14.1

(a) $(2e^{-100t}+e^{-200t})\sin 2000t$

From Euler's identity,

$$\sin 2000t = \frac{1}{2j}(e^{j2000t}-e^{-j2000t})$$

$$(2e^{-100t}+e^{-200t})\sin 2000t = \frac{1}{2j}(e^{j2000t}-e^{-j2000t})(2e^{-100t}+e^{-200t})$$

$$= 2e^{(-100+j2000)t}-2e^{(-100-j2000)t}-\frac{1}{2}je^{(-200+j2000)t}+\frac{1}{2}je^{(-200-j2000)t}$$

$$= k_1e^{s_1t}+k_2e^{s_2t}+k_3e^{s_3t}+k_4e^{s_4t}$$

Therefore the complex frequencies are:

$S_1 = (-100+j2000)\,s^{-1}$

$S_2 = S_1^* = (-100-j2000)\,s^{-1}$

$S_3 = (-200+j2000)\,s^{-1}$

$S_4 = S_3^* = (-200-j2000)\,s^{-1}$

(b) $(2-e^{-10t})\cos(4t+\phi)$

From Euler's identity,

$$\cos(4t+\phi) = \frac{1}{2}[e^{j(4t+\phi)}+e^{-j(4t+\phi)}]$$

$$(2-e^{-10t})\cos(4t+\phi) = \frac{1}{2}(2-e^{-10t})[e^{j(4t+\phi)}+e^{-j(4t+\phi)}]$$

$$= e^{j(4t+\phi)}+e^{-j(4t+\phi)}-\frac{1}{2}e^{-10t+j(4t+\phi)}-\frac{1}{2}e^{-10t-j(4t+\phi)}$$

$$= e^{j4t}\cdot e^{j\phi}+e^{-j4t}\cdot e^{-j\phi}-\frac{1}{2}e^{(-10+j4)t}\cdot e^{j\phi}-\frac{1}{2}e^{(-10-j4)t}\cdot e^{j\phi}$$

$$= k_1e^{s_1t}+k_2e^{s_2t}+k_3e^{s_3t}+k_4e^{s_4t}$$

Therefore the complex frequencies are:

$S_1 = (j4)\,s^{-1}$

$S_2 = S_1^* = (-j4)\,s^{-1}$

$S_3 = (-10+j4)\,s^{-1}$

$S_4 = S_3^* = (-10-j4)\,s^{-1}$

(c) $e^{-10t}\cos 10t \sin 40t$

From Euler's identity,

$$\sin 40t = \frac{1}{2j}[e^{j40t} - e^{-j40t}]$$

$$\cos 10t = \frac{1}{2}[e^{j10t} + e^{-j10t}]$$

$$e^{-10t}\cos 10t\sin 40t = e^{-10t}\left(\frac{e^{j10t}+e^{-j10t}}{2}\right)\left(\frac{e^{j40t}-e^{-j40t}}{2j}\right)$$

$$= \frac{e^{-10t}}{4j}(e^{j50t} - e^{-j50t} + e^{j30t} - e^{-j30t})$$

$$= \frac{1}{4j}e^{(-10+j50)t} - \frac{1}{4j}e^{(-10-j50)} + \frac{1}{4j}e^{(-10+j30)t} - \frac{1}{4j}e^{(-10-j30)t}$$

$$= k_1e^{s_1t} + k_2e^{s_2t} + k_3e^{s_3t} + k_4e^{s_4t}$$

Therefore the complex frequencies are:

$s_1 = (-10+j50)\,s^{-1}$

$s_2 = s_1^* = (-10-j50)\,s^{-1}$

$s_3 = (-10+j30)\,s^{-1}$

$s_4 = s_3^* = (-10-j30)\,s^{-1}$

14.3

$$f(t) = -6e^{-2t}[u(t+3) - u(t-2)]$$

(a) For two-sided $F(s)$,

$$F(s) = \int_{-\infty}^{\infty} f(t)e^{-st}\,dt = \int_{-\infty}^{\infty} -6e^{-2t}[u(t+3)-u(t-2)]e^{-st}\,dt$$

$$= -6\left[\int_{-\infty}^{\infty} e^{-(s+2)t}u(t+3)\,dt - \int_{-\infty}^{\infty} e^{-(s+2)t}u(t-2)\,dt\right]$$

Since $u(t+3) = \begin{cases} 0, & t<-3 \\ 1, & t>-3 \end{cases}$,

$u(t-2) = \begin{cases} 0; & t<2 \\ 1; & t>2 \end{cases}$.

