Xi'an Jiaotong-Liverpool University
西交利物浦大学

2020/21 Semester 1 - Assignment(Final)

Bachelor Degree - Year 2

Electrical Circuit 1

Time Allowed : 7 Days

Student ID	**Name**
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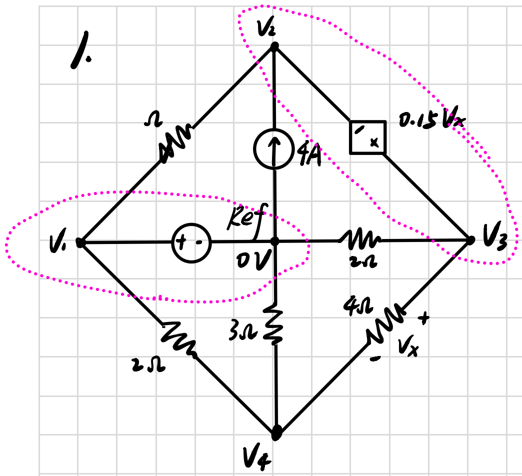
The table below is for the record of each question score and the total mark.

Question	**Mark**
1	
2	
3	
Total Mark	

1.



(a) Apply KCL to 3-4 supernode,

$$\frac{V_1-V_2}{1}+4+\frac{-V_3}{2}+\frac{V_4-V_3}{4}=0$$

$$-4V_1+4V_2+3V_3-V_4=16 \quad \cdots\cdots ①$$

Since there is a voltage source between node 2 and node 3,

$$V_3 - V_2 = 0.15V_x$$

Moreover, $V_x = V_3 - V_4$

Therefore, $V_3 - V_2 = 0.15(V_3 - V_4)$

$$-V_2+0.85V_3+0.15V_4=0 \quad \cdots\cdots ②$$

Apply KCL to 1-Ref supernode,

$$\frac{V_2-V_1}{1}+\frac{V_4-V_1}{2}+\frac{V_3}{2}-4=0$$

$$-9V_1+6V_2+3V_3+5V_4=24 \quad \cdots\cdots ③$$

Apply KCL to node 4,

$$\frac{V_1-V_4}{2}+\frac{V_3-V_4}{4}+\frac{-V_4}{3}=0$$

$$6V_1+3V_3-13V_4=0 \quad \cdots\cdots ④$$

Combine ①②③④, we have

$$\begin{cases} -4V_1+4V_2+3V_3-4V_4=16 \\ -9V_1+6V_2+3V_3+5V_4=24 \\ 6V_1+3V_3-13V_4=0 \\ -V_2+0.85V_3+0.15V_4=0 \end{cases}$$

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These four equations are the required equations.

(b) $\begin{cases} -4V_1 + 4V_2 + 3V_3 - 4V_4 = 16 \\ -9V_1 + 6V_2 + 3V_3 + 5V_4 = 24 \\ 6V_1 + 3V_3 - 13V_4 = 0 \\ -V_2 + 0.85V_3 + 0.15V_4 = 0 \end{cases}$

For the simultaneous equations, applying matrix inversion, we have

$$\begin{bmatrix} -4 & 4 & 3 & -4 & 16 \\ -9 & 6 & 3 & 5 & 24 \\ 6 & 0 & 3 & -13 & 0 \\ 0 & -1 & 0.85 & 0.15 & 0 \end{bmatrix}$$

Solve the matrix, the results are:

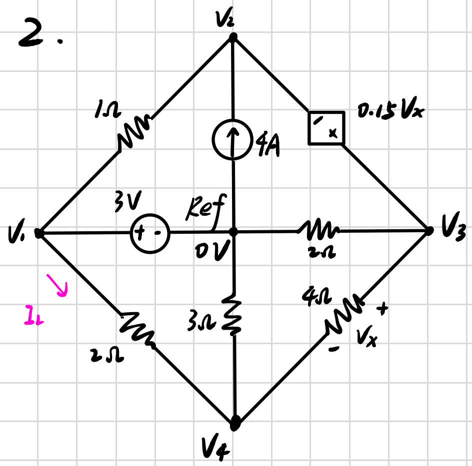
$$\begin{cases} V_1 = \frac{205}{126} V_4 - \frac{20}{21} \\ V_2 = \frac{269}{252} V_4 + \frac{34}{21} \\ V_3 = \frac{68}{63} V_4 + \frac{40}{21} \end{cases}$$

Since there are infinate possible values for V_4, there are infinate solutions to the simultaneous equations.

