Demystifying the Convolutions in PyTorch

Lecture Notes on Deep Learning

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Preamble

On the face of it, the convolution is one of the simplest ideas in computer vision and image processing.

All that's meant by a convolution is that you sweep an image with a flipped kernel (which is assumed to be smaller in size than the image), you sum the product of the two at each position of the kernel, and report the value calculated to the output.

But, as with so many things in life, this simplicity can be deceptive — especially so in the context of deep learning.

Hidden behind the simplicity is the fact that calculating a convolution calls for making assumptions about what to do at the border of the input. While the consequences of those assumptions can be ignored in computer vision and image processing, that's not so easily done in DL where the resolution hierarchies can be deep and, at the top of a resolution pyramid, each pixel may represent a significant chunk of the image at the bottom.

Preamble (contd.)

Other issues regarding convolutions in DL relate to the role played by the channels. How do M channels in the input go into N channels at the output for literally arbitrary values for M and N?

And what about the "groups" option when you call PyTorch's functions for convolutions? What does that do?

Finally, what about the fact that DL convolutions are really not convolutions, but cross-correlations?

Here is a "NOTE" on the doc page for torch.nn.Conv2d:

"Depending on the size of your kernel, several (of the last) columns of your input might be lost because it is a valid cross-correlation and not a full-correlation. It is up to the user to add proper padding."

What exactly does that mean — considering especially the fact that padding involves making assumptions about imagined pixels outside the image array?

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2D Convolution

 The following snippet of Python code nicely says it all as far as the definition of 2D convolution is concerned:

```
def convo2d(input, kernel):
    H,W = input.shape
    M,N = kernel.shape
    out = numpy.zeros((H-M+1,W-N+1), dtype=float)
    kernel = numpy.flip(kernel)
    for i in range(H-M+1):
        for j in range(W-N+1):
            out[i,j] = numpy.sum( input[i:i+M,j:j+N] * kernel)
    return out
```

• If you are a beginner Python programmer, pay attention to the role of numpy.flip() in the script.,

[NOTE: Note my use of the "mnemonics" for the variables \mathbb{H} for the "height" and \mathbb{W} for the "width" of the input pattern. This is to help cope with possible mental confusion when you are also using the PIL library in the same program. The image related functions in that library are based on the notion of (x,y) coordinates, with 'x' standing for the horizontal axis and 'y' for the vertical axis.]

2D Convolution

Let's now define an input array for the convolutions:

• Next we need to define a kernel:

2D Convolution (contd.)

 Applying the convolution function to the input arr and to the kernel ker returns:

- The size of our input array went down from (8,8) to (6,6).
- There will be almost nothing left of this "image" after 3 or 4 application so this convolutional operator.
- Now think of a 256×256 image and think of a convolutional processing chain that uses several rounds of $k \times k$ kernels. You may not be left with much of the image at the output of the chain.

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Using scipy.signal.convolve2d()

• Let's now try the popular scipy.signal.convolve2d() for 2D convolutions. In the following function call, we use the same 8×8 input array arr and the same 3×3 kernel that were defined earlier.:

 Pay particular attention to the option string mode='valid' in the call to the convolution function. What that means is that we want the convolution function to only use valid pixels — and not any hallucinated pixels outside the array just for the sake of returning an 8 × 8 array.

Comparing My convo2d() with scipy.signal.convolve2d()

- So you see that my implementation of 2D convolution on Slide 6 is the same as what's produced by scipy.signal.convolve2d() when the latter is used in the "valid" mode.
- We could have tried using scipy.signal.convolve2d() with a different set of options as in

```
output = scipy.signal.convolve2d(arr, ker, mode='full', boundary='symm')
```

But that would have required making assumptions about the pixels outside the input array — which *may not* always be a good thing to do.

• So what we have here is a dilemma: Either we choose to accept a shrunk convolutional output that uses only valid pixels, or we choose an output that is of the same size as the input, but is based on hallucinations at the imagined pixels beyond the border.

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Input Specs for PyTorch's

torch.nn.functional.conv2d()

 This PyTorch function only works on input tensors whose shape corresponds to:

```
(batch_size, num_input_channels, image_height, image_width)
```

- Depending on how we define our input initially, this may call for "repacking" the input tensors as you will soon see.
- Having to convert a numpy representation of the input into a tensor representation on the fly in a custom data loader is not an unlikely scenario. I can easily imagine an application where the multi-dimensional input data comes in the form of numpy arrays. Consider, for example, DL being used on the volumetric data produced by a weather radar (Re: Prof. Robin Tanamachi).

Input Specs for torch.nn.functional.conv2d() (contd.)

• Converting the input 8×8 number array arr into a tensor:

```
tensor arr = torch.from numpv( arr )
print(tensor_arr)
                         tensor([[4., 4., 4., 4., 1., 1., 1., 1.],
                                 [4., 4., 4., 4., 1., 1., 1., 1.]
                                 [4., 4., 4., 4., 1., 1., 1., 1.]
                                 [4., 4., 4., 4., 1., 1., 1., 1.],
                                 [4., 4., 4., 4., 1., 1., 1., 1.],
                                 [4., 4., 4., 4., 1., 1., 1., 1.]
                                 [4., 4., 4., 4., 1., 1., 1., 1.],
                                 [4., 4., 4., 4., 1., 1., 1., 1.], dtype=torch.float64)
print(tensor arr.shape)
                                                      ## torch.Size([8, 8])
print(tensor arr.size())
                                                      ## torch.Size([8, 8])
print(type(arr))
                                                      ## <type 'numpy.ndarray'>
print(type(arr[0,0]))
                                                      ## <type 'numpy.float64'>
```

- As to why the data in the tensor returned by the torch.from_numpy()
 converter is of type torch.float64, note the datatype in the numpy
 array.
- All of the different tensor data types are listed at:

Kernel Specs for torch.nn.functional.conv2d()

 The kernels that you feed into torch.nn.functional.conv2d() must be tensors of shape:

```
(out_channels, in_channels, kernel_height, kernel_width)
```

- You'll notice that the shape specification for a kernel is NOT the same as for the input.
- ullet In order to get there, let's first convert the 3 imes 3 kernel into a tensor:

• Notice again the datatype for the elements of the kernel tensor.

