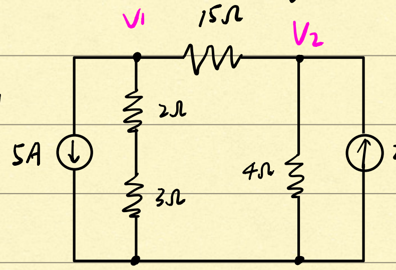
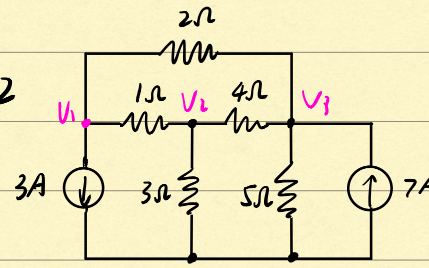
After-class Assignment 2

4.1



4.2



Apply KCL to node 1,

$$-5 = \frac{V_1}{2+3} + \frac{V_1 - V_2}{15}$$

$$-75 = 3V_1 + V_1 - V_2$$

$$-75 = 4V_1 - V_2 \quad \text{——} ①$$

Apply KCL to node 2,

$$2 = \frac{V_2}{4} + \frac{V_2 - V_1}{15}$$

$$120 = 15V_2 + 4V_2 - 4V_1$$

$$120 = 19V_2 - 4V_1 \quad \text{——} ②$$

Solve the equations,

$$① + ② \Rightarrow 45 = 18V_2$$

$$V_2 = \frac{5}{2}\ V$$

$$V_1 = -\frac{145}{8}\ V$$

Thus V_1 is $\frac{5}{2}$ V and V_2 is $-\frac{145}{8}$ V.

Apply KCL to node 1,

$$-3 = \frac{V_1 - V_2}{1} + \frac{V_1 - V_3}{2}$$

$$-6 = 2V_1 - 2V_2 + V_1 - V_3$$

$$-6 = 3V_1 - 2V_2 - V_3 \quad \text{—} ①$$

Apply KCL to node 2,

$$\frac{V_2}{3} + \frac{V_2 - V_1}{1} + \frac{V_2 - V_3}{4} = 0$$

$$4V_2 + 12V_2 - 12V_1 + 3V_2 - 3V_3 = 0$$

$$0 = -12V_1 + 19V_2 - 3V_3$$

Apply KCL to node 3,

$$7 = \frac{V_3}{5} + \frac{V_3 - V_2}{4} + \frac{V_3 - V_1}{2}$$

$$140 = 4V_3 + 5V_3 - 5V_2 + 10V_3 - 10V_1$$

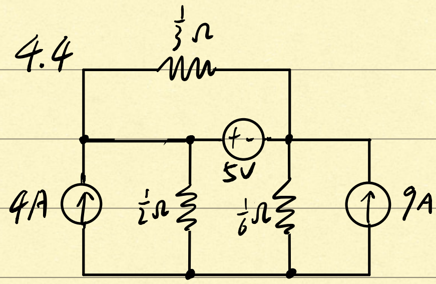
$$140 = -10V_1 - 5V_2 + 19V_3 \quad \text{——} ③$$

Combine ①②③, solve the equations.

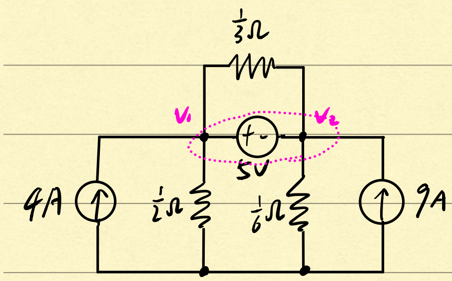
$$\begin{cases} -6 = 3V_1 - 2V_2 - V_3 \\ 0 = -12V_1 + 19V_2 - 3V_3 \\ 140 = -10V_1 - 5V_2 + 19V_3 \end{cases} \Rightarrow \begin{cases} V_1 = 5.235\ V \\ V_2 = 5.1175\ V \\ V_3 = 11.47\ V \end{cases}$$

Thus, V_{3A} is 5.235 V and V_{7A} is 11.47 V.