If we add an additional condition $V_1 = 3V$, the solution will be

$V_1 = 3V$, $V_2 = 4.212V$, $V_3 = 4.527V$, $V_4 = 2.429V$ #

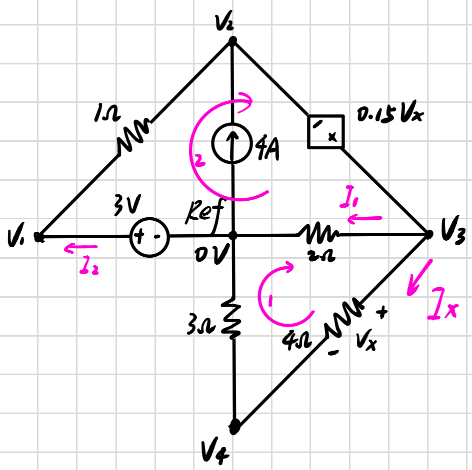
2.



Since the circuit is the same as the circuit in Question 1, and $V_1 = 3V$, the values of voltage is solved as $V_2 = 4.212V$, $V_3 = 4.527V$, $V_4 = 2.429V$.

Therefore, the current through the load resistor is

$$I_L = \frac{V_1 - V_4}{2} = \frac{3 - 2.429}{2} = 0.2855A$$



Rewrite the circuit by removing the load resistor to obtain Thevenin-equivalent-voltage.

From the circuit, the current through 4Ω resistor is $I_x = \frac{V_x}{4}A$

Apply KVL to mesh 1 made of V_4, Ref. and V_3.

$$\frac{V_x}{4}\cdot(3+4) - 2I_1 = 0$$

$$I_1 = \frac{7}{8}V_x \quad \cdots\cdots ①$$

Apply KCL to node Ref.

$$I_1 + \frac{V_x}{4} = 4 + I_2$$

$$I_2 = \frac{9}{8}V_x - 4 \quad \cdots\cdots ②$$

Apply KVL to mesh 2 made of V_1, V_2, V_3, Ref.

$$2I_1 - 3 + I_2 - 0.15V_x = 0 \quad \cdots\cdots ③$$

Combine ①, ②, ③,

$$2\times\frac{7}{8}V_x - 3 + \frac{9}{8}V_x - 4 - 0.15V_x = 0 \qquad V_x \approx 2.569V$$

Apply KVL to mesh made of V_1, V_4, and Ref.

$$3 - V_{TH} - 3 \cdot \frac{V_4}{4} = 0$$

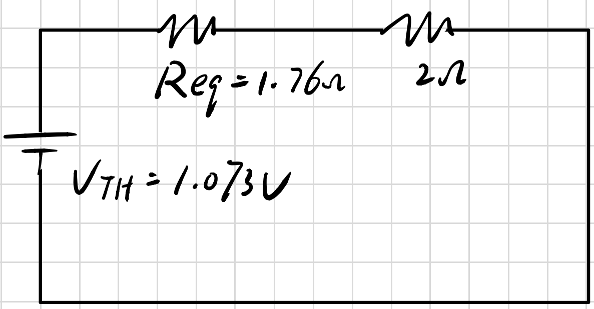
$$3 - V_{TH} - 3 \cdot \frac{2.569}{4} = 0$$

$$V_{TH} \approx 1.073 \ V$$

Therefore, Thevenin-equivalent-resistor is

$$R_{eq} = \frac{V_{TH}}{I_L} - 2 = \frac{1.073}{0.2855} - 2 \approx 1.76 \ \Omega$$

Thus, the Thevenin-equivalent-circuit is



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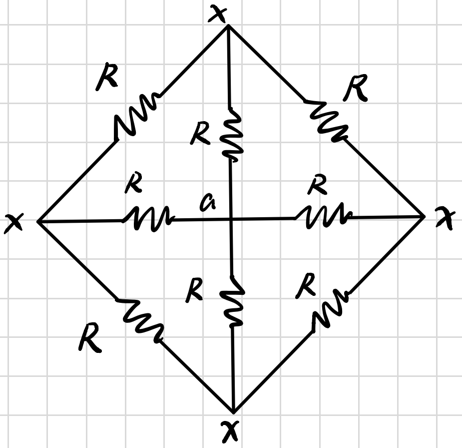
3. (a) By the discription, the circuit is drawn as follow.