Repackaging the Input and the Kernel Tensors

Now we must reformat the input and the kernel tensors so that they
correspond to the input specifications for the
torch.nn.functional.conv2d() function. Here is a link to the doc
page where you will see the shape specifications for the two tensors:

```
https://pytorch.org/docs/stable/nn.functional.html
```

 BTW, that doc page also has a very important link related to generating reproducible results (important while you are debugging your code):

```
https://pytorch.org/docs/stable/notes/randomness.html
```

• Summary of the two shape specifications:

```
input: (batch_size, num_input_channels, image_height, image_width)
kernel: (out_channels, in_channels, kernel_height, kernel_width)
```

Repackaging the Input Tensor

- That raises the following very important question: How to reshape the input tensor so that its "axis 0" corresponds to the batch size and its "axis 1" to the number of input channels?
- This is how you do it in PyTorch:

- This now has the correct shape for the input tensor.
- But what is the call torch.unsqueeze(tsr_arr, 0) actually doing?

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Reshaping a Tensor with reshape() and view()

But First Let's Talk About How to Talk About the Shape of a Tensor

- Let's say you are thinking out loud as you are staring at a tensor whose shape is (1, 1, 16, 16). How would you describe such a data object to yourself?
- If you said to yourself that you were dealing with a 4-dimensional data object, you are in deep trouble — a bad place to be in this age of deep learning.
- If you are one of those who thinks of a 10-element vector as a 10-dimensional data object and a 10×10 array as a 2-dimensional data object, you are again in deep trouble.
- Thinking of a 10-element vector as a 10-dimensional data object and a 10×10 array as a 2-dimensional data object can create cognitive dissonance (which, if not properly dealt with, could result in mental anguish and, eventually, mental-health problems).

Talking About the Shape of a Tensor (contd.)

- When a tensor has shape (1, 1, 16, 16), we say that the tensor has FOUR AXES. The first entry in the shape is for Axis 0, the second entry for Axis 1, the third for Axis 2, and the last for Axis 3.
- For the same data object, whose shape is (1,1,16,16), we say that the dimensionality along Axis 0 is 1, the dimensionality along Axis 1 is also 1, the dimensionality along Axis 2 is 16, and the dimensionality along Axis 3 is also 16.
- Now we can refer to a vector as a data object of a single axis, a
 matrix as a data object of two axes, an RGB image as a data object
 with 3 axes, and so on.
- When a convolutional layer is meant for RGB images of size 256×256 , and the layer produces 64 channels at its output, the input to the layer is of shape (3, 256, 256) and its output of shape (64, 256, 256), both the input and the output data objects being of 3

Talking About the Shape of a Tensor (contd.)

- In your own mind's eye, can you now visualize the difference between the following three data objects: one of shape (16,16), the other of shape (1,16,16), and the last of shape (1,1,16,16)?
- Think of the first one as an image in the XY-plane, the second the same image in the same plane but in the XYZ-space, and the last as the same image in the same plane but in the XYZW-space.
- Let's say we are given a data object, an image actually, of shape (16,16) and we want it to be processed by a function that expects its inputs of shape, say, (m,16,16) where the integer m stands for the number of channels associated with the image.
- Assume that ours is a grayscale image, implying that m=1. So before we can invoke the function, we need to convert the image into the shape (1,16,16). We can do that by calling on the unsqueeze() function as you saw earlier.

You Can Call torch.unsqueeze() on Different Axes

In order to convert

```
tensor_x = torch.tensor([1,2,3,4])
print(x.shape) ## torch.Size([4])

into the tensor
    tensor_y = torch.tensor([[1,2,3,4]])
    print(y.shape) ## torch.Size([1, 4])

I'd need to call unsqueeze() on Axis 0 of tensor_x:

    tensor_y = torch.unsqueeze(tensor_x,0)
    print(tensor_y) ## tensor([[1, 2, 3, 4]])
    print(tensor_y,shape) ## torch.Size([1, 4])
```

• In general, through, you can call unsqueeze() on any axis of a tensor. While a call to unsqueeze() on axes other than 0 may yield results that at first sight look like a strange rearrangement of the data, you never lose any data in the process. That is, the overall data content in the augmented space is the same as in the original space.

Calling torch.unsqueeze() on Different Axes (contd.)

Here is applying unsqueeze() to Axis 1 of the same tensor:

- Applying unsqueeze() to Axis 1 made the dimensionality along that axis to 1. If you want to associate the matrix imagery with what happened, it turned a row vector into a column vector.
- The next slide shows a slightly more elaborate example of applying unsqueeze(). Do you understand the data rearrangement caused by the function?

