Many Body Localized Hidden Born Machine

Massachusetts Institute of Technology
HARVARD UNIVERSITY
IBM

Weishun Zhong, Xun Gao, Susanne Yelin, Khadijeh Najafi

The Model

Born Machine $$p_{\text{model}}(\mathbf{z}) = \frac{|\langle \mathbf{z}|\psi\rangle|^2}{\mathcal{N}}$$

hidden Born Machine $$p_{\text{model}}(\mathbf{v}) = \frac{\text{Tr}\rho_v \hat{Z}}{\mathcal{N}}$$

$$\rho_v = \text{Tr}_h |\psi\rangle\langle\psi|$$

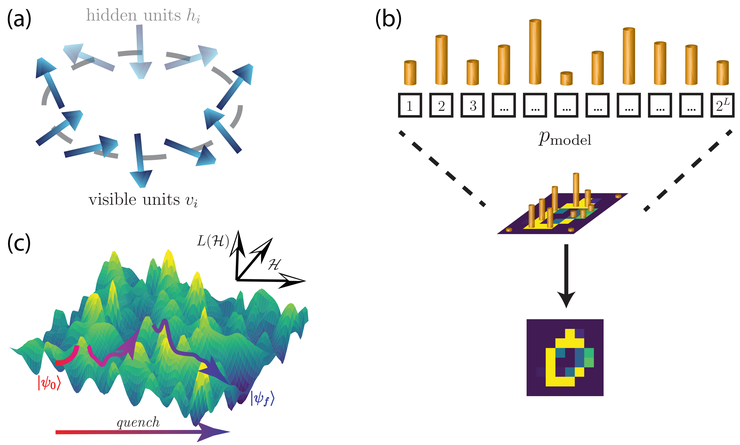


Fig.1 Illustrations of the model.
(a) A spin chain in periodic boundary condition. Spins in transparent color are hidden units, spins in solid color are visible units.
(b) The probability distribution of finding individual states in the z-basis represents the model distribution for the generative model.
(c) The training is done by optimizing disorder configurations in the Hamiltonian during each quantum quench, which is then used to evolve the quantum state over sucessive epoches of quenches.

MBL Ansats

$$\hat{\mathcal{H}}_{\text{total}} = \hat{\mathcal{H}}_{\text{XXZ}} + \hat{\mathcal{H}}_{\text{quench}}(\mathbf{\Theta}_m)$$

$$\hat{\mathcal{H}}_{\text{quench}} = \sum_i h_i^m \hat{S}_i^z$$ Disorder to be optimized

$$\hat{\mathcal{H}}_{\text{XXZ}} = \sum_i \frac{J_{xy}}{2}(\hat{S}_{i+1}^+\hat{S}_i^- + \hat{S}_{i+1}^-\hat{S}_i^+) + \sum_i J_{zz}\hat{S}_i^z\hat{S}_{i+1}^z$$

$$|\psi_{m+1}^{(n)}\rangle = \hat{\mathcal{U}}(\mathbf{\Theta}_m^{(n)})|\psi_m\rangle \text{ with } \hat{\mathcal{U}} = \hat{\mathcal{T}}\exp\left(-i\int_0^T dt\hat{\mathcal{H}}_{\text{total}}\right)$$

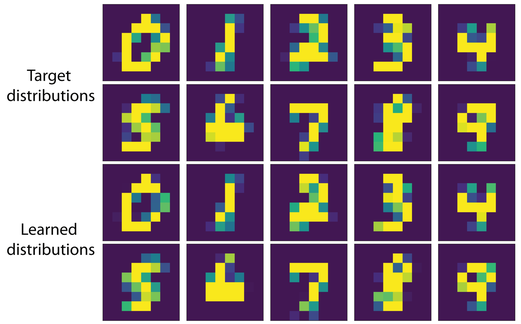
Training & Outcome

Summary of the algorithm
1. Uniformly sample N disorder realizations from the interval $[-h_d, h_d]$;
2. Independently evolve the state obtained from the previous quench with each of the disorder realizations;
3. Evaluate the model distribution in each realization, and compute its corresponding loss on the validation set;
4. Pick out the optimal disorder configuration among the candidates and use the corresponding state as the starting point for our next quench.

Loss function

$$\mathcal{L}_{MMD}(p_{\text{model}}, q_{\text{data}}) = \left|\left|\sum_x p(x)\phi(x) - \sum_x q(x)\phi(x)\right|\right|^2$$

Kernel functions



Learning quantum states

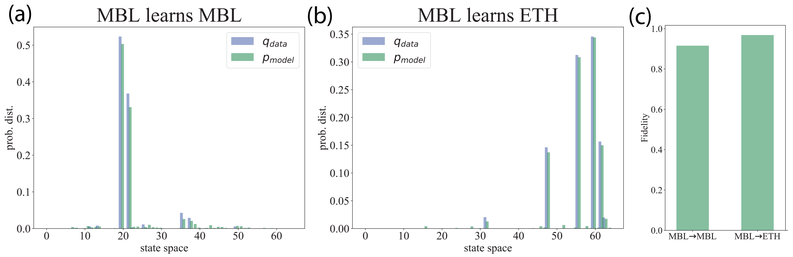


Fig.2 Learning MBL/ETH states.
(a)-(b) Model trained in MBL phase learns MBL/ETH state.
(c) Fidelity of the resulting state.

Model comparison

Theorem1: *For any given loss function L, comparing the hidden Born Machine and the vanilla Born Machine, we have $L_{\text{hBM}} \leq L_{\text{BM}}$*
Theorem2: *The XXZ model in 2D subject to quenches in z-direction can be reduced to an Ising model, whose expressibility has quantum computational advantage*

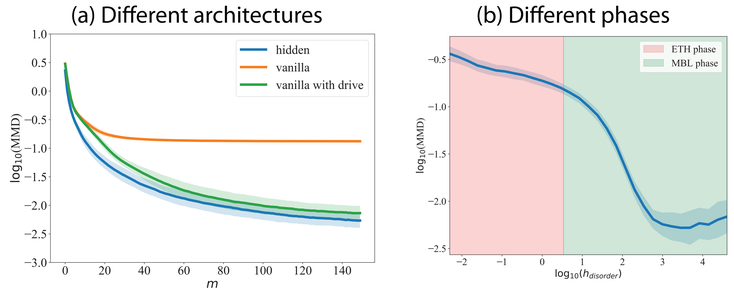


Fig.3 Model comparison.
(a) Train the vanilla model, hidden model, and the randomly driven vanilla model on MNIST dataset. The hidden model shows overall best performance.
(b) The hidden model in thermal/MBL phase. The MBL phase learns better.