(b) Rewrite the circuit.

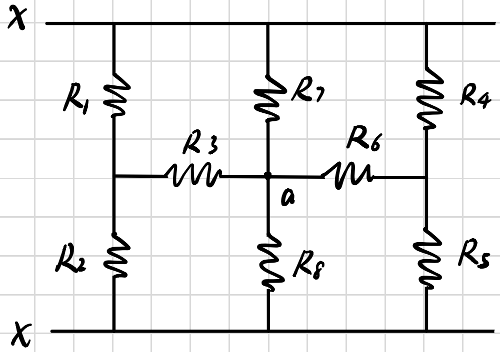


Figure 3.1

Applying Y-Δ conversion,

convert R_1, R_2, R_3 into R_a, R_b, R_c

and R_4, R_5, R_6 into R_d, R_e, R_f

$R_a = R_b = R_c = R_d = R_e = R_f$

$= \dfrac{R_1R_2 + R_2R_3 + R_3R_1}{R_2} = \dfrac{3R^2}{R} = 3R$

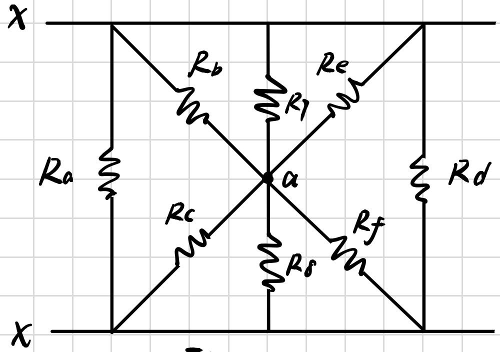


Figure 3.2

Rewrite the circuit as shown in figure 3.2

Rewrite the circuit as shown in figure 3.3.

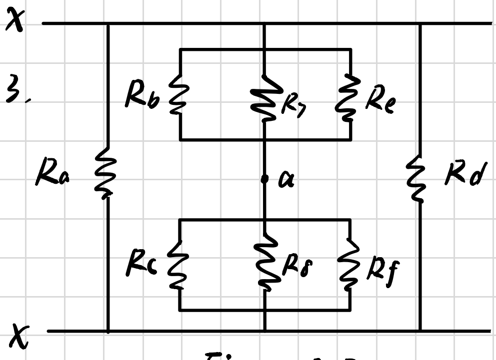


Figure 3.3

$Req_1 = R_b \| R_7 \| R_e$

$= 3R \| R \| 3R$

$= \frac{3}{4}R \| 3R$

$= \frac{3}{5}R$

$Req_2 = Req_1 = \frac{3}{5}R$

$Req_3 = R_a \| R_d$

$= 3R \| 3R$

$= \frac{3}{2}R$

Therefore, the total resistance of a-x pairs is

$R_{T1} = Req_1 \| (Req_2 + Req_3)$

$= \frac{3}{5}R \| \frac{21}{10}R$

$= \frac{7}{15}R$ #

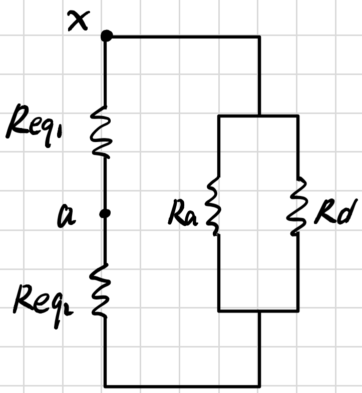


Figure 3.4

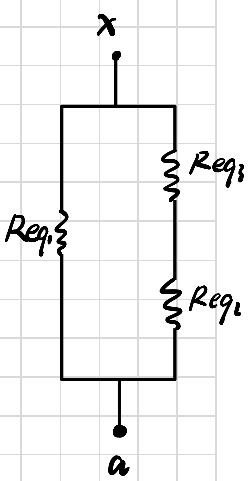


Figure 3.5

(C) Rewrite the circuit,

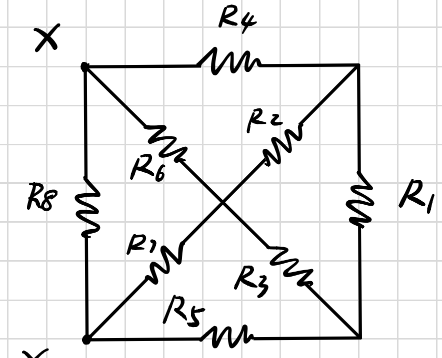


Figure 3.6

Apply Δ - Y conversion.

