XI'AN JIAOTONG-LIVERPOOL UNIVERSITY
西交利物浦大学

COURSEWORK SUBMISSION COVER SHEET

Name	Li (Surname)	Zhongnan (Other Names)
Student Number	1931254	
Programme	Mechatronics and Robotic Systems	
Module Title	Solid and Structure	
Module Code	CEN103	
Assignment Title	Coursework 1 truss and beams	
Submission Deadline	3nd May 2021	
Module Leader	Charles Loo	

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Feedback on the strength of the work

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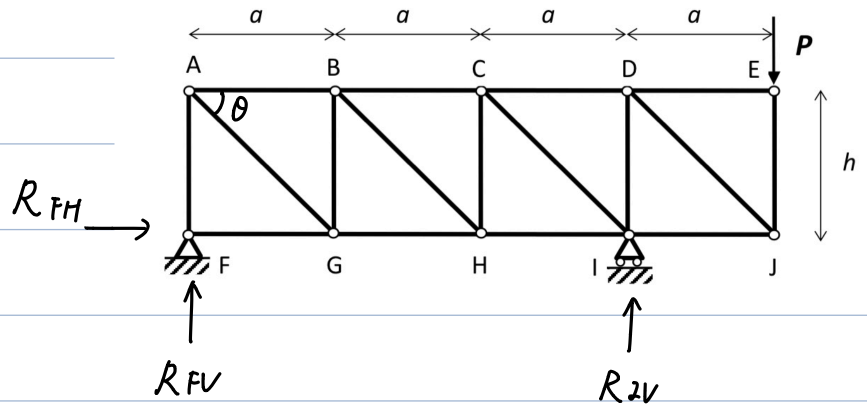
Students: *Please start your assignment on the next page.*

Coursework 1

Part A



Q1 Solution:

Solving reactions:

$\Sigma F_H = 0$; $R_{FH} = 0$

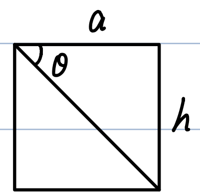
$\Sigma M_F = 0$; $P \times 4a - R_{2V} \times 3a = 0$

$R_{2V} = \frac{4}{3}P$

$\Sigma F_V = 0$; $R_{FV} + R_{2V} = P$

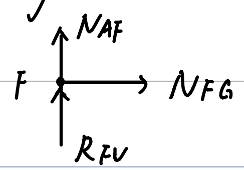
$R_{FV} = -\frac{1}{3}P$

Solving forces in each members:



$\sin\theta = \frac{h}{\sqrt{a^2+h^2}}$, $\cos\theta = \frac{a}{\sqrt{a^2+h^2}}$, $\tan\theta = \frac{h}{a}$

① At joint F

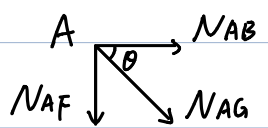


$\Sigma F_H = 0$; $N_{FG} = 0$

$\Sigma F_V = 0$; $N_{AF} + R_{FV} = 0$

$N_{AF} = \frac{P}{3}$

② At joint A



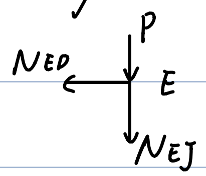
$\Sigma F_V = 0$; $N_{AF} + N_{AG}\sin\theta = 0$

$N_{AG} = -\frac{P}{3\sin\theta}$

$$\Sigma F_H = 0; \quad N_{AB} + N_{AG}\cos\theta = 0$$

$$N_{AB} = \frac{P}{3\tan\theta}$$

③ At joint E

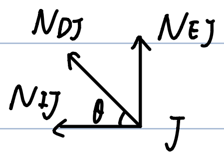


$$\Sigma F_V = 0; \quad P + N_{EJ} = 0$$

$$N_{EJ} = -P$$

$$\Sigma F_H = 0; \quad N_{ED} = 0$$

④ At joint J



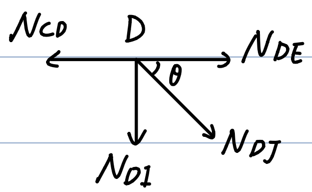
$$\Sigma F_V = 0; \quad N_{EJ} + N_{DJ}\sin\theta = 0$$

$$N_{DJ} = \frac{P}{\sin\theta}$$

$$\Sigma F_H = 0; \quad N_{IJ} + N_{DJ}\cos\theta = 0$$

$$N_{IJ} = -\frac{P}{\tan\theta}$$

⑤ At joint D



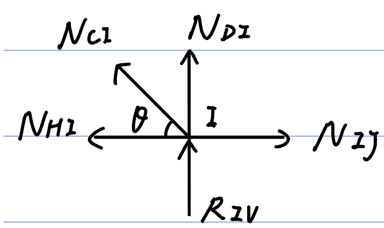
$$\Sigma F_V = 0; \quad N_{DI} + N_{DJ}\sin\theta = 0$$

$$N_{DI} = -P$$

$$\Sigma F_H = 0; \quad N_{CD} = N_{DE} + N_{DJ}\cos\theta$$

$$N_{CD} = \frac{P}{\tan\theta}$$

⑥ At joint I



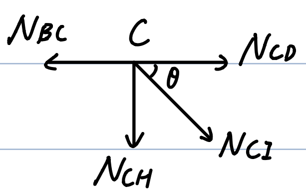
$$\Sigma F_V = 0; \quad N_{DI} + N_{CI}\sin\theta + R_{IV} = 0$$

$$N_{CI} = -\frac{P}{3\sin\theta}$$

$$\Sigma F_H = 0; \quad N_{HI} + N_{CI}\cos\theta = N_{IJ}$$

$$N_{HI} = -\frac{2P}{3\tan\theta}$$

⑦ At joint C



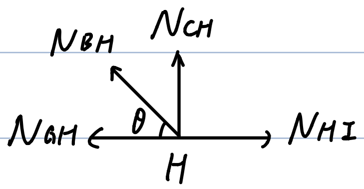
$$\Sigma F_V = 0; \quad N_{CH} + N_{CI}\sin\theta = 0$$

$$N_{CH} = \frac{P}{3}$$

$$\Sigma F_H = 0; \quad N_{BC} = N_{CD} + N_{CI}\cos\theta$$

$$N_{BC} = \frac{2P}{3\tan\theta}$$

⑧ At joint H



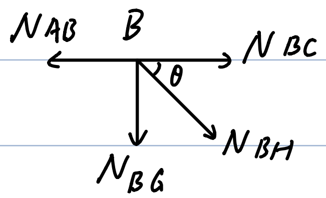
$\Sigma F_V = 0;\quad N_{CH} + N_{BH}\sin\theta = 0$

