Coursework 1

Lecturer responsible: Charles Loo

Date set: 01/04/2021

Date of submission: 03/05/2021 (5pm on Leaning Mall)

Weight: 10% of final mark

Part A: Truss Problem:

The truss shown in Fig. 1.1 is statically determinate, supported at nodes F and I, and loaded at node E.

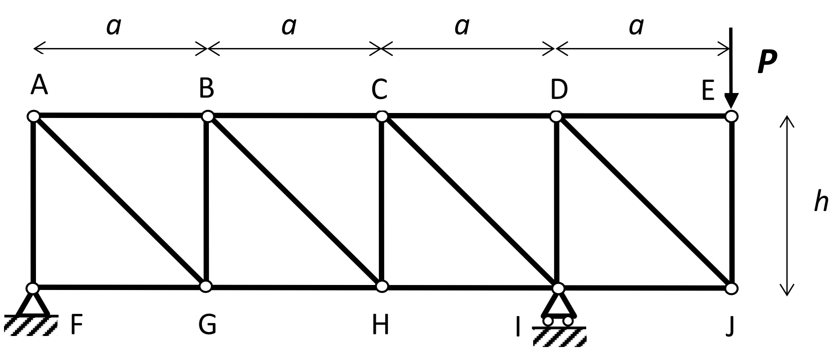
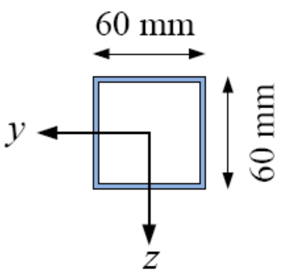


Fig. 1.1 *Statically determinate truss, geometry and loading.*

Q1: Determine the reactions at points F and I (show your definition of positive directions of the reaction forces). Determine the forces in all the members.

Q2: Show by color coding in a drawing which truss members are subjected to tension (**blue**) and which members are subjected to compression (**red**). Characterize the drawing in words (keywords: Top chord, bottom chord, verticals, diagonals).

All truss members are here assumed to have the same cross section. Though this may be inefficient from a structural point of view, it may be justified for other reasons such as economy (it often makes fabrication easier if all members are the same) or aesthetics. The members are here assumed to be hollow quadratic tubes as shown in Fig. 1.2.



$$A = 882\ \text{mm}^2$$
$$I = I_y = I_z = 459 \cdot 10^3\ \text{mm}^4$$

Fig. 1.2. *Cross section geometry and properties.*

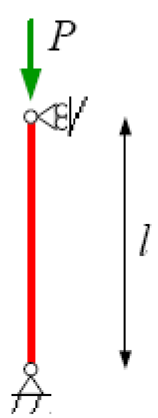
Truss members subjected to tension can be assumed to fail when the normal stress, σ, reaches the tensile strength, f_t, of the material. The maximum normal force, $N_{t,max}$, which a tension member in a truss can sustain is thus:

$$N_{t,max} = A \cdot f_t \quad (1)$$

Truss members subjected to compression can fail in two (or more) ways. Compression members may either fail when the normal stress, σ, reaches the compression strength, f_c, of the material or they may fail due to instability (buckling).

(Buckling is not a part of the curriculum for CEN103, but will be taught in CEN203. For those interested in the topic, a presentation may be found in e.g. the recommended textbook D. Gross et al., Engineering Mechanics 2, Chapter 7. You will not at the present stage be able to fully understand the topic since it requires some knowledge on basic beam theory, but you may get an impression of the nature of buckling phenomena).

The simplest theory for buckling of a member subjected to compression is the so-called Euler theory (Leonhard Euler – Swiss mathematician and physicist, 1707-1783). The theory assumes that the member is initially a mathematically straight line. This assumption never holds true in real life since structural members always have some geometrical imperfections. The Euler theory therefore tends to overestimate the buckling load. For real design of structures, we need to take into account imperfections and other issues, but we will here for simplicity assume that buckling occurs due to the Euler theory.
The critical Euler buckling load, N_E, for a compression member which is simply supported (pinned connections) at both ends is given by Eq. (2).



$$N_E = \frac{\pi^2 EI}{l^2} \quad (2)$$

where l is the geometrical length of the member, E is the modulus of elasticity, and I is the moment of inertia. (You will learn how to calculate the moment of inertia later in CEN103).

The maximum normal force, $N_{c,max}$, which a compression member in a truss can sustain is therefore;

$$N_{C,max} = min\begin{cases} A \cdot f_c \\ \frac{\pi^2 EI}{l^2} \end{cases} \tag{3}$$

Assume the following material properties and geometry:

E = 150,000 MPa, $f_t = f_c$ = 190 MPa, a = 2.0 m, h = 2.0 m, A and I as given in Fig. 1.2.