An Example of Calling torch.unsqueeze() on Axis 1

```
tensor_x = torch.tensor([[[1,2,1,2], [3,4,3,4], [1,3,1,3]], [[5,6,5,6], [6,7,6,7], [8,9,8,9]]])
print(tensor_x)
                            tensor([[[1, 2, 1, 2],
                                     [3, 4, 3, 4],
                                     [1, 3, 1, 3]],
                                    [[5, 6, 5, 6].
                                     [6, 7, 6, 7],
                                     [8, 9, 8, 9]]])
print(tensor_x.shape)
                               ## torch.Size([2, 3, 4])
tensor_y = torch.unsqueeze(tensor_x, 1)
print(tensor_v)
                            tensor([[[[1, 2, 1, 2],
                                      [3, 4, 3, 4].
                                      [1, 3, 1, 3]]],
                                    [[[5, 6, 5, 6],
                                      [6, 7, 6, 7],
                                      [8, 9, 8, 9]]]])
print(tensor_v.shape)
                              ## torch.Size([2, 1, 3, 4])
```

The Function squeeze()

- Function squeeze() drops all axes whose dimensionality is just 1.
- This example illustrates the idea:

• What if the axis whose dimensionality is 1 is in the middle?

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Finally, Using torch.nn.functional.conv2d()

 Now that you have seen how to repackage the input tensor so that its shape is as specified on Slide 16, we must do the same for the kernel:

At long last, we are ready to use torch.nn.functional.convo2d():

• Compare this convolution output to what was produced by scipy.signal.convolution2d() on Slide 10. The signs of the nonzero entries are reversed!!!!!!!!!!!

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The Class torch.nn.Conv2d and its Callable Instances

- First of all, note that, unlike what we have dealt with so far, torch.nn.Conv2d is NOT a function. It is a Python class. You construct instances of this class and, because they are callable, you can use them like functions.
- Because you are now dealing with a class, you now have two types of specifications to deal with: The parameter structure for calling the constructor and the parameter structure for invoking the callable instance.
- The three required arguments for calling the constructor of torch.nn.Conv2d are:

```
(in_channels, out_channels, kernel_size)
```

The last argument, for kernel_size, can be scalar when using a square kernel, or a tuple for non-square kernels.

The Class torch.nn.Conv2d and its Callable Instances (contd.)

 A callable instance of torch.nn.Conv2d is subsequently called with an input tensor as its argument. This input tensor must be of the following shape:

```
(batch_size, in_channels, H, W)
```

where H and W refer to the height and the width of the image array.

- But Where is the Kernel Specified for torch.nn.Conv2d?
- By default, the kernel is implicit. Since the kernel is a learnable entity, the user need not worry about its precise specification except, of course, for its shape.

Using an Instance of torch.nn.Conv2d on Random Input

 Here we construct an instance of torch.nn.Conv2d for the constructor parameters in_channels=1, out_channels=1, and kernel_size=3

```
conop = nn.Conv2d(1, 1, 3, bias=False) ## assumes a stride of 1
```

• In order to become more familiar with torch.nn.Conv2d, let's first construct a random input for the callable conop:

Using an Instance of torch.nn.Conv2d on Random Input (contd.)

• Here is the convolution with the input shown on the previous slide with a 3×3 kernel:

• Note that whereas the input was of size 8×8 , the output is of size 6×6 . That is the because, by default, the convolution is carried in the "valid" mode that you saw earlier in these slides.

Accessing the Kernel Used for the Convolution

- About the kernel that conop used for the convolution output shown on the previous slide, what if we have an uncontrollable desire to eat pizza? Sorry! I must be hungry. Meant to say what if we have an uncontrollabe desire to see what exactly was used for the kernel. Is it possible to do that?
- Yes, by accessing the weight attribute of the conop instance:

• Note the two special "things" about the weight attribute we printed out: It is of type Parameter and its requires_grad property is set.

Using an Instance of torch.nn.Conv2d on a Deterministic Input and With Our Own Kernel

• Let's apply the conop instance to the same deterministic input that you saw earlier on Slide 17. Here is that input:

 We note that the datatype of our input tensor is not the same as what torch.nn.Conv2d uses for its internally generated kernel. So we do the type conversion:

```
tensor_arr = tensor_arr.float()
print(tsr_arr.type()) ## torch.FloatTensor
```

With Our Own Input and Kernel (contd.)

Now we specify our kernel:

And, define the convolutional operator and set its weight attribute:

• Finally, we are ready to carry out the convolution:

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Specifying the Input and the Kernel for the Convovs. Correlation Test

• Here is the input:

- In this case, I have specified the input in one go by directly calling torch.zeros() function.
- Since in PyTorch programming, it is not uncommon to go back and forth between numpy and torch, note that the syntax for numpy.zeros() is NOT identical to that for torch.zeros(). The former takes a maximum of three args.

Specifying the Input and the Kernel for the Test (contd.)

• And here is the kernel:

• Do you understand why I have constructed the input and the kernel in the manner shown? If not, you need to think about what is different between a convolution and a cross-correlation.

Aquí Está el Resultado de la Prueba

• Now we are ready to do the convolution:

 Can you tell that only a cross-correlation could have produced this output?

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Questions Related to Multi-Channel Convolutions

- The first convolutional layer in a network meant for processing color images is likely to have 3 input channels, one for each color channel.
- And it would not be uncommon for the same first layer to have 64 or 128 or even 256 output channels.
- And, again, it would not be uncommon for there to be another convolutional layer further up the stack whose input channels and output channels would both equal, say, 128.
- That raises the following sorts of questions:
 - What is the shape of the kernel tensor for in_ch=3 and out_ch=128?
 - And what might the kernel tensor look like for the case in_ch=128 and out_ch=128?
 - When in_ch == out_ch, is it possible to set up, say, a 1-1 convolutional connection between the input and the output channels? That is, can we tell PyTorch that exists a 1-1 correspondence between the input channels and the output channels?