$$N_{BH} = -\frac{P}{3\sin\theta}$$

$\Sigma F_H = 0;\quad N_{GH} + N_{BH}\cos\theta = N_{HI}$

$$N_{GH} = -\frac{P}{3\tan\theta}$$

⑨ At joint B



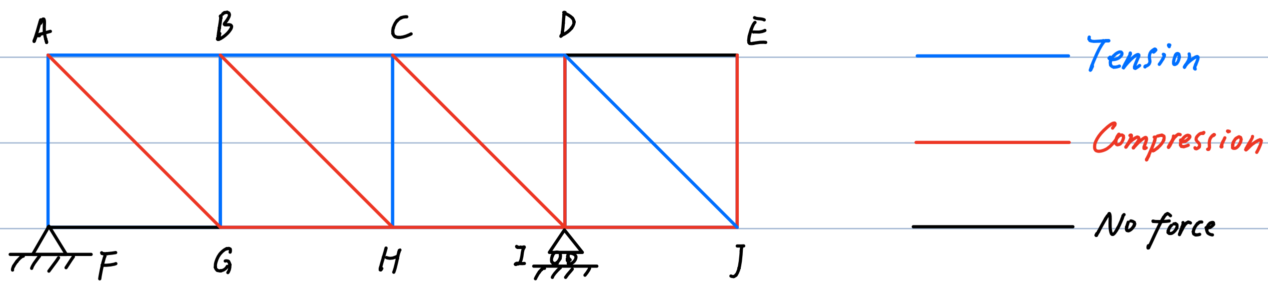
$\Sigma F_V = 0;\quad N_{BG} + N_{BH}\sin\theta = 0$

$$N_{BG} = \frac{P}{3}$$

Therefore, we have forces of all members:

Member:	AB	BC	CD	DE	FG	GH	HI	IJ	
Force:	$\frac{Pa}{3h}$	$\frac{2Pa}{3h}$	$\frac{Pa}{h}$	0	0	$-\frac{Pa}{3h}$	$-\frac{2Pa}{3h}$	$-\frac{Pa}{h}$	
Member:	AF	BG	CH	DI	EJ	AG	BH	CI	DJ
Force:	$\frac{P}{3}$	$\frac{P}{3}$	$\frac{P}{3}$	$-P$	$-P$	$-\frac{\sqrt{a^2+h^2}}{3h}P$	$-\frac{\sqrt{a^2+h^2}}{3h}P$	$-\frac{\sqrt{a^2+h^2}}{3h}P$	$\frac{\sqrt{a^2+h^2}}{h}P$

Q2 Solution:



Top chord: Truss members AB, BC, CD are subjected to tension.
There is no force on DE.

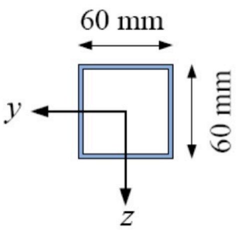
Bottom chord: Truss members GH, HI, IJ are subjected to compression.
There is no force on FG.

Verticals: Truss members AF, BG, CH are subjected to tension.

Truss members DI, EJ are subjected to compression.

Diagonals: Truss members AG, BH, CI are subjected to compression.

Truss member DJ is subjected to tension.



$A = 882\ \text{mm}^2$

$I = I_y = I_z = 459 \cdot 10^3\ \text{mm}^4$

Q3 Solution:

① For top chord, bottom chord and verticals,

$l_1 = a = h = 2\,m = 2\times10^3\,mm$

Thus, $N_{c,max} = \min \begin{cases} Af_c = 882\times190 = 167580\,N = 167.58\,kN \\ \dfrac{\pi^2 EI}{l_1^2} = \dfrac{\pi^2\times150\times10^3\times459\times10^3}{(2\times10^3)^2} \approx 169.88\,kN \end{cases}$

$= 167.58\,kN$

$N_{t,max} = A\cdot f_t = 882\,mm^2\times190\,MPa = 167.58\,kN$

② For diagonals, $l_2 = \sqrt{a^2+h^2} = \sqrt{2^2+2^2} = 2\sqrt{2}\,m = 2\sqrt{2}\times10^3\,mm$

$N_{c,max} = \min \begin{cases} Af_c = 882\times190 = 167580\,N = 167.58\,kN \\ \dfrac{\pi^2 EI}{l_2^2} = \dfrac{\pi^2\times150\times10^3\times459\times10^3}{(2\sqrt{2}\times10^3)^2} \approx 84.940\,kN \end{cases}$

$= 84.940\,kN$

$N_{t,max} = A\cdot f_t = 882\,mm^2\times190\,MPa = 167.58\,kN$

Thus, the table below shows the $N_{c,max}$ and $N_{t,max}$ for each members.

Members	AB	BC	CD	DE	FG	GH	HI	IJ	
Nc,max (KN)	167.58	167.58	167.58	167.58	167.58	167.58	167.58	167.58	
Nt,max (KN)	167.58	167.58	167.58	167.58	167.58	167.58	167.58	167.58	
Members	AF	BG	CH	DI	EJ	AG	BH	CI	DJ
Nc,max (KN)	167.58	167.58	167.58	167.58	167.58	84.94	84.94	84.94	84.94
Nt,max (KN)	167.58	167.58	167.58	167.58	167.58	167.58	167.58	167.58	167.58

Q4 Solution:

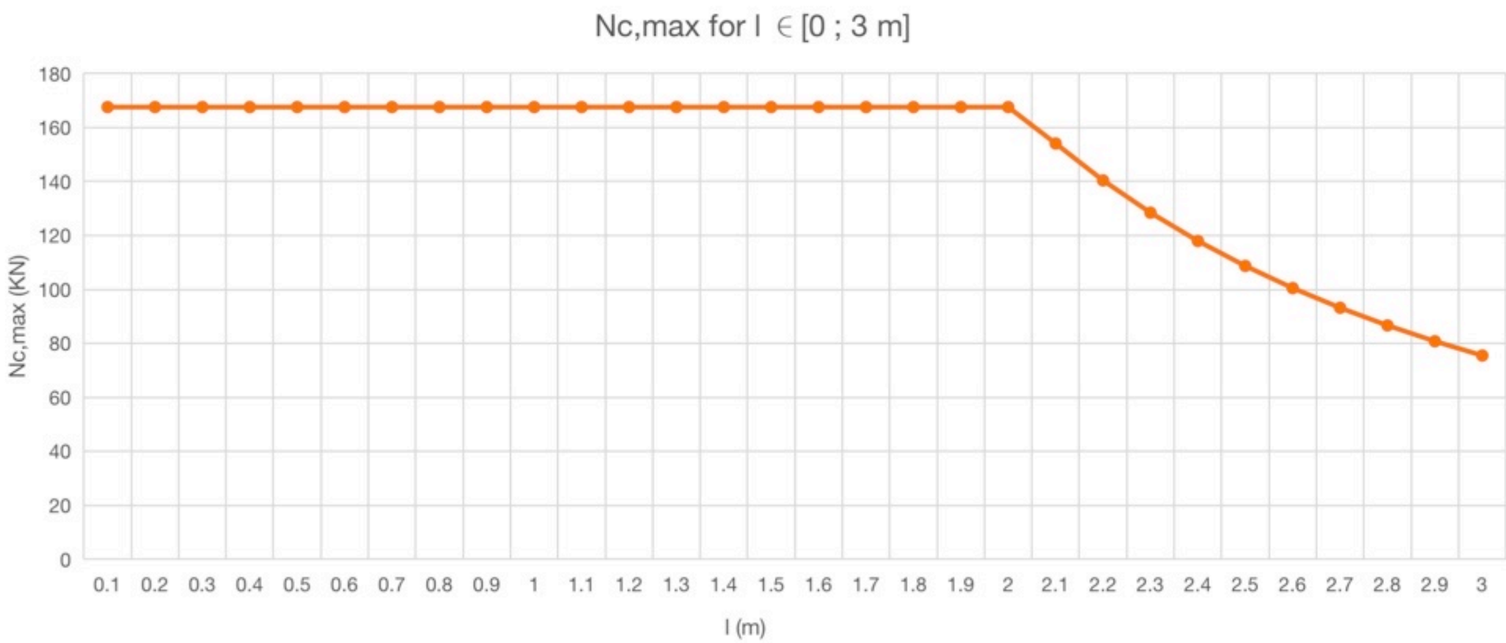
$$N_{c,max} = \min \begin{cases} A f_c = 882 \times 190 = 167580\ N = 167.58\ kN \\ \dfrac{\pi^2 E I}{l^2} = \dfrac{\pi^2 \times 150 \times 10^3 \times 459 \times 10^3}{(l \times 10^3)^2} \approx \dfrac{679.52}{l^2}\ kN \end{cases}$$

