Discrete Mathematics

Assigntest	final	Due	06/21/24 23:50	Scale	100	Attachment	
Student	D10755505 Bei Jiayi	SubmittedAt	06/22/24 09:22(9h32m Delay)	Score	68	Notes	auto scanned

1 For the recurrence problem $a_n = 3a_{n-1} - 2a_{n-2}, a_0 = 1, a_1 = 0$,
characteristic equation is x^2 - 2x + 2 = 0
General solution with C1 and C2 is C1=2 ; C2=-1
After solving the system of equations for C1 and C2, the solution to the recurrence problem is $a_n = 2 - 2^n$

Submitted x^2 - 2x + 2 = 0 / C1=2 ; C2=-1 / $a_n = 2 - 2^n$
Reference $r^2 - 3r + 2 = 0\ C1 + C2 2^n\ 2 - 2^n$
Comments
Time Spent: 2663" Score: 1.4+1.4+1.4=4 / 4

2 A graph with 5 vertices and 10 edges is guaranteed to have:
a. A cycle
b. A tree
c. A planar graph
d. A bipartite graph

Submitted a A
Reference A
Comments ✓
Time Spent: 3179" Score: 4 / 4

3 The chromatic number of a graph is:
a. The average number of colors needed to color the graph
b. The maximum number of colors needed to color the graph
c. The minimum number of colors needed to color the graph
d. The number of vertices in the graph

Submitted c The minimum number of colors needed to color the
Reference C
Comments ✓
Time Spent: 3050" Score: 4 / 4

4 Which of the following is an example of a linear Diophantine equation?
a. x^2 + y^2 = 25
b. x/y = 2

c. x · y = 5
d. 2x + 3y = 5

Submitted a x^2 + y^2 =
Reference D
Comments ✓
Time Spent: 17" **Score:** 4 / 4

5 **The additive counting principle states that:**
a. If there are m ways to do one task and n ways to do another task, then there are mn ways to do both tasks
b. If there are m ways to do one task and n ways to do another task, then there are m+n ways to do both tasks
c. If there are m ways to do one task and n ways to do another task, then there are m-n ways to do both tasks
d. If there are m ways to do one task and n ways to do another task, then there are m/n ways to do both tasks

Submitted b If there are m ways to do one task and n ways to do another task, then there are m+n ways to do both
Reference B
Comments ✓
Time Spent: 3240" **Score:** 4 / 4

6 **A function f: A → B is surjective if:**
a. Every element in A is mapped to every element in B
b. Every element in A is mapped to at most one element in B
c. Every element in B is mapped to at least one element in A
d. Every element in B is mapped to at most one element in A

Submitted c Every element in B is mapped to at least one element in
Reference C
Comments ✓
Time Spent: 17" **Score:** 4 / 4

7 **The generating function for the recurrence relation**
$a_n = 2a_{n-1} + a_{n-2}, a_0 = 1, a_1 = 0$ **is** $G(x) = \frac{1}{1-2x} - \frac{1}{1-x}$

Submitted $\frac{1}{1-2x} - \frac{1}{1-x}$
Reference $\frac{1-2x}{1-2x-x^2}$
Comments
Time Spent: 223" **Score:** 0 / 4

8 The coefficient of x^5 in the binomial expansion of (x + 2)^10 is the closest to:
a. 252
b. 504
c. 1001
d. 2002

Submitted a
Reference B
Comments ✗
Time Spent: 37" **Score:** 0 / 4

9 In graph theory, a bipartite graph is a graph whose vertices (or nodes) can be divided into two
disjoint sets X and Y such that every edge connects a vertex in X to one in Y.

Submitted bipartite
Reference bipartite
Comments
Time Spent: 9" **Score:** 4 / 4

10 The Brooks' Theorem states that:
a. A graph with maximum degree δ has chromatic number $\delta + 1$
b. A graph with maximum degree δ has chromatic number at least δ
c. A graph with maximum degree δ has chromatic number exactly δ
d. A graph with maximum degree δ has chromatic number at most δ

Submitted d A graph with maximum degree δ has chromatic number at most δ
Reference D
Comments ✓
Time Spent: 109" **Score:** 4 / 4

11 Using contrapositive to state p → q: $p \rightarrow q$

Submitted $p \rightarrow q$
Reference ¬q → ¬p
Comments
Time Spent: 1477" **Score:** 0 / 4

12 The number of permutations of choosing 3 from 5 distinct objects is 60

Submitted 60
Reference 60

Comments
Time Spent: 5" Score: 4 / 4

13 A graph with 7 vertices and 12 edges has a minimum degree of at least 2 vertex(vertices)
Submitted 2
Reference 2
Comments
Time Spent: 2" Score: 4 / 4

14 The degree of the vertex is the sum of all incident edges. A graph has an Euler circuit if and only if the
degree of every vertex is even, and A graph has an Euler trail if and only if there are at most
2(1/2/3/4) vertices having odd degree
Submitted incident / even / 2 / odd
Reference incident even 2 odd
Comments
Time Spent: 6" Score: 1+1+1+1=4 / 4