Specifying a Two-Channel Input

 As you'll recall for the input specification for an instance of torch.nn.Conv2d, it must have four axes, the first is for the batch size, the second for the number of input channels, the third for the height of the image, and the last for the width of the image:

- One of the annoying aspects of PyTorch tensors is that the datatype float in a declaration actually means double, which gives you 64-bit floats.
- However, with its default settings, the torch.nn.Conv2d class likes
 to do its thing with 32-bit floats. So, you may need to cast the tensor
 to a 32-bit float and you do that by calling float().

Specifying a Two-Channel Input

Let's look at the input we constructed on the previous slide:

```
print(input)
                          tensor([[[[0.. 0.. 1.. 0.. 0.. 0.. 0.. 0.]
                                    [0.. 0.. 1.. 0.. 0.. 0.. 0.. 0.].
                                    [0., 0., 1., 0., 0., 0., 0., 0.],
                                    [0., 0., 1., 0., 0., 0., 0., 0.],
                                    [0.. 0.. 1.. 0.. 0.. 0.. 0.. 0.].
                                    [0., 0., 1., 0., 0., 0., 0., 0.],
                                    [0., 0., 1., 0., 0., 0., 0., 0.],
                                    [0.. 0.. 1.. 0.. 0.. 0.. 0.. 0.]].
                                   [[0., 0., 0., 0., 0., 0., 1., 0.],
                                    [0.. 0.. 0.. 0.. 0.. 0.. 1.. 0.].
                                    [0., 0., 0., 0., 0., 0., 1., 0.].
                                    [0., 0., 0., 0., 0., 0., 1., 0.],
                                    [0.. 0.. 0.. 0.. 0.. 0.. 1.. 0.].
                                    [0., 0., 0., 0., 0., 0., 1., 0.].
                                    [0., 0., 0., 0., 0., 0., 1., 0.],
                                    [0., 0., 0., 0., 0., 1., 0.]]]])
print(input.shape)
                                                        ## torch.Size([1, 2, 8, 8])
```

• I intentionally chose these two patterns for the two input channels in order to see how the input channels would contribute to the output channels. You can do that for small (say, 3×3) convolutional operators, as you will see.

Examining the Kernel for in_ch=2 **and** out_ch=1

• Here is an instance of torch.nn.Conv2d for in_ch=2 and out_ch=1:

```
conop = nn.Conv2d(2, 1, 3, bias=False)
                                                 ## ker size 3x3 and assumes a convo stride of 1
ker = conop.weight
print(ker)
                         tensor([[[[ 0.0624, -0.0712, -0.0463],
                                   [-0.2252, -0.1561, -0.0972],
                                   [ 0.0087, 0.0932, 0.1414]],
                                  [[-0.1598, -0.1026, 0.0856],
                                   [ 0.1957, -0.0485, 0.1764],
                                   [-0.0380, 0.0249, 0.2134]]]], requires grad=True)
                                  ## torch.Size([1, 2, 3, 3])
print(ker.shape)
                                                                   [This is in keeping with the kernel
                                  ##
                                                                    specs at the bottom of Slide 16]
output = conop(input)
                                  ## input is as shown on the previous slide
print(output)
                         tensor([[[-0.0021, -0.1342, -0.1541, 0.0000, 0.4754, -0.1262],
                                   [-0.0021, -0.1342, -0.1541, 0.0000, 0.4754, -0.1262],
                                   [-0.0021, -0.1342, -0.1541, 0.0000, 0.4754, -0.1262],
                                   [-0.0021, -0.1342, -0.1541, 0.0000, 0.4754, -0.1262],
                                   [-0.0021, -0.1342, -0.1541, 0.0000, 0.4754, -0.1262],
                                   [-0.0021, -0.1342, -0.1541, 0.0000, 0.4754, -0.1262]]]]
                                grad fn=<MkldnnConvolutionBackward>)
print(output.shape)
                                ## torch.Size([1, 1, 6, 6])
```

• Do you understand why the kernel is shaped the way it is?

Another Look at the Output for in_ch=2 and out_ch=1

- In the output shown on the previous slide, can you visualize the relationship between the output, the two 3 × 3 operators in the kernel, and the two input channels? [Regarding the output shown on the previous slide, by running the first 3 × 3 operator in the kernel over the first channel of the input, you can tell that the first 3 columns of the output are a result of the convolution produced by those two. Similarly, you can verify that the last three columns in the output are a result of the second 3 × 3 operator in the kernel convolving with the the second channel.]
- To make it more convenient to visualize, let's discretize the output shown on the previous slide:

Examining the Kernel for in_ch=2 **and** out_ch=3

Here is an instance of torch.nn.Conv2d for in_ch=2 and out_ch=3:

```
conop = nn.Conv2d(2, 3, 3, bias=False)
                                                ## args: in_channel out_channel ker_size
ker = conop.weight
print(ker)
                         tensor([[[[-0.2187, -0.1484, -0.0597],
                                  [-0.0919, 0.2036, -0.1528],
     For the first
                                  [-0.1085, -0.1647, -0.2207]],
     out channel
                                  [[-0.1376, 0.2026, 0.1052],
                                  [ 0.1142, 0.0124, -0.1208].
                                  [ 0.0399, -0.2201, -0.1703]]],
                                 [[-0.1215, 0.1487, 0.1382],
                                  [-0.1045, -0.0085, 0.1507],
                                  [ 0.2343, 0.0935, 0.0318]],
     For the second |
     out channel
                                  [[ 0.1580, -0.1388, 0.0439],
                                  [-0.1827, -0.1634, -0.1218],
                                  [ 0.1066, 0.0948, -0.1396]]],
                                 [[ 0.0712, 0.1294, -0.0297],
                                 [ 0.0090, 0.0546, 0.1462].
                                  [ 0.2263, -0.1816, -0.0864]],
     For the third
     out channel
                                  [[ 0.0926, 0.1953, 0.2051].
                                  [ 0.2080, 0.0469, -0.2050],
                                  [ 0.0217, -0.1475, -0.2197]]]], requires_grad=True)
```

The in_ch=2 and out_ch=3 Case (contd.)