Let $\dfrac{679.52}{l^2} = 167.58$, $l = 2.0137\ m$

Therefore, $0 \le l \le 2.0137\ m$, $N_{c,max} = A \cdot f_c = 167.58\ kN$

$2.0137\,m \le l \le 3\,m$, $N_{c,max} = \dfrac{679.52}{l^2}\ kN$

The graph for $N_{c,max}$ as a function of l for $l \in [0; 3m]$ is shown below.



Q5 Solution:

Members	AB	BC	CD	DE	FG	GH	HI	IJ	
Force	P/3	2P/3	P	0	0	-P/3	-2P/3	-P	
Members	AF	BG	CH	DI	EJ	AG	BH	CI	DJ
Force	P/3	P/3	P/3	-P	-P	-√2P/3	-√2P/3	-√2P/3	√2P

Q6 Solution:

For tension member, the maximum tension force is $N_{DJ} = \sqrt{2} P$

$\sqrt{2} P < N_{t,max}$

$P < 118497\ N$

For compression member, $N_{DI} = -P$, $N_{AG} = -\dfrac{\sqrt{2}}{3} P$

$$|-P| < N_{c,max} = 167.58\,kN$$

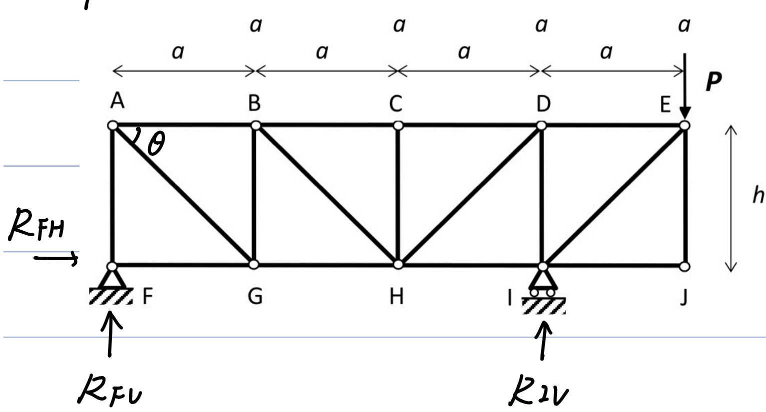
$$P < 167580\,N$$

$$\left|-\frac{\sqrt{2}}{3}P\right| < N_{c,max} = 84.94\,kN$$

$$P < 180185\,N$$

Therefore, the maximum load can be applied without failure $P_{max} = 118497\,N$.

Q7 Solution:



Solving reactions:

$\Sigma F_H = 0$; $R_{FH} = 0$

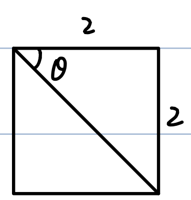
$\Sigma M_F = 0$; $P \times 4a - R_{2V} \times 3a = 0$

$$R_{2V} = \frac{4}{3}P$$

$\Sigma F_V = 0$; $R_{FV} + R_{2V} = P$

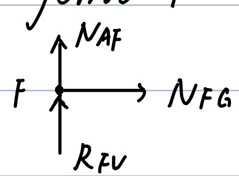
$$R_{FV} = -\frac{1}{3}P$$

Solving forces in each members:



$\sin\theta = \frac{\sqrt{2}}{2}$, $\cos\theta = \frac{\sqrt{2}}{2}$, $\tan\theta = 1$

① At joint F

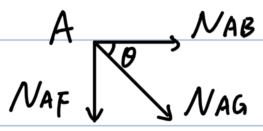


$\Sigma F_H = 0$; $N_{FG} = 0$

$\Sigma F_V = 0$; $N_{AF} + R_{FV} = 0$

$$N_{AF} = \frac{P}{3}$$

② At joint A



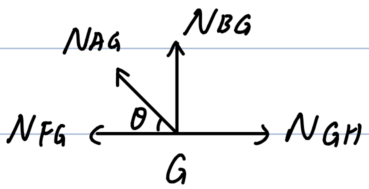
$\Sigma F_V = 0$; $N_{AF} + N_{AG} \sin\theta = 0$

$$N_{AG} = -\frac{\sqrt{2}P}{3}$$

$\Sigma F_H = 0$; $N_{AB} + N_{AG} \cos\theta = 0$

$$N_{AB} = \frac{P}{3}$$

③ At joint G



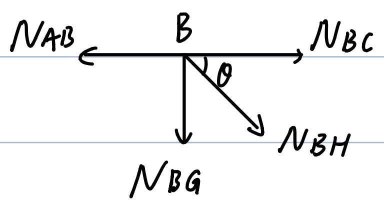
$\Sigma F_V = 0$; $N_{BG} + N_{AG} \sin\theta = 0$

$$N_{BG} = \frac{P}{3}$$

$\Sigma F_H = 0$; $N_{FG} + N_{AG} \cos\theta = N_{GH}$

$$N_{GH} = -\frac{P}{3}$$

④ At joint B



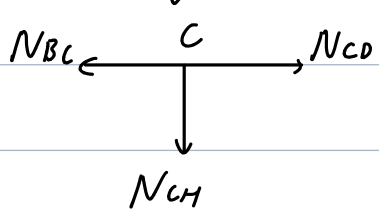
$\Sigma F_V = 0$; $N_{BG} + N_{BH} \sin\theta = 0$

$$N_{BH} = -\frac{\sqrt{2}P}{3}$$

$\Sigma F_H = 0$; $N_{AB} = N_{BC} + N_{BH} \cos\theta$

$$N_{BC} = \frac{2P}{3}$$

⑤ At joint C

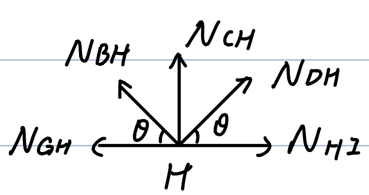


$\Sigma F_V = 0$; $N_{CH} = 0$

$\Sigma F_H = 0$; $N_{CD} = N_{BC}$

$$N_{CD} = \frac{2P}{3}$$

⑥ At joint H



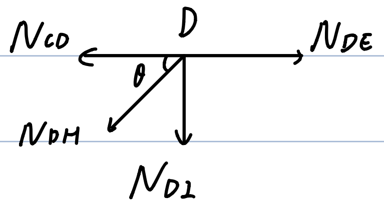
$\Sigma F_V = 0$; $N_{BH} \sin\theta + N_{DH} \sin\theta = 0$