convert R_1, R_2, R_3 into R_a, R_b, R_c.

$$R_a = R_b = R_c = \frac{R_1 R_2}{R_1 + R_2 + R_3} = \frac{R^2}{3R} = \frac{R}{3}$$

Rewrite the circuit as figure 3.7 shows.

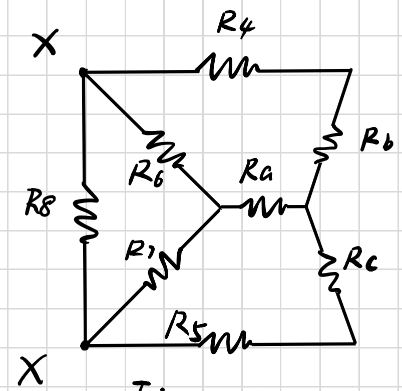


Figure 3.7

$$R_d = R_4 + R_b = R + \frac{R}{3} = \frac{4}{3}R$$

$$R_e = R_5 + R_c = R + \frac{R}{3} = \frac{4}{3}R$$

Rewrite the circuit, we have figure 3.8

Apply Δ - Y conversion.

convert R_d, R_6, R_a into R_o, R_p, R_q

$$R_o = \frac{R_d R_6}{R_d + R_6 + R_a} = \frac{\frac{4}{3}R \cdot R}{\frac{4}{3}R + R + \frac{R}{3}} = \frac{1}{2}R$$

$$R_p = \frac{R_d R_a}{R_d + R_6 + R_a} = \frac{\frac{4}{3}R \cdot \frac{1}{3}R}{\frac{8}{3}R} = \frac{1}{6}R$$

$$R_q = \frac{R_a R_6}{R_d + R_6 + R_a} = \frac{\frac{1}{3}R \cdot R}{\frac{8}{3}R} = \frac{1}{8}R$$

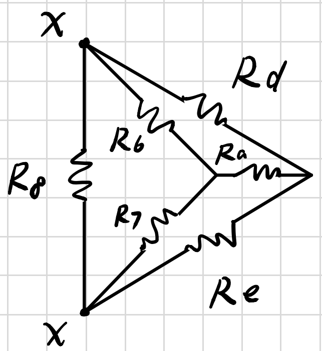


Figure 3.8

Rewrite the circuit as figure 3.9 shows.

$$R_m = R_7 + R_q = R + \frac{1}{8}R = \frac{9}{8}R$$

$$R_n = R_e + R_p = \frac{4}{3}R + \frac{1}{6}R = \frac{3}{2}R$$

Therefore, the total resistance of x-x pairs is

$$R_{T_2} = R_8 \parallel (R_o + R_m \parallel R_n)$$

$$= R \parallel (\tfrac{1}{2}R + \tfrac{9}{8}R \parallel \tfrac{3}{2}R)$$

$$= \frac{8}{15}R$$

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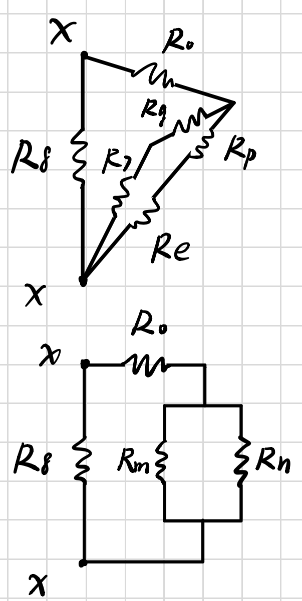
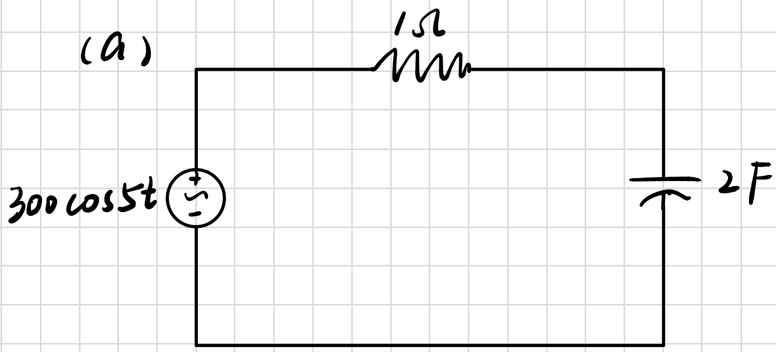