$$F(s) = -6\left[\int_{-3}^{\infty} e^{-(s+2)t}\,dt - \int_{2}^{\infty} e^{-(s+2)t}\,dt\right]$$

$$= -6\left[\frac{e^{-(s+2)t}}{-(s+2)}\bigg|_{-3}^{\infty} - \frac{e^{-(s+2)t}}{-(s+2)}\bigg|_{2}^{\infty}\right]$$

$$= -6\left[\frac{0-e^{3s+6}}{-(s+2)} - \frac{0-e^{-2s-4}}{-(s+2)}\right]$$

$$= \frac{6}{s+2}(e^{-2s-4} - e^{3s+6})$$

(b) For one-sided $F(s)$,

$$F(s) = \int_{0^-}^{\infty} e^{-st} f(t)\, dt$$

$$= \int_{0^-}^{\infty} e^{-st} \{-6e^{-2t}[u(t+3) - u(t-2)]\}\, dt$$

$$= -6\left[\int_{0^-}^{\infty} e^{-(s+2)t} dt - \int_{2}^{\infty} e^{-(s+2)t} dt\right]$$

$$= -6\left[\frac{e^{-(s+2)t}}{-(s+2)}\Big|_0^{\infty} - \frac{e^{-(s+2)t}}{-(s+2)}\Big|_2^{\infty}\right]$$

$$= \frac{6}{s+2}(0 - 1 - 0 + e^{-2(s+2)})$$

$$= \frac{6}{s+2}(e^{-2s-4} - 1)$$

14.6

$$H(s) = \frac{2}{s} - \frac{4}{s^2} + \frac{3.5}{(s+10)(s+10)}$$

By the Linearity theorem,

$$h(t) = \mathcal{L}^{-1}\{H(s)\} = \mathcal{L}^{-1}\left\{\frac{2}{s} - \frac{4}{s^2} + \frac{3.5}{(s+10)(s+10)}\right\}$$

$$= \mathcal{L}^{-1}\left\{\frac{2}{s}\right\} - \mathcal{L}^{-1}\left\{\frac{4}{s^2}\right\} + \mathcal{L}^{-1}\left\{\frac{3.5}{(s+10)(s+10)}\right\}$$

By the Homogeneity property,

$$h(t) = 2\mathcal{L}^{-1}\left\{\frac{1}{s}\right\} - 4\mathcal{L}^{-1}\left\{\frac{1}{s^2}\right\} + 3.5\mathcal{L}^{-1}\left\{\frac{1}{(s+10)(s+10)}\right\}$$

Since

$$u(t) \Leftrightarrow \frac{1}{s}$$

$$tu(t) \Leftrightarrow \frac{1}{s^2}$$

$$te^{-\alpha t}u(t) \Leftrightarrow \frac{1}{(s+\alpha)^2},$$

$$h(t) = 2u(t) - 4tu(t) + 3.5te^{-10t}u(t)$$

$$= (2 - 4t + 3.5te^{-10t})u(t)$$

Therefore, $h(t) = (2 - 4t + 3.5te^{-10t})u(t)$.

14.7

$$Q(s) = \frac{3s^2 - 4}{s^2} = 3 - \frac{4}{s^2}$$

$$q(t) = \mathcal{L}^{-1}\{Q(s)\} = \mathcal{L}^{-1}\left\{3 - \frac{4}{s^2}\right\}$$

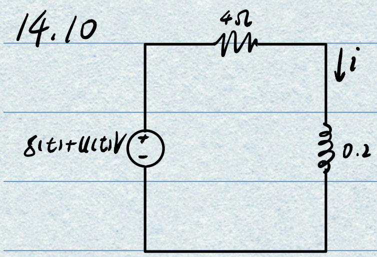
By the Linearity theorem and the Homogeneity property,

$$q(t) = \mathcal{L}^{-1}\{3\} - \mathcal{L}^{-1}\{\tfrac{4}{s^2}\} = 3\mathcal{L}^{-1}\{1\} - 4\mathcal{L}^{-1}\{\tfrac{1}{s^2}\}$$

Since $\delta(t) \Leftrightarrow 1$, $tu(t) \Leftrightarrow \frac{1}{s^2}$,

$$q(t) = 3\delta(t) - 4tu(t).$$

14.10



Since inductor is short circuited when $t=0$,

$$i(0^-) = 0 \text{ A}$$

Apply KVL to the circuit,

$$\delta(t) + u(t) = 4i(t) + 0.2\frac{di(t)}{dt}$$

Take the Laplace transform of each term,

$$1 + \frac{1}{s} = 4I(s) + 0.2[sI(s) - i(0^-)]$$

$$1 + \frac{1}{s} = 4I(s) + 0.2sI(s)$$

$$I(s) = \frac{1+\frac{1}{s}}{4+0.2s} = \frac{s+1}{(0.2s+4)s} = \frac{5s+5}{s(s+20)}$$

$$= \frac{5}{s(s+20)} + \frac{5}{s+20}$$

Apply the method of residues,

$$\frac{5}{s(s+20)} = \frac{A}{s} + \frac{B}{s+20}$$

$$A = \frac{5}{s+20-s} = \frac{5}{20} = 0.25$$

$$B = \frac{5}{s-(s+20)} = -\frac{5}{20} = -0.25$$

Therefore $I(s) = \dfrac{0.25}{s} - \dfrac{0.25}{s+20} + \dfrac{5}{s+20}$

$$= \frac{0.25}{s} + \frac{4.75}{s+20}$$

Thus $i(t) = \mathcal{L}^{-1}\{\frac{0.25}{s}\} + \mathcal{L}^{-1}\{\frac{4.75}{s+20}\} = (0.25 + 4.75e^{-20t})u(t)$ A.