$$N_{DH} = \frac{\sqrt{2}P}{3}$$

$\Sigma F_H = 0$; $N_{GH} + N_{BH} \cos\theta = N_{HI} + N_{DH} \cos\theta$

$$N_{HI} = -P$$

⑦ At joint D



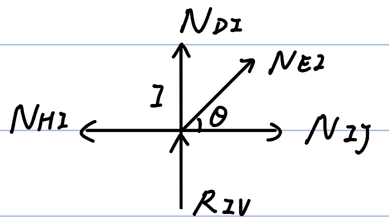
$\Sigma F_V = 0$; $\quad N_{DI} + N_{DH}\sin\theta = 0$

$$N_{DI} = -\frac{P}{3}$$

$\Sigma F_H = 0$; $\quad N_{CD} + N_{DH}\cos\theta = N_{DE}$

$$N_{DE} = P$$

⑧ At joint I



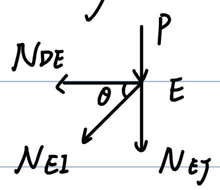
$\Sigma F_V = 0$; $\quad N_{DI} + N_{EI}\sin\theta + R_{EI} = 0$

$$N_{EI} = -\sqrt{2}P$$

$\Sigma F_H = 0$; $\quad N_{HI} = N_{IJ} + N_{EI}\cos\theta$

$$N_{IJ} = 0$$

⑨ At joint E



$\Sigma F_V = 0$; $\quad N_{EJ} + N_{EI}\sin\theta + P = 0$

$$N_{EJ} = 0$$

$\Sigma F_H = 0$; $\quad N_{DE} + N_{EI}\cos\theta = 0$

$$N_{DE} = P$$

For tension member, the maximum tension force is $N_{DE} = P$

$$P < N_{t,max}$$

$$P < 167580\,N$$

For compression member, $N_{HI} = -P$, $N_{EI} = -\sqrt{2}P$

$$|-P| < N_{c,max} = 167.58\,kN$$

$$P < 167580\,N$$

$$|-\sqrt{2}P| < N_{c,max} = 84.94\,kN$$

$$P < 60062\,N$$

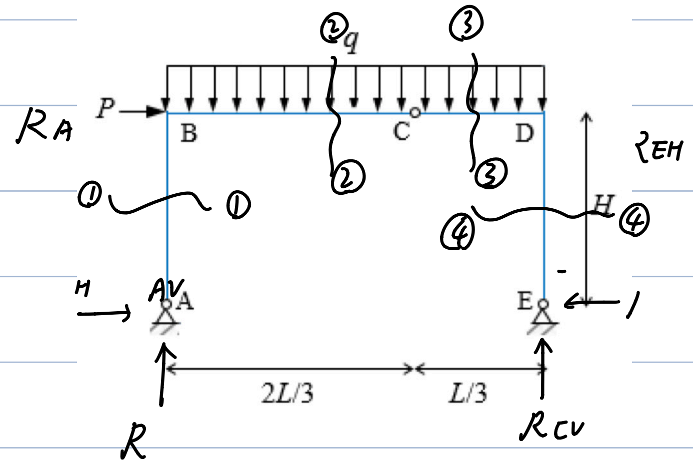
Therefore, the maximum load can be applied without failure

$P_{max} = 60062\,N$

Part B

Q2.1 Solution:

For P acting alone,



$\Sigma M_E = 0$; $P \cdot H = R_{EV} \cdot L$

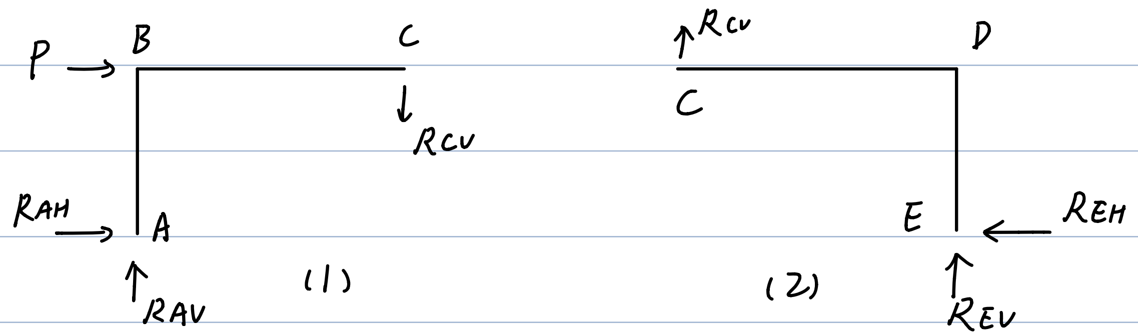
$R_{EV} = \dfrac{PH}{L}$

$\Sigma F_V = 0$; $R_{AV} + R_{EV} = 0$

$R_{AV} = -\dfrac{PH}{L}$

$\Sigma F_H = 0$; $R_{AH} + P = R_{EH}$ ①

Now, divide into two parts (1), (2).



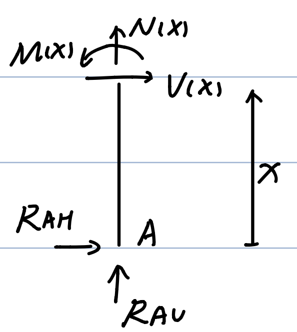
For (1), $\Sigma F_V = 0$; $R_{CV} = R_{AV} = -\dfrac{PH}{L}$

