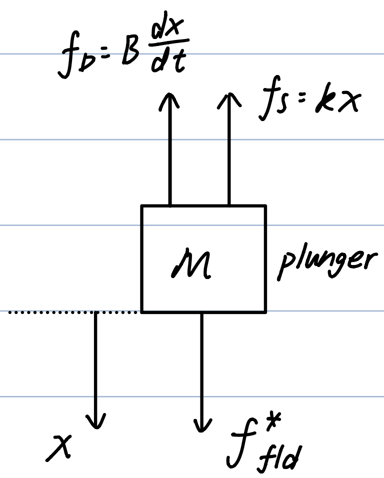
$$= iR_c + L(x)\frac{di}{dt} + i\frac{dL(x)}{dx}\cdot\frac{dx}{dt}$$

The equation of motion with x-coordinate system is:

$$M\frac{d^2x}{dt^2} = f^*_{fld} - B\frac{dx}{dt} - kx$$

$$M\frac{d^2x}{dt^2} + B\frac{dx}{dt} + kx = f^*_{fld}$$

$$f^*_{fld} = \frac{\mu_0 \pi R^2 N^2}{2\left[\delta + g\left(\frac{R}{2h}\right)\right]^2} i^2, \text{ with } \delta = \delta_0 - x$$



The free-body diagram of the plunger is shown left. f^*_{fld} is in x-coordinate system which is opposite to f_{fld} in δ-coordinate system.

Let $x = x_1$, $x_2 = \frac{dx_1}{dt}$, $x_3 = i$, then

$$\dot{x}_1 = x_2,$$

$$\dot{x}_2 = \frac{1}{M}\left(f^*_{fld} - Bx_2 - kx_1\right)$$

$$\dot{x}_3 = \frac{1}{L}\left(V_{in} - R_c x_3 - \frac{dL}{dx}\cdot x_3 \cdot x_2\right)$$

with $$f^*_{fld} = \frac{\mu_0 \pi R^2 N^2}{2[(\delta_0 - x) + g(\frac{R}{2h})]^2} x_3^2$$

$$\frac{dL}{dx} = \frac{\mu_0 \pi R^2 N^2}{[(\delta_0 - x) + g(\frac{R}{2h})]^2}$$

Solve the equations with

Inputs: $V_{in}(t) = 6V$

Initial conditions: $x_1(0) = x_2(0) = x_3(0) = 0$

The MATLAB programming part is shown below:

/Users/li/Desktop/归档/Question_d.m

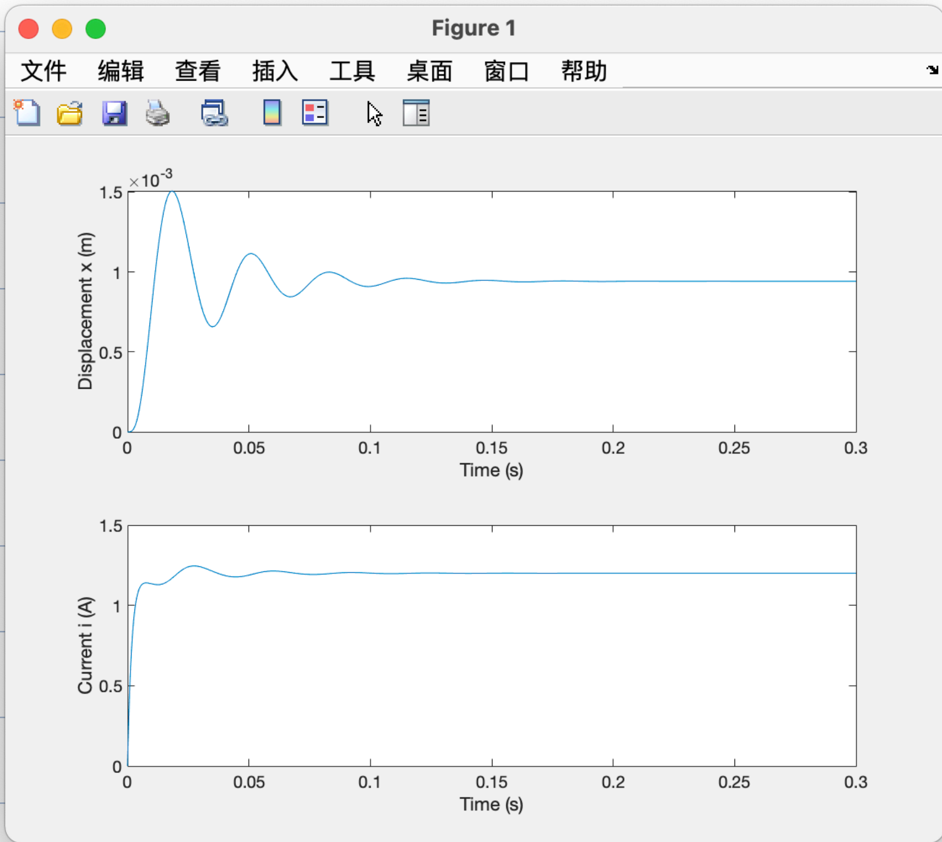
```
clear all
close all
clc

[t, x] = ode45(@Solenoid_valve, [0 0.3], [0 0 0]);
subplot(2,1,1);
plot(t, x(:,1));
xlabel('Time (s)');
ylabel('Displacement x (m)');
subplot(2,1,2);
plot(t, x(:,3));
xlabel('Time (s)');
ylabel('Current i (A)');

function [xdot] = Solenoid_valve(t, x)

R = 0.0064;
h = 0.0016;
g = 0.002;
delta_0 = 0.0015;
K = 1800;
B = 0.5;
M = 0.03;
N = 550;
R_c = 5;
mu_0 = 4*pi*10^(-7);
V_t = 6;

xdot = zeros(3,1);
f_fld = (mu_0*pi*R^2*N^2*x(3)^2) / (2*((delta_0-x(1))+(g*R/(2*h)))^2);
L_x = (mu_0*pi*R^2*N^2) / ((delta_0-x(1))+(g*R/(2*h)));
dLdx = (mu_0*pi*R^2*N^2) / (((delta_0-x(1))+(g*R/(2*h)))^2);
xdot(1) = x(2);
xdot(2) = f_fld/M-(B/M)*x(2)-(K/M)*x(1);
xdot(3) = V_t/L_x-(R_c/L_x)*x(3)-(dLdx/L_x)*x(3)*x(2);
end
```



Stead displacement

$x \approx 0.95$ mm

$I_{stead} = 1.2A$

The respond is good since both graph becomes to stable after a short time (0.15s) and the peak oscillation on displacement is small.

(e) We can increase the frictional coefficient B to increase the system dynamic performance to reduce

{ the response time (settle down time)
the peak oscillation on displacement

For example, with $B = 5kg/s$, compare the dynamic response with original design $B = 0.5 kg/s$.

Empirely, the mechanical system is expected to have good performance with damping ratio in the range of $0.4 - 0.7$.

⇒ Defination of the damping ratio. for a 2nd order system.