- Regarding the shape of the kernel shown on the previous slide torch.Size([3, 2, 3, 3]) Axis 0 corresponds to the output channels, Axis 1 to the input channels, Axis 2 to H, and Axis 3 to W. Again, this is in keeping with the kernel specs shown at the bottom of Slide 16. [By this time you should have gotten used to the fact that the order in which you specify the number of input channels and the number of output channels is opposite for the nn.Conv2d constructor and for the kernel it needs.]
- After descretization in the same manner as before, here is the output for this convolution:

```
output = conop(input)
                                          ## input is the same as shown on Slide 43
output = output * 10 + 0.5
output = output.type(torch.int8)
print(output)
                         tensor([[[[-3, 0, -3, 0, -1, 0],
                                   [-3, 0, -3, 0, -1, 0].
                                   [-3, 0, -3, 0, -1, 0],
                                   [-3, 0, -3, 0, -1, 0],
                                   [-3, 0, -3, 0, -1, 0]],
                                    3, 2, 0, 0, -1, -1],
                                  [[ 0, 0, 3, 0, -1, 1],
                                    0, 0, 3, 0, -1, 1],
                                    0, 0, 3, 0, -1, 1],
                                    0, 0, 3, 0, -1, 1],
                                   [ 0, 0, 3, 0, -1, 1], [ 0, 0, 3, 0, -1, 1]]]], dtype=torch.int8)
```

Using the groups Option in the torch.nn.Conv2d Constructor Call

- The default value for this option is 1. So if you set groups=1 when you construct an instance of torch.nn.Conv2d, you get the same kernel structure that you saw on the previous slide: Each output channel is produced by summing the outputs from ALL input channels.
- However, If you use an integer value for groups greater than 1, you
 have to be careful. In this case, both in_ch and out_ch must be
 divisible by groups.
- What that implies is that a constructor call like

```
conop = nn.Conv2d(2, 3, 3, groups=2, bias=False)
would be illegal.
```

• In the next slide, we will consider the case of groups=4 when both in_ch and out_ch are also equal to 4.

Using groups=4 with in_ch=4 and out_ch=4

- When both in_ch and out_ch are the same and the value of groups is also the same number, you basically have a 1-1 connection between the input channels and the output channels.
- What that means, a single kernel operator will work on a single input channel and the result will be the corresponding output channel. Here is an example:

```
conop = nn.Conv2d(4, 4, 3, groups=4, bias=False)
                                                          # args: in ch. out ch. ker size
ker = conop.weight
print(ker)
                         Parameter containing:
                         tensor([[[[ 0.0675, 0.2119, 0.3157],
                                                                             ## The 3x3 op for that maps
                                    [ 0.2117, 0.3165, -0.0241],
                                                                             ## the first input ch to the
                                    [-0.2994, -0.1580, 0.2270]]].
                                                                             ## first output ch
                                  [[-0.0022, -0.1657, -0.2554],
                                                                             ## The 3x3 op that maps the
                                    [-0.3120, -0.2813, -0.0676],
                                                                             ## second input ch to the
                                    [ 0.1828, 0.1802, -0.3215]]],
                                                                             ## second output ch
                                 [[[ 0.2079, -0.2608, -0.0705],
                                                                             ## The 3x3 op that maps the
                                    [-0.1352, -0.0642, -0.0654]
                                                                             ## third input ch to the
                                    [-0.2991, -0.2878, -0.0522]]].
                                                                             ## third output ch
                                 [[] 0.0043, -0.1514, 0.1256].
                                                                             ## The 3x3 op that maps the
                                   [-0.3000, -0.0225, 0.2931],
                                                                             ## fourth input ch to the
                                    [-0.1360, 0.3010, 0.1207]]]],
                                                                             ## fourth output ch
                                requires grad=True)
                                                ## torch.Size([4, 1, 3, 3])
print(ker.shape)
```

Using groups=2 with in_ch=4 and out_ch=4

- When the value of groups divides both in_ch and out_ch, you have groupings of input channels sending information to the corresponding groupings of the output channels.
- For example, when in_ch=16, out_ch=64, and groups=4, and consider, for illustration, the case when the kernel size is 3 × 3. In this case, there will exist four 3 × 3 operators for each input channel, with each operator sending its output to one of the channels in a grouping of 4 output channels. In this case, you will have a total of 64 3 × 3 operators in the kernel.
- The next slide shows an example of the kernel tensor when groups=2 and with in_ch=4 and out_ch=4. [In this case, each grouping of 2 channels at the input will contribute to each grouping of two channels at the output. Within each group-to-group connections, you will need four 3x3 operators. Each 3x3 operator will take one input channel to all the output channels within each group. That calls for a total of eight 3x3 operators that are shown below.]

Using groups=2 with in_ch=4 and out_ch=4 (contd.)