$\Sigma M_B = 0$; $R_{AH} \cdot H = R_{CV} \cdot \frac{2}{3} L$

$R_{AH} = -\dfrac{2P}{3}$ ②

Combine ① ②, we have $R_{EH} = P - \dfrac{2P}{3} = \dfrac{P}{3}$

Section 1-1



$\Sigma F_V = 0$; $N(x) + R_{AV} = 0$

$N(x) = \dfrac{PH}{L}$

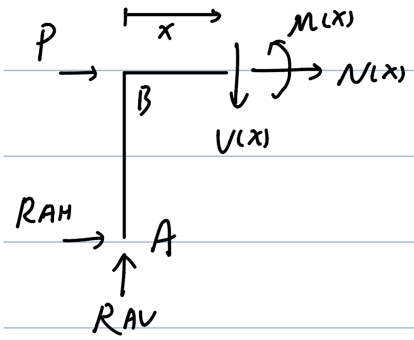
$\Sigma F_H = 0$; $V(x) + R_{AH} = 0$

$V(x) = \dfrac{2P}{3}$

$\Sigma M = 0$; $M(x) + R_{AH} \cdot x = 0$

$M(x) = \dfrac{2P}{3} x$

Section 2-2



$\Sigma F_H = 0;\quad P + R_{AH} + N(x) = 0$

$N(x) = -\frac{P}{3}$

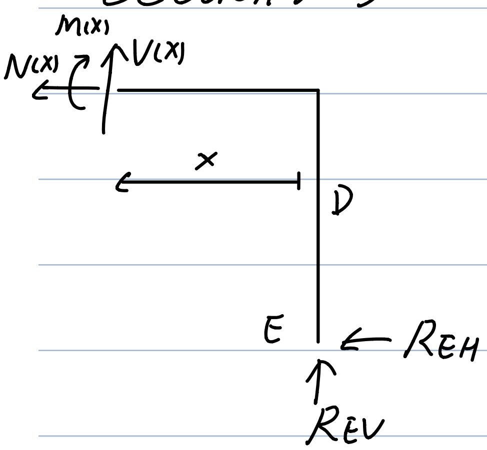
$\Sigma F_V = 0;\quad V(x) = R_{AV}$

$V(x) = -\frac{PH}{L}$

$\Sigma M = 0;\quad M(x) + R_{AH} \cdot H = R_{AV} \cdot x$

$M(x) = -\frac{PH}{L}x + \frac{2PH}{3}$

Section 3-3



$\Sigma F_H = 0;\quad N(x) + R_{EH} = 0$

$N(x) = -\frac{P}{3}$

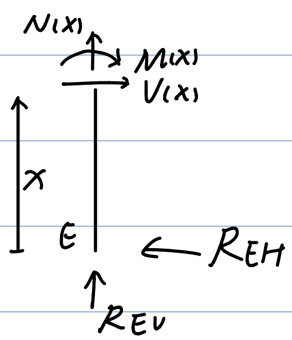
$\Sigma F_V = 0;\quad V(x) + R_{EV} = 0$

$V(x) = -\frac{PH}{L}$

$\Sigma M = 0;\quad M(x) + R_{EH} \cdot H = R_{EV} \cdot x$

$M(x) = \frac{PH}{L}x - \frac{PH}{3}$

Section 4-4



$\Sigma F_V = 0;\quad N(x) + R_{EV} = 0$

$N(x) = -\frac{PH}{L}$

$\Sigma F_H = 0;\quad V(x) = R_{EH}$

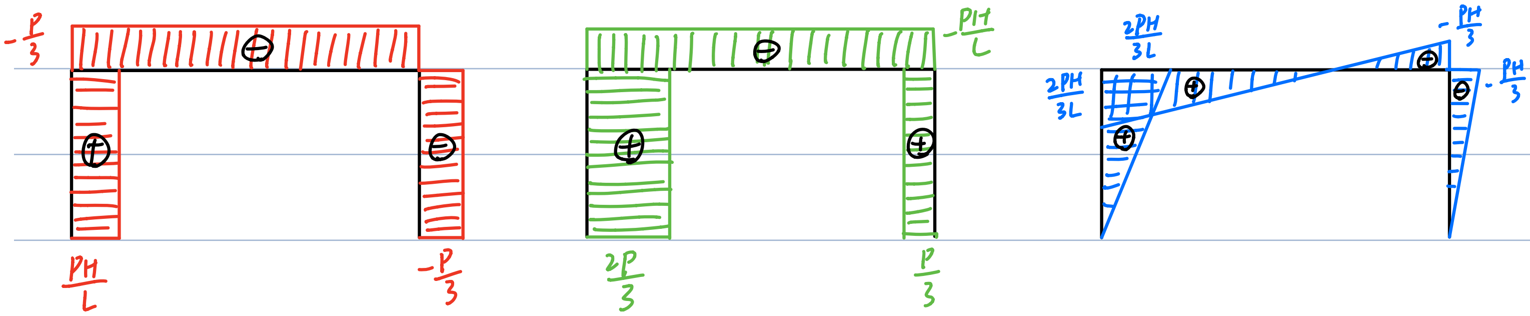
$V(x) = \frac{P}{3}$

$\Sigma M = 0;\quad M(x) + R_{EH} \cdot x = 0$

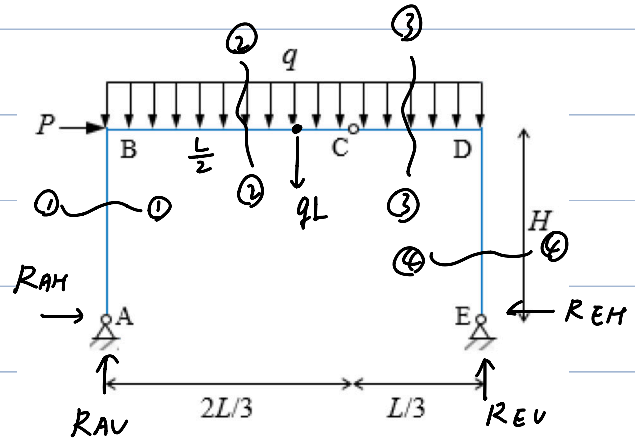
$M(x) = -\frac{P}{3}x$

Therefore, when P acts alone,

NFD SFD BMD



Q2.2 Solutions:



For q acting alone,

$\Sigma M_A = 0$;

$$qL \cdot \frac{L}{2} = R_{EV} \cdot L$$

$$R_{EV} = \frac{qL}{2}$$

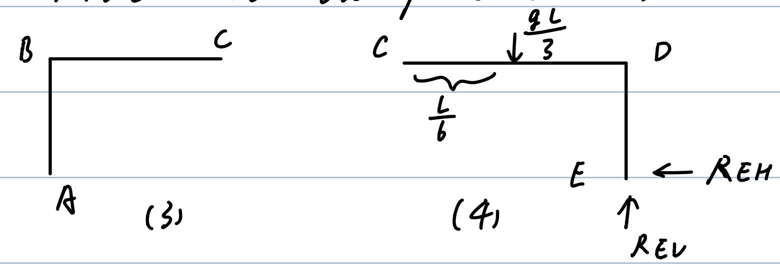
$\Sigma F_V = 0$;

$$qL = R_{AV} + R_{EV}$$

$$R_{AV} = \frac{qL}{2}$$

$\Sigma F_H = 0$; $R_{AH} = R_{EH}$

Divide into two parts (3) (4)



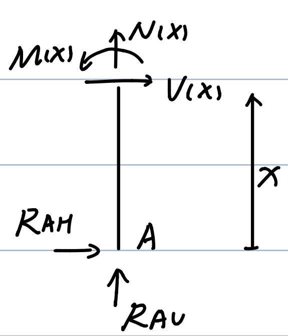
For (4), $\Sigma M_C = 0$;

