assuming an incompressibility condition: density of material points does not change with time;

$$\frac{D\rho}{Dt} = \frac{\partial \rho}{\partial t} + \vec{v}\,\mathrm{grad}\,(\rho) = 0.$$

it is valid in case:

pressure and temperature changes are not very large and n t phase transformations leading to volume changes occur in the medium

incompressible continuity equation

$$\mathrm{div}(\vec{v}) = 0.$$

An alternative form of the equation, which is useful for numerical analysis is the Lagrangian continuity equation:

$$\frac{D\rho}{Dt} \equiv \frac{\partial \rho}{\partial t} + u_j \frac{\partial \rho}{\partial x_j} = -\rho \frac{\partial u_j}{\partial x_j}, \tag{1.3}$$

which can also be written in Eulerian form:

$$\frac{\partial \rho}{\partial t} = -\frac{\partial}{\partial x_j}(\rho u_j). \tag{1.4}$$

For an incompressible fluid, the equation of continuity reduces to:

$$\frac{\partial u_j}{\partial x_j} = 0, \text{ because } \frac{\partial \rho}{\partial t} + u_j \frac{\partial \rho}{\partial x_j} = \frac{D\rho}{Dt} = 0. \tag{1.5}$$