 Shown below is the constructor call for the case when in_ch=4 and out_ch=4 and groups=2 and the resulting kernel tensor:

```
conop = nn.Conv2d(4, 4, 3, groups=2, bias=False)
                                                                 # args: in_ch, out_cha, ker_size
ker = conop.weight
print(ker)
                         Parameter containing:
                         tensor([[[[ 0.1885, -0.1905, 0.0253],
                                   [-0.0493, 0.1683, 0.0658]
                                   [ 0.1133, 0.0832, -0.0567]],
                                  [[-0.0496, -0.1942, 0.1277],
                                  [ 0.1871, 0.1613, -0.1663]
                                  [ 0.0105, -0.1662, -0.1298]]].
                                 [[[-0.1373, 0.0806, -0.1405],
                                    -0.0051, 0.0099, 0.15197
                                   [-0.1782, -0.1618, -0.1369]],
                                  [[ 0.1650, -0.0847, 0.1988],
                                   [ 0.0852, 0.0298, -0.0018]
                                   [-0.0466, 0.0296, -0.0538]]],
                                 [[[-0.0017, 0.0301, -0.1844],
                                   Γ-0.1235, 0.1903, -0.1913].
                                   [-0.0169, 0.2332, 0.0851]],
                                  [ 0.0067, -0.2043, 0.1168].
                                   [-0.1679, -0.0669, -0.0791]
                                   [-0.0349, 0.0026, 0.1944]]],
                                 [[[ 0.0294, 0.2111, 0.1442],
                                   [-0.1490, 0.1057, -0.1666]
                                   [-0.0999, 0.0693, 0.0778]],
                                  [[ 0.1768, -0.0759, 0.0004],
                                     0.1213, -0.2279, 0.17041,
                                   [-0.1949, 0.0032, -0.0401]]]], requires_grad=True)
print(ker.shape)
                     ## torch.Size([4, 2, 3, 3]) ## Axis 0: out_ch Axis 1: in_ch Axis 2: H Axis 3: W
```

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Reshaping Tensors

- It is not at all uncommon that your implementation of a deep learning algorithm would call for reshaping a tensor before it is input to the next layer. This is especially the case when the last layer is fully-connected and the penultimate layer is convolutional. When such is the case, the output of the penultimate layer is likely to be of shape (C, H, W), which would be a tensor with 3 axes (not including batching), while the shape of the tensor at the input to the fully-connected layer will need to have just one axis (again not including batching).
- The two most commonly used functions for reshaping a tensor in PyTorch are view() and reshape().
- While these two functions seem to do the same thing most of the time, they are not identical.
- The goal of this section is to present examples of tensor reshaping with view() and reshape() and to explain the difference between the

Purdue Haviorsit of these two important functions.

Introducing reshape() and view()

- On Slide 57, in order to demonstrate how you can carry out tensor reshaping with reshape() and view(), I start by constructing in line (A) a 2-axis random tensor of shape (3, 4).
- Subsequently, in line (C) we call on view() to reshape the originally (3,4) tensor into a (2,6) tensor. And, we do the same thing in line (E) but by calling on reshape().
- As you can see from the results displayed in line (D) and (F), both view() and reshape() are doing exactly the same thing.
- Just to illustrate that the tensors that are returned by <code>reshape()</code> and <code>view()</code> are independent of the original tensor <code>x</code>, in line (G) we delete the tensor reference held by the variable <code>x</code> on which we had invoked the two reshaping functions. As shown in lines (H) and (I), deleting <code>x</code> does not affect the tensor references held by the variables <code>y</code> and <code>z</code>.

Using -1 as a Reshaping Argument

- Continuing with the explanation of the reshape1.py script, the syntax shown in lines (J) through (M) is more for fun than anything else. Despite the fact that we already know that the y and z tensors are identical (since they are both obtained from the same underlying tensor with exactly the same reshaping operation), we nonetheless test them for equality in a couple of different ways.
- The main goal of the code in lines (N) through (V) is to illustrate the use of "-1" as one of the arguments to reshape() and view(). Using "-1" means that you want PyTorch to figure out what the value should be at that position in the call syntax. PyTorch can easily figure that out since any reshaping must use all of the data in the original tensor.
- For example, in the call to reshape() in line (Q), it is trivial to conclude that shape value to use where "-1" is indicated is 5. That is because the original tensor has a total of 10 numbers in it and the call in line (Q) wants to reshape the original tensor into a tensor of two axes with

Purthe stipulation that the dimensionality along the first axis is set to $\frac{2}{5}$

Using -1 as a Reshaping Argument (contd.)

- Obviously, then, the dimensionality along the "-1" in the call in line (Q) must be 5. Exactly the same logic applies to the call to view() in line (U).
- It is important to understand why the tensor reshaping calls in lines (W) and (Y) elicit run-time errors. The $_{\rm x}$ tensor has 10 data elements in it. Since 3 is a not a factor of 10, there is no way to reshape $_{\rm x}$ into a tensor of two axes with one of the axes having dimension 3.

