1 习题

假设随机变量 $X_1, \ldots, X_4$ 相互独立且同分布，$P(X_i = 1) = 0.6$, $P(X_i = 0) = 0.4$。求行列式 $\begin{vmatrix} X_1 & X_2 \\ X_3 & X_4 \end{vmatrix}$ 的概率分布。

解：注意到 $\begin{vmatrix} X_1 & X_2 \\ X_3 & X_4 \end{vmatrix} = X_1X_4 - X_2X_3$

先计算 $X_iX_j \sim \begin{pmatrix} 0 & 1 \\ 1-0.6^2 & 0.6^2 \end{pmatrix} = \begin{pmatrix} 0 & 1 \\ 0.64 & 0.36 \end{pmatrix}$

再计算

$$
\begin{aligned}
P(X_1X_4 - X_2X_3 = -1) &= P(X_1X_4 = 0, X_2X_3 = 1) \\
&= P(X_1X_4 = 0) \cdot P(X_2X_3 = 1) \\
&= 0.64 \times 0.36 = 0.2304
\end{aligned}
$$

$$
\begin{aligned}
P(X_1X_4 - X_2X_3 = 1) &= P(X_1X_4 = 1, X_2X_3 = 0) \\
&= \cdots\cdots = 0.2304
\end{aligned}
$$

$$
\begin{aligned}
P(X_1X_4 - X_2X_3 = 0) &= 1 - P(\ldots = -1) - P(\cdots = 1) \\
&= 0.5392
\end{aligned}
$$

$\therefore X_1X_4 - X_2X_3 \sim \begin{pmatrix} -1 & 0 & 1 \\ 0.2304 & 0.5392 & 0.2304 \end{pmatrix}$ #

2 习题

设二维随机变量 (X,Y) 的联合概率密度函数为

$$f(x,y)=\begin{cases}\frac{1+xy}{4}, & |x|<1, |y|<1\\ 0, & \text{otherwise}\end{cases}$$

求证：X 与 Y 不独立，但 X^2 与 Y^2 独立。