Q3: Calculate $N_{c,max}$ and $N_{t,max}$ for each member in the truss. Present the results in a table.

Q4: Show a graph for $N_{c,max}$ as a function of l for $l \in$ [0 ; 3 m]. Use Excel.

Q5: Use your results from Q1 to determine all member forces for the given geometry (a = 2.0 m, h = 2.0 m) as a function of P. Present the results in a table. Use Dr. Frame to check your results for P = 12 kN. Place the Dr. Frame printouts in an Appendix called "Appendix Q5".

Q6: Determine the maximum load, P_{max}, which can be applied at point E without causing failure in any member. You do this by considering each member of the truss. You have in Q5 determined the force in each member as a function of P. For each compression member in the truss set the member force equal to the $N_{c,max}$ for that member as determined in Q3 and determine the value of P. Correspondingly, for each tension member in the truss set the member force equal to $N_{t,max}$ and determine P. The minimum value of P taken for all truss members is the maximum load, P_{max}, which can be applied without causing failure.

Consider the truss shown in Fig. 1.3.

Q7: What is the failure load, P_{max}, for the truss shown in Fig. 1.3? Assume again the following material properties and geometry: E = 150.000 MPa, $f_t = f_c$ = 190 MPa, a = 2.0 m, h = 2.0 m, A and I as given in Fig. 1.2.

Q8: What is the difference between the two configurations? Which of the trusses shown in Figs. 1.1 and 1.3 leads to the highest failure load? Comment on you answers.

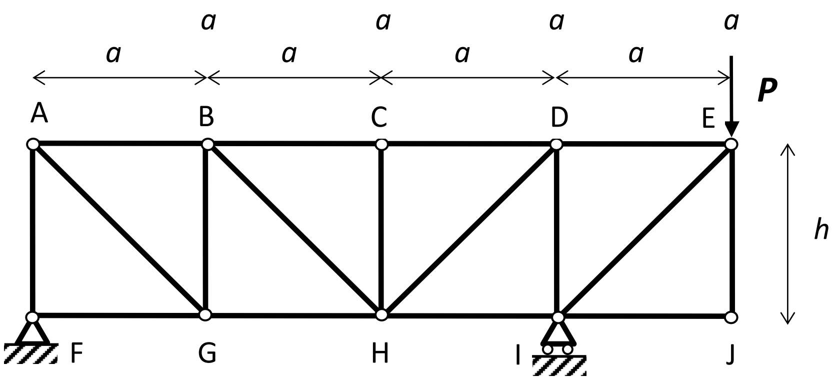


Fig. 1.3. *Alternative truss.*

Part B: Beam Problem:

A 3-hinged portal frame as shown in Fig. 2.1 is considered. The supports at points A and E are hinged (or pinned). Further, there is a hinge at point C. Due to the fact that there are three hinges, the frame is statically determinate. The moment is zero at the hinges.

All the beam parts AB, BC, CD, and DE have the same and constant cross section. The cross section area is A, the shear area is A_s, the moment of inertia is I_y, the modulus of elasticity (or Young's modulus) is E and the shear modulus (or modulus of rigidity) is G.

The height of the frame is H and the span is L.

The frame is in general loaded by a horizontal force, P, acting in point B and a vertical uniformly distributed load, q, (units: N/m) acting on the beam parts BC and CD. The load P is a wind load and q is due to self-weight of the structure, snow, and wind.