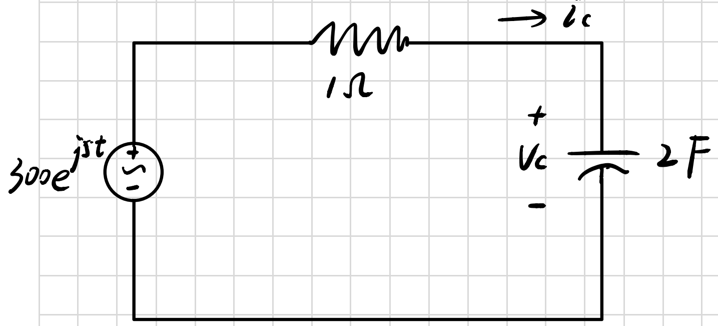
Figure 3.9

Since $R_{T_1} = \frac{7}{15}R \neq R_{T_2}$, the total resistance is not similar for all possible vertex pairs.

4. (a)



(b) Replace the real source with a complex source $300e^{j5t}$ V, and rewrite the circuit.



Apply KVL to the circuit,

$$-300e^{j5t} + 1 \cdot i_c + V_c = 0$$

$$-300e^{j5t} + \frac{dV_c}{dt} + V_c = 0$$

We anticipate a steady-state response of the same form as source, what means,

$$V_c = V_m e^{j5t}$$

Therefore
$$-300e^{j5t} + \frac{d\,V_m e^{j5t}}{dt} + V_m e^{j5t} = 0$$

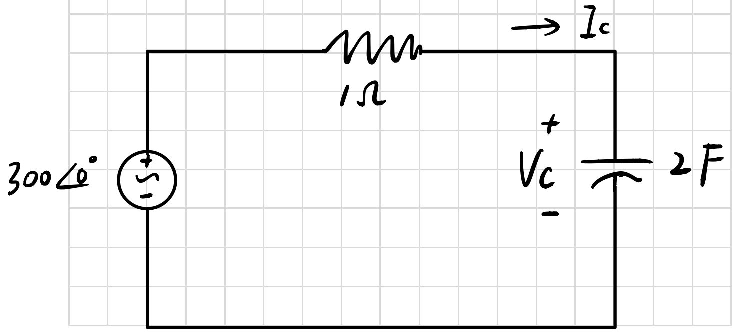
$$-300e^{j5t} + j10 V_m e^{j5t} + V_m e^{j5t} = 0$$

$$V_m = \frac{300}{1+j10} = \frac{300}{\sqrt{1+10^2}} \angle -\tan^{-1}\left(\frac{10}{1}\right) = 29.85\,e^{-j84.3^\circ}$$

The steady-state capacitor voltage is #

$$Re\{V_c\} = Re\{29.85e^{-j84.3^\circ} e^{j5t}\ V\} = 29.85\cos(5t - 84.3^\circ)\ V$$

(c) Replace the real source with a complex source in frequency-domain $300\angle 0^\circ$ V, and rewrite the circuit.