4.4



Redraw the circuit diagram.



Apply KCL to 1-2 supernode.

$$4+9=\frac{V_1}{\frac{1}{2}}+\frac{V_2}{\frac{1}{6}}$$

$$2V_1+6V_2=13 \quad \text{——①}$$

Since there is a 5V voltage between nodes 1 and 2,

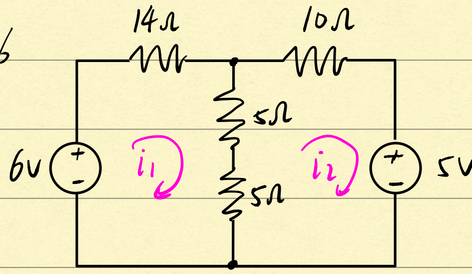
$$V_1-V_2=5 \quad \text{——②}$$

Combine ① and ②, solve the equations.

$$\begin{cases} 2V_1+6V_2=13 \\ V_1-V_2=5 \end{cases} \Rightarrow \begin{cases} V_1=5.375\ V \\ V_2=0.375\ V \end{cases}$$

Thus V_{4A} is 5.375V and V_{9A} is 0.375V.

4.6



Apply KVL to mesh 1,

$$-6+14i_1+(5+5)(i_1-i_2)=0$$

$$24i_1-10i_2=6 \quad \text{——①}$$

Apply KVL to mesh 2,

$$(5+5)(i_2-i_1)+10i_2+5=0$$

$$-10i_1+20i_2=-5 \quad \text{——②}$$

Solve both equations.

① × 2 + ②, we get

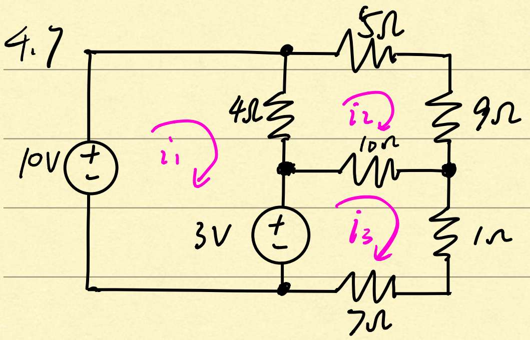
$$38i_1=7$$

$$i_1=\frac{7}{38}\approx 0.1842A=184.2mA$$

$$i_2=-\frac{3}{19}\approx -0.1579A=-157.9\ mA$$

Thus i_1 is 184.2mA and i_2 is −157.9mA.

4.7



The three mesh are assigned as indicated in the diagram.

Apply KVL to mesh 1,

$$-10 + 4(i_1 - i_2) + 3 = 0$$

$$4i_1 - 4i_2 = 7 \quad \text{— ①}$$

Apply KVL to mesh 2,

$$5i_2 + 9i_2 + 10(i_2 - i_3) + 4(i_2 - i_1) = 0$$

$$-4i_1 + 28i_2 - 10i_3 = 0$$

$$-2i_1 + 14i_2 - 5i_3 = 0 \quad \text{— ②}$$

Apply KVL to mesh 3,

$$i_3 + 7i_3 - 3 + 10(i_3 - i_2)$$

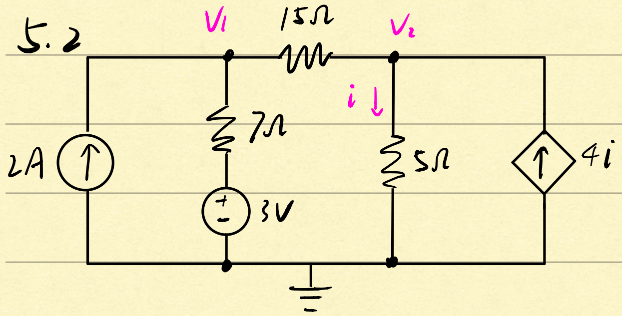
$$-10i_2 + 18i_3 = 3 \quad \text{— ③}$$

Combine ①,②,③, solve the equations.