Damping ratio $\xi = \frac{\alpha}{X_0}$

where Damping coefficient $\alpha = \frac{B}{2M}$

Resonant frequency $\omega_0 = \sqrt{\frac{K}{M}}$

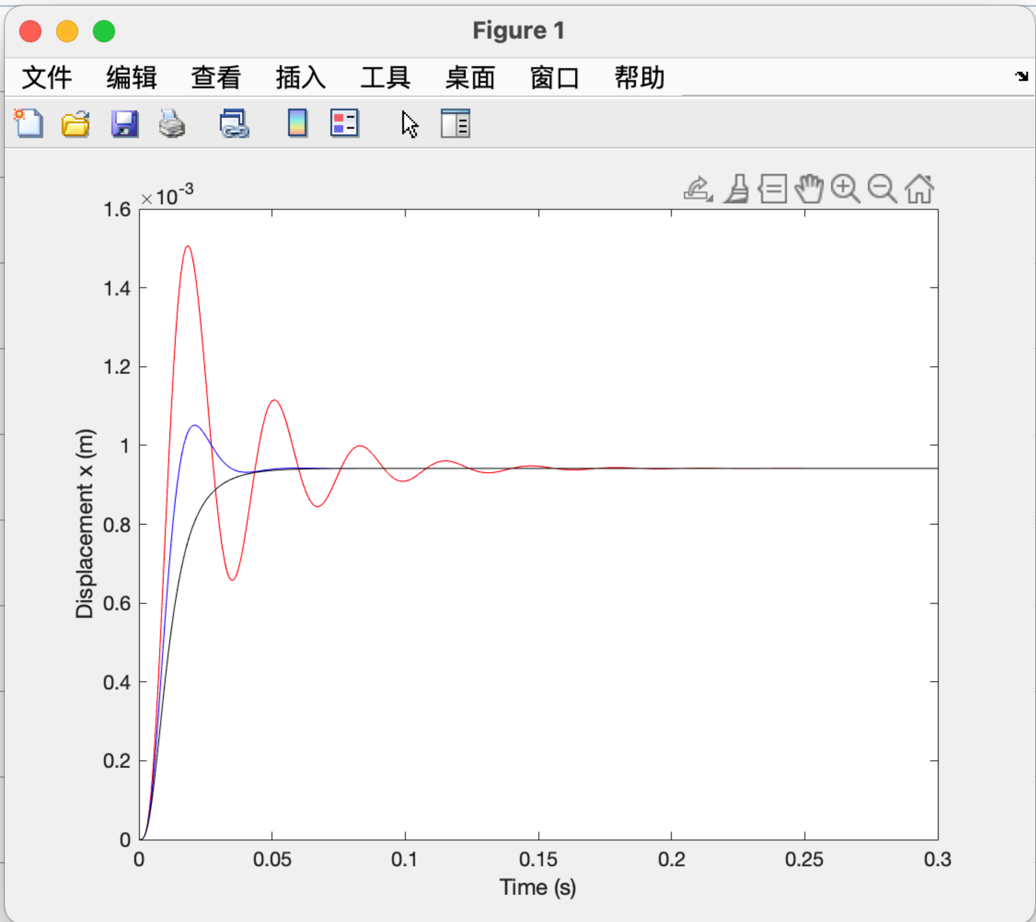
For the solenoid magnet of this question, try to find the value of B, by keeping K and M, for $\xi = 0.5$.

$\Rightarrow \omega_0 = \sqrt{\frac{4000}{0.03}} = 365.1$ rad/s

$\alpha = \frac{B}{6}$ 1/s

$\xi = \frac{\alpha}{\omega_0} = \frac{B/6}{365.1} = 0.5$

$\Rightarrow B = 11$



System response can be improved by increasing the frictional coefficient B

→ this can stablize the motion, but not that friction cannot be too large.

However, further increasing B will resulting in too much friction, then slow down the system response time to settle down.

→ this is over-damping of the system, which is not always good.

→ large friction increases the system energy loss, which should be avoided.

(f) Working principle:

When the coil is electrified, it will produce the electromagnetic force to lift the plunger from valve seat and valve will open. When the power is cut down, the electromagnetic force will disappear, and the spring push the plunger to the valve seat and valve will close.

Aspects:

① Elastic coefficent of the spring: the larger elastic coefficient, the greater the force on the spring. and the faster the response speed of solenoid valve.

② Friction: the smaller the friction in the system, the higher the energy conversion efficiency, the smaller the resistance of the plunger movement, the faster the response speed.

③ Electromagnetic force: the greater the electromagnetic force, the greater the attraction to the plunger, the faster the response time, the larger the displacement.