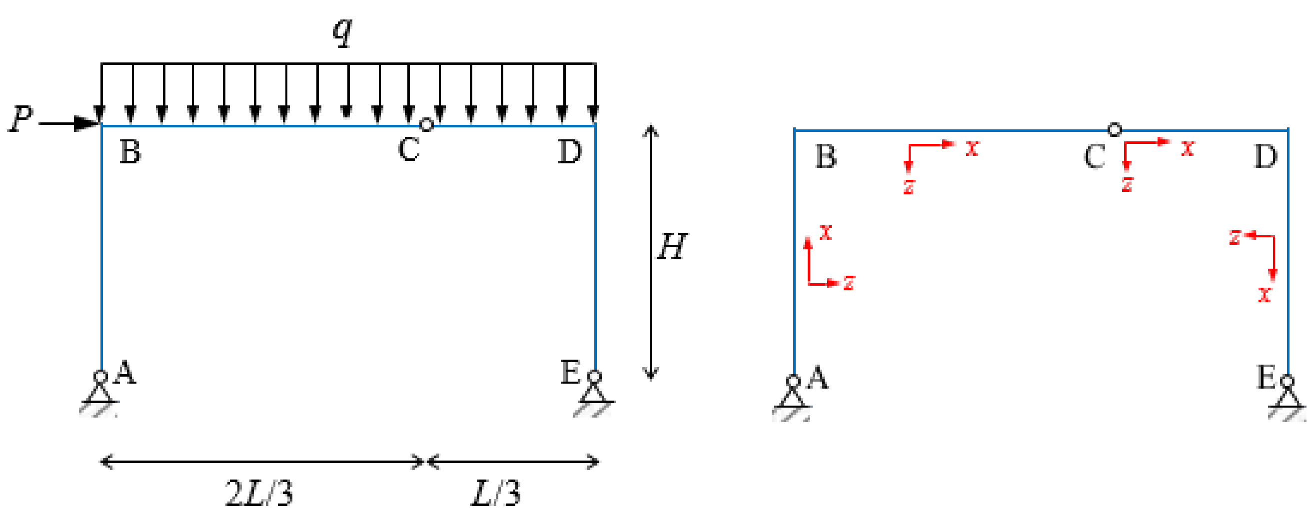


Fig. 2.1 *Geometry, loading, support conditions, and specified positive directions for 3-hinged portal frame*

Normal force, shear force, and moment diagrams:

Q2.1: Draw the normal force, shear force, and moment diagrams for *P* acting alone

Q2.2: Draw the normal force, shear force, and moment diagrams for *q* acting alone

Q2.3: Draw the normal force, shear force, and moment diagrams for *P* and *q* acting simultaneously

Cross section analysis:

The cross section of the frame is built up using a steel plate 8×400 mm^2 and four hollow steel tubes with outer dimensions 100×50 mm^2, see Fig. 2.2. The cross section area of a tube is $A_T = 1810$ mm^2 and the moments of inertia about the local principal y_T-axis and z_T-axis are $I_{yT} = 4.44\cdot10^6$ mm^4 and $I_{zT} = 0.76\cdot10^6$ mm^4 respectively.

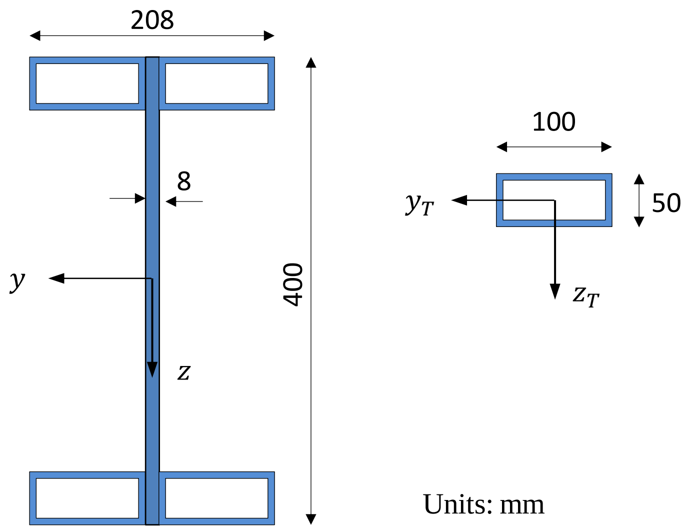


Fig. 2.2 *Cross section geometry*

Q2.4: Determine the cross section area, *A*, and the moment of inertia, I_y, about the principal *y*- axis for the cross section shown in Fig. 2.2

Stress analysis:

Assume that the frame in Fig. 2.1 has the height $H = 5$ m, the span $L = 10$ m and the cross section shown in Fig. 2.2.

The steel used for the frame has the same tensile strength, f_t, and compression strength, f_c, i.e. $f = f_t = f_c = 250$ MPa (= 250 N/mm^2).

The horizontal load on the frame as shown in Fig. 2.1 is $P = 20$ kN, $q = 8$ kN/m.

Q2.5: Consider the normal force and moment diagrams in Q2.3. Which points in the frame may be critical when considering the normal stress? Which combinations (N, M) are you going to consider?

Q2.6: For all the (N, M)-combinations mentioned in Q2.6 show the normal stress distribution over the cross section for $P = 20$ kN, $q = 8$ kN/m. (Show the normal stress distribution stemming from the normal force N, show the normal stress distribution stemming from the moment M, and show the resulting normal stress distribution for N and M acting simultaneously).

Software analysis:

Q2.7: Use DR FRAME to check your answer to Q2.6. Place the print-outs from your DR FRAME in an appendix called Appendix Q2.7.