$$\begin{cases} 4i_1 - 4i_2 = 7 \\ -2i_1 + 14i_2 - 5i_3 = 0 \\ -10i_2 + 18i_3 = 3 \end{cases} \Rightarrow \begin{cases} i_1 \approx 2.220 \\ i_2 \approx 0.470 \\ i_3 \approx 0.428 \end{cases}$$

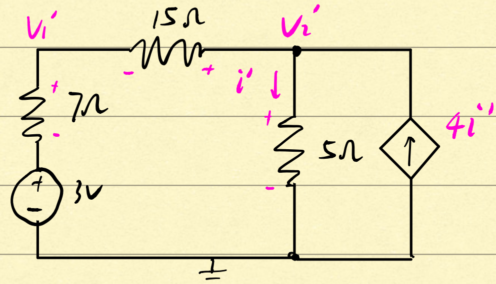
Thus i_1 is 2.220A and i_2 is 470.0mA.

5.2



Apply superposition.

1° Set current source to 0,



Apply KCL to node,

$$4i' = i' + \frac{V_2' - V_1'}{15\Omega} \quad —①$$

Thus the current on left part of the circuit is $3i'$.

Apply KVL to the left mesh.

$$-3 + (7+15)(-3i') + 5i' = 0$$

$$i' = -\frac{3}{61} \quad —②$$

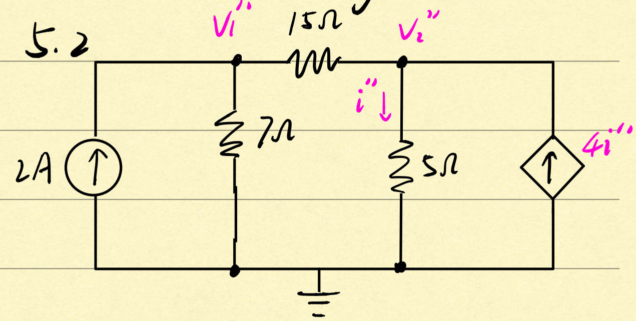
At the middle circuit, $\frac{V_2'}{5} = i'$ —③

Combine ②③, $V_2' = -\frac{15}{61} \approx 0.246V$ —④

Combine ①②④, $V_1' = \frac{120}{61} \approx 1.967V$

2° Set voltage source to 0,

5.2



Apply KCL to node 1,

$$-2 + \frac{V_1'' - V_2''}{15} + \frac{V_1''}{7} = 0 \quad —⑤$$

Apply KCL to node 2,

$$-4i'' + i'' + \frac{V_2'' - V_1''}{15} = 0 \quad —⑥$$

From 1° we have, $\frac{V_2''}{5} = i''$ —⑦

Combine ⑤⑥⑦, solve the equation,

$$\begin{cases} i' = -\frac{14}{61} \approx 0.230\,A \\ V_1' = \frac{560}{61} \approx 9.180\,V \\ V_2' = -\frac{70}{61} \approx -1.148\,V \end{cases}$$