Using reshape() and view() in Code

```
## reshape1.pv
import torch
torch.manual_seed(0)
x = torch.randn(3,4)
                                                                                                       ## (A)
                                                                                                       ## (B)
print(x)
                       ## tensor([[ 1.5410, -0.2934, -2.1788, 0.5684],
                                  [-1.0845, -1.3986, 0.4033, 0.8380],
                                  [-0.7193, -0.4033, -0.5966, 0.1820]])
                       ##
y = x.view(2,6)
                                                                                                       ## (C)
print(v)
                       ## tensor([[ 1.5410, -0.2934, -2.1788, 0.5684, -1.0845, -1.3986],
                                                                                                       ## (D)
                                  [ 0.4033, 0.8380, -0.7193, -0.4033, -0.5966, 0.1820]])
z = x.reshape(2,6)
                                                                                                       ## (E)
print(z)
                       ## tensor([[ 1.5410, -0.2934, -2.1788, 0.5684, -1.0845, -1.3986],
                                                                                                       ## (F)
                                  [ 0.4033, 0.8380, -0.7193, -0.4033, -0.5966, 0.1820]])
del x
                                                                                                       ## (G)
print(v)
                       ## tensor([[ 1.5410, -0.2934, -2.1788, 0.5684, -1.0845, -1.3986],
                                                                                                       ## (H)
                                  [ 0.4033, 0.8380, -0.7193, -0.4033, -0.5966, 0.1820]])
                       ##
print(z)
                       ## tensor([[ 1.5410, -0.2934, -2.1788, 0.5684, -1.0845, -1.3986],
                                                                                                       ## (I)
                                  [ 0.4033, 0.8380, -0.7193, -0.4033, -0.5966, 0.1820]])
                       ##
print(torch.eq(v.z))
                       ## tensor([[True, True, True, True, True, True],
                                                                                                       ## (J)
                       ##
                                  [True, True, True, True, True, True]])
print(y.eq(z))
                       ## tensor([[True, True, True, True, True, True],
                                                                                                       ## (K)
                       ##
                                  [True, True, True, True, True, True]])
```

(Continued on the next slide)

Using reshape() and view() in Code (contd.)

(..... continued from the previous slide)

```
## An interesting construct from stackoverflow.com:
print(torch.all(torch.lt(torch.abs(torch.sum(y - z)), 1e-12)))
                                                                     ## tensor(True)
                                                                                                       ## (I.)
print(torch.allclose(y,z))
                                                                                                      ## (M)
                                                                     ## True
## A second reshaping example to illustrate the use of "-1" as an arg:
x = torch.randn(10)
                                                                                                       ## (N)
z = x.reshape(2,5)
                                                                                                      ## (0)
print(z)
                       ## tensor([[-0.8567, 1.1006, -1.0712, 0.1227, -0.5663],
                                                                                                      ## (P)
                                  [ 0.3731, -0.8920, -1.5091, 0.3704, 1.4565]])
                       ##
z = x.reshape(2,-1)
                                                                                                      ## (Q)
print(z)
                       ## tensor([[-0.8567, 1.1006, -1.0712, 0.1227, -0.5663],
                                                                                                      ## (R.)
                                  [ 0.3731, -0.8920, -1.5091, 0.3704, 1.4565]])
                       ##
z = x.view(2.5)
                                                                                                      ## (S)
print(z)
                       ## tensor([[-0.8567, 1.1006, -1.0712, 0.1227, -0.5663],
                                                                                                      ## (T)
                                  [ 0.3731, -0.8920, -1.5091, 0.3704, 1.4565]])
                       ##
z = x.view(2,-1)
                                                                                                      ## (II)
print(z)
                       ## tensor([[-0.8567, 1.1006, -1.0712, 0.1227, -0.5663],
                                                                                                      ## (V)
                       ##
                                  [ 0.3731, -0.8920, -1.5091, 0.3704, 1.4565]])
# z = x.reshape(3, -1) ## RuntimeError: shape '[3, -1]' is invalid for input of size 10
                                                                                                      ## (W)
# z = x.view(3, -1) ## RuntimeError: shape '[3, -1]' is invalid for input of size 10
                                                                                                      ## (Y)
```

But view() and reshape() Are Not the Same

- The code shown in the previous script, reshape1.py, might create the impression that the functions view() and reshape() possess identical behaviors. With the help of the next script, flatten.py, I'll now show that that's NOT the case.
- But first a bit on what is meant by flattening a tensor. Flattening means reshaping a tensor into a single-axis data object a vector really. Think about flattening a matrix by scanning one row at a time and arranging all the elements in a single linear structure. We can extend this idea to any tensor by invoking reshape(-1) or view(-1), as shown in lines (E) through (H) on the next slide.
- However, flattening a tensor with view(-1) works only if a tensor is contiguous. As to what that means will be explained shortly. Suffice it to say here that whereas the tensor y created originally in line (C) is contiguous, the tensor w, obtained by transposing y, is NOT.
- When we try to flatten w by calling wiew(-1) in line (P), we run into a Purchantimersity or from the system.

Flattening a Tensor

```
## flatten.py
import torch
torch.manual seed(0)
                                                                                                          ## (A)
x = torch.arange(10)
                                     ## tensor([0, 1, 2, 3, 4, 5, 6, 7, 8, 9])
print(x)
                                                                                                          ## (B)
y = x.view(2,5)
                                                                                                          ## (C)
                                         tensor([[0, 1, 2, 3, 4],
print(v)
                                                                                                          ## (D)
                                     ##
                                                  [5, 6, 7, 8, 9]])
z1 = y.reshape(-1)
                                                                                                          ## (E)
                                     ## tensor([0, 1, 2, 3, 4, 5, 6, 7, 8, 9])
print(z1)
                                                                                                          ## (F)
z2 = y.view(-1)
                                                                                                          ## (G)
print(z2)
                                     ## tensor([0, 1, 2, 3, 4, 5, 6, 7, 8, 9])
                                                                                                          ## (H)
z3 = v.view(1,-1)
                                                                                                          ## (T)
print(z3)
                                     ## tensor([[0, 1, 2, 3, 4, 5, 6, 7, 8, 9]])
                                                                                                          ## (J)
w = v.T
                                                                                                          ## (K)
print(w)
                                         tensor([[0, 5],
                                                                                                          ## (I.)
                                                  ſ1. 6].
                                     ##
                                                  [2, 7],
                                     ##
                                                  [3, 8],
                                     ##
                                     ##
                                                  [4, 9]])
z3 = w.reshape(-1)
                                                                                                          ## (M)
print(z3)
                                     ## tensor([0, 5, 1, 6, 2, 7, 3, 8, 4, 9])
                                                                                                          ## (N)
\#z4 = w.view(-1)
                                          RuntimeError: view size is not compatible
                                                                                                          ## (P)
```

with input tensor's size

What is a Contiguous Tensor?