Apply KVL to the circuit,

$$-300\angle 0^\circ + 1 \cdot i_c + V_c = 0$$

Since in frequency-domain, the voltage-current express is

$V = \frac{1}{j\omega C} I$, the equation is

$$-300\angle 0^\circ + 1 \cdot I_C + \frac{1}{j\omega C} I_C = 0$$

$$I_C = \frac{300\angle 0^\circ}{1 + \frac{1}{j\omega C}} = \frac{300\angle 0^\circ}{1 + \frac{1}{j10}}$$

Since in frequency-domain, $Z_C = \frac{1}{j\omega C} = \frac{1}{j10}\ \Omega$,

$$V_C = I_C \cdot Z_C$$

$$= \frac{300\angle 0^\circ}{1 + \frac{1}{j10}} \cdot \frac{1}{j10}$$

$$= \frac{1}{j10 + 1} \cdot 300\angle 0^\circ$$

$$= \frac{300}{j10 + 1} \cdot \angle 0^\circ$$

$$= 29.85\angle -84.3^\circ \cdot \angle 0^\circ$$

$$= 29.85\angle -84.3^\circ$$

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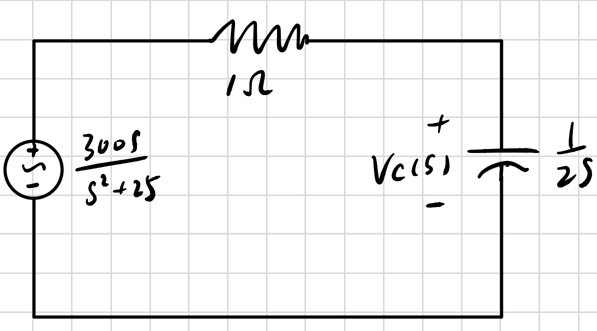
Therefore the steady-state capacitor voltage is $29.85\cos(5t - 84.3^\circ)$

(d) Replace the real source with a complex source in s-domain $\frac{300s}{s^2+25}$ V, and rewrite the circuit.

By Laplace transform, $Z_C(s) = \frac{1}{sC} = \frac{1}{2s}$



$$I(s) = \frac{V(s)}{1 + Z_C(s)} = \frac{\frac{300s}{s^2+25}}{1 + \frac{1}{2s}}$$

$$V_C(s) = I(s) \cdot Z_C(s)$$

$$= \frac{300s}{s^2+25} \cdot \frac{\frac{1}{2s}}{1+\frac{1}{2s}}$$

$$= \frac{150s}{(s^2+25)(s+\frac{1}{2})}$$

$$= \frac{300}{101}\left(\frac{s+50}{s^2+25} - \frac{1}{s+\frac{1}{2}}\right)$$

$$= \frac{300}{101}\frac{s}{s^2+25} + \frac{3000}{101}\frac{5}{s^2+25} - \frac{300}{101}\frac{1}{s+\frac{1}{2}}$$

Since $\cos wt\, u(t) \Rightarrow \frac{s}{s^2+w^2}$

$\sin wt\, u(t) \Rightarrow \frac{w}{s^2+w^2}$

$e^{-\alpha t} u(t) \Rightarrow \frac{1}{s+\alpha}$

By the Linearity theorem and the Homogeneity property.

$$V_C(t) = \mathcal{L}^{-1}\{V_C(s)\}$$

$$= \mathcal{L}^{-1}\left\{\frac{300}{101}\frac{s}{s^2+25} + \frac{3000}{101}\frac{5}{s^2+25} - \frac{300}{101}\frac{1}{s+\frac{1}{2}}\right\}$$

$$= \frac{300}{101}\mathcal{L}^{-1}\left\{\frac{s}{s^2+25}\right\} + \frac{3000}{101}\mathcal{L}^{-1}\left\{\frac{5}{s^2+25}\right\} - \frac{300}{101}\mathcal{L}^{-1}\left\{\frac{1}{s+\frac{1}{2}}\right\}$$

$$= \frac{300}{101}\cos 5t\, u(t) + \frac{3000}{101}\sin 5t\, u(t) - \frac{300}{101}e^{-\frac{1}{2}t}u(t)$$

$$= \frac{300}{\sqrt{101}}\left(\frac{1}{\sqrt{101}}\cos 5t + \frac{10}{\sqrt{101}}\sin 5t\right)u(t) - \frac{300}{101}e^{-\frac{1}{2}t}u(t)$$

$$= \frac{300}{\sqrt{101}}\cos[5t - \tan^{-1}(10)]\,u(t) - \frac{300}{101}e^{-\frac{1}{2}t}u(t) \quad \#$$

Therefore the steady-state capacitor voltage is $29.85\cos(5t-84.3^\circ)$