Thus $V_1 = V_1' + V_1''$

$= 1.967 + 9.180$

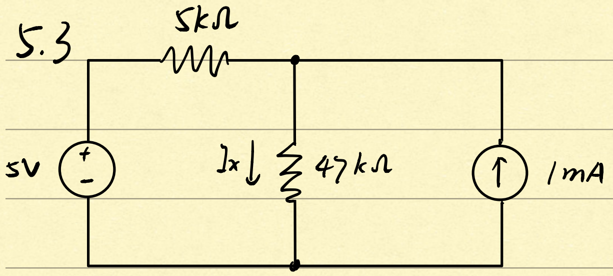
$= 11.147\,V$

$V_2 = V_2' + V_2''$

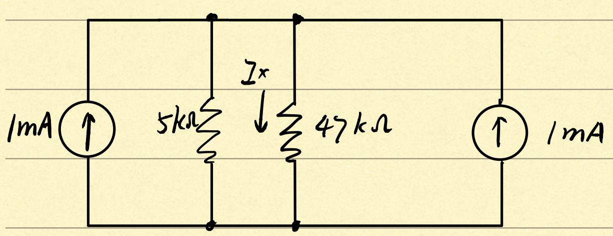
$= 0.246 - 1.148$

$= -1.394\,V$

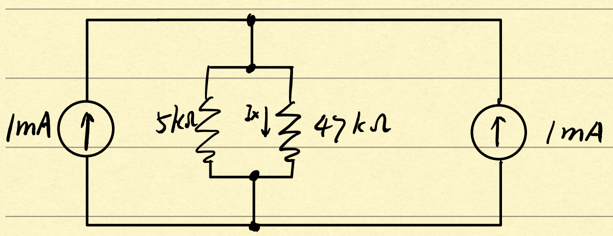
5.3



(a)



(b)



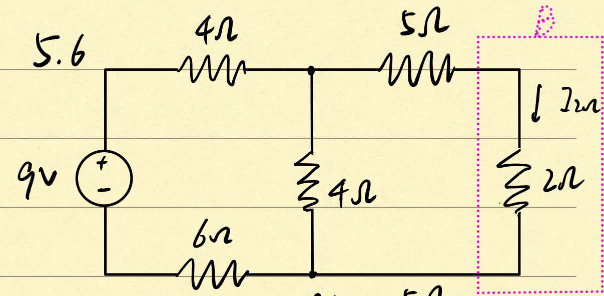
(c)

For diagram (a), Transforming voltage source to current source, as shown as diagram (b) the current is $\frac{5V}{5k\Omega} = 1mA$.
Redrawing the circuit diagram.
Apply the current diversion.

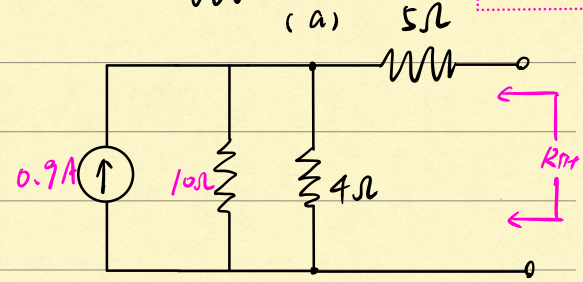
$$I_x = (1mA + 1mA) \times \frac{5}{47+5}$$
$$= \frac{10}{52}$$
$$\approx 0.192\ mA$$
$$= 192\ \mu A$$

Thus I_x is $192\ \mu A$.

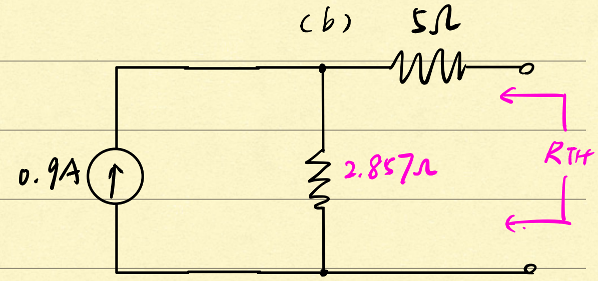
5.6



(a)



(b)



(c)