$$\frac{qL}{3} \times \frac{L}{6} + R_{EH} \cdot H = R_{EV} \cdot \frac{L}{3}$$

$$R_{EH} = \frac{qL^2}{9H}$$

Thus, $R_{AH} = \frac{qL^2}{9H}$

Section 1-1



$\Sigma F_V = 0$; $N(x) + R_{AV} = 0$

$$N(x) = -\frac{qL}{2}$$

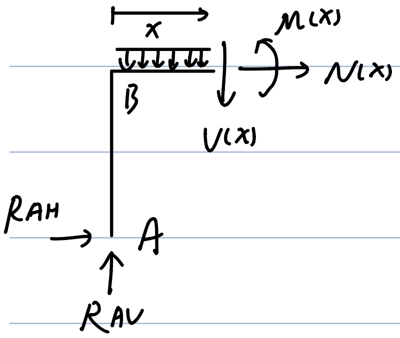
$\Sigma F_H = 0$; $V(x) + R_{AH} = 0$

$$V(x) = -\frac{qL^2}{9H}$$

$$\Sigma M = 0; \quad M(x) + R_{AH} \cdot x = 0$$

$$M(x) = -\frac{qL^2}{9H} x$$

Section 2-2



$$\Sigma F_H = 0; \quad N(x) + R_{AH} = 0$$

$$N(x) = -\frac{qL^2}{9H}$$

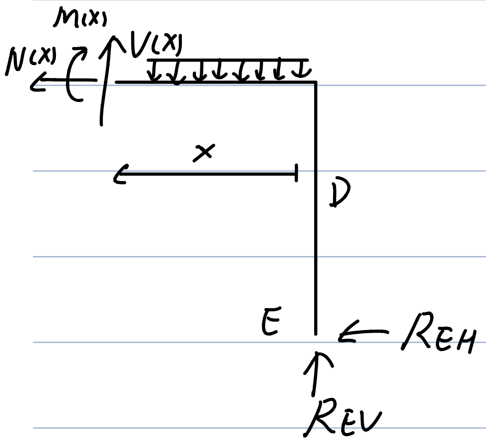
$$\Sigma F_V = 0; \quad V(x) + qx = R_{AV}$$

$$V(x) = -qx + \frac{qL}{2}$$

$$\Sigma M = 0; \quad M(x) + R_{AH} \cdot H + qx \cdot \frac{x}{2} = R_{AV} \cdot x$$

$$M(x) = -\frac{q}{2}x^2 + \frac{qL}{2}x - \frac{qL^2}{9}$$

Section 3-3



$$\Sigma F_H = 0; \quad N(x) + R_{EH} = 0$$

$$N(x) = -\frac{qL^2}{9H}$$

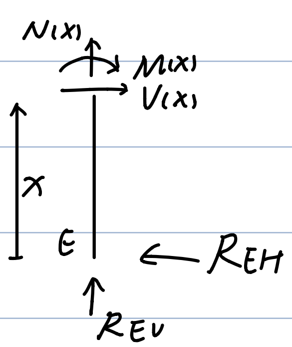
$$\Sigma F_V = 0; \quad V(x) + R_{EV} = qx$$

$$V(x) = qx - \frac{qL}{2}$$

$$\Sigma M = 0; \quad M(x) + R_{EH} \cdot H + qx \cdot \frac{x}{2} = R_{EV} \cdot x$$

$$M(x) = -\frac{q}{2}x^2 + \frac{qL}{2}x - \frac{qL^2}{9}$$

Section 4-4



$$\Sigma F_V = 0; \quad N(x) + R_{EV} = 0$$

$$N(x) = -\frac{qL}{2}$$

$$\Sigma F_H = 0; \quad V(x) = R_{EH}$$

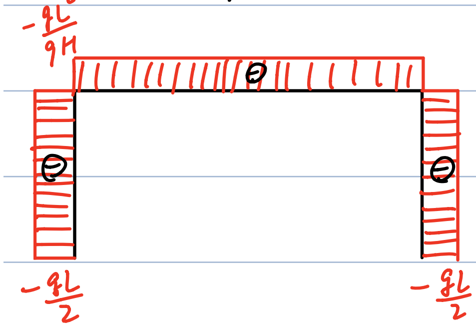
$$V(x) = \frac{qL^2}{9H}$$

$$\Sigma M = 0; \quad M(x) + R_{EH} \cdot x = 0$$

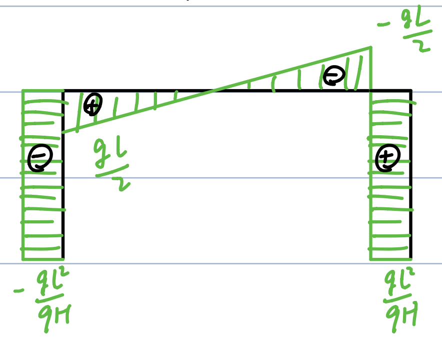
$$M(x) = -\frac{qL^2}{9H}x$$

Therefore, when P acts alone,

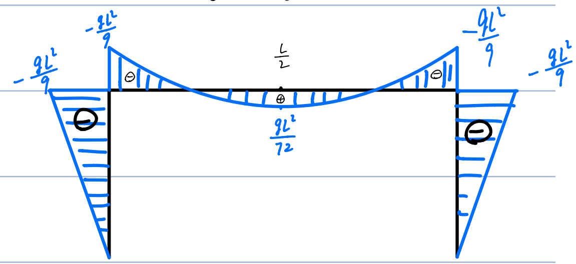
NFD



SFD

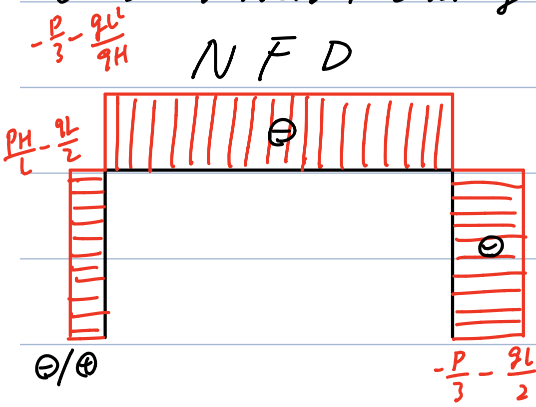


BMD

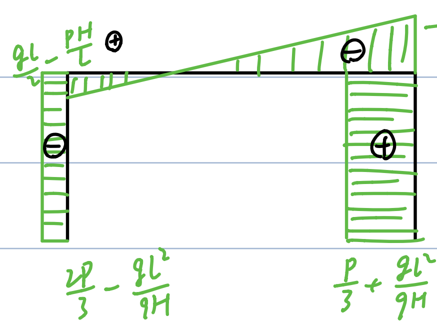


Q2.3 When P and q acts simultaneously,

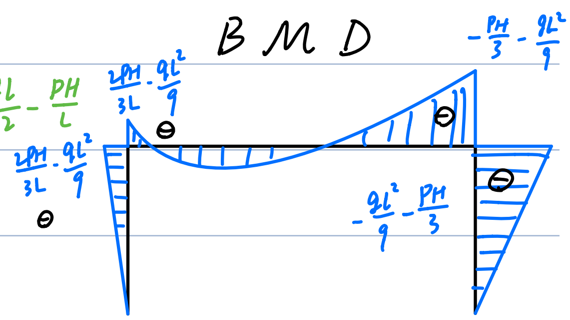
NFD



SFD



BMD



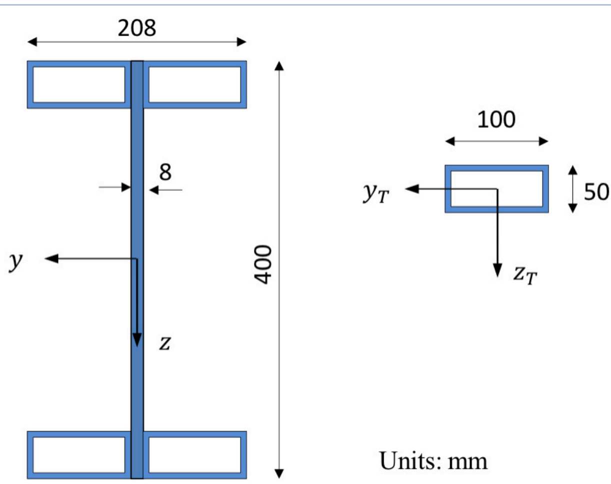
Q2.4 Solution:

$$A = 8\times 400 + 4\times 1810 = 10440\ mm^2$$

$$I_y = \sum_{i=1}^{5}(I_{yi} + z_{oi}^2 A_i)$$

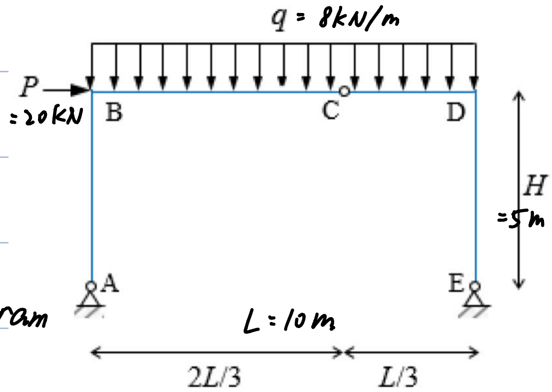
$$= \frac{1}{12}\times 8\times 400^3 + 4\times(4.44\times 10^6 + 175^2\times 1810)$$

$$\approx 282151667\ mm^4$$

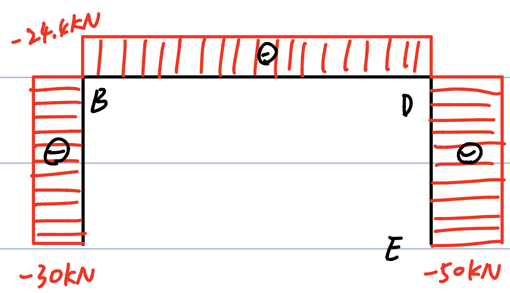


Q2.5 Solution:

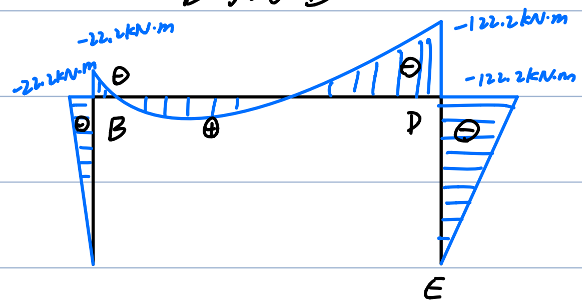
Since $H = 5m$, $L = 10m$, $P = 20kN$, $q = 8kN/m$, the normal force diagram and the moment diagram are shown below.



NFD



BMD



From the diagram, $|N|_{max} = 50kN$ and $|M|_{max} = 122.2\ kN\cdot m$ at point D. So point D is critical when considering the normal stress.

On BD, $\sigma_{D1} = \frac{N(x)}{A} + \frac{M_y(x)}{I_y} z$

$= \frac{24.4kN}{10440mm^2} + \frac{-122.2kN/m}{282151667mm^4} z$

$= 2.3372\ N/mm^2 - 0.4331\ N/mm^3 \cdot z$

$|\sigma_{D1,max}| = |2.3372\ N/mm^2 - 0.4331\ N/mm^3 \cdot (-\frac{1}{2} \times 400mm)|$

$= |88.9572\ N/mm^2|$

$= |88.9572\ MPa|$

$= 88.9572\ MPa$

On DE, $\sigma_{D2} = \frac{N(x)}{A} + \frac{M_y(x)}{I_y} z$

$= \frac{-50\ kN}{10440mm^2} + \frac{-122.2kN/m}{282151667mm^4} z$

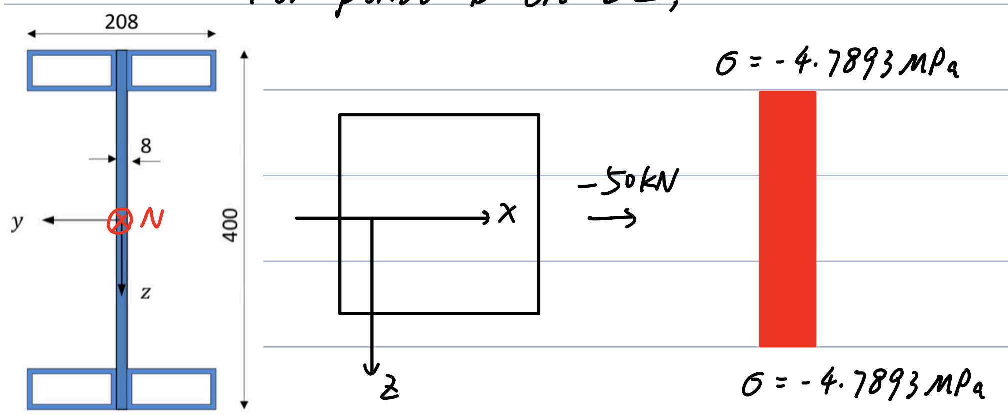
$= -4.7893\ N/mm^2 - 0.4331\ N/mm^3\ z$

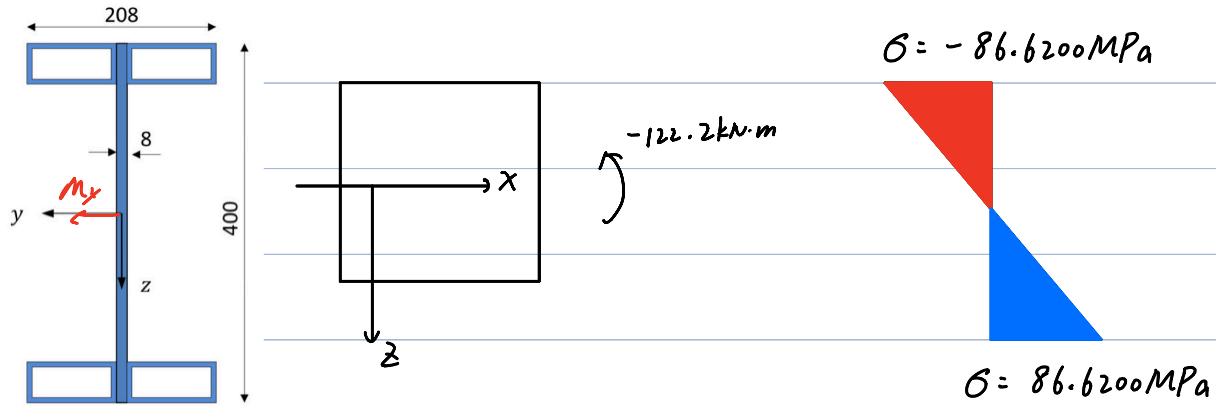
$$|\sigma_{D2,max}| = |-4.7893\,N/mm^2 - 0.4331\,N/mm^3 \times \tfrac{1}{2} \times 400mm|$$
$$= |-91.4093\,MPa|$$
$$= 91.4093\,MPa$$