15 Along an Euler's trail, you visit each edge exactly once, while along a Hamilton trial, you visit
each vertex exactly once
Submitted edge / vertex / once
Reference edge vertex once
Comments
Time Spent: 4" Score: 1.4+1.4+1.4=4 / 4

16 The number of bit strings of length 64 with exactly 3 ones is 54740 (an integer in its simplest format)
Submitted 54740
Reference 41664
Comments
Time Spent: 5" Score: 0 / 4

17 What is the definition of the chromatic number of a graph?
a. The maximum number of colors needed to color the vertices of a graph such that no two adjacent vertices share the same color.
b. The minimum number of colors needed to color the vertices of a graph such that no two adjacent vertices share the same color.
c. The maximum number of colors needed to color the edges of a

graph such that no two adjacent edges share the same color.
d. The minimum number of colors needed to color the edges of a graph such that no two adjacent edges share the same color.

Submitted b The minimum number of colors needed to color the vertices of a graph such that no two adjacent vertices share the same color.

Reference B

Comments ✓

Time Spent: 81" **Score:** 4 / 4

18 The mathematical formula, typeset by LaTex, for calculating the number of ways for distributing k candies to n kids is $\binom{n+k-1}{k}$. If each kid must receive at least 1 candy, this formula beco mes $\binom{k-1}{n-1}$.

Submitted $\binom{n+k-1}{k}$ / $\binom{k-1}{n-1}$

Reference $\binom{k+n-1}{n-1}$ $\binom{k-1}{n-1}$

Comments

Time Spent: 57" **Score:** 2+2=4 / 4

19 The Chinese Go game board has 19 marking lines in each direction that make total 361 cross intersections.
Suppose the coordinate of the left-upper corner is (0,0), left-bottom (0, 18), right-upper (18,0) and right-bottom (18,18). Furthermore, assume $n < m, k < l$. If an ant wants walk from point (n, k) to (m, l) by following the marking lines and keeping right and down direction, how many way does it have?
If there are multiple answers, write the one with minimum number of m. 252

Submitted 252

Reference $\binom{m-n+l-k}{l-k}$

Comments

Time Spent: 177" **Score:** 0 / 4

20 In a Chinese Go game board, how many sub-squares, including itself, are there? Format the answer to its simplest format (never use comma or other unnecessary punctuation marks) 2470

Submitted 2470

Reference 2109

Comments

Time Spent: 5" **Score:** 0 / 4

21 **We want to solve the three-unknow Diophantine equation $10x + 11y + 12z = 123$ for x, y, z in the positive integer set. How many solutions are there? 2**

Submitted 2

Reference 5

Comments

Time Spent: 60" **Score:** 0 / 4

22 **After initializing solution count to 0 (count:0;), if you are allowed to write one line of maxima code using selection and loops, what would you write? for x:1 thru 10 do (for y:1 thru (123-10*x)\11 do (if (mod(123-10*x-11*y,12)=0 and 123-10*x-11*y > 0) then count:count+1));**

Submitted for x:1 thru 10 do (for y:1 thru (123-10*x)\11 do (if (mod(123-10*x-11*y,12)=0 and 123-10*x-11*y > 0) then count:count+1));

Reference for x:1 thru 10 do (for y:1 thru 10 do (for z:1 thru 10 do(if (10*x+11*y + 12*z =123) then count:count+1)));

Comments

Time Spent: 17" **Score:** 0 / 4

23 **If there are only 5-cent stamps and 9-cent stamps sold in a post office, what is the maximum amount of postage in cents that can not be made up? (obviously, 1-cent,2-cent,3-cent,4-cent,6-cent... can not be made up, but postage of package costs lot more than these nowadays) 31-cent**
If the postage is 5 dollars, the minimum number of stamps required is 56

Submitted 31-cent / 56

Reference 31 56

Comments

Time Spent: 14" **Score:** 2+2=4 / 4

24 **What method should be used to prove $\sqrt{5}$ is irrational? contradiction. Suppose otherwise, $\sqrt{5}$ is rational, we would have $m, n \in N, m > n$ such that gcd(m,n) = 1 and**
$sqrt5$ = m/n
This would imply m^2 is divisible by 5. Furthermore, n^2 is also divisible by 5 as well, which is contradicted to the assumption on the beginning.

Submitted contradiction / $ \ sqrt {5}$ = m/n / ${m}^{2}$ / ${n}^{2}$ / contradicted

Reference contradiction $\sqrt{5} = \frac{m}{n}$ m n contradict

Comments

Time Spent: 86" **Score:** 0.8+0.8+0.8+0.8+0.8=4 / 4

25 **Make truth table for** $P \wedge Q \rightarrow P \vee Q$

P	Q	$P \wedge Q \rightarrow P \vee Q$
T	T	T
T	F	T
F	T	T
F	F	T

Submitted T / T / T / T
Reference T T T T
Comments
Time Spent: 63" **Score:** 1+1+1+1=4 / 4

Counter Q=25 **Drop Lowest D=4** **Sum S=68** **Leaving Time T=0** **Formula F=S**