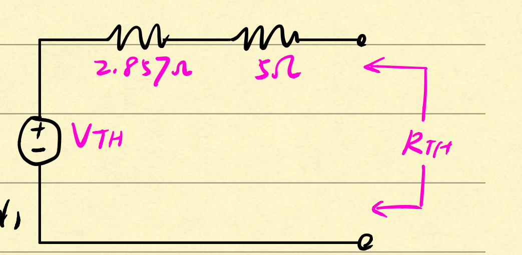
Designate the 2Ω resistor as network B.
For diagram (a), Transforming voltage source to current source, as shown as diagram (b)

$$I = \frac{9}{10} = 0.9\ A$$

Then redraw the diagram, two resistors can be drawn as (c)

$$R_{eq} = \frac{10 \times 4}{10+4} \approx 2.857\Omega$$

(d)



For diagram (c), Transforming current source to voltage source, as shown as diagram (d)

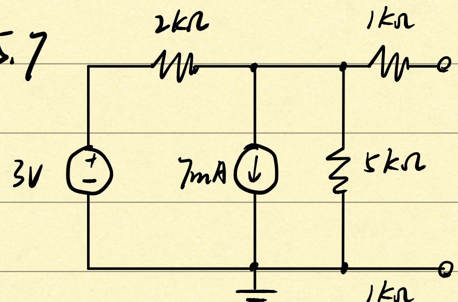
$V_{TH} = 2.857 \times 0.9 \approx 2.571$ V

$R_{TH} = 2.857 + 5 = 7.857\,\Omega$

$I_{2\Omega} = \frac{V_{TH}}{R_{TH}+2} = \frac{2.571}{7.857+2} \approx 0.2608A = 260.8mA$

Thus the current through 2Ω is 260.8mA.

5.7

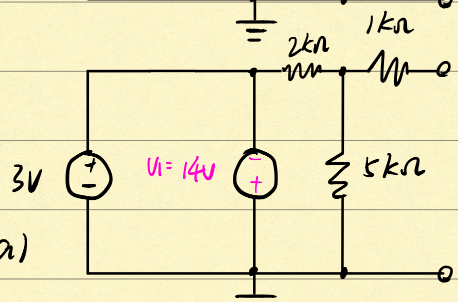


Apply the transformation on 7mA current source by replacing it with a voltage source V_1 as shown in diagram (a)

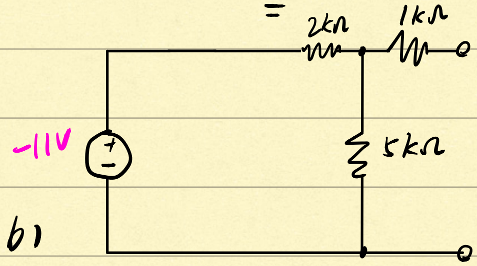
$V_1 = 2k\Omega \times 7mA = 14V$

$V_{Total} = 3 + (-14) = -11$ V

Redraw the diagram as (b)



(a)

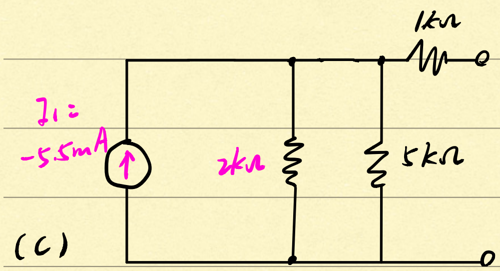


(b)

Apply the transformation on -11 V voltage source by replacing it with a current source I_1 and a $2k\Omega$ resistor in diagram (c)

$I_1 = \frac{-11V}{2k\Omega} = -5.5mA$

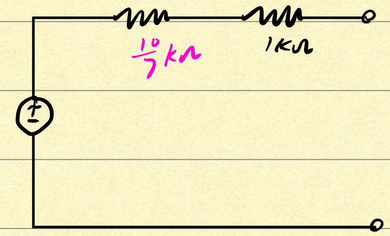
Therefore, $R_{eq} = \frac{2 \times 5}{2+5} = \frac{10}{7} k\Omega$



(c)

Apply the transformation on

(d)



current source I_1 and $\frac{10}{7}\Omega$ resistor by replacing them with a voltage source V_{TH} and $\frac{10}{7}$ kΩ resistor in series in diagram (d).