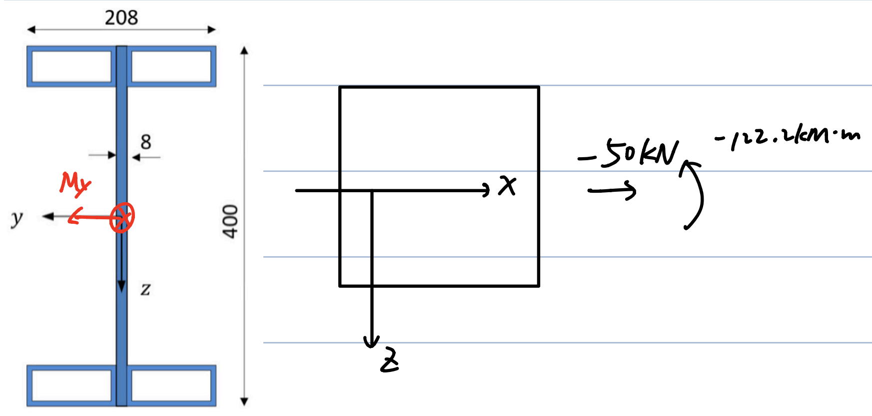
Thus $|\sigma_{D2,max}| > |\sigma_{D1,max}|$, so the combination of $(-50kN, -122.2kN{\cdot}m)$ will be considered.

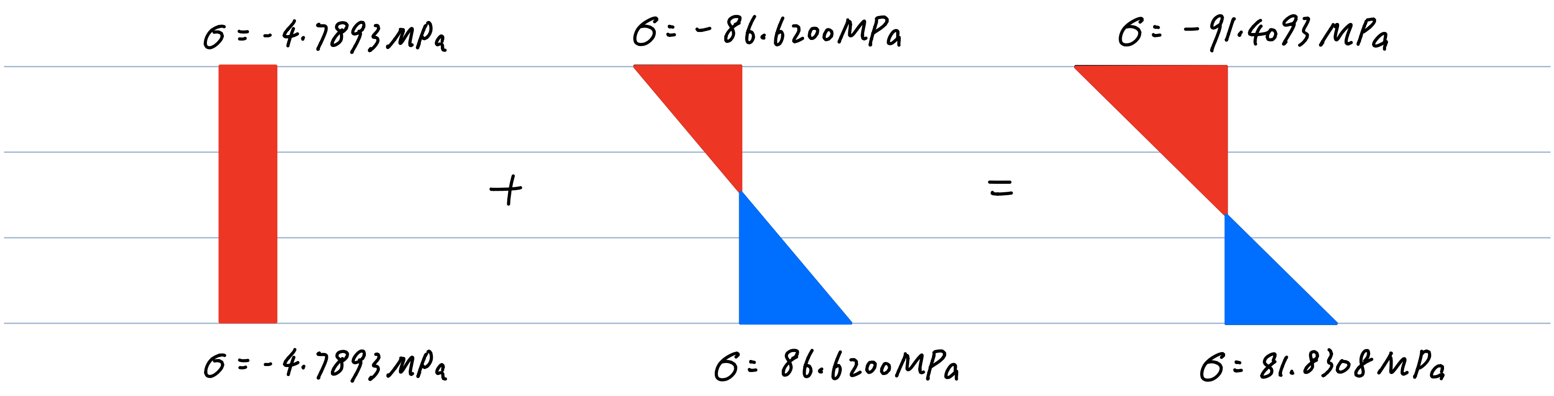
Q2.6 Solution:

For point D on DE,

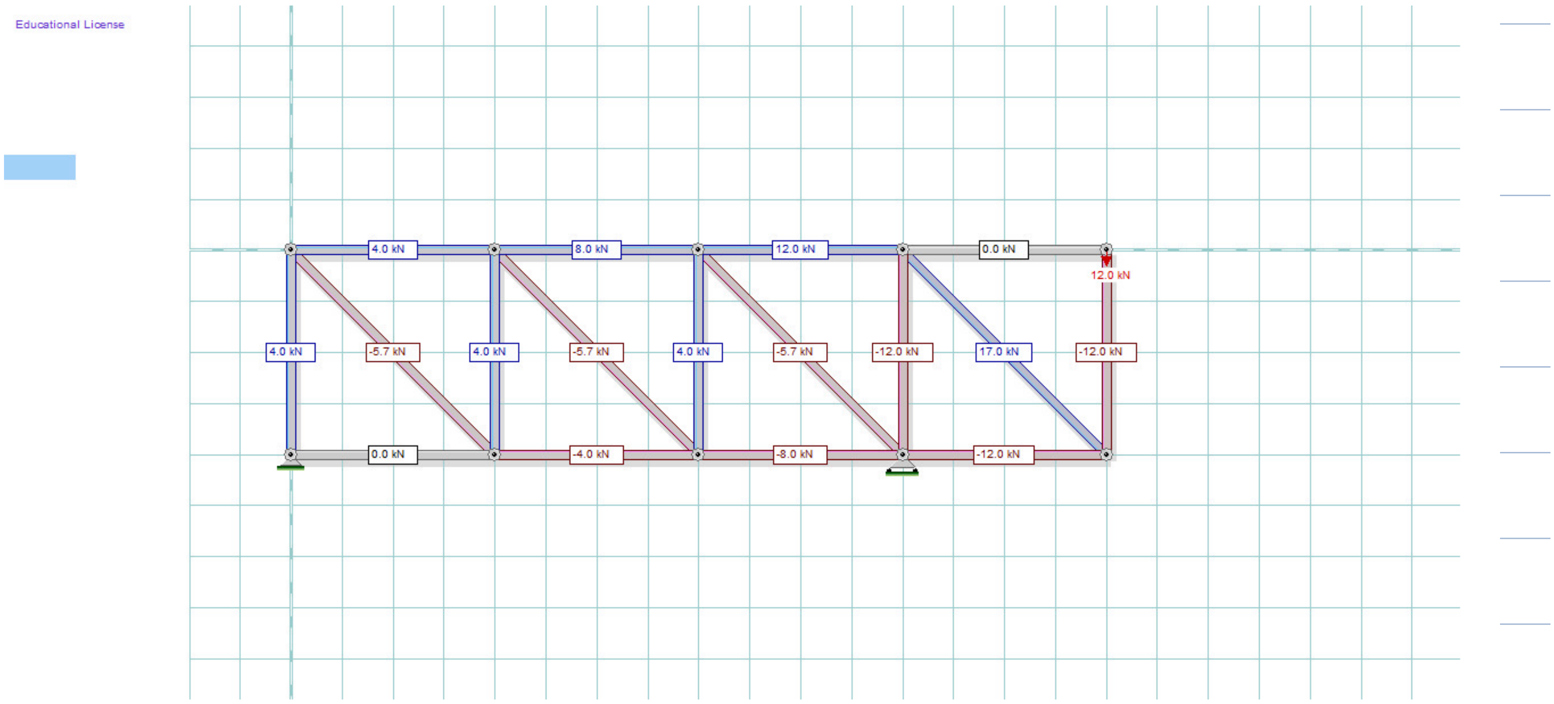








Appendix Q5



Appendix Q2.7