- The call to $_{view(-1)}$ in line (P) of the script on the previous slide does not work because the tensor $_{v}$ there is NOT contiguous. In order to explain what that means, I'll start with the 2×5 tensor $_{x1}$ created in line (C) of the next script, $_{reshape2.py}$, shown on Slide 64.
- Note the output produced by the statements in lines (D) and (E) of the next script. Line (D) tells us that the "strides" associated with x1 are 5 and 1, arranged in the order of the axes, and line (E) says that x1 is contiguous. For an array of 2 axes, contiguous means that the elements of the array are stored in a row-major format — that is, if you wanted to walk through all the data elements sequentially, first you will see the first row, followed by the second row, and so on. And the stride value of 5 means that you have to take five steps to go from one row to the next, whereas the stride value of 1 means that. in the same row, you have to take only one step to go to the next column entry.

- Lines (H) and (I) show the same properties of the tensor y which is the transpose of the tensor x1. The tensor y is NOT contiguous and its strides are given by (1,5).
- The tensor y is not contiguous because the transpose operation in line (F) leaves unchanged the storage of the actual data elements that were placed there when x1 was defined. All that is different for y is that its element access logic knows how to convert its row and column index values into where those elements are actually stored in the memory. This is reflected in the stride values for y in line (I). Now jumping from one row to the next requires only a one-step hop. On the other hand, scanning along each row requires making 5-step jumps as you go from one data element to the next.
- As to how x1 is stored may be referred to as the *natural order* for its elements. So whereas the data in x1 is stored in its natural order, the data elements in the transpose y are not.

- As shown in lines (J) and (K), we can flatten y by invoking reshape() on
 it. That is because the contract of reshape() alters the underlying
 storage of the data elements so that they are relocated in the
 memory in their natural order.
- On the other hand, the contract of view() does not allow it to automatically alter how the data elements are actually stored in the memory. So it is unable to flatten y as evidenced by the invocation in line (N) and the resulting error message in line (O).
- What is interesting is that you are allowed to convert a non-contiguous tensor into a contiguous tensor, as shown by the call in line (P). So even though both y and y2 as returned by the call in line (P) get displayed as identical tensors, the former is non-contiguous whereas the latter is.
- The tensor y2 being contiguous, we can now flatten it with a call to Purdie thives; before, as shown in line (T).

```
## reshaping2.py
import torch
torch.manual_seed(0)
x = torch.arange(10)
                                                            tensor([0, 1, 2, 3, 4, 5, 6, 7, 8, 9])
print(x)
                                                                                                          ## (A)
print("\nIs x contiguous: ", x.is_contiguous())
                                                        ## True
                                                                                                          ## (B)
x1 = x.view(-1,5)
                                                                                                          ## (C)
print(x1)
                                                        ## tensor([[0, 1, 2, 3, 4],
                                                                    [5, 6, 7, 8, 9]])
print("\nStride for x1: ", x1.stride())
                                                            (5, 1)
                                                                                                          ## (D)
                                                                                                          ## (E)
print("\nIs x1 contiguous: ", x1.is_contiguous())
                                                            True
y = x1.T
                                                                                                          ## (F)
print(y)
                                                            tensor([[0, 5],
                                                                                                          ## (G)
                                                                    ſ1, 6],
                                                                    [2, 7],
                                                                    [3, 8],
                                                                    [4, 9]])
print("\nIs v contiguous: ", v.is contiguous())
                                                                                                          ## (H)
                                                            False
print("\nStride for y: ", y.stride())
                                                           (1.5)
                                                                                                          ## (T)
y1 = y.reshape(-1)
                                                                                                          ## (.I)
print(y1)
                                                        ## tensor([0, 5, 1, 6, 2, 7, 3, 8, 4, 9])
                                                                                                          ## (K)
                                                                                                          ## (I.)
print("\nIs v1 contiguous: ", v1.is_contiguous())
                                                        ## True
print("\nStride for v1: ", v1.stride())
                                                        ## (1.)
                                                                                                          ## (M)
```

(Continued on the next slide)

(..... continued from the previous slide)

```
try:
   v2 = v.view(-1)
                                                                                                         ## (N)
except RuntimeError as e:
   print(e)
                                    ## view size is not compatible with input tensor's size
                                                                                                         ## (D)
   v2 = v.contiguous()
                                                                                                         ## (P)
   print(y2)
                                                              tensor([[0.5].
                                                                                                         ## (Q)
                                                                       [1, 6],
                                                                       [2, 7],
                                                                       [3, 8].
                                                                       [4. 9]])
   print("\nIs y2 contiguous: ", y2.is_contiguous())
                                                          ## True
                                                                                                         ## (R.)
   print("\nStride for v2: ", v2.stride())
                                                          ## (2.1)
                                                                                                         ## (S)
   y3 = y2.view(-1)
                                                                                                         ## (T)
   print(v3)
                                                          ## tensor([0, 5, 1, 6, 2, 7, 3, 8, 4, 9])
                                                                                                         ## (U)
   print("\nIs y3 contiguous: ", y3.is_contiguous())
                                                          ## True
                                                                                                         ## (V)
   print("\nStride for y3: ", y3.stride())
                                                          ## (1.)
                                                                                                         ## (V)
```