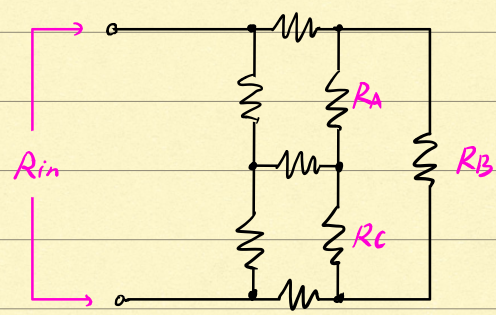
Therefore. $V_{TH} = \frac{10}{7}\,k\Omega \times (-5.5)\,mA \approx -7.857\,V$

$R_{TH} = \frac{10}{7} + 1 \approx 2.429\,k\Omega$

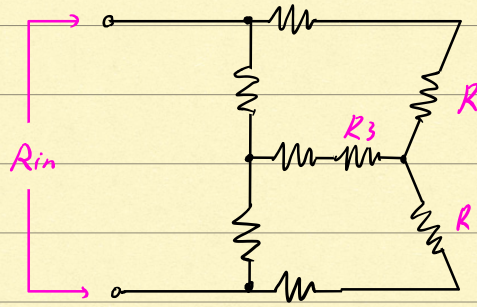
$I_{TH} = \frac{V_{TH}}{R_{TH}} = \frac{-7.857\,V}{2.429\,k\Omega} \approx -3.235\,mA$

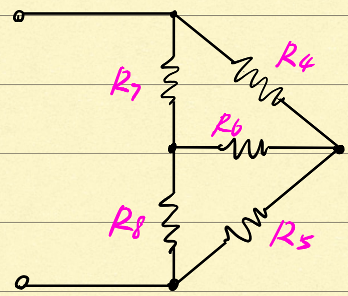
Thus the V_{TH} is $-7.857V$, R_{TH} is $2.429\,k\Omega$, I_{TH} is $-3.235mA$.

5.11



Apply Δ−Y conversion, convert R_A, R_B, R_C into R_1, R_2, R_3.

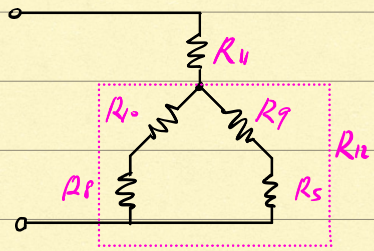




$R_1 = R_2 = R_3 = \frac{10 \times 10}{10+10+10} = \frac{10}{3}\,\Omega$

Therefore $R_4 = R_5 = R_6 = 10 + \frac{10}{3} = \frac{40}{3}\,\Omega$

Apply Δ−Y conversion, convert R_4, R_6, R_7 into R_9, R_{10}, R_{11}.



$$R_9 = \frac{R_4 R_6}{R_4 + R_6 + R_7} = \frac{\frac{40}{3} \times \frac{40}{3}}{\frac{40}{3} + \frac{40}{3} + 10} = \frac{160}{33}\,\Omega$$

$$R_{10} = R_{11} = \frac{\frac{40}{3} \times 10}{\frac{40}{3} + \frac{40}{3} + 10} = \frac{40}{11}$$

$$R_{12} = \frac{(R_8 + R_{10})(R_9 + R_5)}{R_8 + R_{10} + R_9 + R_5} = \frac{(10 + \frac{40}{11})(\frac{160}{33} + \frac{40}{3})}{10 + \frac{40}{11} + \frac{160}{33} + \frac{40}{3}} = \frac{600}{77}$$

$$R_{eq} = R_{12} + R_{11} = \frac{600}{77} + \frac{40}{11} = \frac{80}{7} \approx 11.43\,\Omega$$

Thus Thévenin equivalent resistance is $11.43\,\Omega$.